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Time-dependent laser linewidth : beat-note digital acquisition and numerical analysis

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Abstract: We revisit and improve the optical heterodyne technique for the measurement of the laser coherence, by digital acquisition of the beat-note and numerical analysis of the resulting signal. Our main result is that with the same experimental setup we reach the very "short-time linewidth" with the highest accuracy as well as the frequency noise spectrum.

OCIS codes: (140.0140) Lasers and laser optics; (030.1640) Coherence; (300.6310) Spectroscopy, heterodyne.

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1. Introduction

It is a well known fact that the fundamental linewidth limit of most lasers originates from the spontaneous emission coupled to the lasing mode. This limit is called the "Shawlow-Townes linewidth" [1], enhanced by Henry in the case of semiconductor lasers [2, 3]. This topic of research has been widely discussed in the scientific literature during the 1980-2000 years [4–6] thanks to the development of single frequency laser diodes [7–10], encouraged itself by the rise of fiber optic telecommunications for the world-wide internet network [11]. The laser diodes linewidth is indeed a crucial parameter for the optical fiber telecommunication systems themselves, as it obviously impacts the purity of the signal phase, as well as the intensity noise at the optical receiver [12, 13].

But beyond that, in a more fundamental perspective, these laser components exhibit strong advantages for studying the linewidth, compared to other laser technologies. First of all, they do not exhibit mode-hopping in time, making practical, for the first time, the experimental study of this fundamental limit of laser noise. Moreover, in many cases, their spontaneous emission level is strong, masking the contributions of most other technical fluctuations (cavity length or index variations) to the linewidth. This motivated the production of a large set of studies and data on these devices [14–25].

As a result, because the noise of these components originates mainly from the spontaneous emission, which is theoretically a pure white noise, the value of the linewidth is usually considered to be stable whatever the time during which its measurement occurs. However, lasers emitting a small amount of spontaneous emission (such as edge-emitting extended cavity lasers [26, 27], fiber lasers [28, 29], vertical external cavity surface emitting lasers [30, 31], and other kinds ...) lead to laser spectra dominated by other sources of noise (that we call here "technical noise sources": these are typically mechanical, thermal or carrier-induced).

Unfortunately, in that case, the linewidth depends strongly on the observation time. This fact raises theoretical problems as well as experimental ones, which are usually incompletely addressed in the literature. Nowadays, this issue has to be addressed regarding the increasing number of narrow linewidth lasers developed, motivated by new kinds of applications, mainly in the field of optical sensors [32–38].

In this paper, we intend to provide a better understanding as well as experimental techniques to overcome the limitations of the standard metrology.

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2. Limits of the standard metrology of the linewidth

Concerning the experimental techniques, the metrology of the linewidth classically relies on the so-called heterodyne set-up: the beams of two identical lasers are superimposed spatially, and the frequency of the lasers is adjusted to generate a beating at a frequency which is low enough to be observable with radio-frequency (RF) instrumentation. The resulting beam then falls on a photodetector that generates a beat-note in the radio-frequency domain which is in turn analyzed thanks to a sweeping (superheterodyne) RF spectrum analyzer [39] (see Fig.1).

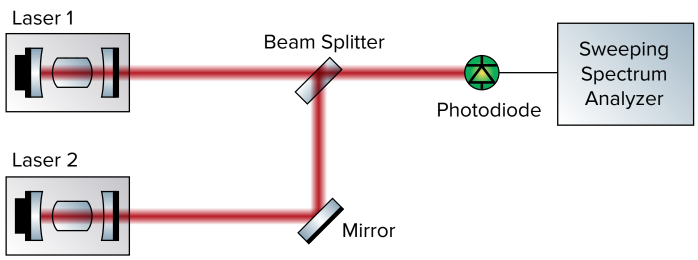

Fig. 1. Basic Heterodyne Set-up.

Unfortunately, these spectrum analyzers do not allow to study easily the time dependency of the linewidth, because of the time required to sweep across the RF spectrum, which cannot usually be quicker than 1 *ms* in practical cases. There is an additional problem: using this device, the data obtained cannot be considered to be a snapshot of the laser spectrum, because all the points of such spectrum are not evaluated accurately at the same time due to the sweeping of the resolution filter [39] (Fig. 2).

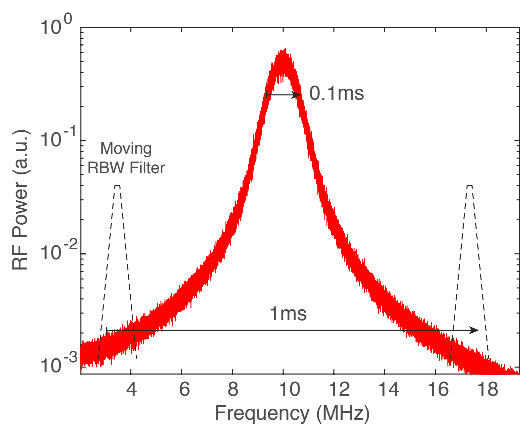

Fig. 2. Sweeping (superheterodyne) RF spectrum analyzer at work: the resolution filter sweeps in time across all the frequencies to obtain the whole spectrum. The spectrum does not reflect a specific timeframe.

This comes from the fact that acquisitions obtained with this kind of analyzer assume that the spectrum remains stable during the whole sweeping, which is not true with laser lines perturbed by technical noise sources. Thus, the profile acquired is distorted by the highly non-linear transfer function of the spectrum analyzer, loosing accuracy on the spectral shape itself.

Finally, it is clear that in some cases, the time-limited linewidth may change at each acquisition, even for a constant timeframe. A telling example is the case of frequency fluctuations originating

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from mechanical vibrations, which are typically sine-wave shaped in time (they usually find their origin in mechanical resonances). One will easily see that the linewidth is not constant between repeated experiments, given that the RMS frequency fluctuation intercepted in the corresponding time window changes, as depicted in Fig. 3.

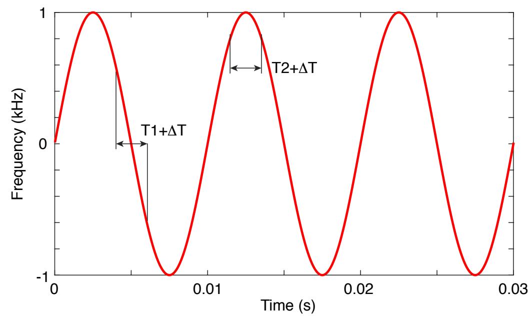

Fig. 3. Frequency variations induced by typical mechanical noise. In this case, the RMS value is not stable as a function of the starting time T_i of the experiment.

