Chapter 5:
Backing-Off Language Models

5.1. Unknown words and unseen unigrams

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Goal of this section

- How to define a vocabulary for language tasks
- Notion of “out-of-vocabulary” words

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Example „Crime and Punishment“

- Training corpus: 11 000 tokens
- Test corpus:
  - 3000 tokens
  - 108 of them not seen in training corpus

- Usually training corpus defines vocabulary
- Unseen words in test corpus are hence not part of the vocabulary

OOV-words: “out-of-vocabulary” words (tokens)

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Example „Crime and Punishment“

- Training corpus: 11 000 tokens
- Test corpus:
  - 3000 tokens
  - 108 of them not seen in training corpus

Definition: $$\text{OOV-rate} = \frac{\#\,\text{unseen tokens in test corpus}}{\#\,\text{of tokens in test corpus}}$$

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OOV-Rate on "Crime and Punishment"

- OOV-Rate = 108/3000 = 0.036 = 3.6%

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How to define a vocabulary

- Count the frequency of each word in a corpus
- Use the most frequent N words for the vocabulary

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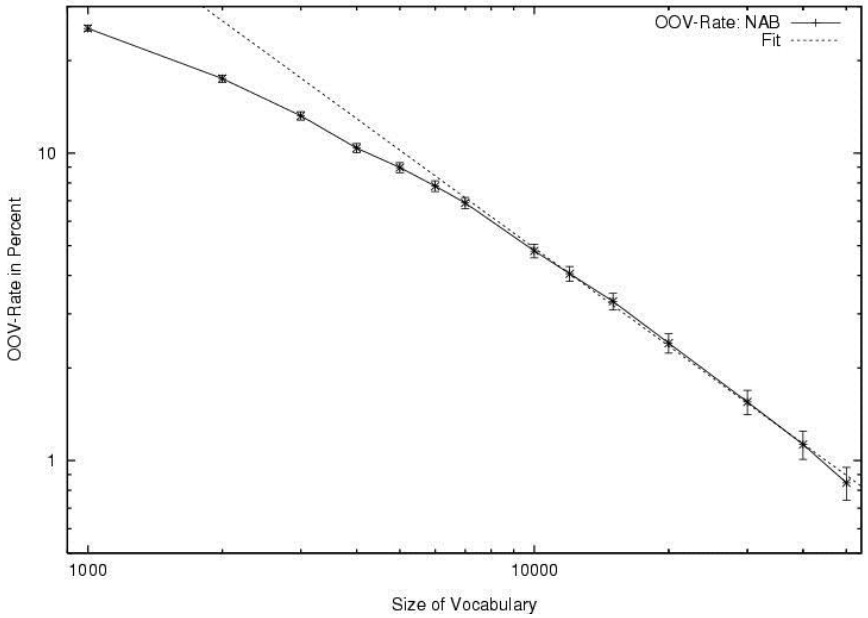
OOV-Rate vs. Vocabulary



- OOV rate depends on vocabulary size
- Morphologically rich languages have higher OOV rates for the same vocabulary size
- Specifically the relation is a power law

$$\text{OOV - Rate} \propto \frac{1}{(\text{Size of Vocabulary})^{\alpha}}$$

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Corresponding Problem with sequences of words (M-Gram)

- Size of vocabulary: N
- Number of sequences of length M with N words:

$$N^M$$

- Numerical example: N=64000 and M=3
  - $N^M = 3\ 10^{14}$
  - No realistic possibility to observe all trigrams in a training corpus

$\mapsto$ need methods to account for unseen events

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5.2 Maximum Likelihood Estimate of Probabilities

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Goal of this section

- Simple way to determine a language model P(w|h)

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From Perplexity to Likelihood

$$PP = P(w_1 \ldots w_N)^{-1/N}$$

$$= \exp\left(-\sum_{w,h} f(w,h) \log\left(P(w \mid h)\right) \right)$$

$$-\log(PP) = \sum_{w,h} f(w,h) \log\left(P(w \mid h)\right) = \frac{1}{N} \sum_{w,h} N(w,h) \log\left(P(w \mid h)\right)$$

Define likelihood $F = \sum_{w,h} N(w,h) \log\left(P(w \mid h)\right)$

Minimize perplexity $\Leftrightarrow$ Maximize likelihood

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Maximum-Likelihood Estimation of Probabilities

Maximize

$$F = \sum_{w,h} N(w,h) \log\left(P(w \mid h)\right)$$

Using the normalization constraint

$$\sum_{w} P(w \mid h) = 1 \quad \text{for all h}$$

Modified likelihood using Lagrange multiplier

$$F' = \sum_{w,h} N(w,h) \log\left(P(w \mid h)\right) + \sum_{h} \lambda(h) \left[1 - \sum_{w} P(w \mid h)\right]$$

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Maximum-Likelihood Estimation of Probabilities (cont.)

