Experimental validation of a simple approximation to determine the linewidth of a laser from its frequency noise spectrum

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Laser frequency fluctuations can be characterized either comprehensively by the frequency noise spectrum or in a simple but incomplete manner by the laser linewidth. A formal relation exists to calculate the linewidth from the frequency noise spectrum, but it is laborious to apply in practice. We recently proposed a much simpler geometrical approximation applicable to any arbitrary frequency noise spectrum. Here we present an experimental validation of this approximation using laser sources of different spectral characteristics. For each of them, we measured both the frequency noise spectrum to calculate the approximate linewidth and the actual linewidth directly. We observe a very good agreement between the approximate and directly measured linewidths over a broad range of values (from kilohertz to megahertz) and for significantly different laser line shapes.

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1. Introduction

The spectral coherence of a laser constitutes an important characteristic in various applications, such as atomic physics, coherent optical communications, or high-resolution spectroscopy, to name a few. The linewidth, i.e., the full width at half-maximum (FWHM) of the optical line shape, is commonly used to characterize the spectral properties of a laser, as this single parameter is convenient to compare different laser sources in a simple and straightforward manner. However, it gives only poor information about the spectral distribution of the laser frequency noise. Fitting the laser line shape by a Voigt profile allows to extract the Lorentzian and Gaussian contributions, thus to get some information about the respective contribution of white frequency noise and flicker noise to the laser spectrum [1], but the information obtained in this way remains incomplete. Finally, any free-running laser suffers from flicker noise that diverges at low frequency, leading to a linewidth that depends on the observation time. All these points make the linewidth improper as a figure of merit of the laser spectral properties.

On the opposite, a full picture of the laser frequency noise is given by the frequency noise power spectral density (PSD). It represents the spectral density of the laser frequency fluctuations and shows those noise spectral components that contribute to the laser linewidth. The frequency noise PSD can be measured using a frequency discriminator to convert the frequency fluctuations of the laser into measurable voltage fluctuations. An optical frequency discriminator, such as the side of an atomic/molecular transition [2,3] or of a Fabry–Perot resonance [4], can be used to measure directly the laser frequency fluctuations in the optical domain. Alternatively, a radio-frequency (RF) discriminator can be used to analyze the frequency fluctuations of

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the heterodyne beat signal between the laser under test and a reference laser in the electrical domain [5]. Both approaches are fully equivalent.

The frequency noise PSD contains the complete information about the laser frequency noise. The optical line shape, and thus the linewidth, may in principle be calculated from the frequency noise spectrum, while the reverse process is not possible. However, the exact determination of the linewidth from the frequency noise spectral density is not straightforward in most cases and involves a two-step numerical integration procedure [6–9], which we will briefly recapitulate at the beginning of Section 2.

Recently, we proposed a very simple approximation to determine the linewidth of a laser from an arbitrary frequency noise spectrum based on theoretical considerations [10]. Here, we present the first experimental validation of this simple formula. Using state-of-the-art femtosecond fiber and solid-state lasers as test signals, we compare the linewidth calculated from the measured frequency noise PSD by our simple approach with the actual linewidth independently measured. We used femtosecond lasers as test signals because they offer an easy and convenient means to modify the frequency noise spectrum and consequently to vary the corresponding linewidth over a broad range. The experimental validation of our simple approximation of the linewidth is presented here over three decades, covering linewidths spanning from kilohertz to megahertz.

The paper is organized as follows: Section 2 recalls the theoretical background of the relation between laser frequency noise and linewidth, with a special emphasis on our simple approximate expression of the linewidth. Section 3 presents the experimental setup and methods used to compare the linewidth calculated from the measured frequency noise PSD and the actual linewidth independently determined. Section 4 shows the experimental results of this comparison and a conclusion is presented in Section 5.

