Lecture 20: Applied Related Rates (ARR)

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Autumn 2021

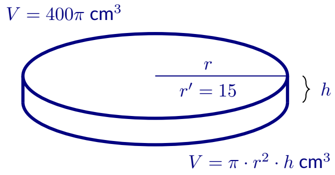
Applied Related Rates Problems

General procedures

1. **Introduce variables and identify the given and unknown rates.** Assign a variable to each quantity that changes in time.
2. **Draw a picture.** If possible, draw a schematic picture with all the relevant information.
3. **Find equations.** Write equations that relate all relevant variables.
4. **Differentiate with respect to time t.** Here we will often use implicit differentiation and obtain an equation that relates the given rate and the unknown rate.
5. **Evaluate.** Evaluate each quantity at the relevant moment.
6. **Solve.** Solve for the unknown rate at that moment.

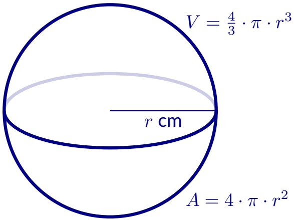
Example 1. (Cylindrical geometry)

A hand-tossed pizza crust starts off as a ball of dough with a volume of 400π cm^3. First, the cook stretches the dough to the shape of a cylinder of radius 12 cm. Next the cook tosses the dough.
If during tossing, the dough maintains the shape of a cylinder and the radius is increasing at a rate of 15 cm/min, how fast is its thickness changing when the radius is 20 cm?



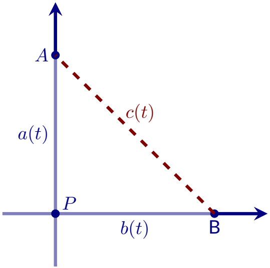
Example 2. (Spherical geometry)

Consider a melting snowball. We will assume that the rate at which the snowball is melting is proportional to its surface area. Show that the radius of the snowball is changing at a constant rate.



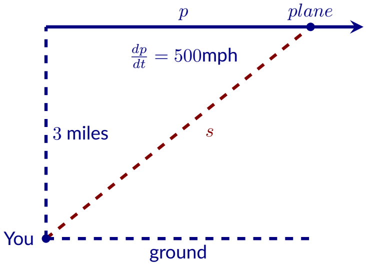
Example 3. (Right triangles)

A road running north to south crosses a road going east to west at the point P. Cyclist A is riding north along the first road, and cyclist B is riding east along the second road. At a particular time, cyclist A is 3 kilometers to the north of P and traveling at 20 km/hr, while cyclist B is 4 kilometers to the east of P and traveling at 15 km/hr. How fast is the distance between the two cyclists changing at that time?



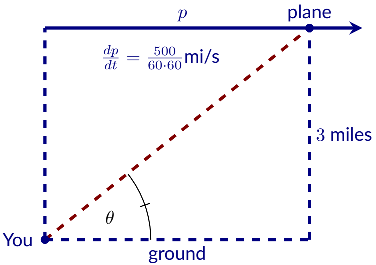
Example 4. (Right triangles)

A plane is flying at an altitude of 3 miles directly away from you at 500 mph. How fast is the plane's distance from you increasing at the moment when the plane is flying over a point on the ground 4 miles from you?



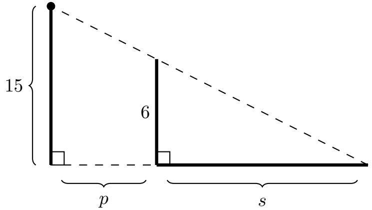
Example 5. (Angular rates)

A plane is flying at an altitude of 3 miles directly away from you at 500 mph . Let θ be the **angle of elevation** of the plane, i.e., the angle between the ground and your line of sight to the plane. How fast (in radians per second) is the angle θ decreasing at the moment when the plane is flying over a point on the ground 4 miles from you?



Example 6. (Similar triangles)

It is night. Someone who is 6 feet tall is walking away from a street light at a rate of 3 feet per second. The street light is 15 feet tall. The person casts a shadow on the ground in front of them. How fast is the length of the shadow growing when the person is 7 feet from the street light?



Example 7. (Similar triangles)

Water is poured into a conical container at the rate of 10 cm^3/s. The cone points directly down, and it has a height of 30 cm and a base radius of 10 cm. How fast is the water level rising when the water is 4 cm deep?