Formally written, the value of:

$$\sqrt{\frac{1}{\Delta T}\int_{T_i}^{T_i+\Delta T}\sin^2(2\pi f_M t)dt} \quad (1)$$

is obviously not constant as a function of T_i nor as a function of ΔT (Here, T_i is the time at the beginning of the experiment, ΔT the duration of the experiment, and f_M the frequency of the considered mechanical oscillation). For that reason, such linewidth can only be given in statistical terms over repeated measurements, such as average value and standard deviation.

Due to these metrological limits, the experimenters try to estimate the laser linewidth from frequency noise measurements. It is an interesting idea because thanks to a frequency noise spectrum, they can pinpoint the characteristic time associated with each noise source and obtain much more information than with a single-shot heterodyne measurement. In this field, various works are reported [7, 40], but the most useful and interesting one, to our knowledge, has been performed by Di Domenico et al. by applying the β-separation line theory to the study of the laser linewidth [41, 42]. This is a method of calculating a time-dependent linewidth from a power density spectrum of frequency noise. Whereas it is reliable in many practical cases, this approach has two drawbacks: first of all, it relies on frequency noise measurements, which require quite complex set-ups and are not so easy to calibrate accurately. Secondly, the computation of the linewidth through this process requires approximations that can lead to inaccurate results.

This can be explained by the fact that linewidth metrology and the associated signal analysis were imagined when the available RF instrumentation was far less developed than they are today. It is now possible to acquire the heterodyne beat-note in the time domain thanks to the emergence of high-speed data acquisition systems at quite low cost (compared to RF spectrum analyzers). This beat-note contains obviously all the noise information traditionally obtained by indirect ways. In the work described here, we propose to use modern widespread data acquisition systems (high performance samplers) and numerical signal analysis to perform a more accurate and more powerful study of the beat-note obtained with the heterodyne set-up (Fig. 4), enabling to overcome the already-discussed drawbacks of the methods proposed in the state-of-the-art.

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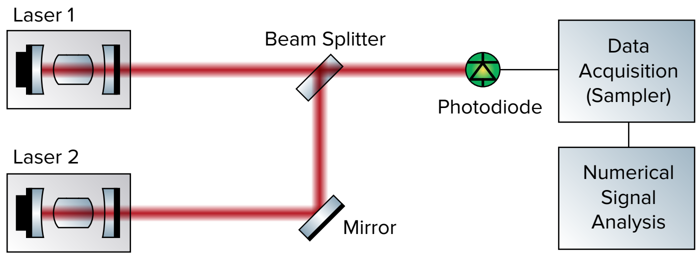

Fig. 4. Heterodyne Set-up proposed in this work: use of a sampler and numerical signal analysis.

3. Data treatment through signal analysis: an overview

In contrast to what is usually reported in the scientific literature, our approach consists in performing numerical processing to the acquired time-domain beat-note signal in order to extract the phase fluctuation information in terms of power spectral density and linewidth. The associated theoretical description is summarized in Fig. 5, that displays the attainable physical quantities as well as the mathematical relationships that exist between them. The numerical acquisition of the time-domain beat-note is the central entry point. These mathematical relationships are not new and are very well understood in the signal analysis literature. However, in the context of laser study, they are generally taken as a pure theoretical basis and not used in practice under their numerical form in order to be applied directly on recorded signals: this is precisely what we intend to propose here.

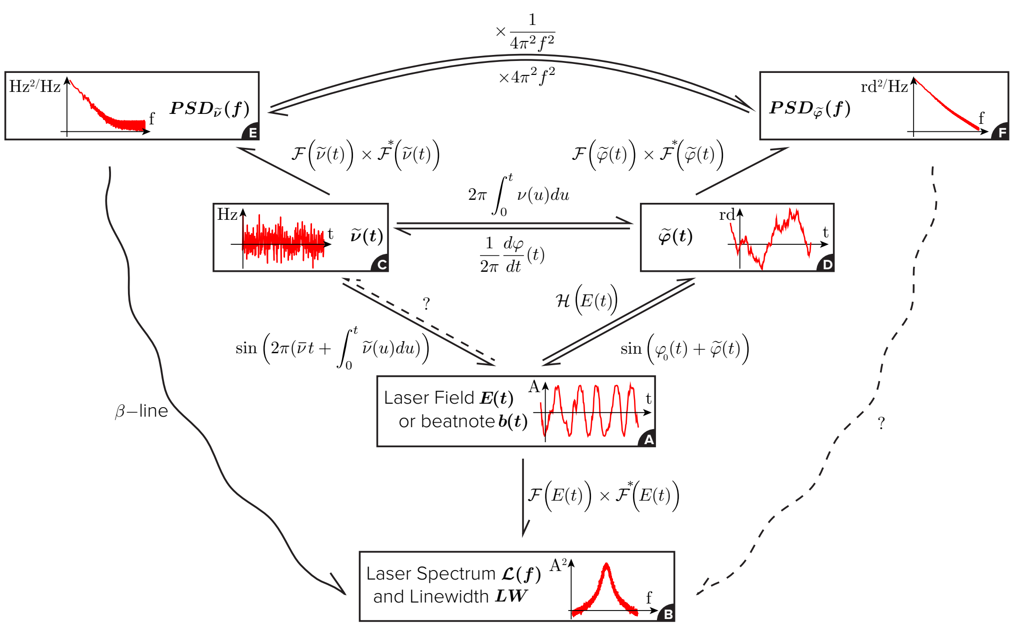

Fig. 5. Summary of the relevant signals and the mathematical relationships between them. No arrow is for no possible path. Dashed arrow is for no conceptual impossibility but not reported in literature. Wavy arrow is for possible path with approximations.

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Frequency Noise: By sampling the beat-note, it is possible to extract the instantaneous phase in the time-domain thanks to the numerical Hilbert transform $\mathcal{H}$. It opens the study of the frequency noise and related spectral densities with the classical approach, by following the path A→D→C→E in Fig. 5. Thanks to this procedure, we get more comfortable experimental conditions than with traditional frequency noise measurement systems, because the heterodyne set-up offers all the benefits of a coherent detection: we work with a high magnitude signal (the beat-note), which is weakly sensitive to the background noise of the instrumentation or to the RIN (Relative Intensity Noise) of the laser itself, which are usual limits of the methods exploiting frequency discriminators. Moreover, the calibration of the frequency noise spectrum does not rely anymore on the evaluation of the frequency discriminator slope, and there is thus no requirement for any calibration process.

