Lecture 18: Computation of Derivatives (Review)

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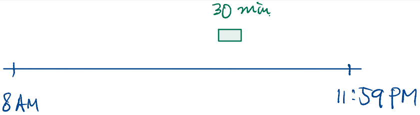
Autumn 2021

Computational Test



- Monday, October 11

  (8 AM ~ 11:59 PM)

- 8 questions & 30 min time limit
- One attempt ; no retake
- Closed-book, closed-note

  Writing on iPad or other tablet device Not Allowed !
- Proctorio — Computer ~~iPad~~

  Chrome ~~Safari~~, ~~Firefox~~, ~~..~~ . .
- Under 18 ? Contact me!

Instructions

Compute the derivative of each of the following functions.

- You do not need to simplify.
- You do not need to show steps.
- No calculator is allowed.
- Be extremely careful with notations, signs, parentheses, etc.

Handy Ones

1. $f(x) = \sqrt{x}$

$$f'(x) = \frac{1}{2\sqrt{x}} \quad \text{or} \quad \frac{1}{2}x^{-1/2}$$

2. $f(x) = \dfrac{1}{x}$

$$f'(x) = -\frac{1}{x^2} \quad \text{or} \quad -x^{-2}$$

3. $f(x) = \ln(3x)$

$$f'(x) = \frac{1}{x} \quad \text{or} \quad \frac{1}{3x} \cdot 3$$

Let $u(x)$ is some function.

- $$\frac{d}{dx}\sqrt{u(x)} = \frac{u'(x)}{2\sqrt{u(x)}}$$
- $$\frac{d}{dx}\frac{1}{u(x)} = -\frac{u'(x)}{[u(x)]^2}$$
- $$\frac{d}{dx}\ln(u(x)) = \frac{u'(x)}{u(x)}$$

Do You Really Need To?

- PR (works, but slow)
- "teenager"
- Pro approach: $\underbrace{\frac{d}{dx}\ln(u(x)) = \frac{u'(x)}{u(x)}}$ & $\underbrace{\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}}$.

1. $f(x) = 9\ln\left(\frac{1}{x}\right)$

$$f'(x) = 9\,\frac{-1/x^2}{1/x} = -\frac{9}{x}$$

- QR (works, but slow)
- "teenager"
- Pro approach: $\frac{d}{dx}\frac{1}{u(x)} = -\frac{u'(x)}{(u(x))^2}$

2. $f(x) = -\frac{2}{x^2+1}$ $\quad u(x)$

$$f'(x) = \frac{4x}{(x^2+1)^2}$$

3. $f(x) = \frac{\sin^{-1}(2x)}{6} = \underbrace{\frac{1}{6}}_{\text{teenage}}\sin^{-1}(2x)$

$$f'(x) = \frac{1}{6}\,\frac{2}{\sqrt{1-4x^2}} = \frac{1}{3\sqrt{1-4x^2}}$$

Yet another way to solve (1)

$$f(x) = 9\ln\left(\underset{\substack{\| \\ x^{-1}}}{1/x}\right) = -9\ln(x)$$

$$f'(x) = -\frac{9}{x}$$

Confusing Constants

Classify the following expressions. Which of the following are power functions/exponential functions/constants?

1. e^x — exp.
2. x^e — pwr.
3. π^x — exp.
4. $x^{\sqrt{\pi}}$ — pwr.
5. e^π — const.
6. 7^e — const.
7. e^e — const.

	form	examples
• exp. func:	$(\#)^{(\text{var})}$	e^x, 2^{x^2-1}
• power func:	$(\text{var})^{(\#)}$	x^2, $(2x+9)^{1/2} = \sqrt{2x+9}$, ...
• tower func:	$(\text{var})^{(\text{var})}$	x^x, ...

Confusing Constants (cont')

$$\frac{x^e}{x^{1/2}} = x^{e-1/2}$$

❶ $f(x) = \frac{7^\pi}{\sqrt[4]{x}} + \frac{x^\pi}{\sqrt[4]{7}} + \frac{7^x}{\sqrt[4]{7}}$

$$f'(x) = -\frac{7^\pi}{\sqrt{x}} \cdot \frac{1}{4} x^{-3/4} + \frac{\pi\, x^{\pi-1}}{\sqrt[4]{7}} + \frac{7^x \cdot \ln 7}{\sqrt[4]{7}}$$

❷ $f(x) = \frac{e^x}{\sqrt{e}} + \frac{x^e}{\sqrt{x}} + \frac{e^{\sqrt{3}}}{\sqrt{5}}$

$$f'(x) = \frac{e^x}{\sqrt{e}} + (e - \tfrac{1}{2})\, x^{e-3/2}$$

❸ $f(x) = \csc(x)\cot(3) + \csc(3)\cot(x) + \csc(x)\cot(x)$

$$f'(x) = -\csc(x)\cot(x)\cot(3) - \csc(3)\csc^2(x) - \csc(x)\cot^2(x) - \csc^3(x)$$

Lengthy Calculations

$(x^{1/2})^e = x^{e/2}$

1. $f(x) = \dfrac{2x\cot^3(x^2-4)}{e^{\sqrt{x}} + \sqrt{x}^e}$

$$f'(x) = \frac{\left(e^{\sqrt{x}} + \sqrt{x}^e\right)\left(2\cot^3(x^2-4) + 2x\cdot 3\cot^2(x^2-4)(-\csc^2(x^2-4))(2x)\right) - \left(2x\cot^3(x^2-4)\right)\left(\frac{e^{\sqrt{x}}}{2\sqrt{x}} + \frac{e}{2}x^{\frac{e}{2}-1}\right)}{\left(e^{\sqrt{x}} + \sqrt{x}^e\right)^2}$$

√

2. $f(x) = 2^\pi \ln(\sqrt{x}) + 2^{3x}\sqrt{\ln(x)} + 2^{\ln\sqrt{x}}$

$\ln(x^{1/2}) = \frac{1}{2}\ln(x)$

$\frac{1}{2}\ln(x)$

$$f'(x) = \frac{2^{\pi-1}}{x} + \left(2^{3x}\cdot\ln 2\cdot 3\sqrt{\ln x} + 2^{3x}\frac{1/x}{2\sqrt{\ln x}}\right) + 2^{\frac{1}{2}\ln x}\cdot\ln 2\cdot\frac{1}{2x}$$

Weird Tower Functions and Log Differentiation

1 $f(x) = \boxed{x^{e^x}} + e^{x^e}$

LD

$$y = x^{e^x}$$

$$\ln y = e^x \ln x$$

$$\frac{y'}{y} = e^x \ln x + \frac{e^x}{x}$$

$$y' = \underbrace{x^{e^x}}_{y} \underbrace{e^x\left(\ln x + \frac{1}{x}\right)}_{\text{previous RHS}}$$

$$f'(x) = x^{e^x} e^x \left(\ln x + \frac{1}{x}\right) + e^{x^e} \cdot e\, x^{e-1}$$

2 $f(x) = \boxed{x^{x^e}} + e^{e^x}$

LD

$$y = x^{x^e}$$

$$\ln y = x^e \ln x$$

$$\frac{y'}{y} = e x^{e-1} \ln x + \frac{x^e}{x}$$

$// \ x^{e-1}$

$$y' = \underbrace{x^{x^e}}_{y} \underbrace{x^{e-1}\left(e \ln x + 1\right)}_{\text{previous RHS}}$$

$$f'(x) = x^{x^e} x^{e-1} \left(e \ln x + 1\right)$$