Lecture 7: An Application of Limits (AAOL)

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Average velocity

- Let $s(t)$ denote the position of an object moving along a vertical (or a horizontal) line at time t.
- The **average velocity** of the object on the time interval $[a, b]$ is given by

$$v_{\text{avg}} = \frac{\text{change in position}}{\text{change in time}} = \frac{s(b) - s(a)}{b - a} \,.$$

Example

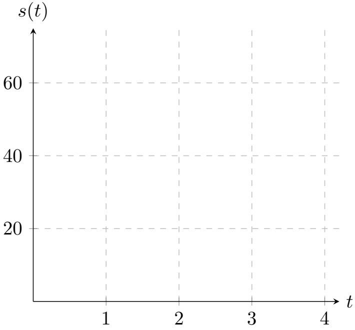
Suppose you are throwing a ball straight upward into the air with velocity 64 ft/sec. Its height (in feet) after t seconds is given by

$$s(t) = 64t - 16t^2\,.$$

Answer the following questions.

Questions.

1. Sketch the graph of $s(t)$.



2. When will it hit the ground?

3. Compute the average velocity of the ball on the time interval $[1.5, 3]$.

4. Compute the average velocity of the ball on the time interval $[t, 3]$ for $0 < t < 3$.

5. Finally, do the same with $[3, t]$ for $3 < t < 4$.

Instantaneous velocity

- An average velocity over a shorter time interval yields a better approximation of the **instantaneous velocity** at a moment contained in the time interval.
- This statement can be made precise using limits:

$$\underbrace{v(a)}_{\text{inst. vel.}} = \lim_{t\to a} \underbrace{\frac{s(t)-s(a)}{t-a}}_{\text{avg. vel.}}$$

where $v(a)$ is the (instantaneous) velocity at time a.

6. Using the results of parts 4 and 5, compute the velocity of the ball at $t = 3$.