Time-Dependent Linewidth: Beyond that, the acquisition of the beat-note opens the access to a range of meaningful information. The laser spectrum may now be assessed over different timeframes, simply by computing the Fourier transform $\mathcal{F}$ over sub-samples of various lengths. Now it is simply the A→B path in Fig. 5 that we have to follow. This triggers the extraction of valuable statistics about the linewidth, which may now be calculated from various sub-samples, as depicted in Fig. 6. In particular, the average, as well as the associated standard deviation, can be brought to light.

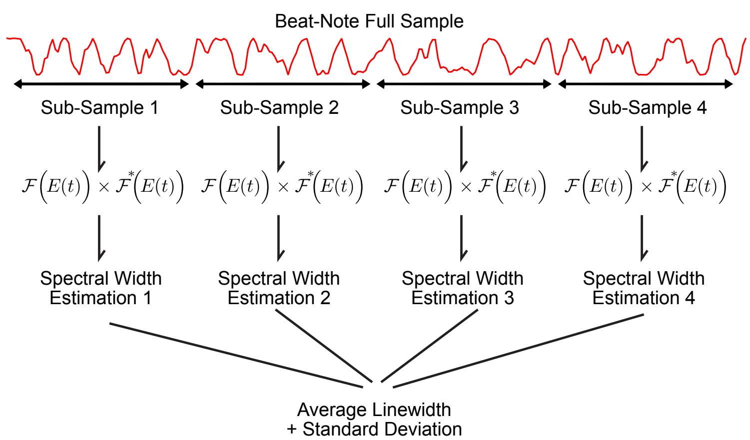

Fig. 6. Beat-note sampling and short-time linewidth estimation. Thanks to the change of the length of the sub-sample, a time-dependent linewidth study is possible.

Such way of exploiting signal analysis on the same beat-note sample makes available in one go various informations that would normally require different experimental set-ups. Moreover, because all these processings are performed on the same data source, the results can only be consistent between them.

About the acquisition of the beat-note itself, in numerous practical cases, it is difficult to maintain the beating of two lasers over long durations in a too-confined spectral span. Thus, the sampled beat-note must be set at a quite high frequency, which requires a high sampling frequency, typically in the 100 $MHz - 1\,GHz$ range. To access the statistical studies (averages of power-spectral densities, average and standard deviation of the linewidth, etc), long duration time-samples are required. For that reason, a wide amount of memory is mandatory, usually 10^8 to 10^9 samples, which is possible today with a modern desktop computer embedding some *Go* of RAM.

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4. Experimental work: configurations under study

4.1. Configuration 1: DFB-SC for atomic-clock systems

The first configuration used to check the validity of our approach consists in studying the beating of two similar "Distributed FeedBack Semiconductor laser diodes (DFB-SC)". The noise profile of the optical field emitted by these devices is generally affected by two main components.

The first one is linked to the –cold– optical cavity linewidth through the Schawlow-Townes limit, enhanced by a multiplying factor α_H known as the Henry factor [2, 3]. This correction arises from the perturbation of the refractive index of the gain medium through the gain change with carrier density fluctuations in this medium. The resulting laser lineshape is Lorentzian [1].

The second component of the noise spectral density is related to the electrical nature of the pumping process. A very fundamental consequence of this is the existence of a carrier noise occurring in various locations of the component. The most fundamental part of this noise (which is thermal noise due to the equivalent electrical resistance of the device) does not impact the optical field. Unfortunately, some additional transport mechanisms, due to inhomogeneities in the semiconductor crystal, impact the noise at low frequencies [43]. Even with the cleanest fabrication processes, the "$1/f$ type" noise seems to be unavoidable and is always observed [44]. Its contribution to the linewidth is known to be Gaussian [7, 45]. The laser spectrum resulting from these two contributions has a "Voigt Profile", which is the result of the convolution between a Lorentzian and a Gaussian spectrum.

As a case study, compact modules emitting at 894nm – which corresponds to the $D1$ transition line of the Cesium 133 – can be used to realize the Optical Cesium Frequency Standard, a type of compact atomic clock which allows better short-term stability than its magnetic counterpart [46]. In such apparatus, the stability of the clock is known to depend on the noise characteristics of the pumping laser. In the present case, the GaAs DFB-SC of $2\,mm-$long cavities are used at a $40\,mW$ output at the Cesium $D1$ line. The measurements are done on a packaged version of the diodes that includes thermal monitoring and regulation. In particular, the beat-note is realized with two lasers of equivalent first-order characteristics.

During the experiments, the two laser diodes are biased using a set of batteries and low-noise resistors to perform the voltage to current conversion. The noise of the resulting laser driver is much lower than the intrinsic electrical noise of the laser diode itself [47, 48]: this ensures that the noise sources observed during the experiments are intrinsic to the laser components and are not due to some environmental perturbations. The noise analysis coming from these components is thus designed to give a reference measurement using a class of components which is well known in the literature. Also, it enables an accurate estimation of the impact of the $1/f$ noise on the linewidth.

4.2. Configuration 2: DFB-FL and ECDL

To expand this study, we investigated the noise coming from the beating between a DFB Fiber Laser (DFB-FL) [28] and a single-frequency External Cavity Diode Laser (ECDL) [49], both operating in the telecommunication window ($1530\,nm$ – $1560\,nm$ for the ECDL, $1553\,nm$ for the DFB-FL). Here, the DFB-FL spectrum can be identified as a Dirac peak compared to the ECDL, as its frequency noise is known to be ultra low, exhibiting FWHM (Full Width Half Maximum) linewidth close to $10\,kHz$.

The DFB-FL involved here is a commercial device by IxFiber. It is $\approx 5\,cm$ long and is pumped by a battery-biased low power EDFA (Erbium-Doped Fiber Amplifier) pump ($< 300\,mW$); the emitted power is close to $100\,\mu W$ for $150\,mW$ pump power launched. The packaging ensures a very small contribution of mechanical vibrations on the linewidth.

The ECDL is a refurbished OSICS ECL Model 1560 biased through the electricity network. It is an interesting kind of laser to study because its spontaneous emission noise is known to be

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quite low (compared to DFB-SC) but due to its external cavity, it may experience some cavity length fluctuations originating from mechanical noise, leading to frequency variations in time. The emitted power is in the $10\,mW$ range over the whole tuning capability of the laser.