$$F' = \sum_{w,h} N(w,h)\log\left(P(w|h)\right) + \sum_{h} \lambda(h)\left[1 - \sum_{w} P(w|h)\right]$$

$$\frac{\partial F'}{\partial P(w'|h')} = \frac{N(w',h')}{P(w'|h')} - \lambda(h') = 0 \qquad \Rightarrow P(w'|h') = \frac{N(w',h')}{\lambda(h')}$$

Constraint

$$1 = \sum_{w} P(w|h) = \sum_{w'} \frac{N(w',h')}{\lambda(h')} = \frac{N(h')}{\lambda(h')} \qquad \Rightarrow \lambda(h') = N(h')$$

$$P(w|h) = \frac{N(w,h)}{N(h)}$$

Maximum likelihood estimate results in relative frequencies

Issues with Maximum Likelihood Estimate

- Unseen events

$\mapsto$ vanishing probability estimate

$\mapsto$ this sequence cannot be found in speech recognition

$\mapsto$ IR: this document does not cover all query terms, it cannot be relevant

Need other modeling scheme

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5.3 Absolute discounting

(Kneser-Ney-Smoothing Part 1)

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Goal of this section

- Discuss smoothing techniques that avoid vanishing probabilities

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Backing-off language model

$$P(w \mid h) = \begin{cases} \dfrac{N(w,h)}{N(h)} + \\ 0 \end{cases}$$

für $N(w,h) > 0$

Backing-off weight $\alpha(h)$
Backing-off distribution $\beta(w|h)$ (normalized!)
Discounting parameter d

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Backing-off language model (without animation)

$$P(w \mid h) = \begin{cases} \dfrac{N(w,h) - d}{N(h)} + \alpha(h)\ \beta(w \mid h) & \text{für } N(w,h) > 0 \\ \alpha(h)\ \beta(w \mid h) \end{cases}$$

Backing-off weight α(h)
Backing-off distribution β(w|h) (normalized!)
Discounting parameter d

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Calculating the backing-off weight α(h)

Use normalization

$$1 = \sum_w P(w|h) = \sum_{w:N(w,h)>0} \frac{N(w,h)-d}{N(h)} + \sum_w \alpha(h)\ \beta(w|h)$$

$$= 1 - \frac{\sum_{w:N(w,h)>0} d}{N(h)} + \alpha(h)$$

Solve for α(h)

$$\alpha(h) = \frac{\sum_{w:N(w,h)>0} d}{N(h)} = \frac{d \sum_{w:N(w,h)>0} 1}{N(h)} = \frac{d\,R(h)}{N(h)} \qquad \text{with } R(h) = \sum_{w:N(w,h)>0} 1$$

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Calculating the backing-off weight $\alpha(h)$

Backing-off weight

$$\alpha(\mathrm{h}) = \frac{d\,R(h)}{N(h)}$$

with $R(h) = \sum_{w:N(w,h)>0} 1$

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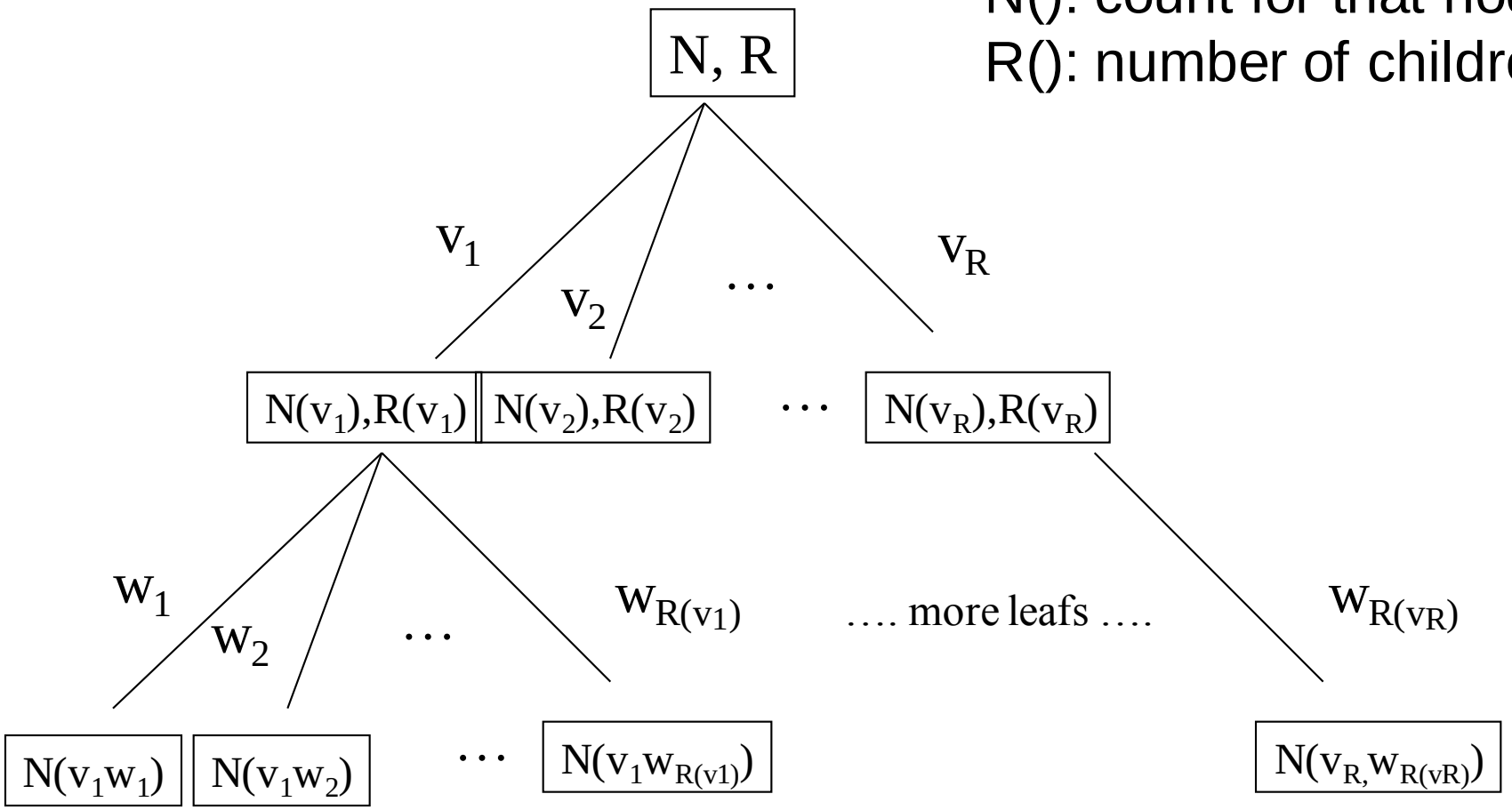
Storing the counts (Example: Bigram v,w)