2. Theoretical Background

The universal method for the calculation of the laser optical line shape from its frequency noise PSD $S_{\delta\nu}(f)$ has been derived by Elliot *et al.* [6]. The PSD of the laser optical field $S_E(\nu)$ is given by

$$S_E(\nu) = 2\int_{-\infty}^{\infty} e^{-i2\pi\nu\tau}\left[E_0^2 e^{i2\pi\nu_0\tau} \times \exp\left(-2\int_0^{\infty} S_{\delta\nu}(f)\frac{\sin^2(\pi f\tau)}{f^2}\mathrm{d}f\right)\right]\mathrm{d}\tau. \quad (1)$$

This expression is the Fourier transform of the autocorrelation function of the laser electric field represented by the term in the square brackets. The only situation in which Eq. (1) can be analytically solved is the ideal case of a pure white frequency noise [7], which leads to the well-known Lorentzian line shape described by the Schawlow–Townes–Henry linewidth [11,12]. In the real case of an experimental frequency noise spectrum, Eq. (1) has to be numerically integrated and some care is required in the implementation of this procedure to obtain the correct laser optical line shape without numerical artifact. This includes the evaluation of the autocorrelation function of the laser electric field over an ensemble of correlation times τ, i.e., the first integral (in the parentheses) has to be calculated many times for different values of τ. Using improper values for either the overall range of τ [which determines the resolution of the line shape spectrum calculated from Eq. (1)] or the sampling rate of the autocorrelation function (which determines the Nyquist frequency of the calculated line shape) may lead to an incorrect calculated laser optical line shape. As the line shape (linewidth) to be retrieved is not known *a priori*, the choice of the τ values is not trivial and may require an iterative process to achieve the correct linewidth. Furthermore, an experimental frequency noise spectrum covers a finite frequency range, so that the first integration cannot be performed between zero and infinity as defined in the general Eq. (1). Therefore, one has to restrict the integration over a narrower interval, but from the double integration of Eq. (1), it is not obvious to determine which parts of the frequency noise spectrum contribute to the linewidth and which parts do not. We recently introduced the concept of the β-separation line in the frequency noise spectrum that provides a straightforward metric to identify those spectral components that contribute to the linewidth [10].

Based on the concept of the β-separation line, we also proposed a simple approximate formula to determine the linewidth of a laser from its frequency noise PSD [10]. This approximation, based on a geometrical separation of the frequency noise PSD into two areas, applies to any type of noise and avoids the two-step integration procedure required in the exact line shape calculation previously discussed. Here, we briefly review the principles of this approximation. In the next sections, we present an experimental validation of this formula and assess the accuracy of this approximation using real experimental laser spectra.

In our theoretical study, we introduced the β-separation line, defined as $S_{\delta\nu}(f) = \frac{8\ln 2}{\pi^2}f$, which geometrically separates the frequency noise PSD in two regions with a significantly different influence on the optical line shape (see Fig. 1).

Only the slow frequency modulation area, where $S_{\delta\nu}(f) > \frac{8\ln 2}{\pi^2}f$, contributes to the linewidth of the signal. In the fast frequency modulation area, where $S_{\delta\nu}(f) < \frac{8\ln 2}{\pi^2}f$, the frequency fluctuations are too fast to affect the laser linewidth and only contribute to the wings of the line shape. Our simple approximation of the laser linewidth is obtained from the surface A of the slow modulation area of the PSD spectrum displayed in Fig. 1:

$$\mathrm{FWHM} = \sqrt{(8\ln 2)A}. \quad (2)$$

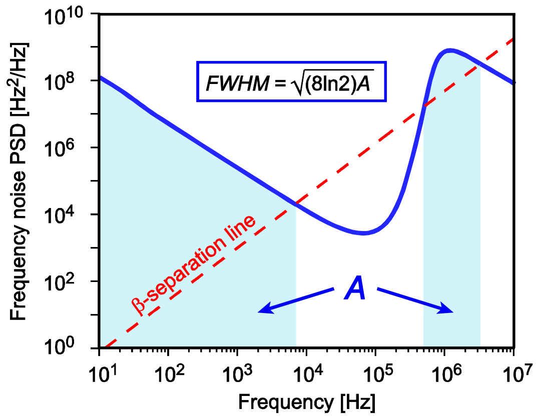

Fig. 1. (Color online) Graphical representation of the simple approximation proposed to determine the linewidth of a laser from its frequency noise PSD $S_{\delta\nu}(f)$, calculated from the surface A of the slow modulation area [10]. The shadowed areas on this schematized frequency noise spectrum represent the surface A that encloses all spectral components for which $S_{\delta\nu}(f)$ exceeds the β-separation line $S_{\delta\nu}(f) = \frac{8\ln 2}{\pi^2}f$ (dashed line).