4.3. Configuration 3: two DFB-FL

Some additional results are obtained with the beating of two DFB-FL identical to the one described in the previous section. Once again, the pumps of these two lasers were biased using the set of low-noise batteries and resistors already described in section 4.1.

5. Frequency noise

The first part of the study is the frequency noise of each laser configuration, obtained from the beat-note and the application of the process described in §3.

5.1. Theoretical Background

As mentioned above, the starting point is the time domain sample of the beat-note signal, $b(t)$. It is a real-valued function and can be written as:

$$b(t) = a(t) \times \cos(\varphi(t)) \tag{2}$$

where, $a(t)$ and $\varphi(t)$ are the instantaneous amplitude and phase of the signal.

The so-called analytic signal associated with the real-valued signal $b(t)$ can be constructed using the Hilbert transform $\mathcal{H}$ as follows [50]:

$$b_a(t) = b(t) + j\mathcal{H}[b(t)] \tag{3}$$

the quantities $a(t)$ and $\varphi(t)$, accounting for the demodulated magnitude and phase of $b(t)$, are then given by, respectively: $a(t) = \sqrt{b(t)^2 + \mathcal{H}[b(t)]^2}$ and $\varphi(t) = \arctan[\mathcal{H}[b(t)]/a(t)]$. Using this representation, the rate of change of $\varphi(t)$ is the instantaneous frequency of the beat-note. Without loss of generality, it can be written as [51]:

$$\begin{aligned} \nu(t) &= \frac{1}{2\pi} \times \frac{\mathrm{d}\varphi(t)}{\mathrm{d}t} && (4) \\ &= \bar{\nu} + \widetilde{\nu}(t) && (5) \end{aligned}$$

where $\bar{\nu}$ is the central frequency of the beat-note, and $\widetilde{\nu}(t)$ represents the noise contribution. Hence, the power spectral density (PSD) of the frequency noise of the laser can be calculated thanks to the Fourier transform of the autocorrelation function:

$$PSD_\nu(f) = \mathcal{F}\left(\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} \widetilde{\nu}^*(t)\widetilde{\nu}(t+\tau)\,dt\right) \tag{6}$$

For computation time reasons, it may be better not to evaluate the autocorrelation function directly. Fortunately, this expression can find another form, entirely based on the Fourier transform [52]:

$$PSD_\nu(f) \propto \mathcal{F}\left(\widetilde{\nu}(t)\right) \times \mathcal{F}^*\left(\widetilde{\nu}(t)\right) \tag{7}$$

where * is the conjugate complex operator. This expression behaves well in the numerical world, as the time required for the autocorrelation computation (Eq. 6) is proportional to N^2 for a $N-$samples long signal, while the one required for the Fast-Fourier-Transform (FFT) based expression in (Eq. 7) is only proportional to $N \times \log_2(N)$ [52] for the same signal length. Averaging a given amount of such spectra with a properly scaled FFT transform [53] leads to the expected $PSD_\nu(f)$ estimation with the required magnitude accuracy.

5.2. Experimental validation of our technique

In order to validate our approach, we compared the results obtained with our technique to the ones recorded through a direct demodulation interferometer, here a fiber optic Michelson, depicted in Fig. 7. The experiments were carried out with the two DFB-SC lasers (configuration 1) developed for the Cesium atomic clock.

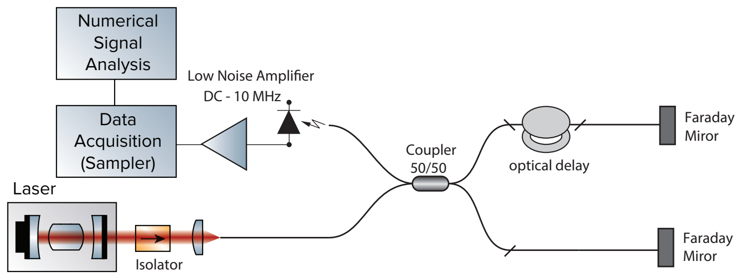

Fig. 7. Michelson-Based Frequency Noise set-up.

As the beat-note contains the noise contributions of the two lasers, the frequency noises of the two DFB-SC sources were measured directly and then added to be compared to the output given by the Hilbert demodulation. The result is shown in Fig. 8. A very good matching between the two techniques is observed.

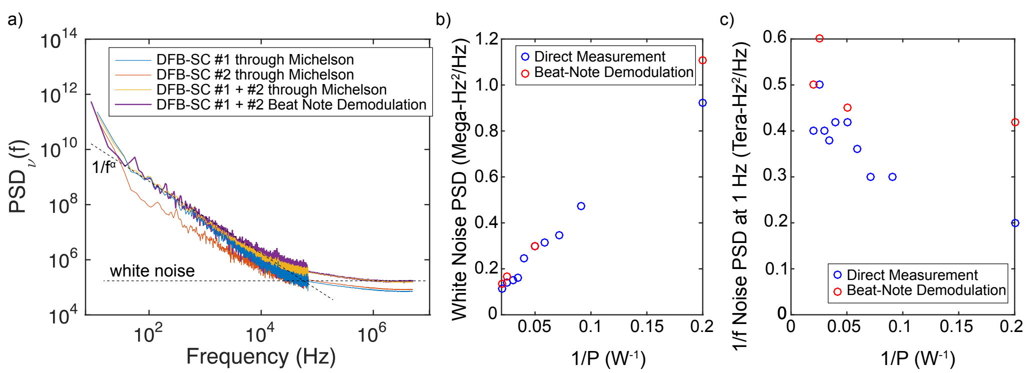

Fig. 8. Frequency noise spectrum obtained from Michelson experiments and beating experiments for the DFB-SC (each close to $40mW$). The $1/f^{\alpha}$ noise part usually comes from electrical transport phenomena [43], whereas the white noise part comes from the spontaneous emission (Shawlow-Townes-Henry).

5.3. ECDL and DFB-FL results

The result of the beat-note demodulation process for these lasers is given in Fig. 9. Beyond the fact that this operation can be done on a variety of lasers, these data will be useful for a better understanding of the linewidth results to come.

As expected, the strongest contribution in the case of DFB-FL is the low-frequency $1/f^{\alpha}$ noise originating from the thermal fluctuations induced by the pump intensity noise, with some feeble mechanical contributions in the 100 Hz-1 kHz range.