N(): count for that node
R(): number of children



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Examples

- Example of backing-off code
- Example of language model

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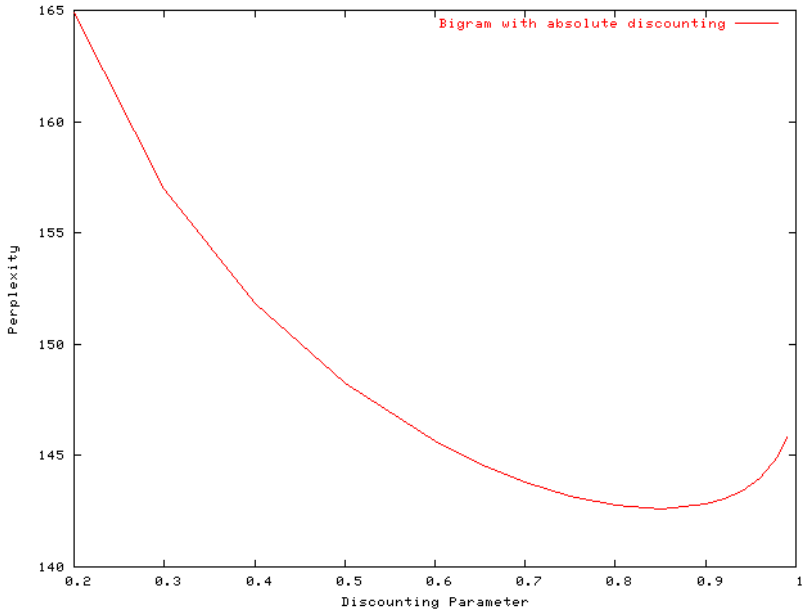
Influence of Discounting Parameter



- Discounting parameter has a significant impact on perplexity
- Values of between 0.7 and 0.9 are typically good choices

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5.4 Leaving-one-out estimate of the discounting parameters

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Goal

- How to get a good estimate for the discounting parameter without tuning it explicitly?

