

Long Optical Paths of Large Aperture

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THE measurement of the vapor phase spectra of compounds having high boiling points presents an experimental problem that may be solved either by heating the absorption cells or by making them very long. In the infra-red region radiation from the hot gases in heated cells decreases the accuracy of absorption measurements. If only a small amount of sample is available, the only possibility is to use an optical system in which the radiation goes back and forth through the same volume a large number of times. Several designs for such systems have been published recently^{1,2} but none of them permits the use of large angular apertures at points off the optic axis. In this paper an absorption cell is described in which the light traverses a small volume a large and arbitrarily variable number of times, and in which the angular aperture of the mirrors is not occulted either on or off the optical axis. The design gives very high light transmission and can be used for observing spectra that are very weak, or that belong to high boiling point compounds or to compounds obtainable only in very low concentrations. It can be used for any liquids or gases that do not injure the mirror surfaces, with which they are directly in contact.

The essential parts of the equipment are three spherical, concave mirrors that all have the same radius of curvature. These are set up as shown in Fig. 1 with two mirrors *A* and *A'* close together at one end of the absorption cell, and the third mirror *B* at the other end. The centers of curvature of *A* and *A'* are on the front surface of *B*, and the center of curvature of *B* is halfway between *A* and *A'*. This arrangement establishes a system of conjugate foci on the reflecting surfaces of the mirrors, by which all the light leaving any point on *A* is brought to a focus by *B* at the corresponding point on *A'*, and all the light leaving this point on *A'* is focused back again to the

original point on *A*. Similarly, all the light leaving any point on *B* and going to either *A* or *A'* is focused back to a new point on *B* that is somewhat offset to one side of the original one.

Figure 1 illustrates the way these properties are used to obtain very long optical paths. Light enters through a slit close to one end of *B*, whence it goes to *A*, from there to *B*, then to *A'*, back to *B*, to *A*, and so on, back and forth between *B* and *A'* or *A* alternately. The positions of successive images can all be located by the rule that object and image points near the center of curvature of a spherical mirror always lie on a straight line whose midpoint falls on the center of curvature. Thus mirror *A* forms an image 1 of the entrance slit on the surface of *B* as far from *A*'s center of curvature as the entrance slit is from it. Then, since the center of curvature of mirror *B*, shown by a circle, is halfway between *A* and *A'*, *B* forms an image of *A* on *A'*. In the same way, mirror *A'* forms on *B* a second image 2 of the slit, whose position is determined by the distance between 1 and the center of curvature of *A'*. Mirror *B* forms an image of *A'* on *A*, and *A* forms another image 3 of the slit on *B*, which is again returned to *A*, and so on. Each successive image of the slit on *B* is offset to one side or the other of the preceding one until finally the last one falls beyond the end of *B*. The different images fall in order at the points marked 1, 2, 3, and 4 in Fig. 1. The large angular aperture obtainable off the optic axis is easily explained. Since all the light in the first image of the slit that is formed on *B* is focused on *A'*, and since all the light falling on *A'* is returned to the second image on *B*, and so on, no light is lost off the edges of the mirrors.

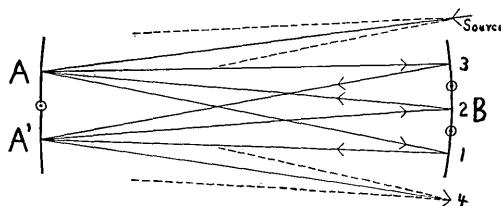


FIG. 1.

¹ H. D. Smith and J. K. Marshall, *J. Opt. Soc. Am.* **30**, 338 (1940).

² H. R. Kratz and J. E. Mack, *Phys. Rev.* **57**, 1059A (1940).

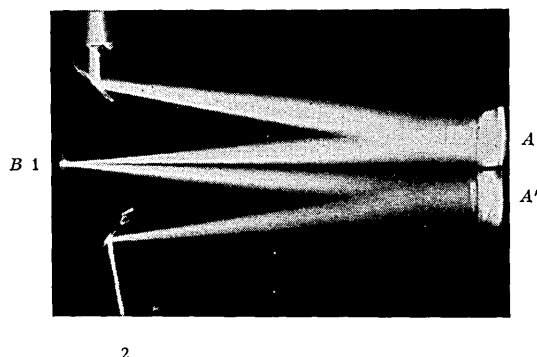


FIG. 2. Smoke photograph of optical path 2.5 meters long.

The only way intensity can be lost is by absorption or scattering on the reflecting surfaces.