In the case of the ECDL/DFB-FL beating, the noise levels exhibited lead to the expected conclusion that the DFB-FL noise level can be neglected, and that the beating is representative of the ECDL alone. Concerning the ECDL, Fig. 9 shows that the strongest contribution to the noise is due to mechanical variations in the 100 Hz-1 kHz range. Additional pollution comes in the 10kHz-1 MHz range from the switching power supply. At higher frequencies, the white noise coming from spontaneous emission is quite low ($\approx 10^4 Hz^2/Hz$).

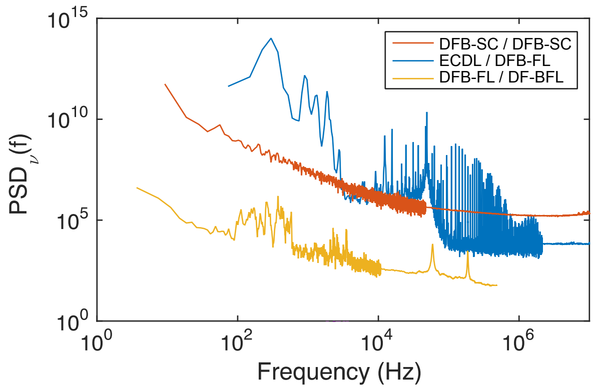

Fig. 9. Frequency Noise spectra obtained from DFB-SC, ECDL and DFB-FL beating.

6. Time dependent linewidth of the beat-note

6.1. Theoretical background

The linewidth is obtained from the power spectrum of the beat-note, and computed thanks to the Fourier transform of the autocorrelation function:

$$\mathcal{L}(f) = \mathcal{F}\left(\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{+T} b^*(t)b(t+\tau)\,dt\right) \tag{8}$$

This spectrum is only defined for an infinite observation time, which is not compatible with realistic experimental conditions. We define then a short-time spectrum, computed from a beat-note $b_{T_i,\Delta T}(t)$ existing over a time-window ΔT starting at the time T_i:

$$\mathcal{L}_{T_i,\Delta T}(f) = \mathcal{F}\left(\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{+T} b^*_{T_i,\Delta T}(t)b_{T_i,\Delta T}(t+\tau)\,dt\right) \tag{9}$$

Again, as it is better to avoid the evaluation of the autocorrelation function numerically, we use another form of this expression:

$$\mathcal{L}_{T_i,\Delta T}(f) = \mathcal{F}\left(b^*_{T_i,\Delta T}(t)\right) \times \mathcal{F}\left(b_{T_i,\Delta T}(t)\right) \tag{10}$$

We now have to find an estimator for the spectral width. Unfortunately, exploiting the FWHM, as it is very usual in the literature, is not relevant here. Indeed, if we have a look to Fig. 10(b) showing the spectrum of an ECDL obtained over a quite long integration time ($\approx ms$), we can easily assert that the strongest part of the laser line fluctuation is not contained in the FWHM but certainly in the pedestal, and thus the FWHM indicator does not accurately report the real size of this signal in the Fourier space.

The question of the quantification of the width of a signal has been very often considered in the scientific literature, and in various domains. In the field of lasers, we can notice that A.E. Siegman examines the problem of the width of the spatial (transverse) modes in a very interesting document [54]. At the end of his article, Siegman concludes that it is "maybe" relevant

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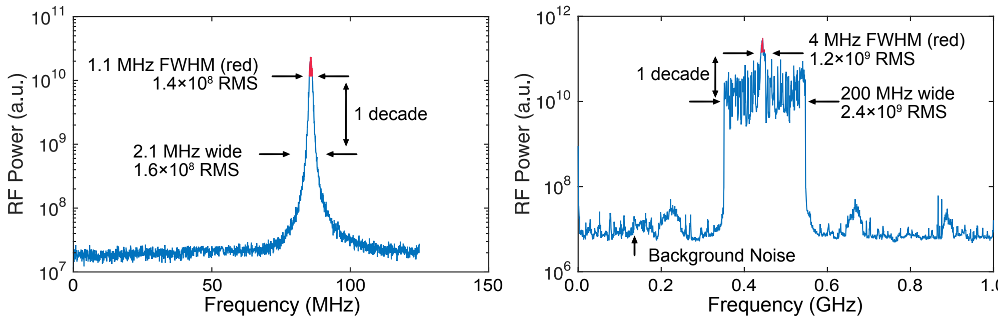

Fig. 10. Relevance (a) and irrelevance (b) of the FWHM depending on the spectral shape. a) case of a semiconductor DFB: the width and the associated RMS value do not change too much over one decade of magnitude. b) case of an ECDL: the width and associated RMS value changes dramatically in the same conditions.

to use the second order momentum instead of the FWHM. In the case of spatial modes, which are usually combinations of Gaussian distributions, it works very well. Unfortunately, in the case of laser spectra, such a definition cannot be used. Indeed, a laser lineshape has always a Lorentzian contribution [22] and none of the high order moments (>1) can be evaluated for such case.

As the goal is to compare very different laser technologies, we propose here to use the spectral width definition derived from the coherence time, as described in the fundamental work of Mandel and Wolf on the optical coherence [55, 56] :

$$\Delta\nu_{T_i,\Delta T} = \frac{\left(\displaystyle\int_0^{+\infty} \mathcal{L}_{T_i,\Delta T}(f)df\right)^2}{\displaystyle\int_0^{+\infty} \mathcal{L}^2_{T_i,\Delta T}(f)df} \tag{11}$$

Contrary to the FWHM, this definition depends on the whole spectrum and does not neglect the impact of some spectral components. Moreover, it shows sturdiness because it is defined for all the usual spectral shapes (Lorentzian, Gaussian, Voigt, Rectangular, ...).

In the numerical treatment performed here, we compute, for each targeted ΔT, the average :

$$\overline{\Delta\nu_c} = \frac{1}{M}\sum_{i=1}^{M} \Delta\nu_{T_i,\Delta T} \tag{12}$$

over the M available sub-samples, as well as the uncertainty using the standard-deviation:

$$\Delta\nu_c = \overline{\Delta\nu_c} \pm \frac{1}{2}\sqrt{\frac{1}{M}\sum_{i=1}^{M}(\Delta\nu_{T_i,\Delta T} - \overline{\Delta\nu_c})^2} \tag{13}$$

by exploiting all the possible sub-samples from the signal $b(t)$. To facilitate the comparisons with existing data, Table 1 links $\Delta\nu_c$ to the FWHM for usual spectral shapes.