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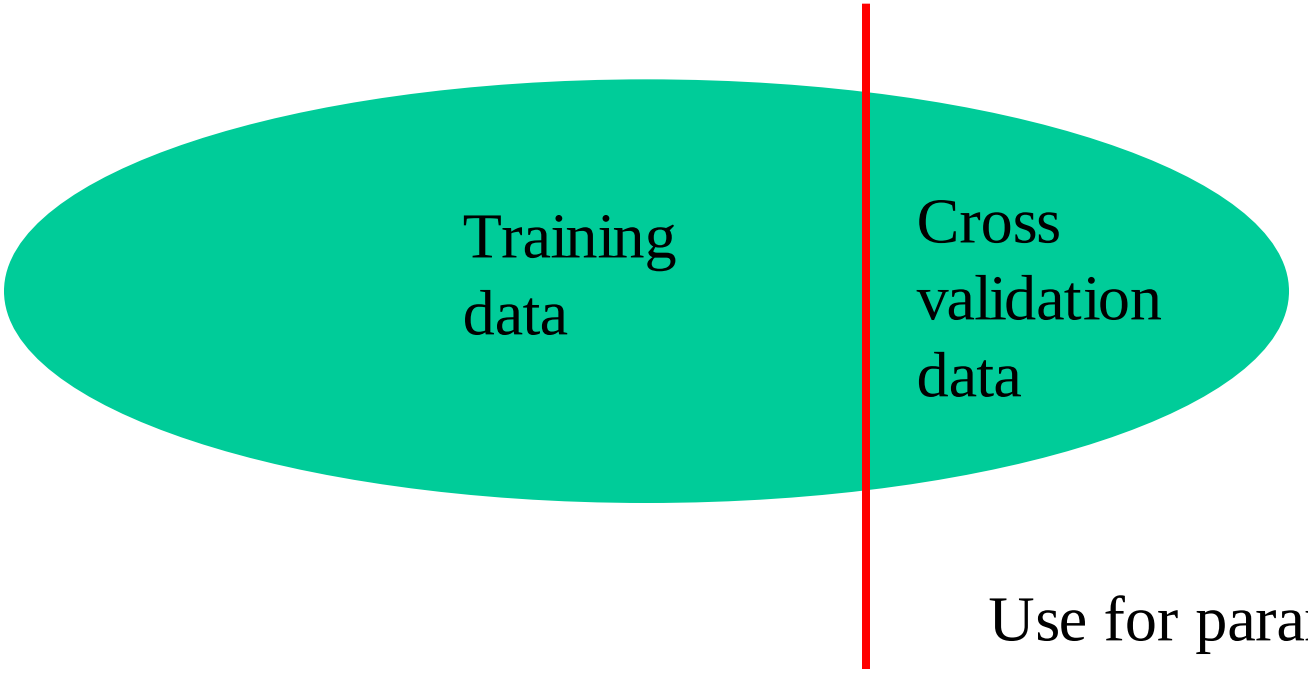
Closed form optimization solution for discounting parameter

- Idea of cross validation: split training data



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Variants of cross validation

- Suppose you want to train a unigram language model:
  - What is the smallest possible cross validation corpus?
  - How do you avoid fluctuations?

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Leaving-one-out

- Each unigram of the corpus can be the cross validation corpus
- Calculate average of all possible cross validation corpora

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Change of counts for leaving one out

- $w_1\ w_2\ w_3\ w_4\ w_5\ \cancel{w_6}\ w_7\ w_8\ w_9\ \ldots\ w_N$

$$N(w_6) \quad \mapsto \quad N(w_6)\text{-}1$$

$$N \quad \mapsto \quad N\text{-}1$$

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Leaving-one-out likelihood

$$F_{LOO} = \sum_{n=1}^{N} \log P_{LOO}(w_n) = \sum_{w \in W} N(w) \log P_{LOO}(w)$$

$$= \sum_{w \in W : N(w) > 1} N(w) \log \left[\frac{N(w) - 1 - d}{N - 1} + \frac{Rd}{N - 1} \beta \right]$$

$$+ \sum_{w \in W : N(w) = 1} \log \left[\frac{(R - 1)d}{N - 1} \beta \right]$$

Calculate $\frac{\partial F_{LOO}}{\partial d} = 0$

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Maximizing Leaving-one-out Likelihood

Calculate $\frac{\partial F_{LOO}}{\partial d} = 0$

- Skip calculating the derivative
- Result

$$0 = \sum_{r=2}^{\infty} n_r \, r \frac{R\beta - 1}{r - 1 - d + R\beta d} + \frac{n_1}{d}$$

with

n_1 : number of words seen once (singeltons)

n_r : number of words that have a count of r (count-of-counts)

$\mapsto$ no closed form solution to this euqation

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Iterative solution

Split off terms in n_1 and n_2

Solve for d and use as an iterative equation

$$d_{i+1} = \frac{n_1}{(1-R\beta)\left(n_1 + 2n_2 + \sum_{r=3}^{\infty} \frac{n_r r(1-d_i(1-R\beta))}{r-1-d_i(1-R\beta)}\right)}$$

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Approximate solution

Just use first iteration and initialize with $d_0=0$

$$d_1 = \frac{n_1}{(1-R\beta)\left(n_1 + 2n_2 + \sum_{r=3}^{\infty} \frac{n_r r}{r-1}\right)}$$

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Approximate solution

Usually $R\beta<1$

Ignore the sum (only a small change to discounting parameter)

$$d \approx \frac{n_1}{n_1 + 2n_2}$$

Works very well for all practical applications

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Demonstrate Approximate Solution

