Au 21 Math 1151

$\boxed{\text{Lecture 42, the last one}}$

Announcements

- **Final Exam** on Monday, December 13, 6:00 ~ 7:45 PM.
  - Part 1 (Carmen Quiz) : 20 minutes, must be completed by 6:45 PM
  - Part 2 (Written) : Gradescope. 40 minutes (6:50 PM ~ 7:30 PM)
- **Extra office hours**
  - Friday, December 10 : 10 AM ~ noon
  - Monday, December 13 : 10 AM ~ noon

- Schedule conflicts w/ other final exams

  → contact me ASAP

Problem 3. (Properties and techniques of integration)

Suppose that $\int_1^3 f(x)\,dx = 4$.

(a) Evaluate the following integrals.

i. $\int_1^9 \frac{3f(\sqrt{x})}{\sqrt{x}}\,dx$ $\quad \frac{dx}{\sqrt{x}} = 2\,du$ $\quad = \int_1^3 3f(u)\cdot 2\,du = 6\underbrace{\int_1^3 f(u)\,du}_{= 4} = 6\cdot 4 = \boxed{24}$

Set $u = \sqrt{x}$

$\left\{ du = \frac{1}{2\sqrt{x}}\,dx \Rightarrow \frac{1}{\sqrt{x}}\,dx = 2\,du \right.$

Limits: $x = 1 \longrightarrow u = \sqrt{1} = 1$

$x = 9 \longrightarrow u = \sqrt{9} = 3$

ii. $\int_0^{\sqrt{2}} 3x f(x^2+1)\,dx$ $\quad u \quad \frac{1}{2}du$ $\quad = \int_1^3 3f(u)\frac{1}{2}\,du = \frac{3}{2}\underbrace{\int_1^3 f(u)\,du}_{= 4} = \frac{3}{2}\cdot 4 = \boxed{6}$

Set $u = x^2+1$

$\left\{ du = 2x\,dx \rightarrow x\,dx = \frac{1}{2}\,du \right.$

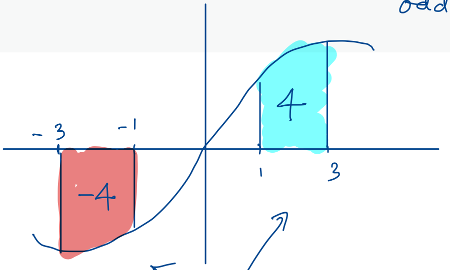
Limits: $x = 0 \longrightarrow u = 0^2+1 = 1$

$x = \sqrt{2} \longrightarrow u = \sqrt{2}^2+1 = 3$

odd.

(b) Assume additionally that f is odd. Evaluate $\int_{-1}^{-3} f(x)\,dx$.

$$= -\underbrace{\int_{-3}^{-1} f(x)\,dx}_{\shortparallel \atop -4} = -(-4) = \boxed{4}$$



two congruent region w/ same geom area but opposite signs

(c) Find f_{avg}, the average value of f, on the interval $[1, 3]$.

$$f_{avg} = \frac{1}{3-1}\int_1^3 f(x)\,dx$$

$$= \frac{1}{2}\cdot 4 = \boxed{2}$$

Problem 4. (Accumulation function)

Let g be defined on $[0, 10]$ by

$$g(x) = \begin{cases} x - 2 & 0 \le x < 4 \\ 2 & 4 \le x \le 10 \end{cases}.$$

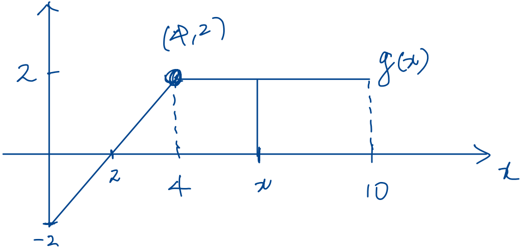
Define A by

$$A(x) = \int_0^x g(t)\,dt, \quad \text{for } 0 \le x \le 10.$$

Evaluate:

(a) $A(4)$

(b) $A'(4)$ $= \left.\dfrac{dA}{dx}\right|_{x=4}$



(a) $A(4) = \int_0^4 g(t)\,dt$

$$= \int_0^4 (t-2)\,dt$$

$$\overset{FTC2}{=} \left[\frac{t^2}{2} - 2t\right]_0^4 = \left(\overset{8}{\cancel{\frac{16}{2}}} - 8\right) - \overset{0}{\left(\frac{0^2}{2} - 2\cdot 0\right)} = \boxed{0}$$

(b) Note by FTC1,

$$\frac{d}{dx}A(x) = \frac{d}{dx}\int_0^x g(t)\,dt \overset{FTC1}{=} g(x)$$

$$\Rightarrow A'(4) = g(4) = \boxed{2}$$

Q. Determine where the graph of A is concave up.

A. $A''(x) > 0$ where?

Since $A'(x) = g(x)$, need to look for where $g'(x) > 0$, i.e., where graph of $g(x)$ is increasing.

$g'(x) = A''(x)$

which happens on $(0, 4)$.

Problem 5. (Initial value problems)

Answer the following questions.

(a) Graph several functions that satisfy the differential equation $f'(x) = 3x^2 - 1$. Then find and graph the particular solution that satisfies the initial condition $f(2) = 1$. (This was one of Midterm 3 review problems.)

(b) Find and graph the function $A(x) = \int_0^x (3t^2 - 1)\, dt$. Does the function A satisfy the differential equation in the previous part? Explain. Compute $A(2)$. Does the function A satisfy the initial condition given above?

See Solutions.

IVP & L'Hopital

Suppose $f(x)$ is the soln of

$$\begin{cases} f'(x) = (x-2)\sin(\pi x) \\ f(2) = 1 \end{cases}$$

Evaluate

$$\lim_{x\to 2} \frac{f(x)-1}{e^{x-2}+1-x}$$

Since $\lim_{x\to 2}(f(x)-1) = 1-1=0$

and $\lim_{x\to 2}(e^{x-2}+1-x) = e^{0}+1-2=0$,

the given limit is in "$\frac{0}{0}$" form.

So let's use L'H.

$$(\text{Given}) \overset{\text{L'H}}{=} \lim_{x\to 2} \frac{f'(x)}{e^{x-2}-1}$$

$$= \lim_{x\to 2} \frac{(x-2)\sin(\pi x) \to 0}{e^{x-2}-1 \to 0}$$

This is again in "$\frac{0}{0}$" form

$$\overset{\text{L'H}}{=} \lim_{x\to 2} \frac{\sin(\pi x) + (x-2)\pi\cos(\pi x)}{e^{x-2}}$$

$$= \frac{\sin(2\pi) + 0\cdot\pi\cos(2\pi)}{e^{0}}$$

$$= \frac{0}{1} = \boxed{0}$$

As we wrap up ...

$$\text{Life} = \int_{\text{birth}}^{\text{death}} \text{choice}(t)\, dt$$

$$\downarrow$$

$$\left(\begin{array}{c}\text{Rest of} \\ \text{your life}\end{array}\right) = \int_{\underline{\text{today}}}^{\text{death}} \text{choice}(t)\, dt$$

Thank you

and go Bucks!