6.2. *Experimental Results*

We display in Fig. 11 the evolution of $\Delta\nu_c$ calculated thanks to the process described in §6.1 for the beating between an ECDL and a DFB-FL. This experiment is the most iconic of our work because the ECDL noise contains all the usual contributions (mechanical, $1/f$ and white noise) to the linewidth (Fig. 9).

This graph shows three main trends:

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Table 1. Relations between standard FWHM and $\Delta\nu_c$

Shape	FWHM
Lorentzian	$\pi\Delta\nu_{T_i,\Delta T}$
Gaussian	$\Delta\nu_{T_i,\Delta T}/\sqrt{2\ln 2\pi}$
Rectangular	$\Delta\nu_{T_i,\Delta T}$
sinc^2	$\approx \Delta\nu_{T_i,\Delta T}/1.7$

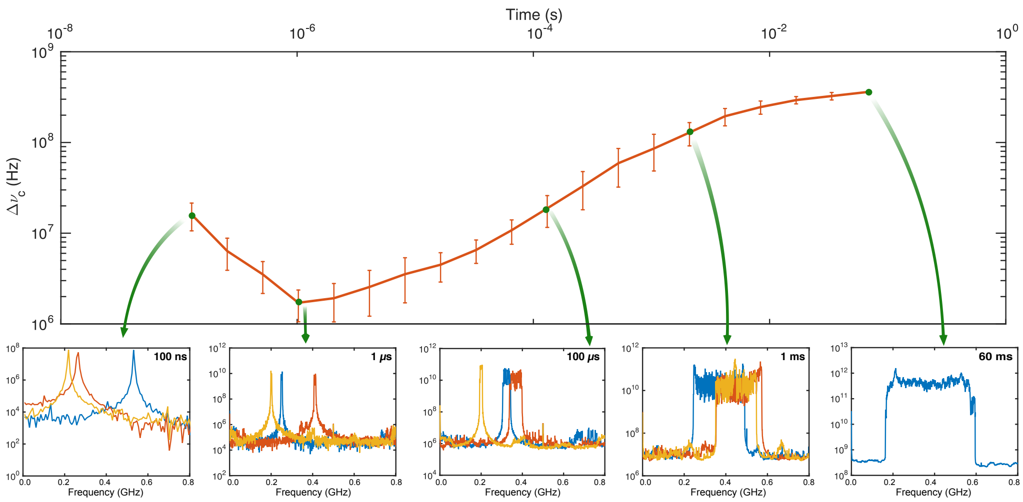

Fig. 11. Estimated $\Delta\nu_c$ for the ECDL/DFB-FL experiments as a function of the beat-note time sample length. The main curve is the average value and the error bars are the standard deviation over 128 samples up to 1 ms. Above this duration, the beat-note sample size imposes the amount of averages to decrease, down to a single full-size sample for the last point at 60 ms. In the insets, various spectra have been displayed to show the instability of the central frequency and of the linewidth due to mechanical fluctuations; the vertical scale of the insets is the RF power in arbitrary unit.

- for the shortest integration times ($< 1\ \mu s$), the linewidth estimation is dominated by the Fourier limit: the time window is small and the contribution of the frequency noise is much lower than the Fourier uncertainty. In that case, all the linewidth spectra are intrinsically convoluted with a sinc^2 which $\Delta\nu_c$ width evolves like $1.77/\Delta T$, where ΔT is the sample length in time. As the time window increases, the associated linewidth decreases.
- for longer times, the frequency noise of the laser becomes much more significant. The linewidth increases quickly up to the $\sim 0.1\,ms$ observation time, mainly because this time window allows to integrate an increasing part of the $1/f$ and mechanical contributions, which characteristic time is also in the ms range(see Fig. 9). This behavior can be understood from the insets: for times below $10\,\mu s$, the lineshape is always Lorentzian, demonstrating that the spontaneous emission noise dominates. Starting at $100\,\mu s$, the lineshape contains some top-hat contribution, which renders the strong mechanical fluctuations. In fact, this shape is close to the one of the Fourier Transform of the sum of Bessel functions of the first kind. Such form arises from the fact that the laser sine wave function

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is modulated in frequency by additional mechanical sine waves:

$$b(t) = A\cos(2\pi f_b t + \beta\sin(2\pi f_M t)) = A\sum_{n=-\infty}^{+\infty} J_n(\beta)\cos(2\pi(f_p + nf_M)t).$$

where f_b is the beating frequency and $J_n(\beta)$ the n^{th} Bessel function of the first kind.

- Then, for times $> 0.1\,ms$, the huge mechanical contribution is fully integrated and the linewidth begins to saturate as a function of the integration time. For even longer times, the linewidth may keep on increasing due to $1/f$ noise (due to memory limitations in the data acquisition card, we were not able to explore further).

To explore the impact of $1/f$ noise, we also focused on the DFB-SC beating configuration, as shown in Fig. 12. For these experiments, we calculated $\Delta\nu_c$ and extracted from the same data the Lorentzian and Gaussian linewidth FWHM (resp. $\Delta\nu_L$ and $\Delta\nu_G$) thanks to a robust automated fitting process exploiting an analytical accurate approximation of the Voigt function to reduce the computation time [57]. Just as for $\Delta\nu_c$, the average and standard deviation over a large number of samples were performed. The corresponding Voigt profile FWHM was calculated thanks to the empirical formula: $\Delta\nu_V \approx 0.5346\Delta\nu_L + \sqrt{(0.2166\Delta\nu_L^2 + \Delta\nu_G^2)}$ according to [58].

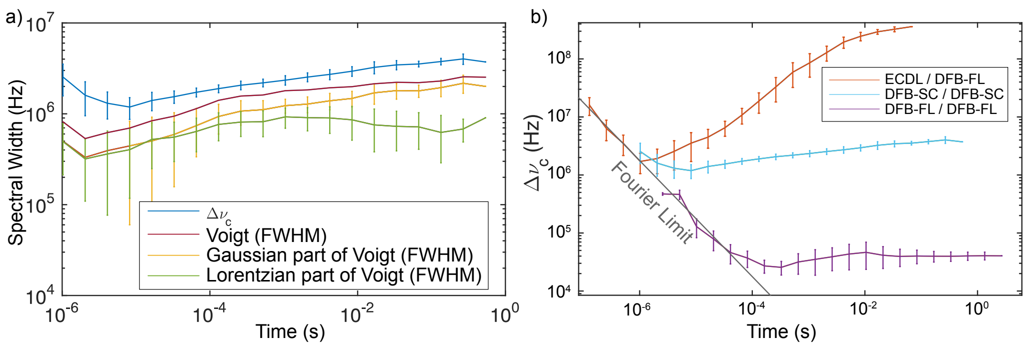

Fig. 12. a) Spectral width of the DFB-SC beating and comparison of the $\Delta\nu_c$ criterion with the Voigt profile fitting. b) $\Delta\nu_c$ for configurations 1, 2 and 3.

