Lecture 19: More Than One Rate (MTOR)

Tae Eun Kim, Ph.D.

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Related rates problems

- Suppose two variables x and y are both dependent on time t.
- Moreover, assume that these two are related to each other.
- In this context, the rate of change of y with respect to time is expected to be *related* to that of x;
- when one of the rates is known and the other is to be found, we have a **related rates** problem.

Key idea.

If y is written in terms of x and we are given $\dfrac{dx}{dt} = x'(t)$, then we can find $\dfrac{dy}{dt} = y'(t)$ using the chain rule:

$$\frac{dy}{dt} = y'(x(t)) \cdot x'(t)\,.$$

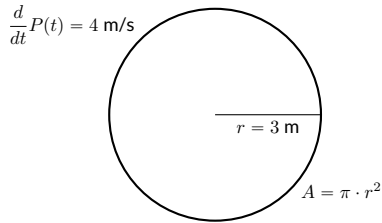
Problem-solving strategies

General procedure

1. **Draw a picture.** If possible, draw a schematic picture with all the relevant information.
2. **Find equations.** We want equations that relate all relevant functions.
3. **Differentiate the equations.** Here we will often use implicit differentiation.
4. **Evaluate.** Evaluate each quantity at the relevant moment.
5. **Solve.** Solve for the relevant rate at the relevant moment.

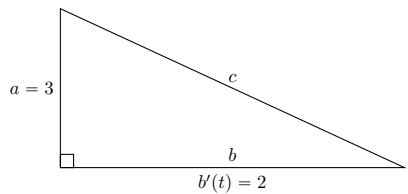
Example 1. (Circular geometry)

Imagine an expanding circle. If we know that the perimeter is expanding at a rate of 4 m/s, what rate is the area changing when the radius is 3 meters?



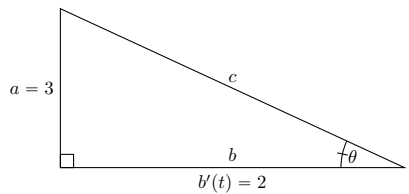
Example 2. (Right triangles)

Imagine an expanding right triangle. If one leg has a fixed length of 3 m, one leg is increasing with a rate of 2 m/s, and the hypotenuse is expanding to accommodate the expanding leg, at what rate is the hypotenuse expanding when both legs are 3 m long?



Example 3. (Angular rates)

Imagine an expanding right triangle. If one leg has a fixed length of 3 m, one leg is increasing with a rate of 2 m/s, and the hypotenuse is expanding to accommodate the expanding leg, at what rate is the angle opposite the fixed leg changing when both legs are 3 m long?



Example 4. (Similar triangles)

Imagine two right triangles that share an angle. If x is growing from the vertex with a rate of 3 m/s, what rate is the area of the smaller triangle changing when $x = 5$ m?