Lecture 15: Logarithmic Differentiation (LD)

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Introduction

Let's recall:

Properties of logarithms

Let $b > 0$ and $b \neq 1$; let $x, y > 0$.

- $\log_b(xy) = \log_b(x) + \log_b(y)$
- $\log_b(x/y) = \log_b(x) - \log_b(y)$
- $\log_b(x^y) = y \log_b(x)$

Logarithmic differentiation

A key point of the logarithmic differentiation is the following application of the chain rule:

$$\frac{d}{dx}\ln(f(x)) = \frac{f'(x)}{f(x)}\,.$$

Illustration of method

To differentiate $y = f(x)$, i.e., to find $\frac{dy}{dx} = y'$:

1. Take the logarithm of $y = f(x)$: $\ln y = \ln f(x)$
2. Differentiate implicitly: $y'/y = f'(x)/f(x)$
3. Solve for y'.

Question. Compute

$$\frac{d}{dx}\frac{x^9 e^{4x}}{\sqrt{x-4}}.$$

Question. For $x > 0$, compute

$$\frac{d}{dx} x^x \, .$$

This function is an example of a *tower function*. [1]

[1] **Note.** Make sure you are able to distinguish the following functions:

$$a^x \, , \qquad x^a \, , \qquad x^x \, .$$

Question. Compute the derivative

$$\frac{d}{dx} \ln\left(|x|\right).$$

Use the result to compute the derivative

$$\frac{d}{dx} \ln\left(|f(x)|\right).$$

Question. Using logarithmic differentiation, compute the derivative.

$$\frac{d}{dx}\left(\frac{\sin x}{x}\right).$$

The Power Rule Revisited

Theorem (The power rule)

For any real number n and a positive real number x,

$$\frac{d}{dx}x^n = nx^{n-1}.$$