Here, A represents the overall surface under the portions of $S_{\delta\nu}(f)$ that exceed the β-separation line, which is mathematically obtained by integrating the product between the frequency noise PSD and the Heaviside step function $H(x) = 0$ if $x < 0$, $H(x) = 1$ if $x \geq 0$:

$$A = \int_{1/T_0}^{\infty} H\left(S_{\delta\nu}(f) - \frac{8\ln 2}{\pi^2}f\right)S_{\delta\nu}(f)\mathrm{d}f. \quad (3)$$

A cut-off frequency $1/T_0$, where T_0 is the observation time, is introduced here to prevent the divergence of the integral, and consequently of the linewidth, in presence of $1/f$ noise. However, integration down to zero frequency (infinite observation time) is possible in absence of diverging low-frequency noise, for instance when the laser flicker noise is being suppressed by an active stabilization.

In comparison with the two-step integration procedure, the approximation based on the β-separation line requires only the evaluation of the area of a bounded surface below the frequency noise spectrum $S_{\delta\nu}(f)$. This numerical integration is quite trivial and not prone to numerical artifacts, making our method very robust and easy to apply.

3. Experimental Setup

The validation of our theoretical approximation of the linewidth requires the availability of different test signals, covering a wide range of linewidths. An appropriate test signal for this study would be a laser source or an optical beat signal of variable linewidth. From our activities on the stabilization of optical frequency combs, we learned that the carrier-envelope offset (CEO) beat of a frequency comb, which corresponds to the heterodyne optical beat between comb lines located at the two extreme edges of an octave-spanning comb spectrum [13], can be changed in a wide range of linewidths. For this convenience, we opted for the CEO beat of a frequency comb as a test signal for the experimental validation of our approximate formula.

A. Laser Sources

A frequency comb is generated from an ultrafast laser with femtosecond pulses. It comprises tens to hundreds of thousands of equidistant spectral lines that form a frequency ruler over a broad spectrum [14,15]. The carrier-envelope offset results from a phase shift between the laser pulse envelope and the carrier field. In the frequency domain, it manifests as a uniform shift f_{CEO} of all frequency comb lines from exact harmonics of the laser repetition rate f_{rep} ($\nu_N = f_{\mathrm{CEO}} + Nf_{\mathrm{rep}}$).

In time and frequency metrology, a frequency comb provides a direct link between optical and microwave frequencies and is generally stabilized to a microwave reference. This is accomplished by phase locking the two comb parameters (f_{CEO} and f_{rep}) to a highly stable external reference. For the stabilization of the CEO in our combs, the phase fluctuations between the CEO beat and the external reference are detected in a digital phase detector (Menlo DXD200). After amplification by a servo controller, the feedback signal is applied to the pump diode of the femtosecond laser. Depending on the gain and bandwidth of the CEO servo loop, the frequency noise of the CEO beat, and thus its linewidth, may be modified. In our experiment, we used two different frequency combs, each covering a different range of linewidths. In each comb, the CEO beat is locked to a 20 MHz reference oscillator.

The first frequency comb is generated from a commercial Er:fiber laser (FC1500 from MenloSystems, Germany) with 250 MHz repetition rate. This system suffers from significant noise as commonly observed in Er:fiber lasers [16], leading to a free-running CEO-beat linewidth of 200–300 kHz. A linewidth ranging from 300 kHz to 2 MHz can be obtained in the stabilized CEO-beat by altering the loop parameters.