- -> Test formula using switchboard bigram

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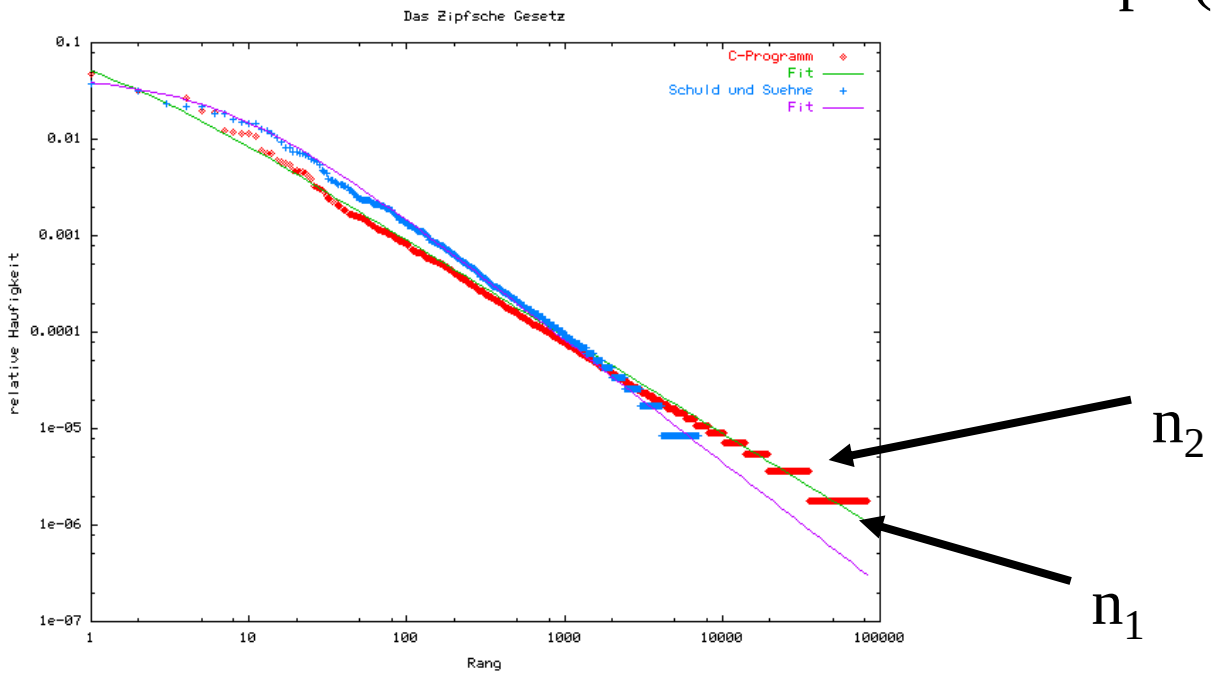
Remember Zipf´s Law



Zipf (simplified):

$$N(r) = \frac{N_0}{r^{\gamma}}$$

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Estimate n_1 and n_2

- Length of the first step in the Zipf-Distribution

$$N(r) = \frac{N_0}{r^{\gamma}} \qquad \Rightarrow \qquad r = \left(\frac{N_0}{N}\right)^{\frac{1}{\gamma}}$$

$$\Rightarrow \qquad n_1 = \left(\frac{N_0}{1/2}\right)^{\frac{1}{\gamma}} - \left(\frac{N_0}{3/2}\right)^{\frac{1}{\gamma}}$$

$$\text{and} \qquad n_2 = \left(\frac{N_0}{3/2}\right)^{\frac{1}{\gamma}} - \left(\frac{N_0}{5/2}\right)^{\frac{1}{\gamma}}$$

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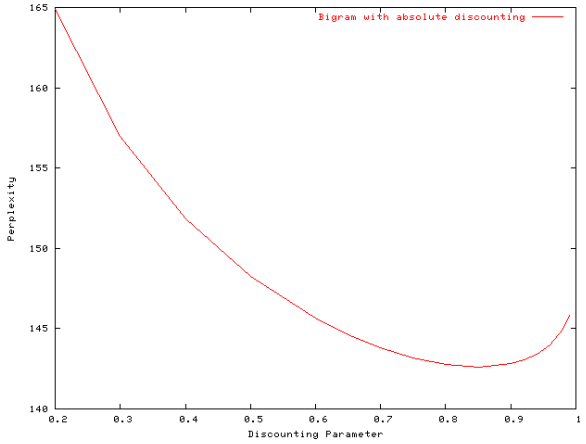
Estimate Discounting Parameter

$$d \approx \frac{n_1}{n_1 + 2n_2} = \frac{1}{1 + 2\frac{n_2}{n_1}}$$

$$= \frac{1}{1 + 2\frac{((2/3)^{1/\gamma} - (2/5)^{1/\gamma})}{(2^{1/\gamma} - (2/3)^{1/\gamma})}}$$

