Mock Final Examination I — Math 1151

Tae Eun Kim Autumn 2019

Practice problems for comprehensive final exam.

Problem 1. (True/false)

(True/False) Circle T if the statement is ALWAYS true; circle F otherwise. No explanation is required.

(a) (T / F) $f(x) = x + 1$ and $g(x) = \dfrac{x^2 - 1}{x - 1}$ are the same functions.

(b) (T / F) If $\displaystyle\lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$ exists, then f is continuous at 3.

(c) (T / F) If f has a vertical asymptote $x = -3$, then $\lim_{x \to -3} f(x) = \infty$.

(d) (T / F) A function may possess three distinct horizontal asymptotes.

(e) (T / F) Let f be continuous on $[1, 3)$. If $f(1) = -2$ and $f(3) = 5$, then the equation $f(x) = 0$ must have a solution between 1 and 3.

Problem 2. (Multiple choice)

Select correct answers. A question may have multiple correct answers. No partial credit is given for this problem.

(a) At what point(s) c does the conclusion of the Mean Value Theorem hold for $f(x) = x^3$ on the interval $[-3, 3]$?

A. $-\sqrt{3}$
B. $-1/\sqrt{3}$
C. 0
D. $1/\sqrt{3}$
E. $\sqrt{3}$
F. None of the above

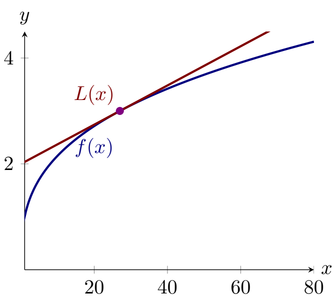
(b) The equation of the line that represents the linear approximation to the function $f(x) = \ln(x)$ at $a = 1$ is

A. $y = x - 1$
B. $y = x + 1$
C. $y = -x - 1$
D. $y = -x + 1$
E. None of the above

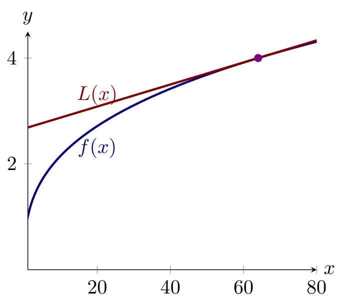
(c) Let $f(x) = \sqrt[3]{x}$ and let $L(x)$ be the linear approximation of $f(x)$ at $a = 64$.

i. Select the figure which includes the correct graph of $L(x)$.

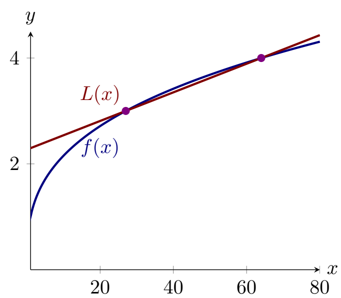
A.



B.



C.



ii. If $L(50)$ is used to approximate $\sqrt[3]{50}$,

A. it gives an overestimate.
B. it gives an underestimate.
C. it gives an exact value of $\sqrt[3]{50}$.
D. it cannot be determined.

Problem 3. (Limit computation)

(a) Evaluate the following limits. You may use L'Hôpital's rule.

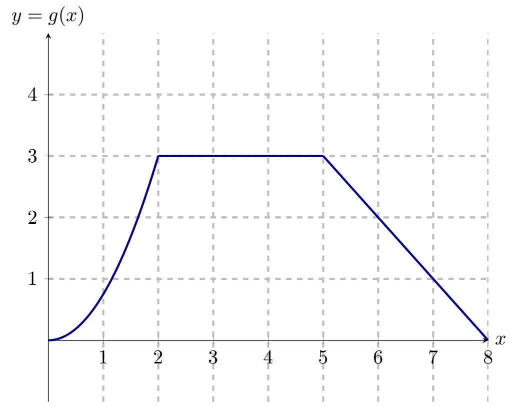
i. $\lim_{x \to 0} \dfrac{e^x - 1 - x}{x^2}$

ii. $\lim_{x \to \infty} \left(\dfrac{x+3}{x} \right)^x$

iii. $\lim_{x \to -\infty} \dfrac{\sqrt{16x^6 + 8x^3 - 4}}{3x^3 - 7x}$

(b) A table of values for $f(x)$ and $f'(x)$, along with a graph of a function $g(x)$ is shown below.

x	$f(x)$	$f'(x)$
1	2	3
2	4	1
3	6	5



Compute the following or state "DNE". There is no partial credit for this problem.

i. $\dfrac{d}{dx}g(x)$ at $x = 5$

ii. $\dfrac{d}{dx}g(f(x))$ at $x = 2$

iii. $f^{-1}(6)$

iv. $\dfrac{d}{dx}f^{-1}(x)$ at $x = 6$

v. $\dfrac{d}{dx}\left[f(x)\,\mathrm{e}^{g(x)}\right]$ at $x = 3$

Problem 4. (Integral exercises)

Compute the following integrals.