The second frequency comb is based on an Er:Yb:glass laser oscillator (ERGO), a femtosecond diode-pumped solid-state laser with 75 MHz repetition rate [17]. Its superior noise properties lead to a free-running CEO beat linewidth of ≈4 kHz (at 10 ms observation time) [18]. A narrow-linewidth range (4 kHz to 20 kHz) was covered with this system by changing the parameters of the CEO servo loop. Moreover, different levels of white noise were added to the piezoelectric transducer controlling the length of the femtosecond laser cavity, which induces additional frequency noise in the CEO beat due to the correlation between the noise of the repetition rate and of the CEO beat generally observed in a frequency comb [19]. In combination with the adjustment of the stabilization loop parameters, this enabled us to further broaden the CEO linewidth in the intermediate range of 20 kHz to 100 kHz.

In both combs, the phase stabilization strongly reduces the low-frequency $1/f$ noise of the free-running

CEO beat. Consequently, only a small portion of the frequency noise spectrum exceeds the β-separation line and contributes to the linewidth, which is moreover independent of the observation time. This allows a true comparison between the linewidth determined from the frequency noise PSD and the actual linewidth observed on a spectrum analyzer, without any experimental artifacts. This also enabled us to make long averaging of the spectra in order to enhance the signal-to-noise ratio and to increase the accuracy of the linewidth determination, thus improving the comparison between the approximate and real linewidths.

Finally, a third different system has been considered in our analysis to demonstrate the universality of our approach. It consists in the heterodyne beat between one line of the Er:fiber femtosecond laser and a 1.55 μm cavity-stabilized ultranarrow linewidth laser [20]. In this case, a frequency noise PSD of a different shape was obtained compared to the CEO beats of the frequency combs, with a corresponding linewidth of ≈170 kHz.

B. Frequency Noise PSD Measurement

The frequency noise PSD of the beat signals was measured using a frequency discriminator, in order to convert the frequency fluctuations into measurable voltage fluctuations [5]. For the ERGO and Er:fiber CEO beats, the digital phase detector DXD200 of the CEO stabilization loop was used, with a measured sensitivity of $\approx 0.02 \times \frac{1}{f}\ \left[\frac{\mathrm{V}}{\mathrm{Hz}}\right]$. We showed in Ref. [5] that this device does not have a constant sensitivity over its entire range of operation, but presents some nonlinear points where the local sensitivity significantly deviates from the average measured value. Even if the phase detector is operated out of these strong nonlinearities, the sensitivity slightly depends on the exact operating point. This makes a very accurate determination of this sensitivity difficult, resulting in a typical uncertainty in the order of 10%. The frequency noise of the beat between the ultrastable laser and the Er:fiber frequency comb was demodulated using an RF frequency discriminator (frequency-to-voltage converter) Miteq FMDM-21.4/4-2 with a sensitivity of $1.25 \times 10^{-6}\ \left[\frac{\mathrm{V}}{\mathrm{Hz}}\right]$. The response of this discriminator is dependent on the amplitude of the input signal [5], which leads to a similar uncertainty of 10% on the discriminator sensitivity.

For each experimental signal, the PSD of the discriminator output voltage was recorded on a fast Fourier transform (FFT) analyzer and afterwards converted into frequency noise PSD using the discriminator sensitivity. In order to achieve a good spectral resolution over the entire considered frequency range, each spectrum was obtained from the combination of five FFT spectra of decreasing spectral resolution (span of 24.4 Hz, 195 Hz, 1.56 kHz, 12.5 kHz, and 100 kHz) after coaveraging at least 300 individual FFT traces. The area of the frequency noise PSD exceeding the β-separation line was determined for each spectrum by Eq. (3) and the approximate linewidth $\mathrm{FWHM_{PSD}}$ was then calculated using Eq. (2). The relative uncertainty on the calculated linewidth lies in the range from 10% to 15% and is mainly due to the uncertainty on the discriminator sensitivity.

