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The Tower of Hanoi

This report covers the infamous puzzle of Tower of Hanoi which has been studied in the problem-solving literature since 1969.In the early stages , various approaches were involved for generating abstraction . A such approach will be discussed that generates abstraction as well as shows an exponential reduction in the search space.

The objective of Tower of Hanoi is to move a pile of disks of different size , smaller stacked on the bigger peg , from one peg to another with the help of an intermediate peg. In this process , only a single disk can be moved at a time and a larger disk can never be placed over a smaller one. The size in the Tower of Hanoi is based on the number of disks involved and the number of possible states for a given puzzle with n disks is 3^n since each disk can be on one of the three pegs , which can be represented as a tree where each node represents the state of the disks in system. When the goal is achieved the tree is terminated and the shortest solution follows the path along the diagonal between the initial and goal states. Different approaches were introduced for solving the Tower of Hanoi in minimum number of moves , one of them being the Hierarchical abstraction , Knoblock 1990 , which provides us with a set of operators and describe a domain, and it produces an abstraction hierarchy by partitioning and ordering the literals in the domain.

The algorithm for producing a hierarchy of abstraction spaces is to first form a directed graph, where the vertices of the graph represent one or more literals , literals being the possible negated atomic formulae (i.e the state of the system defined as a formulae) and the edges represent constraints between literals . A literal class forms the literals as well as the negation of the literal since they directly interact with each other . This removes any cycles in the graph just by fusing both of them into a single literal class. In this directed edge , the first literal class must be higher or at the same level of abstraction as the second one. The algorithm ensures that the addition of a new literal class at a lower level of abstraction will not require any changes in the higher level of abstraction . And finally the partial order of literal classes is transformed into a total order, which forms an abstraction hierarchy. A level 3 Tower of Hanoi , literal class at the highest level of abstraction only includes the use of the largest disk only , the second level of abstractions includes both the largest and the middle disk and the third level of abstraction includes the all three disks . Now , the dramatic reduction is followed and then the effect of the search space reduction on a depth-first search is explored.

As for the application of the discussed algorithm , the problem is solved in steps . The highest level involving only the use of the largest disk i.e. its transfer from the initial peg to the required peg . Doing so , creates two subproblems at the next level where the first subproblem is to get to the state where the abstract operator can be applied, and the second subproblem is to reach the goal state. Therefore , at the second level of hierarchy abstraction a three-step plan is processed which in turn creates four subproblems in the base-level. Solving which leads us to the final step of the problem. The solution in the most abstract space produces a plan that moves the largest disk to the goal peg . The ordering of this hierarchy is really important as the lower levels cannot amend anything to the higher level of abstraction . Thus, the abstraction hierarchy partitions the state space into three smaller spaces and by solving these three spaces a final level consisting of nine equally sized separate state spaces which are solved by doing the same .

The discussed algorithm is a state space reduction technique which reduces the size of the search space from exponential to linear in the length of the solution. The search space size of the original solution was estimated to be O(b^l) , b being the branching factor and l the length of the solution. With the use hierarchy abstraction this can be reduced to linear size estimated to have a search space size as O(l). In order to test the theoretical values , practical solutions were run using breadth-first search, depth-first search. The experiments compared the number of nodes searched and the length of the solutions on problems that range from one to as many as seven disks , respected graphs were plotted against the number of nodes searched and it was observed that breadth-first search always generated the optimal solution.

There are several general conclusions that one can draw from the experiments. First, the degree to which abstraction reduces search depends on the portion of the base-level search space that is explored . These days , the problem of Tower of Hanoi is being studied in the field such as Artificial intelligence and Animal experiments in order to test one's intelligence.

Reference - https://usc-isi-i2.github.io/papers/knoblock90-hanoi.pdf