(a) $\dfrac{d}{dx}\displaystyle\int_0^{\pi/2} \sin^7 t \, dt$

(b) $\displaystyle\int_0^{\pi/2} \frac{d}{dx}(\sin^7 x)\, dx$

(c) $\dfrac{d}{dx}\displaystyle\int_0^{\sin(x)} \ln(t^2+1)\, dt$

(d) $\displaystyle\int_{-1}^{1} \frac{\theta^5 + \sin\theta}{\sqrt{1+\cos^2\theta}}\, d\theta$

(e) $\displaystyle\int (4x-6)\sqrt{x^2-3x}\, dx$

(f) $\displaystyle\int_{0}^{\pi/4} \frac{1+\tan\theta}{\sec\theta}\, d\theta$

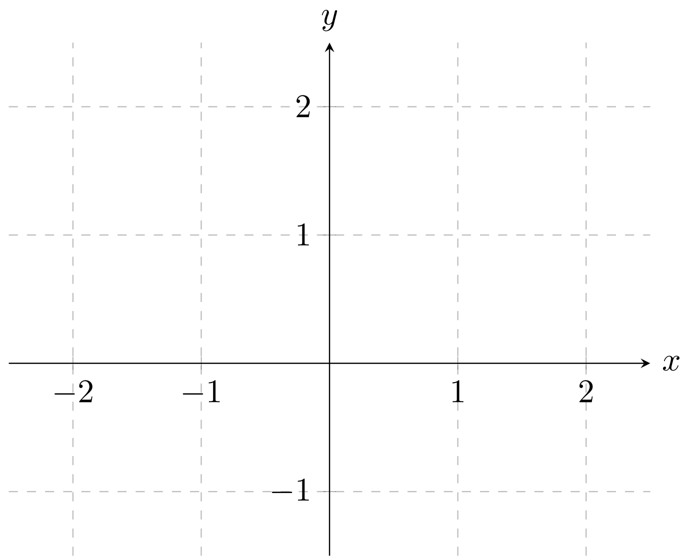
Problem 5. (Squeeze theorem and L'Hôpital)

Consider the three functions, g, f, and h, defined on the interval $(-2, 2)$. Given that

$$g(x) = \cos(\pi x), \quad h(x) = x^2 + 1 \quad \text{and } g(x) \leq f(x) \leq h(x),$$

answer the following questions.

(a) Sketch and label the graph of g and h, and a possible graph of f.



(b) Use the Squeeze Theorem to evaluate $\lim_{x \to 0} f(x)$.

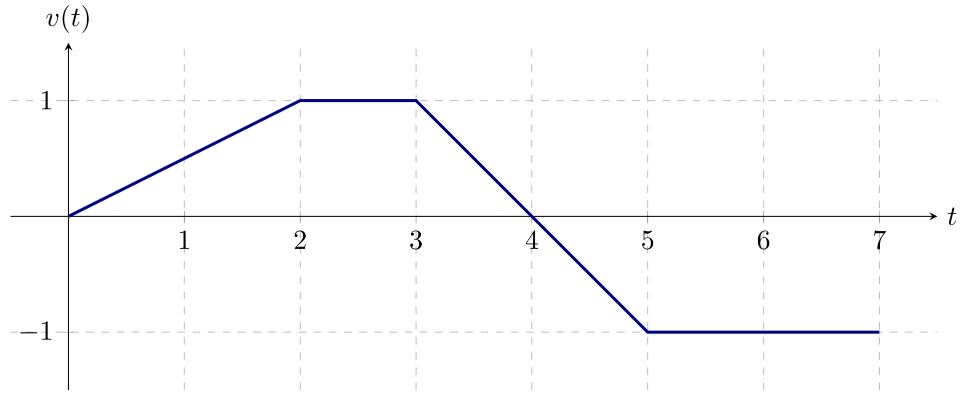
(c) Evaluate

$$\lim_{x \to 0} \frac{g(x) - 1}{h(x) - 1}$$

(Write "does not exist" only if the limit does not exist and is neither $+\infty$ nor $-\infty$.)

Problem 6. (1-D motion)

Consider the motion of a particle moving on a straight line whose velocity v is described in the graph below:



Assume that $s(0) = 0$.

(a) Determine the displacement between $t = 0$ and $t = 7$.

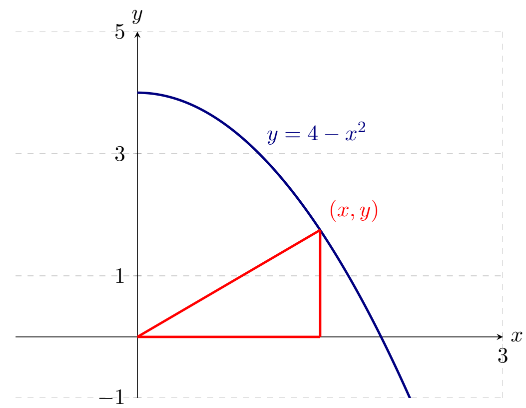
(b) Determine the distance traveled between $t = 0$ and $t = 7$.

(c) Determine the position function, $s(t)$, for $5 \leq t \leq 7$.

(d) Determine the acceleration, $a(t)$, for $5 < t < 7$.

Problem 7. (Optimization)

The figure shows a right triangle in the first quadrant. One side of the triangle is on the x-axis; its hypotenuse runs from the origin to a point on the parabola $y = 4 - x^2$. Find the coordinates that maximize the area of the triangle.



In your solution:

- State explicitly the domain of objective function.
- Be sure to justify that your answer indeed yields the maximal area.

Problem 8. (More integrals)

Suppose that $\int_{-1}^{2} f(x)\,dx = 4$. Assume that f is **odd**.

(a) Evaluate $\int_{1}^{2} f(x)\,dx$.

(b) Which average value of f is larger, the one over $[-1, 2]$ or the one over $[1, 2]$? Explain.

(c) Evaluate $\int_{0}^{2\ln 2} \mathrm{e}^{x} f\left(\mathrm{e}^{x} - 2\right)\,dx$.