C. Line Profile Measurement

The RF heterodyne beat (either CEO beat or laser to comb beat) was separately recorded on an electrical spectrum analyzer (ESA). The absence of $1/f$ noise in the spectra enabled us to improve the signal-to-noise ratio of the recorded signal by coaveraging 200 spectra. The measured power spectra were fitted by a Voigt profile in order to extract the Gaussian and Lorentzian contributions and thus the Voigt linewidth $\mathrm{FWHM_{Voigt}}$. The uncertainty in the Voigt linewidth was determined from the standard deviation of the Lorentzian and Gaussian contributions, using the analytical approximation of the Voigt linewidth given by Oliveiro and Longbothum [21]. Depending on the quality of the signal, the typical relative uncertainty ranges from 1% to 12%.

4. Results

For every test signal, both the line profile and the frequency noise PSD were measured in the same conditions, with the objective to directly compare the measured linewidths with those derived from the frequency noise PSD using our approximate formula. Some representative examples of experimental signals obtained for the three laser systems used in this study are shown in Fig. 2.

The upper row of Fig. 2 shows the frequency noise PSD $S_{\delta\nu}(f)$ measured using the frequency discriminators. The surface A of the spectrum for which $S_{\delta\nu}(f)$ exceeds the β-separation line (i.e., the slow modulation area) is represented by a shadowed area. The linewidth $\mathrm{FWHM_{PSD}}$, calculated from this area by Eq. (2), is displayed in each plot with its corresponding uncertainty, obtained as described in Subsection 3.B and indicated in parentheses. The lower row of Fig. 2 shows the line shape profiles recorded on the ESA, together with the Voigt fit. The small values of the fit residuals shown on top of the line shapes in Fig. 2(d)–2(f), which represent the difference between the fitting function and the measured data, validate the choice of a Voigt profile as a fitting function to extract the FWHM of the measured line shapes. The resulting Voigt linewidth $\mathrm{FWHM_{Voigt}}$ is indicated in each spectrum with the corresponding uncertainty in parentheses, obtained as described in Subsection 3.C.

In the example of the ERGO comb (left column in Fig. 2), the narrow linewidth of 3.9(1) kHz is the result of the contribution of only a tiny portion of the frequency noise PSD (ranging from 300 Hz to 2 kHz). In the displayed example of the Er:fiber comb (right column in Fig. 2), a larger part of the frequency noise PSD contributes to the linewidth, in particular the peak near 40 kHz corresponding to the servo bump

