Lecture 11: Product Rule and Quotient Rule (PRAQR)

Tae Eun Kim, Ph.D.

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Motivation

Question. Let $f(x) = (x^2 + 1)$ and $g(x) = (x^3 - 3x)$. Suppose that you want to compute

$$\frac{d}{dx}[f(x)g(x)]\,.$$

- We can proceed by expanding $f(x)g(x)$ then differentiating the result using the sum rule and power rule.
- This can get very tedious.
- At times, the strategy may not even be applicable.

Product Rule

Theorem (Product Rule)

If f and g are differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + f'(x)g(x)\,.$$

Let's revisit the opening example:

Question. Let $f(x) = (x^2 + 1)$ and $g(x) = (x^3 - 3x)$. Using the product rule, compute

$$\frac{d}{dx}[f(x)g(x)]\,.$$

Question. Compute

$$\frac{d}{dx}(xe^x - e^x)\,.$$

Quotient Rule

Theorem (Quotient Rule)

If f and g are differentiable, then

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}.$$

- Viewing the quotient as a product $f(x)(1/g(x))$, we can use the product rule to derive the above.
- But in order to do that, we need to know what $\frac{d}{dx}(1/g(x))$ is.

Question. Compute:

$$\frac{d}{dx}\frac{x^2+1}{x^3-3x}\,.$$

Question. Compute:

$$\frac{d}{dx}\frac{625 - x^2}{\sqrt{x}}$$

in two ways. First using the quotient rule and then using the product rule.