More Practice Problems for Computational Test — Math 1151

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Each of the following problems can be thought of as a single practice set, which consists of 8 problems just as in the actual test. The following is the generic instructions for the test.

> Compute the derivative of each of the following 8 functions. You do not need to simplify. You do not need to show steps. No calculator is allowed. **No partial credit will be given.** Be extremely careful with notation, signs, parentheses, etc. Please circle or box your final answer for each question. Finish them in **30 minutes**.

Problem 1. (Warm-up)

This is your warm-up set.

(a) $f(x) = x^4 e^{\sqrt{x}} + e^{\sqrt{3}} \cdot (\ln(x))^{\sqrt{4}}$

(b) $f(x) = x^3 - \pi^6 + 6^x$

(c) $f(x) = (7x+1)^{2x}$

(d) $f(x) = \dfrac{3x\cot(x)}{6x + \ln(x)}$

(e) $f(x) = \left(2x + \sin\left(\sqrt{x} + 5\right)\right)^5$

(f) $f(x) = \dfrac{\sec(8)}{\sqrt[5]{x}} + \dfrac{\sec(x)}{\sqrt[5]{8}} + \dfrac{e^6}{\sqrt[3]{6}}$

(g) $f(x) = \cos(x)\left(5x^4 + 2x\right)$

(h) $f(x) = \ln(4)\tan(2x+1) + \csc^5(2x+1)$

(a): $f(x) = x^4 e^{\sqrt{x}} + e^{\sqrt{3}} \cdot (\ln(x))^{\sqrt{4}}$

$$f'(x) = x^4 \cdot \left(e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} \right) + 4x^3 \cdot e^{\sqrt{x}} + e^{\sqrt{3}} \cdot 2(\ln(x)) \cdot \frac{1}{x}$$

(b): $f(x) = x^3 - \pi^6 + 6^x$

$$f'(x) = 3x^2 + 6^x(\ln 6)$$

(c): $f(x) = (7x+1)^{2x}$

$$f'(x) = \left[2x \cdot \frac{7}{7x+1} + 2\ln(7x+1) \right] (7x+1)^{2x}$$

(d): $f(x) = \frac{3x\cot(x)}{6x+\ln(x)}$

$$f'(x) = \frac{(6x+\ln(x))(3\cot(x) - 3x\csc^2(x)) - (3x\cot(x))(6+\frac{1}{x})}{(6x+\ln(x))^2}$$

(e): $f(x) = (2x + \sin(\sqrt{x}+5))^5$

$$f'(x) = 5\left(2x + \sin(\sqrt{x}+5)\right)^4 \cdot \left(2 + \cos(\sqrt{x}+5) \cdot \frac{1}{2\sqrt{x}} \right)$$

(f): $f(x) = \frac{\sec(8)}{\sqrt[5]{x}} + \frac{\sec(x)}{\sqrt[5]{8}} + \frac{e^6}{\sqrt[3]{6}}$

$$f'(x) = \sec(8) \cdot \left(\frac{-1}{5} \right) x^{\frac{-6}{5}} + \frac{\sec(x)\tan(x)}{\sqrt[5]{8}}$$

(g): $f(x) = \cos(x)\left(5x^4 + 2x\right)$

$$f'(x) = (-\sin(x))\left(5x^4+2x\right) + \cos(x)\left(20x^3 + 2\right)$$

(h): $f(x) = \ln(4)\tan(2x+1) + \csc^5(2x+1)$

$$f'(x) = \ln(4)\sec^2(2x+1) \cdot 2 + 5\left(\csc(2x+1)\right)^4 \cdot \left(-\csc(2x+1)\cot(2x+1) \cdot 2\right)$$

Problem 2. (Immersion)

Immerse yourself in the computation of the following derivatives.