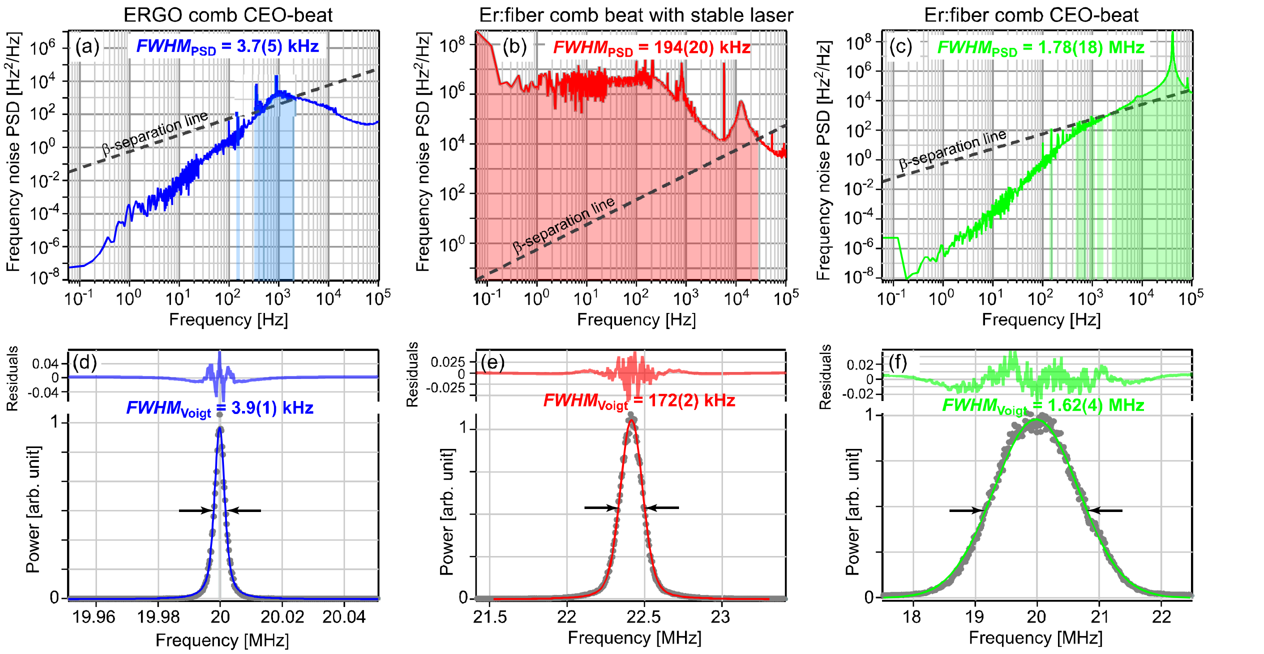

Fig. 2. (Color online) Representative examples of frequency noise PSD (upper row) and corresponding line shapes (lower row) for three different laser systems: CEO beat in the ERGO comb (left), heterodyne beat between one line of the Er:fiber comb and a cavity-stabilized laser (middle), and CEO beat in the Er:fiber comb (right). The linewidth $\mathrm{FWHM}_{\mathrm{PSD}}$ is calculated from the shadowed area for which the frequency noise PSD exceeds the β-separation line (dashed line). The line shapes were recorded using an ESA (grey circles) and were fitted by a Voigt profile (line) to extract the actual linewidth $\mathrm{FWHM}_{\mathrm{Voigt}}$. Fit residuals shown on top of the lower row represent the difference between the values of the fit function and the measured data. The uncertainty on the FWHM is given in parentheses following the FWHM value.

that results from the deliberately increased servo gain. This leads to a larger linewidth of 1.62(4) MHz.

In both comb signals, the low frequency noise does not contribute to the linewidth since it has been completely reduced below the β-separation line by the active CEO stabilization. This leads to a linewidth independent of the observation time. In the beat between the Er:fiber comb and the cavity-stabilized laser, the low-frequency noise is not strongly suppressed and all noise components at $f < 30$ kHz contribute to the linewidth. However, the low-frequency noise remains bounded, i.e., does not diverge, so that the corresponding linewidth of 170(2) kHz is also independent of the observation time.

Such measurements and data processing have been repeated for different experimental conditions in order to cover the widest possible range of linewidths. In total, 26 sets of data have been recorded and the corresponding linewidths have been extracted. They cover linewidth values ranging from 3.9(1) kHz to 1.62(4) MHz (19 points for the ERGO comb, 6 for the Er:fiber comb and one laser-Er:fiber comb beat). Figure 3 summarizes these results, showing the linewidth $\mathrm{FWHM}_{\mathrm{PSD}}$, calculated from the frequency noise PSD (y axis) as a function of the actual linewidth $\mathrm{FWHM}_{\mathrm{Voigt}}$ (x axis) determined from the Voigt fit of the ESA trace, together with the corresponding error bars. One observes that all of the points in Fig. 3, distributed over a broad range of almost three decades, are aligned on the $y = x$ line within the experimental error bars. One also notices a systematic underestimation of ≈10% of the linewidth calculated from the frequency noise PSD using our simple approximation for all points corresponding to a linewidth lower than 100 kHz. All these points have been obtained from the same laser system (CEO beat of the ERGO comb with or without additional noise applied to the comb piezotransducer) using the same digital phase detector. We attribute this systematic error to a bias in the value of the discriminator sensitivity used in the scaling of