The global tendency is the increase of the Gaussian linewidth as a function of time, which is expected since an increasing part of $1/f$ noise is integrated with increasing times.

Finally, if we add the data measured on DFB-FL, we end with the results shown in Fig. 12(b). It states that a wide variety of lasers can be explored using this method, from low to high coherence.

6.3. Discussion: comparison with β-separation line approach

To our knowledge, the most interesting work concerning the short-time linewidth was published by Di Domenico et al. [41], who used the β-separation line theory to the laser linewidth computation. It is a process that defines which part of the noise spectrum has a significant contribution to the FWHM linewidth. It is based upon the identification of the "slow modulation area", defined by comparing the noise level to the β-separation line for each frequency of the spectrum, as shown on an example in Fig. 13(a). Among the points discussed in his article, Di Domenico's work confronts a very challenging problem: the time-dependence of the linewidth in the case of $1/f^\alpha$ $(1 < \alpha < 2)$ noise, and more generally in [42], the time-dependence due to any noise of arbitrary shape. In this approach, the main source of inaccuracy is due an approximation.

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But this approximation was unavoidable because there was no other way than introducing there an assumption to obtain the convergence of the analytical calculus in the case of $1/f^{\alpha}$ noise. The assumption is that, to estimate the linewidth over a duration ΔT, the lowest frequency integrated in the noise spectrum is chosen to be $1/\Delta T$, and thus any event faster than ΔT is not taken into account. Unfortunately, for the reasons illustrated by Eq. 1 and Fig. 3, this choice may lead to strong inaccuracies in the linewidth computation in the case of strong spectral components in the low frequency domain. Following this idea, we compare our numerical approach - which is not based on such an approximation - to the results obtained by the β-line analytical approach, in the case of the three previous and already discussed experimental configurations. This is what is shown in Fig. 13: in part a), the frequency noise spectra and the "slow modulation area" (filled areas) for a fixed ΔT as described in [42] ; part b), the corresponding time-dependent linewidths according to the β-separation line, as well as the one obtained with our numerical treatment.

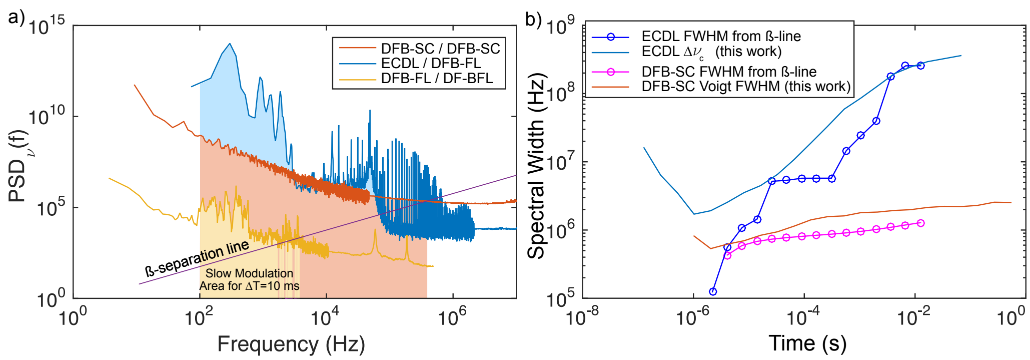

Fig. 13. a) β-separation line illustration, the slow modulation area (filled) is represented for $\Delta T = 1\,ms$. b) Time-dependent linewidth computed thanks to the numerical methods proposed here, compared to the β-separation line approach.

If the β-line computation leads globally to a good estimation of the laser FWHM, we observe three main drawbacks:

- The β-separation line does not take into account the Fourier Limit, and gives quite optimistic results in the case of the shortest times.
- Also, it can lead to strong inaccuracies in the case of lasers dominated by mechanical noise. An example of this fact is given by the ECDL results in Fig. 13(b). In this plot, the $\Delta\nu_c$ and FWHM values can be directly compared for times $> 1\,ms$ (according to Table 1) because the lineshape is close to a top-hat in this range. In this context, we notice that at $0.3\,ms$, the estimation made by the β-separation line process is at least 5 times more optimistic compared to the one obtained with our approach.
- Finally, observing the DFB-SC results in Fig. 13(b) leads to the conclusion that the FWHM estimation through the β-separation line process in the case of lasers dominated by $1/f$ noise leads also to quite optimistic results, with a factor close to 2 on the linewidth.

7. Sensitivity limit

In this section we want to discuss the ultimate sensitivity limits of this set-up. Of course, because it is based on RF instrumentation, it demonstrates ultra-low absolute noise limits compared to usual optical sources.

Firstly, the set-up is limited by the jitter of the acquisition card clock [59]. The main impact is a contribution to the magnitude of the background noise. One of the contributions of this jitter-originated noise is frequency-dependent, and may thus lead to choose optimised conditions for the data acquisition. This noise occurs like an additive noise $\epsilon^\bullet(f)$ appearing on the sampled data. Its spectrum is given by the convolution between the clock jitter spectrum $\Delta T(f)$ and the spectrum of the signal to be sampled :

$$\epsilon^\bullet(f) = j2\pi f \mathcal{F}\left(b(t)\right) * \Delta T(f) \tag{14}$$

Because of the proportionality with f, we should choose the smallest possible beating frequency (but of course high enough to contain the total laser frequency drift during the time-window of the measurement), and thus avoid to work beyond the Shanon frequency. This contribution to background noise of course limits a few the dynamics of the beat-note spectrum.