The optical adjustments of this system are not critical; most of the tests and photographs described here were made manually without screw adjustments. The most important adjustment is the separation of the centers of curvature of the mirrors A and A' . This determines the number of times the light goes through the cell and the uniformity of the separation of the images formed on B . If A and A' are adjusted symmetrically about B and its center of curvature, each image on B is separated from the ones nearest to it by the distance between the centers of curvature of A and A' . The ratio of the length of B to this separation determines the number of times the light passes through the cell. This may be either four times for one image on B , eight times for three images, twelve for five, sixteen for seven, etc. Intermediate numbers are not possible. If A and A' are not symmetrically adjusted horizontally, the images on B occur in pairs rather than uniformly spaced. If A or A' is out of adjustment in the vertical rather than the horizontal angle, alternate images are raised or lowered above the others. In neither case is there any loss of intensity or troublesome cumulative effect of the error. If B is out of adjustment either horizontally or vertically, the first image of A does not fall exactly on A' , and some light is lost around its edges. However, after one reflection there is no further loss, because that part of the light which did fall on A' is reflected back and forth between the same points on A and A' .

This method of obtaining long optical paths in restricted volumes has many advantages over

those of a system using a spherical mirror and a truncated prism or a spherical mirror and a pair of flat mirrors:

I. The full angular aperture of the condensing mirrors is maintained to the extremities of the final image. The length of the image is limited by the size of the mirrors or by astigmatism and coma. The resulting increase in the intensity of the image off the axis can be partially obtained in the other two designs of absorption cell by using concave cylindrical mirrors instead of plane mirrors or by grinding convex cylindrical surfaces on the 45° faces of the truncated prism. In each case such cylindrical surfaces should have radii of curvature 1.414 times the radius of curvature of the spherical concave mirror, and both should have their axes in a plane perpendicular to the slit. Such an arrangement would form an astigmatic image of the concave mirror on itself, and hence greatly reduce the amount of light lost beyond the edges of that mirror. However, the lens defects are cumulative in this arrangement and should be quite serious.

II. The number of traversals of the absorption cell can be changed by a simple adjustment of mirrors A and A' from 4, to 8, or 12, or any reasonable multiple of 4. As neither the position nor the direction of the emergent beam is changed by this, no readjustment need be made in the rest of the optical system.

III. None of the adjustments is difficult to make. The tolerances on all but the horizontal angles of A and A' are usually large, because the principal errors introduced by small inaccuracies are neither troublesome nor cumulative.

IV. Losses of light on the mirror surfaces are kept to the absolute minimum. There are only two reflections, both at normal incidence, for each round trip of the light through the tube. Dust, pinholes, and spots on mirrors A and A' have a less serious effect than ordinarily, because the light from any point on one of these mirrors always goes back to the same point. If there is a hole in one of them, the light falling on the hole is lost, but on the second reflection from that mirror no more light is lost.

V. Except for the entrance and exit windows, there is no transmission of the light through glass or other optical material, with the consequent

reflection losses. For the same reason there are no extraneous images formed by multiple reflections.

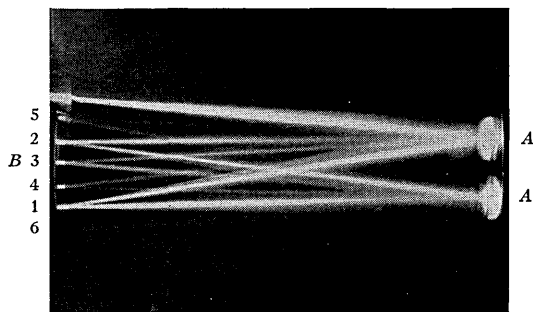
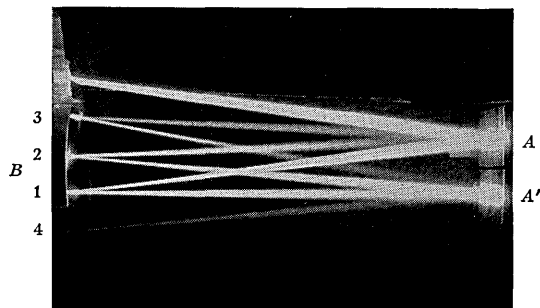
VI. The separation between the images on *B* can ordinarily be made large enough to permit the source and final image to be completely outside the absorption cell. Small, plane mirrors are placed near the ends of mirror *B* to accomplish this.

VII. The space inside the absorption cell is used efficiently. The volume of a large aperture cell designed to give a ten-meter absorption path with sixteen traversals is only 10 liters.

VIII. The image formed by the mirror system after 12 traversals is quite sharp. It shows slight astigmatism, but except for this the individual turns of a common, coiled filament light bulb can be resolved.

IX. The principal disadvantages of this device are that the three concave mirrors must be specially made to have the same focal length and that, like all equipment in which multiple reflections are used from the same mirror, there is bound to be a certain amount of light scattered into the emergent beam without going through the cell the proper number of times.