For $\gamma = 1$: $d \approx \frac{5}{7} \approx 0.71$



Estimate only gives degradation as compared to optimal numerical solution

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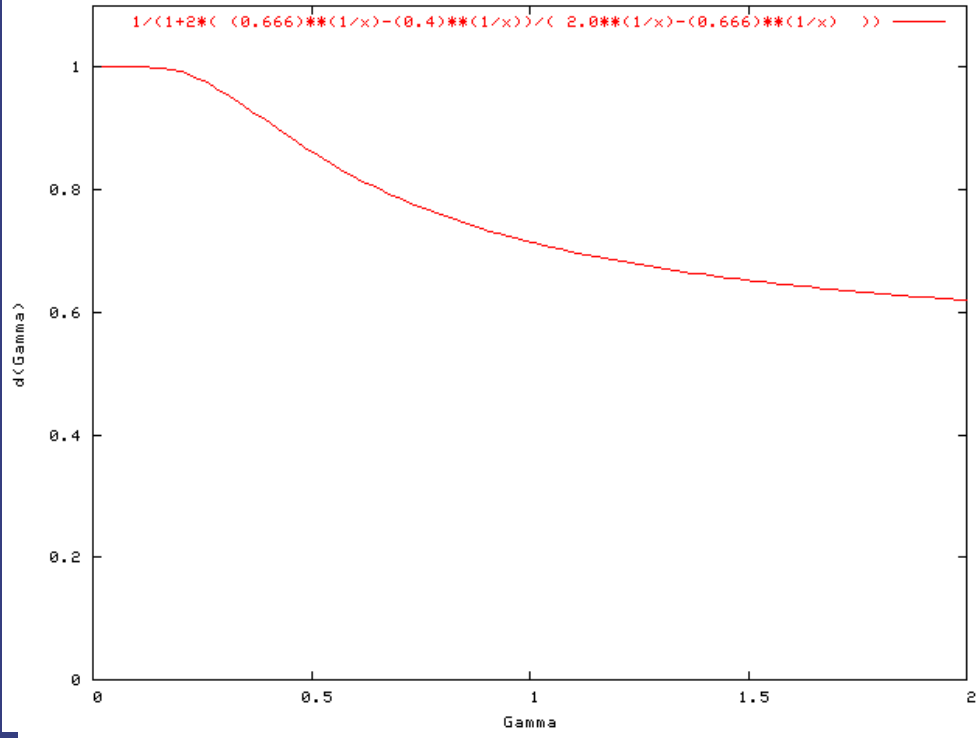
Slope in Zipf vs. Discounting



The larger the slope, the smaller the discounting parameter

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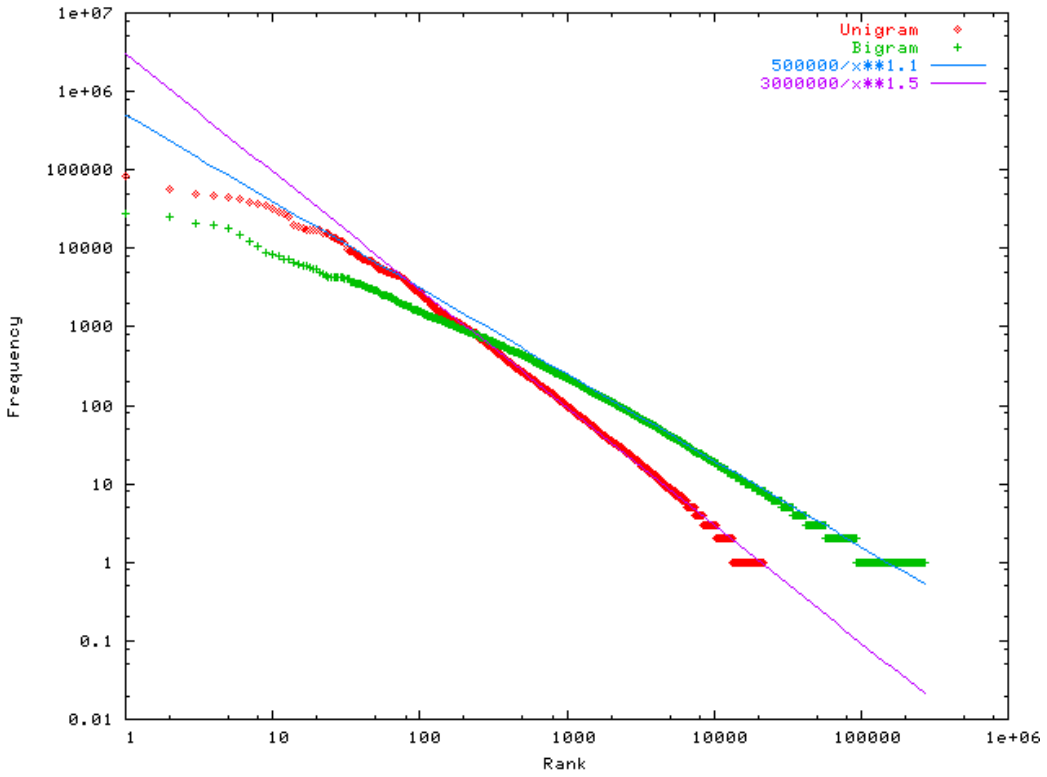
Zipf for Unigram and Bigram



