Lecture 12: Chain Rule (CR)

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Chain Rule

The **chain rule** spells out the method of differentiating composite functions.

Theorem (Chain Rule)

If f and g are differentiable, then

$$\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)\,.$$

Question. Compute $\dfrac{d}{dx}\sin(1+2x)$.

Question. Compute $\dfrac{d}{dx}\sqrt{1+\sqrt{x}}$.

Question. Compute $\dfrac{d}{dx}e^{\sin(x^2)}$.

Question. Derive the quotient rule using the power rule, the product rule, and the chain rule.

Derivatives of trigonometric functions

- At the moment, we only know that $\frac{d}{dx}\sin(x) = \cos(x)$.
- This one fact along with other derivative "shortcuts" will give us the derivative formulas for all other standard trigonometric functions.

Theorem (Derivatives of Trigonometric Functions)

- $\frac{d}{dx}\sin(x) = \cos(x)$.
- $\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$.
- $\frac{d}{dx}\tan(x) = \sec^2(x)$.
- $\frac{d}{dx}\cos(x) = -\sin(x)$.
- $\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$.
- $\frac{d}{dx}\cot(x) = -\csc^2(x)$.

Question. Compute $\left[\dfrac{d}{dx}\cos\left(\dfrac{x^3}{2}\right)\right]_{x=\sqrt[3]{\pi}}$.

Question. Compute $\dfrac{d}{dx}\left(\dfrac{5x\tan(x)}{x^2-3}\right)$.

Question. Compute $\left[\dfrac{d}{dx}\left(\csc(x)\cot(x)\right)\right]_{x=\pi/3}$.