(a) $f(x) = x^3 e^{\sqrt{x}} + e^{\sqrt{3}} \cdot (\ln(x))^{\sqrt{3}}$

(b) $f(x) = x^4 - \pi^4 + 5^x$

(c) $f(x) = (6x+1)^{4x}$

(d) $f(x) = \dfrac{2x\tan(x)}{6x + \ln(x)}$

(e) $f(x) = \left(2x + \cos(\sqrt{x} + 5)\right)^6$

(f) $f(x) = \dfrac{\cot(8)}{\sqrt[5]{x}} + \dfrac{\cot(x)}{\sqrt[5]{8}} + \dfrac{e^5}{\sqrt[3]{5}}$

(g) $f(x) = \sec(x)\left(5x^4 + 2x\right)$

(h) $f(x) = \ln(4)\sin(2x+1) + \csc^5(2x+1)$

Answers written by Nate Clause

(a): $f(x) = x^3 e^{\sqrt{x}} + e^{\sqrt{3}} \cdot (\ln(x))^{\sqrt{3}}$

$$f'(x) = x^3 \cdot \left(e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}\right) + 3x^2 \cdot e^{\sqrt{x}} + e^{\sqrt{3}} \cdot \sqrt{3}(\ln(x))^{\sqrt{3}-1} \cdot \frac{1}{x}$$

(b): $f(x) = x^4 - \pi^4 + 5^x$

$$f'(x) = 4x^3 + 5^x(\ln 5)$$

(c): $f(x) = (6x+1)^{4x}$

$$f'(x) = \left[4x \cdot \frac{6}{6x+1} + 4 \cdot \ln(6x+1)\right](6x+1)^{4x}$$

(d): $f(x) = \frac{2x\tan(x)}{6x+\ln(x)}$

$$f'(x) = \frac{(6x+\ln(x))(2x\sec^2(x) + 2\tan(x)) - (2x\tan(x))(6+\frac{1}{x})}{(6x+\ln(x))^2}$$

(e): $f(x) = (2x + \cos(\sqrt{x}+5))^6$

$$f'(x) = 6\left(2x + \cos(\sqrt{x}+5)\right)^5 \cdot \left(2 - \sin(\sqrt{x}+5) \cdot \frac{1}{2\sqrt{x}}\right)$$

(f): $f(x) = \frac{\cot(8)}{\sqrt[5]{x}} + \frac{\cot(x)}{\sqrt[5]{8}} + \frac{e^5}{\sqrt[3]{5}}$

$$f'(x) = \cot(8) \cdot \left(\frac{-1}{5}x^{\frac{-6}{5}}\right) - \frac{\csc^2(x)}{\sqrt[5]{8}}$$

(g): $f(x) = \sec(x)(5x^4 + 2x)$

$$f'(x) = (\sec(x)\tan(x)(5x^4+2x)) + \sec(x)(20x^3+2)$$

(h): $f(x) = \ln(4)\sin(2x+1) + \csc^5(2x+1)$

$$f'(x) = \ln(4)\cos(2x+1) \cdot 2 + 5(\csc(2x+1))^4 \cdot (-\csc(2x+1)\cot(2x+1) \cdot 2)$$

Problem 3. (Dissemination)

Spend an evening showing your friends how to do these derivatives.

(a) $f(x) = x^5 \cos(6) + 7^x + e^{21} + \dfrac{3}{x}$

$$f'(x) = \boxed{5x^4 \cos(6) + 7^x \ln(7) - \frac{3}{x^2}}$$

(b) $f(x) = \left(2x^5 + 6x\right) \sec(x)$

$$f'(x) = \boxed{(10x^4 + 6)\sec(x) + (2x^5 + 6x)\sec(x)\tan(x)}$$

(c) $f(x) = \dfrac{2x \ln(x^2 - 4)}{e^x + \pi x}$

$$f'(x) = \boxed{\frac{(e^x + \pi x)\left(2\ln(x^2-4) + 2x\frac{2x}{x^2-4}\right) - \left(2x\ln(x^2-4)\right)\left(e^x + \pi\right)}{(e^x + \pi x)^2}}$$

(d) $f(x) = \sec\left(x^4 + \dfrac{4}{x^4}\right)$

$$f'(x) = \boxed{\sec\left(x^4 + \frac{4}{x^4}\right)\tan\left(x^4 + \frac{4}{x^4}\right) \cdot \left[4x^3 - \frac{16}{x^5}\right]}$$

