Review for Final Examination — Math 1151

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Review problems for final exam covering materials from **Antiderivatives and Area** (AAA) till the end. For previous topics, please study the three midterm exams as well as their corresponding review problems.

Problem 1. (Integration exercises, I)

Compute the following:

(a) $\displaystyle\int \frac{\tan^3(\theta)+1}{\cos^2(\theta)}\,d\theta$

(b) $\displaystyle\int (4x-6)\sqrt{x^2-3x}\,dx$

(c) $\displaystyle\int_0^{\pi/2} \frac{d}{dx}(\sin^7 x)\,dx$

(d) $\displaystyle\frac{d}{dx}\int_0^{\pi/2} \sin^7 t\,dt$

(e) $\displaystyle\int_0^{\pi/4} \frac{1+\tan\theta}{\sec\theta}\,d\theta$

Problem 2. (Application of integrals)

Answer the following questions.

(a) Consider an object moving along a straight line with the velocity $v(t) = 12 - 3t$ on $[0, 6]$. Express the distance traveled over the given interval as a sum (or difference) of two definite integrals.

(b) An oil refinery produces oil at a variable rate of $A'(t) = 1000 - 10t$ with $0 \le t \le 40$, where t is measured in days and A is measured in barrels. How many barrels are produced in the first 10 days?

Problem 3. (Properties and techniques of integration)

Suppose that $\int_1^3 f(x)\,dx = 4$.

(a) Evaluate the following integrals.

i. $\displaystyle\int_1^9 \frac{3f(\sqrt{x})}{\sqrt{x}}\,dx$

ii. $\displaystyle\int_0^{\sqrt{2}} 3xf(x^2+1)\,dx$

(b) Assume additionally that f is odd. Evaluate $\displaystyle\int_{-1}^{-3} f(x)\,dx$.

(c) Find f_{avg}, the average value of f, on the interval $[1, 3]$.

Problem 4. (Accumulation function)

Let g be defined on $[0,10]$ by

$$g(x) = \begin{cases} x-2 & 0 \le x < 4 \\ 2 & 4 \le x \le 10 \end{cases}.$$

Define A by

$$A(x) = \int_0^x g(t)\,dt, \quad \text{for } 0 \le x \le 10.$$

Evaluate:

(a) $A(4)$

(b) $A'(4)$

(c) $\displaystyle\int_0^4 |g(t)|\,dt$

Problem 5. (Initial value problems)

Answer the following questions.

(a) Graph several functions that satisfy the differential equation $f'(x) = 3x^2 - 1$. Then find and graph the particular solution that satisfies the initial condition $f(2) = 1$. (This was one of Midterm 3 review problems.)

(b) Find and graph the function $A(x) = \int_0^x (3t^2 - 1)\, dt$. Does the function A satisfy the differential equation in the previous part? Explain. Compute $A(2)$. Does the function A satisfy the initial condition given above?

Determine the following definite integrals.

(a) $\displaystyle\int_0^4 \frac{x-3}{\sqrt{x}}\,dx$

(b) $\displaystyle\int_0^1 \frac{e^x}{e^x + e^{-x}}\,dx$

(c) $\displaystyle\int_{-\pi/4}^{\pi/4} x^4 \tan^9 x\,dx$

Problem 7. (Mean value theorem for integrals)

Let f be given by

$$f(x) = \begin{cases} \dfrac{x}{2} & 0 \le x < 2 \\ 1 & 2 \le x < 3 \\ -x+4 & 3 \le x < 5 \\ -1 & 5 \le x \le 7 \end{cases}.$$

(a) Find the average value of the function f on the interval $[0, 7]$.

(b) Sketch the graph of $f(x)$ and mark the point (or points) c in $[0, 7]$ where the function attains this average value. Draw (best you can) a rectangle whose net area is equal to $\int_0^7 f(t)\,dt$.

(c) Compute $\int_0^7 |f(t)|\,dt$. Find the average value of $|f|$.

Problem 8. (1-D motion of a particle)

Let v be given by

$$v(t) = \begin{cases} \frac{t}{2} & 0 \le t < 2 \\ 1 & 2 \le t < 3 \\ -t+4 & 3 \le t < 5 \\ -1 & 5 \le t \le 7 \end{cases}.$$

Assume that $s(0) = 0$.

(a) Determine the displacement between $t = 0$ and $t = 7$.

(b) Determine the distance traveled between $t = 0$ and $t = 7$.

(c) Determine the position at $t = 3$.

(d) Determine the position at $t = 5$.

(e) Determine the position function, $s(t)$, for $5 \le t \le 7$.

(f) Determine the acceleration, $a(t)$, for $5 < t < 7$.