Longer range models have larger discounting parameters

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5.5 Other smoothing methods

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Goal

- Smoothing techniques that are simpler
- Smoothing techniques that are used for other reasons

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Floor discounting ("add epsilon")

aka additive smoothing; Lidstone smoothing, Laplace Smoothing, …

Add a little bit to each count

$$P(w \mid h) = \frac{1}{Z}\left(N(w,h) + \varepsilon\right)$$

1/Z: Normalisation

$$1 = \sum_{w=1}^{V} P(w \mid h) = \frac{1}{Z}\left(N(h) + \varepsilon V\right) \quad \Rightarrow \quad Z = \left(N(h) + \varepsilon V\right)$$

$$P(w \mid h) = \frac{N(w,h) + \varepsilon}{N(h) + \varepsilon V}$$

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Linear Discounting

$$P(w|h) = (1-\varepsilon)\frac{N(w,h)}{N(h)} + \varepsilon\beta(w|h)$$

ε: determines amount of smoothing

$\beta(w|h)$: "backing-off distribution"

Variant: $$P(w|h) = (1-\varepsilon(h))\frac{N(w,h)}{N(h)} + \varepsilon(h)\beta(w|h)$$

Popular value: $$\varepsilon(h) = \frac{n_1(h)}{N(h)} \quad \text{with } n_1(h) = \sum_{w:N(w,h)=1} 1$$

See example

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Good-Turing Discounting

Adjusted counts:

$$N^*(w,h) = \begin{cases} [N(w,h)+1]\dfrac{n_{N(w,h)+1}}{n_{N(w,h)}} & \text{für } N(w,h) \le 5 \\ N(w,h) & \text{sonst} \end{cases}$$

Model:

$$P(w \mid h) = \frac{N^*(w,h)}{N(h)} + \lambda(h)\beta(w \mid h)$$

Backing-off weight from normalization: $\lambda(h) = 1 - \dfrac{N^*(h)}{N(h)}$

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5.6 Kneser-Ney Smoothing

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Kneser-Ney Smoothing

- Try to find a dedicated backing-off distribution
- Two different variants
  - “Marginal constraint” backing off
  - Singleton backing-off

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Idea

$$P(w \mid h) = \begin{cases} \dfrac{N(w,h) - d}{N(h)} + \alpha(h)\ \beta(w \mid \hat{h}) & \text{für } N(w,h) > 0 \\ \alpha(h)\ \beta(w \mid \hat{h}) \end{cases}$$

With $\hat{h}$ beeing the history shortened by one word

$$p(w \mid \hat{h}) = \sum_{h'} P(w \mid h) P(h' \mid \hat{h}) \qquad (**)$$

with h' beeing the word removed from the history

$\mapsto$ look for backing off distribution such that (**) is satisfied

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Solution

- Direct calculation
- Solution

$$\beta(w \mid \hat{h}) = \frac{N_+(w, \hat{h})}{\sum_w N_+(w, \hat{h})}$$

with

$$N_+(w, \hat{h}) = \sum_{h': N(w, \hat{h}, h') > 0} 1$$

Marginal constraint backing-off

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Alternative Determine Backing-Off Distribution using Leaving-One-Out

$$F = F_0 + \sum_{h'\hat{h}w:N(w\hat{h}h')=1} \log\left(\alpha(h)\beta(w|\hat{h})\right)$$

$$+ \sum_{\hat{h}} \lambda(\hat{h})\left(1 - \sum_{w} \beta(w|\hat{h})\right)$$

F_0 likelihood for events seen more than once

$\lambda(\hat{h})$ Lagange multiplier

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Solution for Leaving-One-Out Approach to Backing-Off Distribution

- Direct calculation of first derivative
- Solution

$$\beta(w \mid \hat{h}) = \frac{N_1(w, \hat{h})}{\sum_w N_1(w, \hat{h})}$$

with

$$N_1(w, \hat{h}) = \sum_{h': N(w, \hat{h}, h')=1} 1$$

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Comparison Marginal-Constraint/Singleton Backing-Off

Backing-off Model	Bigram	Trigram	4-gram
Rel. Frequencies	160.1	146.4	147.8
Marginal Constraint	154.8	138.3	137.9
Singleton BO	156.2	142.6	141.9

