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AIES Theory Assignment

Q4) Assume you are working on a recommendation system for a music streaming platform. The system suggests songs to users based on their music preferences, but it also wants to consider uncertainity in user preferences.

a) Explain why representing uncertainity in user music preferences is important in a recommendation system. Provide a real-life example or scenario to illustrate your point.

Ans] Representing uncertainity in user music preferences is crucial because individual tastes can be dynamic & context-dependent. People may enjoy different genres or artists at various times or under specific circumstances. For instance, if a user might typically listen to upbeat pop music but could be in the mood for calming instrumental tracks after a stressful day. Failing to account for this uncertainity may result in inaccurate recommendations that don't align with the user's current preferences, leading to a less satisfying user experience.

b) Describe one method or technique that can be used to represent uncertainity in user music preferences. How would this method help the recommendation system provide more personalized suggestions to users?

Ans] One method to represent uncertainity is through probabilistic models. These models assign probabilities to different music preferences for a user. For example, a probabilistic model might estimate that there's a 70% chance the user will enjoy pop music & a 30% chance they'll prefer classical. By incorporating probabilities, the recommendation system can offer a diverse set of suggestions, reflecting the uncertainity in the user's preferences at that moment. This enhances the likelihood of presenting music that aligns with the user's mood or context, leading to a more personalized & satisfactory recommendation experience.

Q5) Consider two medical tests, A & B, for a virus. Test A is 95% effective at recognizing the virus when it is present, but has a 10% false positive rate (indicating that the virus is present, when it is not). Test B is 90% effective at recognizing the virus, but has a 5% false positive rate. The two tests use independent methods of identifying the virus. The virus is carried by 1% of all people. Say that a person is tested for the virus using only one of the tests, & that test comes back positive for carrying the virus.

a) If a person is tested using Test A & the result is positive, what is the probability that the person actually has the virus?

Ans] If Test A is positive, the probability that the person actually has the virus can be calculated using Bayes' Theorem:

$$P(\text{Virus} \mid \text{Positive Result A}) = \frac{P(\text{Positive result A} \mid \text{Virus}) \cdot P(\text{Virus})}{P(\text{Positive result A})}$$

$$= \frac{0.95 \cdot 0.01}{0.95 \cdot 0.01 + 0.1 \cdot 0.99}$$

$$\approx 0.086$$

Ans)

b) If a person is tested using Test B & the result is positive, what is the probability that the person actually has the virus?

Ans] Similarly for Test B.

$$P(\text{Virus} \mid \text{Positive result B}) = \frac{P(\text{Positive result B} \mid \text{Virus}) \cdot P(\text{Virus})}{P(\text{Positive result B})}$$

$$= \frac{0.9 \cdot 0.01}{0.9 \cdot 0.01 + 0.05 \cdot 0.99}$$

$$\approx 0.15$$

c) Considering the results of part 'a' & 'b', which test (A or B) seems to be more reliable for correctly identifying individuals who have the virus while minimizing false positives? Justify your answer mathematically.

Ans] Test B seems more reliable for correctly identifying individuals who have the virus while minimizing false positives because the probability of having the virus given a positive result is higher for Test B.

d) If a person is tested using both Test A & Test B (one after the other), & both tests come back positive, what is the probability that the person actually has the virus?

Ans) If both tests are positive, the probability of actually having the virus can be calculated using the joint probability.

$$P(\text{Virus} \mid \text{Positive result } A \& B) = \frac{P(\text{Positive result } A \mid \text{Virus}) \cdot P(\text{Positive result } B \mid \text{Virus}) \cdot P(\text{virus})}{P(\text{Positive result } A) \cdot P(\text{positive result } B)}$$

$$= \frac{0.95 \cdot 0.9 \cdot 0.01}{(0.95 + 0.01 + 0.1 \cdot 0.99) \cdot (0.9 \cdot 0.01 + 0.05 + 0.099)}$$

$$\approx 0.64$$

So, if both tests are positive, the probability of actually having the virus increases significantly.

Q6) Suppose you are given a bag containing n unbiased coins. You are told that $n^{(a-1)}$ of these coins are normal, with heads on one side & tails on the other, whereas one coin is fake, with heads on both sides.

a) Suppose you reach into the bag, pick out a coin at random, flip it, & get a head. What is the (conditional) probability that the coin you choose is the fake coin?

Ans] The probability that the coin chosen is the fake coin, given that a head is obtained, can be found using Bayes' Theorem:

$$P(\text{Fake} \mid \text{Head}) = \frac{P(\text{Head} \mid \text{Fake}) \cdot P(\text{Fake})}{P(\text{Head})}$$

Since the fake coin has heads on both sides, $P(\text{Head} \mid \text{Fake}) = 1$, & $P(\text{Fake}) = \frac{1}{n}$. The probability of getting a head can be expressed as $P(\text{Head}) = P(\text{Head} \mid \text{Fake}) \cdot P(\text{Fake}) + P(\text{Head} \mid \text{Normal}) \cdot P(\text{Normal})$, where $P(\text{Head} \mid \text{Normal}) = \frac{1}{2}$ & $P(\text{Normal}) = \frac{n-1}{n}$. Substituting these values into the formula, we get the conditional probability.

b) Suppose you continue ~~flicking~~ flipping the coin for a total of k times after picking it & see k heads. Now what is the conditional probability that you picked the fake coin?

Ans] After seeing k-heads in a row, the updated probability that the chosen coin is fake can be calculated similarly using Bayes' Theorem. The updated probability is given by:

$$P(\text{Fake} \mid k\text{Heads}) = \frac{P(k\text{ heads} \mid \text{Fake}) \cdot P(\text{Fake})}{P(k\text{ heads})}$$

The probability of getting k heads with the fake coin is $P(k\text{ heads} \mid \text{Fake}) = 1$, & $P(k\text{ heads})$ can be expressed similarly to part (a).

c) Suppose you wanted to decide whether the chosen coin ~~is~~ was fake by flipping it k times. The decision procedure returns fake if all k flips come up heads; otherwise it returns normal. What is the (unconditional) probability that this procedure makes an error?

Ans] The unconditional probability of making an error in deciding whether the chosen coin is fake after k heads can be calculated by considering the cases where the chosen coin is normal but k heads are obtained. This probability is given by:

P(Error) = P(k heads | Normal) . P (Normal)

Since P(k heads | Normal) = $\left(\frac{1}{2}\right)^k$ & P(Normal) = $\frac{n-1}{n}$, substituting these values will give the unconditional probability of making an error.