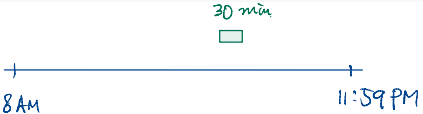
Lecture 18: Computation of Derivatives (Review)

Tae Eun Kim, Ph.D.

Autumn 2021

Computational Test



- Monday, October 11
  (8 AM ~ 11:59 PM)
- 8 questions & 30 min time limit
- One attempt : no retake
- Closed-book, closed-note
  Writing on iPad or other tablet device Not Allowed!
- Proctorio — Computer, Chrome ~~iPad~~ ~~Safari~~, ~~Firefox~~, ... ~~X~~ ...
- Under 18 ? Contact me!

Instructions

Compute the derivative of each of the following functions.

- You do not need to simplify.
- You do not need to show steps.
- No calculator is allowed.
- Be extremely careful with notations, signs, parentheses, etc.

Handy Ones

1. $f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$ or $\frac{1}{2}x^{-1/2}$

2. $f(x) = \frac{1}{x}$

$f'(x) = -\frac{1}{x^2}$ or $-x^{-2}$

3. $f(x) = \ln(3x)$

$f'(x) = \frac{1}{x}$ or $\frac{1}{3x} \cdot 3$

Let $u(x)$ is some function.

- $\frac{d}{dx}\sqrt{u(x)} = \frac{u'(x)}{2\sqrt{u(x)}}$
- $\frac{d}{dx}\frac{1}{u(x)} = -\frac{u'(x)}{[u(x)]^2}$
- $\frac{d}{dx}\ln(u(x)) = \frac{u'(x)}{u(x)}$

Do You Really Need To?

1. $f(x) = 9\ln\left(\frac{1}{x}\right)$

- PR (works, but slow)
- "teenager"
- Pro approach: $\frac{d}{dx}\ln(u(x)) = \frac{u'(x)}{u(x)}$ & $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$.

$f'(x) = 9\,\frac{-1/x^2}{1/x} = -\frac{9}{x}$

2. $f(x) = -\frac{2}{x^2+1}$ $u(x)$

- QR (works, but slow)
- "teenager"
- Pro approach: $\frac{d}{dx}\frac{1}{u(x)} = -\frac{u'(x)}{(u(x))^2}$

$f'(x) = \frac{4x}{(x^2+1)^2}$

3. $f(x) = \frac{\sin^{-1}(2x)}{6} = \frac{1}{6}\sin^{-1}(2x)$

teenage

$f'(x) = \frac{1}{6}\,\frac{2}{\sqrt{1-4x^2}} = \frac{1}{3\sqrt{1-4x^2}}$

Yet another way to solve (1)

$$f(x) = 9\ln\left(\frac{1}{x}\right) = -9\ln(x)$$

$$\frac{1}{x} = x^{-1}$$

$$f'(x) = -\frac{9}{x}$$

Confusing Constants

Classify the following expressions. Which of the following are power functions/exponential functions/constants?

1. e^x — exp.
2. x^e — pwr.
3. π^x — exp.
4. $x^{\sqrt{\pi}}$ — pwr.
5. e^π — const.
6. 7^e — const.
7. e^e — const.

	form	examples
• exp. func:	$(\#)^{(\text{Var})}$	e^x, 2^{x^2-1}
• power func:	$(\text{Var})^{(\#)}$	x^2, $(2x+9)^{1/2}$ ... $= \sqrt{2x+9}$,
• tower func:	$(\text{Var})^{(\text{Var})}$	x^x, --

Confusing Constants (cont')

1. $f(x) = \dfrac{7^{\pi}}{\sqrt[4]{x}} + \dfrac{x^{\pi}}{\sqrt[4]{7}} + \dfrac{7^{x}}{\sqrt[4]{7}}$

2. $f(x) = \dfrac{e^{x}}{\sqrt{e}} + \dfrac{x^{e}}{\sqrt{x}} + \dfrac{e^{\sqrt{3}}}{\sqrt{5}}$

$f'(x) = -\dfrac{7^{\pi}}{\sqrt{x}} \cdot \dfrac{1}{4} x^{-3/4} + \dfrac{\pi\, x^{\pi-1}}{\sqrt[4]{7}} + \dfrac{7^{x} \cdot \ln 7}{\sqrt[4]{7}}$

3. $f(x) = \csc(x)\cot(3) + \csc(3)\cot(x) + \csc(x)\cot(x)$

Lengthy Calculations

1. $f(x) = \dfrac{2x\cot^3(x^2-4)}{e^{\sqrt{x}} + \sqrt{x}^e}$

2. $f(x) = 2^{\pi}\ln(\sqrt{x}) + 2^{3x}\sqrt{\ln(x)} + 2^{\ln\sqrt{x}}$

$\ln(x^{1/2}) = \frac{1}{2}\ln x$ $\qquad$ $= \frac{1}{2}\ln x$

$$f'(x) = \frac{2^{\pi-1}}{x} + \left(2^{3x}\cdot\ln 2\cdot 3\sqrt{\ln x} + 2^{3x}\frac{1/x}{2\sqrt{\ln x}}\right) + 2^{\frac{1}{2}\ln x}\cdot\ln 2\cdot\frac{1}{2x}$$

Weird Tower Functions and Log Differentiation

1. $f(x) = x^{e^x} + e^{x^e}$

2. $f(x) = x^{x^e} + e^{e^x}$