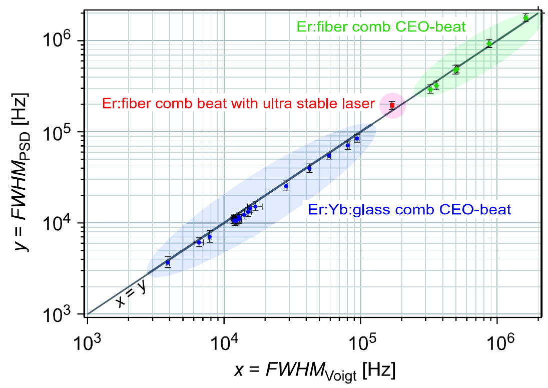

Fig. 3. (Color online) Comparison between the approximate linewidth $\mathrm{FWHM}_{\mathrm{PSD}}$, calculated from the measured frequency noise PSD (y axis), and the actual linewidth $\mathrm{FWHM}_{\mathrm{Voigt}}$ (x axis) determined from the Voigt fit of the ESA trace, obtained over a broad range with three different laser systems. The uncertainty on each point is indicated by the error bars.

the frequency noise PSD, which might result from a slightly different operating point of the phase detector as compared to the conditions used in the calibration of this device [5], as discussed in Subsection 3.B. The point corresponding to a linewidth of ≈170 kHz in Fig. 3 relates to the beat between the Er:fiber comb and the narrow-linewidth laser, which has been measured using a different frequency discriminator (Miteq FMDM21.4/42 frequency-to-voltage converter). The sensitivity of this discriminator is dependent on the amplitude of the input signal and the 14% overestimation of the calculated linewidth, observed in Fig. 3, might also result from a small bias in the considered discriminator sensitivity, due to slightly different signal amplitude used in the experiment. Finally, the points of broadest linewidth in Fig. 3 were obtained from the CEO beat of the Er:fiber comb using the digital phase detector. The frequency noise of this signal being much larger than in the case of the ERGO comb, one cannot exclude that a small influence of the nonlinear response of the phase detector, as discussed in Subsection 3.B, slightly impacts these data points. In such a case, a different frequency noise level could likely lead to a different bias, which could be a reason for the absence of a systematic bias in Fig. 3 in the data points corresponding to the Er:fiber comb.

Nevertheless, the linewidths obtained with our simple approximation are in agreement with the actual linewidths within the experimental error bars of approximately 10%. This result, furthermore obtained with three different laser systems, fully validates our simple approach to determine the linewidth of a laser from its frequency noise spectrum. Moreover, the line shapes considered in this study span from pure Gaussian ($\gamma = 10^{-4}$), to almost an equal weight of Lorentzian and Gaussian contributions ($\gamma = 0.91$), where γ represents the ratio of Lorentzian- to Gaussian-width of the Voigt profile. This characteristic shows that the proposed linewidth approximation based on the β-line applies to various line shape functions and demonstrates its broad applicability.

5. Conclusion

In this work, we have shown an experimental validation of a simple geometrical approximation that we previously introduced to determine the linewidth of a laser from its frequency noise power spectral density. We used state-of-the-art laser light sources (two optical frequency combs and an ultrastable laser) to generate optical test signals of different spectral properties. For each of the test signals, both the frequency noise PSD and the line shape were independently measured. We observed an excellent agreement, within the experimental uncertainties, between the approximate linewidth calculated from the experimental frequency noise PSD using our simple formula and the actual linewidth extracted from a fit of the measured line shape.

This study has demonstrated the applicability of this simple approximation of the linewidth over a broad range of values, spanning almost three decades (from 10^3 Hz to 10^6 Hz), and for a wide variety of line shape functions characterized by a ratio of Lorentzian- to Gaussian-linewidth ranging from 10^{-4} to 0.9.

Altogether, this simple approximation is obtained in an easy and straightforward manner from any experimental frequency noise PSD, avoiding the complicated and time-consuming two-step numerical integration procedure encountered in the exact determination of the laser line shape (and thus linewidth) from the frequency noise PSD. It thus represents a very useful tool for any experimentalist in laser physics as well as in time and frequency metrology.

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