Figures 2, 3, and 4 show the paths of the light beams for different arrangements of the mirrors. They were obtained by blowing smoke between the mirrors and photographing the light scattered from the optical path. Illumination was supplied by an a.c. carbon arc with an orange filter in front of it. Without the filter scattering and reflection losses attenuated the beam so fast that only the shortest optical paths could be photographed, and even with it the exposures were all less than fifteen minutes. The mirrors shown in the photographs have radii of curvature of 62.5



cm, are 8×11 cm and 5.5×15 cm, and can be set up in a vacuum-tight absorption cell of 10-liters volume. The order in which the different images of the slit were formed is shown in each case by the numbers beside them, although the decrease in intensity of the beams can be used as an equally good indication. Figure 2 shows the simplest possible case with one reflection each from *A* and *A'*. The full aperture of the mirrors is used, and two flat mirrors are inserted near *B* to bring the entrant and emergent images outside the body of the absorption cell. These auxiliary mirrors would ordinarily be incorporated in any design using this system. Figure 3 shows the next longer arrangement that gives a five-meter path with eight traversals of the cell. In this photograph the aperture of the entering light beam is cut down to a narrow pencil of rays. Figure 4 shows the case of twelve traversals with the formation of five intermediate images of the slit to give an optical path of 7.5 meters. The aperture of the light beam is here limited by the condensing lens, and the variations in intensity across the beam are due to changes in the arc during the exposure.

The results of preliminary tests of this cell have been most satisfactory. Using the mirrors with 62.5-cm radii of curvature, it was possible to set up the equipment on the top of a table so that the emergent beam had traversed the cell 90 times, corresponding to an optical path 56 meters long. Under these conditions the intensity of the beam was a great deal less than it had been on entering the system. The intensity of the image formed after 12 traversals was measured with a General Electric light meter and compared to that of the image formed after the first reflection. On the

optic axis of the system it was 30 percent as great, and at a point 1 cm from the axis it was 20 percent. These losses are not considered serious in view of the relatively poor condition of the mirrors at the time the measurements were made. The increase of reflectivity with wave-length makes this cell quite satisfactory in the infra-red.

At a wave-length of 3 microns it transmitted through a ten-meter path 70 percent as much light as a simple cell that gave only a 90-cm path.

In conclusion the author wishes to express his thanks to Dr. W. J. Sweeney of the Esso Laboratories for his encouragement and backing in developing and building this absorption cell.

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Resolving Power as a Function of Intensity

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I. OBSERVATIONS WITH A MICROSCOPE

ON the basis of a calculation due to Airy, the Rayleigh criterion gives the limit of resolution for the microscope as

$$X = 0.5 F\lambda/d, \quad (1)$$

where X is the minimum separable distance between two neighboring disk objects, λ is the wave-length of light, F is the focal length of the microscope objective, and d is the diameter of the aperture. Experiments with virus bodies, the size of which was below the limit of resolution according to the above formula, led Merling-Eisenberg to make the following statement: "It has been observed that reduction of intensity shows details that would not be visible in some cases and not resolvable in other cases if the formula was strictly right."¹ In another paper he makes the further observation that "... it becomes clear that the effect of increased resolution by means of reduced intensity is not within the content of the present formula of resolution."² From his experiments he draws the conclusion that the formula for the limit of resolution needs to be altered to include intensity I in the following manner:

$$X = Ik\lambda/A = IkF\lambda/d, \quad (2)$$

where A is the numerical aperture.³ Since the Rayleigh objective resolving power is the reciprocal of the limit of resolution, it must be inversely proportional to intensity if (2) is valid. This conclusion is a matter of more than pedantic interest, for it has led to the development of techniques for resolving virus bodies which are "sub-microscopical" according to the Airy formula. Details of the internal structure of bacterial cells have been clearly resolved; and bacteria have been observed visually and photographically before, during, and after lysis.⁴

There are three respects in which this notable work needs to be extended. In the first place, it is directly applicable only to objective resolving power.⁵ However, the persuasion that spectroscopic and objective resolution are comparable phenomena suggests that they are likely to be related similarly to intensity. But this cannot be assumed without adequate arguments. Secondly, the measurements upon which (2) is based have been in large measure visual rather than photographic. The reason for this has been the extreme smallness of the particles in which Merling-Eisenberg has been interested. His experiments

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¹ K. B. Merling-Eisenberg, *Nature* **139**, 416 (1937).

² K. B. Merling-Eisenberg, J. Quekett *Microscopical Club* **1**, 324-332 (1937).

³ K. B. Merling-Eisenberg, J. Quekett *Microscopical Club* **1**, 311-323 (1937).

⁴ K. B. Merling-Eisenberg, *J. Exper. Pathology* **19**, 338-342 (1938).

⁵ As here used, objective resolving power refers to the ability of an optical instrument to distinguish between two objects radiating the same wave-length, while spectroscopic resolving power refers to the ability to distinguish between two spectral lines from the same source.