Lecture 9: Derivatives as Functions (DAF)

Tae Eun Kim, Ph.D.

Autumn 2021

The derivative as a function

- Recall from last time that the **derivative** of a function f at a point a is given by
$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$
- If we replace a by a variable x, we now have the following function:
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$
- This function gives us the *instantaneous rate of change* at any variable point x. Calculation of this derived function is called **differentiation**.
- **Notation.**
$$f'(x) = \frac{d}{dx} f(x).$$

Differentiability implies continuity

Differentiability is a property that is stronger than continuity.

Theorem (Differentiability implies continuity)

If f is a differentiable function at a, then f is continuous at a.

Notes.

- The contrapositive of the theorem is stated as follows:
  If f is not continuous at a, then f is not differentiable at a.
- Consequently, all differentiable functions are continuous, but not all continuous functions are differentiable.

Question. Which of the following functions are continuous but not differentiable on $\mathbb{R}$?

1. x^2
2. $\lfloor x \rfloor$
3. $|x|$
4. $\dfrac{\sin(x)}{x}$

Question. Consider

$$f(x) = \begin{cases} x^2 & \text{if } x < 3, \\ mx + b & \text{if } x \geq 3. \end{cases}$$

What values of m and b make f differentiable at $x = 3$?