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5.7 Pruning of count trees

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Goal

- Method to decrease size of the tree

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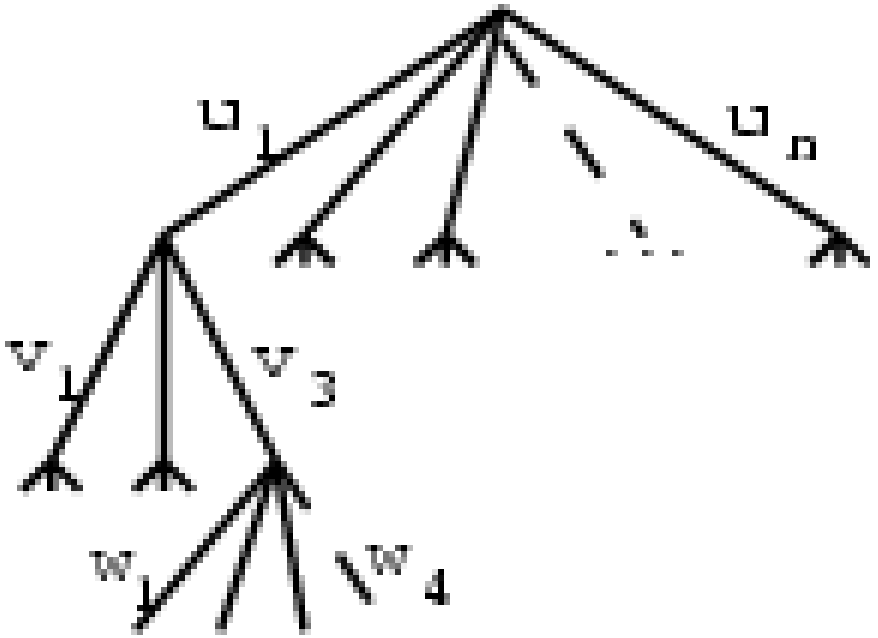
Pruning Count-Trees



Level 1

Level 2

Level 3

Simple criterion for pruning trees:
remove all infrequent nodes

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Improved Criterion: Change in Likelihood

- Metric inspired by maximum likelihood objective
- In case a leaf is pruned, the backing off probability will be used instead

$$\Delta F_{Leaf} = N(w,h)\log P(w|h) - N(wh)\log\left(\alpha(h)\beta(w|\hat{h})\right)$$

$$= N(w,h)\log\left(\frac{P(w|h)}{\alpha(h)\beta(w|\hat{h})}\right)$$

- This is a metric for the leaves of the tree
- Create measure for sub-trees from measures for leaves

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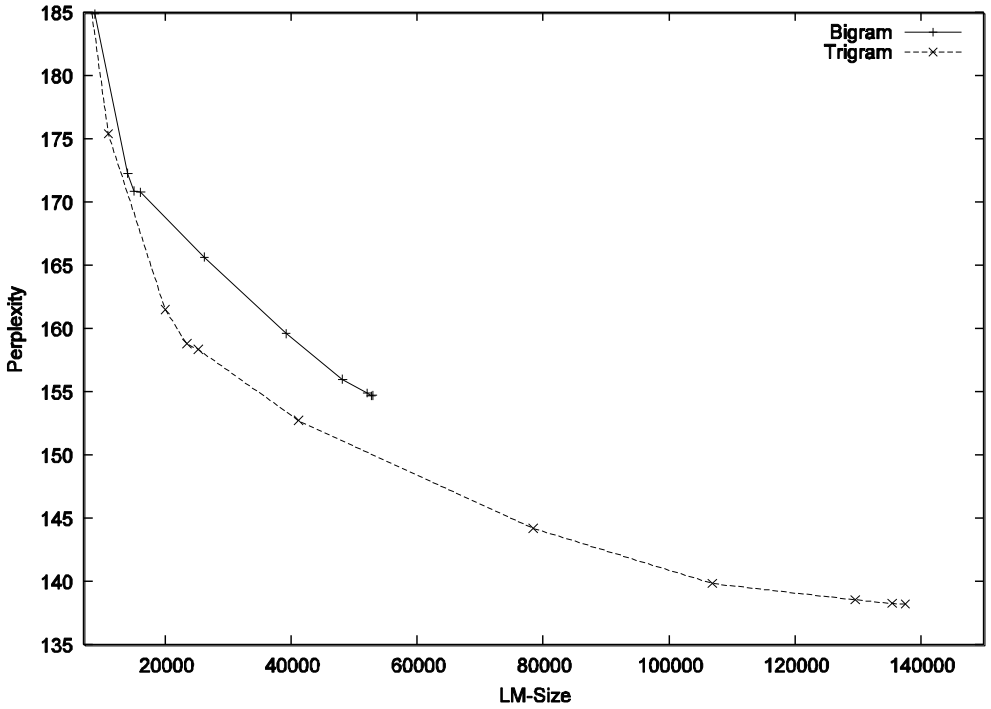
Effect of language model pruning



- Pruned Trigram better than pruned bigram
- For a given size pruning gives you the best LM
- Results generalize to longer N-grams

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Summary

- OOV-words
- Maximum likelihood estimator
- Backing-off language models
- Discounting parameter
- Kneser-Ney Smoothing
- Pruning of trees

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