Secondly, the full span background noise may be a limit for the frequency noise measurement at high Fourier frequencies. According to [60], background intensity noise appears like additionnal phase noise. For a white background voltage noise, the resulting frequency noise is given by :

$$PSD_f^\bullet = 4\pi f^2 \frac{PSD_\epsilon^\bullet}{V_0^2} \tag{15}$$

where $PSD_f^\bullet$ is the frequency noise (in Hz^2/Hz) due to the voltage noise $PSD_\epsilon^\bullet$ (in V^2/Hz), and V_0 the RMS value of the beating voltage. So, the full span background voltage noise spectrum contributes to the measured frequency noise spectrum at high frequencies whatever its origin (laser RIN, amplifier noise, data acquisition card jitter). We have to notice that this limit can be overtaken by the means of a large amount of averages.

Nevertheless, all these expected limits needed to be checked up. That is why we performed additional experiments by using an Agilent 33220A sine wave voltage source working at 1 MHz, which frequency modulation input was seeded with an amplified DC-150 Hz white noise source. This signal was injected into to the Agilent U1084A-001 sampler that was used for the acquisition of most of the data exploited in this work. The result is depicted in Fig. 14 .

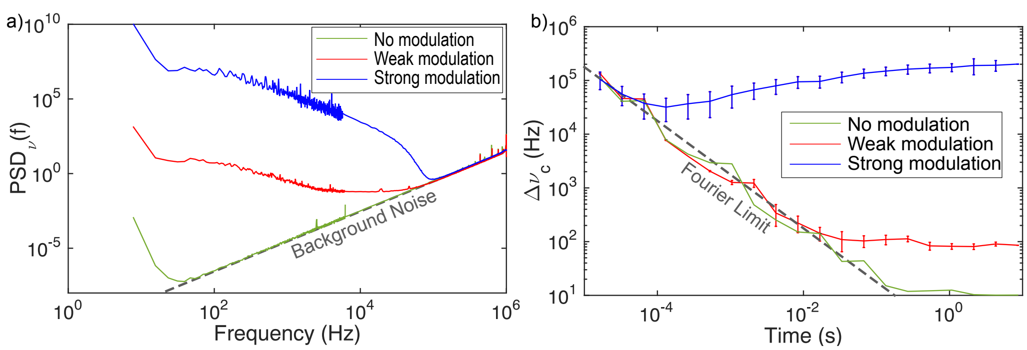

Fig. 14. a) Frequency noise limit measured b) Linewidth limit measured.

These measurements exhibit that the limits are very low compared to available lasers: frequency noise values as low as 1 Hz^2/Hz can be measured over frequency spans as wide as $100kHz$, while linewidth values as narrow as 10 Hz can be extracted. Concerning this last limit, we believe that it is due to the frequency generator and not to the sampler, which time jitter is said to be below 1 ps for $10\mu s$ record length.

8. Extension: intrinsic time dependent linewidth

In the previous experiments, all the results presented were the sum of the frequency noises of two lasers, because they were based on the heterodyne set-up. However, it is often interesting to

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have access to the intrinsic linewidth of a single laser, which cannot be reached in a rigorous way through the β-separation line approach in some cases, as discussed above. In order to compute this quantity, the frequency noise set-up of Fig. 7 was used, but instead of computing the power spectral density of the frequency noise, we performed only the acquisition of the frequency fluctuations as a function of the time. Then, we followed the path C→D→A→B from Fig. 5 to retrieve the laser spectrum as a function of the time. For this purpose, a "virtual" beat-note was generated by injecting the phase fluctuations in time on a deterministic numerical sine wave-shaped carrier, and then the same process as above was carried out to obtain the laser spectrum from the beat-note. The results, grounded on one of the DFB-SC studied in this paper, are displayed in Fig. 15, and illustrate again the spectral broadening due to $1/f$ noise.

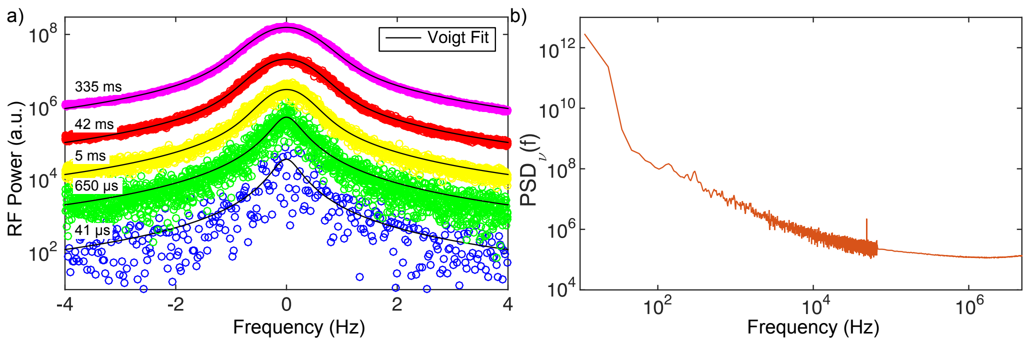

Fig. 15. a) Intrinsic linewidth computed from time-domain frequency noise signal using "virtual" carrier. b) Frequency noise spectrum computed from the time domain frequency noise signal.

9. Conclusion

We have demonstrated in this article the possibilities brought by the sampling of the beat-note obtained from the heterodyne beating interferometry. This process, based on a very simple and low-cost experimental set-up, is associated to a set of numerical processings in order to offer numerous possible derivations, as well as a high accuracy in the analysis of laser lines fluctuations, which have never been developed in the literature to our knowledge. It overcomes the capabilities of more traditional or more complex measurement set-ups, and opens the access to the very "short-time linewidth" that is so often required.

As discussed above, we encourage the reader to use the spectral width indicator $\Delta\nu_c$ associated with Mandel and Wolf's definition of the coherence time, instead of the more basic FWHM, as it can lead to significant errors in the case of lasers dominated by technical noise contributions. We also addressed the complex topic of low frequency noise, including the $1/f^\alpha$ noise, and demonstrated for the first time a method that can really take into account, with the highest accuracy, the impact of these noise sources on the linewidth.

We also mention that other original techniques explored in the scientific literature [61,62], and which rely on more complex RF instrumentation, could take benefit from acquisition systems as simple as the one used in this paper. With adequate signal analysis, it could make them much more available than today, as they depend on so specific instruments.

Finally, we want to mention that the Matlab code that is necessary to reproduce this set-up is open source and available under BSD licence, as additional material associated to this paper (Code 1, Ref. [63]). It takes advantage of the Matlab FFT multiple-core implementation to reduce computation times. For example, computing $\Delta\nu_c$ over a 2×10^8 points sample requires a few minutes on a Xeon E5-1603 2.8 GHz with 32 Go RAM.

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