(e) $f(x) = \underbrace{x^{e^x}}_{\text{tower}} + e^{x^e}$

$\ln x^{e^x} = e^x \ln x = e^x f(x)$

$\downarrow \frac{d}{dx}$

$e^x (f(x) + f'(x))$

$$f'(x) = \boxed{x^{e^x} e^x \left(\ln x + \frac{1}{x}\right) + e^{x^e} \cdot e x^{e-1}}$$

(f) $f(x) = 2^\pi \ln(\sqrt{x}) + 2^{3x}\sqrt{\ln(x)} + 2^{\ln\sqrt{x}}$

$$f'(x) = \boxed{2^\pi \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}} + \left[2^{3x} \cdot \ln 2 \cdot 3\sqrt{\ln x} + 2^{3x}\frac{\frac{1}{x}}{2\sqrt{\ln(x)}}\right] + 2^{\ln\sqrt{x}} \cdot \ln 2 \cdot \frac{\frac{1}{2\sqrt{x}}}{\sqrt{x}}}$$

(g) $f(x) = \csc(3x + 9) - (3x + 9)\cot^5(x)$

$$f'(x) = \boxed{-3\csc(3x+9)\cot(3x+9) - \left[3\cot^5(x) - (3x+9) \cdot 5\cot^4(x)\csc^2(x)\right]}$$

(h) $f(x) = \left(x + \ln(x^5 - 3)\right)^7$

$$f'(x) = \boxed{7\left(x + \ln(x^5-3)\right)^6 \cdot \left(1 + \frac{5x^4}{x^5 - 3}\right)}$$

Problem 4. (Solitude)

Now that you don't have any friends, you should have enough time to yourself to work out the following problems. Enjoy!

(a) $f(x) = (2x^6 + 5x) \cdot \sec(x)$

$$f'(x) = \boxed{(12x^5+5)\sec(x) + (2x^6+5x)\cdot\sec(x)\tan(x)}$$

(b) $f(x) = \left(x + \ln(x^2+3)\right)^4$

$$f'(x) = \boxed{4\left(x+\ln(x^2+3)\right)^3\cdot\left(1+\frac{2x}{x^2+3}\right)}$$

(c) $f(x) = \tan\left(x^4 - \frac{6}{x^3}\right)$

Note the sign flip.

$$f'(x) = \boxed{\sec^2\left(x^4-\frac{6}{x^3}\right)\left(4x^3+\frac{18}{x^4}\right)}$$

(d) $f(x) = \dfrac{\ln(x)\cdot\sqrt[5]{x}}{e^x+4x}$ $\rightsquigarrow x^{\frac{1}{5}}$

$$f'(x) = \boxed{\frac{(e^x+4x)\left(\frac{1}{x}\sqrt[5]{x}+\ln(x)\cdot\frac{1}{5}x^{-\frac{4}{5}}\right)-\ln(x)\sqrt[5]{x}\cdot\left(e^x+4\right)}{(e^x+4x)^2}}$$

(e) $f(x) = (4x+5)^x$ [Log. Diff. : (orig. fnc)$\cdot$(deriv. ln(orig))]

$\ln f(x) = x\ln(4x+5)$

$$f'(x) = \boxed{(4x+5)^x\left[\ln(4x+5)+x\frac{4}{4x+5}\right]}$$

(f) $f(x) = x^7\cdot\sin(6) + 5^x + e^4 + \dfrac{3}{x}$

$$f'(x) = \boxed{7x^6\sin(6)+5^x\ln 5-\frac{3}{x^2}}$$

(g) $f(x) = \dfrac{\sin^4(x)}{\sqrt{x^3+2}}$

$$f'(x) = \boxed{\frac{\sqrt{x^3+2}\cdot 4\sin^3(x)\cos(x)-\sin^4(x)\frac{3x^2}{2\sqrt{x^3+2}}}{x^3+2}}$$

(h) $f(x) = \csc(5x-1) - \cot^3(x)\cdot(5x-1)$

$$f'(x) = \boxed{-5\csc(5x-1)\cot(5x-1)-\left[-3\cot^2(x)\csc^2(x)(5x-1)+5\cot^3(x)\right]}$$

Problem 5. (Defiance)

Who needs all those 30 minutes? Finish the following in 20 minutes.

