Au 21 Math 1151

Lecture 42, the last one

Announcements

- **Final Exam** on Monday, December 13, 6:00 ~ 7:45 PM.
  - Part 1 (Canvas Quiz) : 20 minutes, must be completed by 6:45 PM
  - Part 2 (Written) : Gradescope. 40 minutes (6:50 PM ~ 7:30 PM)
- **Extra office hours**
  - Friday, December 10 : 10 AM ~ noon
  - Monday, December 13 : 10 AM ~ noon

- Schedule conflicts w/ other final exams

  → contact me ASAP

Problem 3. (Properties and techniques of integration)

Suppose that $\int_1^3 f(x)\,dx = 4$.

(a) Evaluate the following integrals.

i. $\int_1^9 \frac{3f(\sqrt{x})}{\sqrt{x}}\,dx$

ii. $\int_0^{\sqrt{2}} 3xf(x^2+1)\,dx$

(b) Assume additionally that f is odd. Evaluate $\int_{-1}^{-3} f(x)\,dx$.

(c) Find f_{avg}, the average value of f, on the interval $[1, 3]$.

Problem 4. (Accumulation function)

Let g be defined on $[0, 10]$ by

$$g(x) = \begin{cases} x - 2 & 0 \le x < 4 \\ 2 & 4 \le x \le 10 \end{cases}.$$

Define A by

$$A(x) = \int_0^x g(t)\, dt, \quad \text{for } 0 \le x \le 10.$$

Evaluate:

(a) $A(4)$

(b) $A'(4)$

Q. Determine where the graph of A is concave up.

Problem 5. (Initial value problems)

Answer the following questions.

(a) Graph several functions that satisfy the differential equation $f'(x) = 3x^2 - 1$. Then find and graph the particular solution that satisfies the initial condition $f(2) = 1$. (This was one of Midterm 3 review problems.)

(b) Find and graph the function $A(x) = \int_0^x (3t^2 - 1)\,dt$. Does the function A satisfy the differential equation in the previous part? Explain. Compute $A(2)$. Does the function A satisfy the initial condition given above?

IVP & L'Hopital

Suppose $f(x)$ is the soln of

$$\begin{cases} f'(x) = (x-2)\sin(\pi x) \\ f(2) = 1 \end{cases}$$

Evaluate

$$\lim_{x \to 2} \frac{f(x) - 1}{e^{x-2} + 1 - x}$$