(a) $f(x) = (4x + \sec(2))^{55}$

$$f'(x) = \boxed{55\left(4x + \sec(2)\right)^{54} \cdot 4}$$

(b) $f(x) = x^6 + 6^x$

$$f'(x) = \boxed{6x^5 + 6^x \ln(6)}$$

Note the sign here!

(c) $f(x) = \dfrac{3x^{10} + x^5}{x^3 + 3}\csc(x)$

$$f'(x) = \boxed{\frac{(x^3+3)(30x^9+5x^4) - (3x^{10}+x^5)\cdot 3x^2}{(x^3+3)^2} \cdot \csc(x) - \frac{3x^{10}+x^5}{x^3+3} \cdot \csc(x)\cot(x)}$$

(d) $f(x) = \ln(5\cot(5x^{\sqrt{3}} + 6))$

$$f'(x) = \boxed{\frac{-5\csc^2\left(5x^{\sqrt{3}}+6\right)\cdot 5\sqrt{3}\,x^{\sqrt{3}-1}}{5\cot\left(5x^{\sqrt{3}}+6\right)}}$$

(e) $f(x) = e^{\sin(x)}\cos(4x)$

$$f'(x) = \boxed{\cos(x)\, e^{\sin(x)} \cdot \cos(4x) - 4e^{\sin(x)}\sin(4x)}$$

(f) $f(x) = \sqrt{\dfrac{4}{x^4} + 2x}$

$$f'(x) = \boxed{\frac{-\frac{16}{x^5} + 2}{2\sqrt{\frac{4}{x^4} + 2x}}}$$

(g) $f(x) = \cos(5x)e^{\sec(x)}$

$$f'(x) = \boxed{-5\sin(5x)\, e^{\sec(x)} + \cos(5x)\sec(x)\tan(x)\, e^{\sec(x)}}$$

(h) $f(x) = \underbrace{\left(\dfrac{7}{x^3}\right)^x}_{\text{L.D.}} + \dfrac{5}{\sqrt[3]{x}} + \dfrac{\sin(x)}{x}$

$$\frac{d}{dx}\left(5x^{-\frac{1}{3}}\right) = 5\cdot\left(-\tfrac{1}{3}\right)x^{-\frac{1}{3}-1} = -\frac{5}{3}x^{-\frac{4}{3}}$$

$$\ln\left(\frac{7}{x^3}\right)^x = x(\ln 7 - 3\ln x)$$

$$f'(x) = \boxed{\left(\frac{7}{x^3}\right)^x\left[(\ln 7 - 3\ln x) + x\left(-\frac{3}{x}\right)\right] + 5\left(-\frac{1}{3}\right)x^{-\frac{4}{3}} + \frac{x\cos(x) - \sin(x)}{x^2}}$$

Problem 6. (Oblivion)

Now that you are an experienced *differentiator*, why not do another set?

(a) $f(x) = \cot^2(x) + \csc^2(2x)$

$$f'(x) = \boxed{-2\cot(x)\cdot\csc^2(x) + 2\csc(2x)\cdot\left(-\csc(2x)\cot(2x)\right)\cdot 2}$$

(b) $f(x) = \dfrac{e^{3x}\sqrt{x}}{\log_3(x)}$

$$f'(x) = \boxed{\frac{\log_3(x)\left(3e^{3x}\sqrt{x} + e^{3x}\cdot\frac{1}{2\sqrt{x}}\right) - e^{3x}\sqrt{x}\cdot\frac{1}{x\ln(3)}}{\left[\log_3(x)\right]^2}}$$

(c) $f(x) = e^{5x}\sqrt{x} - \dfrac{5e^{-2x}}{x^6}$

$$f'(x) = \boxed{5e^{5x}\sqrt{x} + e^{5x}\frac{1}{2\sqrt{x}} - 5\,\frac{x^6(-2)e^{-2x} - e^{-2x}\cdot 6x^5}{x^{12}}}$$

(d) $f(x) = \dfrac{5x}{(\ln(x) + 4x^2)(x^2 - 3e^x)}$

$$f'(x) = \boxed{\frac{5(\ln(x)+4x^2)(x^2-3e^x) - 5x\left((\frac{1}{x}+8x)(x^2-3e^x)+(\ln(x)+4x^2)(2x-3e^x)\right)}{(\ln(x)+4x^2)^2\ (x^2-3e^x)^2}}$$

$g(x)$

$(f(x))$

(e) $f(x) = x^{\cot(x)} + \cot^3(x)$

Notational conv. : $\cot^3(x) = (\cot(x))^3$

$\ln x^{\cot(x)}$

$= \cot(x)\ln x$

$$f'(x) = \boxed{x^{\cot(x)}\left[-\csc^2(x)\ln x + \frac{\cot(x)}{x}\right] + 3\cot^2(x)\cdot\left(-\csc^2(x)\right)}$$

(f) $f(x) = \pi^5 + 7x^6 + x^2e^x$

$$f'(x) = \boxed{42x^5 + \left(2xe^x + x^2e^x\right)}$$

If $f(x) = 7x^6 - x^2e^x$,

$f'(x) = 42x^5 - \left(2xe^x + x^2e^x\right)$.

Note the parentheses are necessary!

(g) $f(x) = \sqrt[4]{3 + e + \csc(x)} = (3+e+\csc(x))^{1/4}$

$$f'(x) = \boxed{\frac{1}{4}(3+e+\csc(x))^{-\frac{3}{4}}\left(-\csc(x)\cot(x)\right)}$$

(h) $f(x) = \tan(\sin(e^x))$

$$f'(x) = \boxed{\sec^2\left(\sin(e^x)\right)\cdot\cos(e^x)\cdot e^x}$$

Problem 7. (Decrescendo)

At last, here comes your cool-down set.

(a) $f(x) = \dfrac{\cos(x)e^{3x}}{e^2}$

$$f'(x) = \boxed{\dfrac{-\sin(x)\,e^{3x} + 3\cos(x)\,e^{3x}}{e^2}}$$

$\frac{1}{2} - 6 = \frac{1}{2} - \frac{12}{2}$

(b) $f(x) = e^{5x} + \dfrac{5\sqrt{x}}{x^6}$ $= e^{5x} + 5\dfrac{x^{\frac{1}{2}}}{x^6} = e^{5x} + 5x^{-\frac{11}{2}}$

$$f'(x) = \boxed{5e^{5x} - \frac{55}{2}x^{-\frac{13}{2}}}$$

(c) $f(x) = (x\sec(x) + 5)\left(x^3 - 2\right)$

$$f'(x) = \boxed{\begin{aligned}&(\sec(x) + x\sec(x)\tan(x))(x^3-2)\\ &+ (x\sec(x)+5)(3x^2)\end{aligned}}$$

(d) $f(x) = x5^{\pi} + \pi^5 + 5x^{\pi} + 5^x$

$$f'(x) = \boxed{5^{\pi} + 5\pi x^{\pi-1} + 5^x\ln(5)}$$

(e) $f(x) = \tan\left(\dfrac{x^{13} - 6x}{2 - 4x^3}\right)$

$$f'(x) = \boxed{\sec^2\left(\frac{x^{13}-6x}{2-4x^3}\right)\cdot\frac{(2-4x^3)(13x^{12}-6)-(x^{13}-6x)(-12x^2)}{(2-4x^3)^2}}$$

(f) $f(x) = \dfrac{\sec^4(x) + \tan(2)}{\sqrt{x^3+1} + 2}$

$$f'(x) = \boxed{\frac{(\sqrt{x^3+1}+2)(4\sec^3(x)\cdot\sec(x)\tan(x)) - (\sec^4(x)+\tan(2))\frac{3x^2}{2\sqrt{x^3+1}}}{(\sqrt{x^3+1}+2)^2}}$$

(g) $f(x) = \ln(x)\cdot\ln\left(\ln(x)\right)$

$$f'(x) = \boxed{\frac{1}{x}\ln(\ln(x)) + \frac{1}{x}}$$

(h) $f(x) = \sqrt{\sqrt{1+5x} - \sqrt{1-5x}}$

$$f'(x) = \boxed{\frac{\dfrac{5}{2\sqrt{1+5x}} + \dfrac{5}{2\sqrt{1-5x}}}{2\sqrt{\sqrt{1+5x} - \sqrt{1-5x}}}}$$