Mock Final Examination II Math 1151

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Practice problems for comprehensive final exam.

Problem 1. (Derivative techniques)

Answer the following questions.

(a) Compute the derivative; you do not need to simplify.

$$\frac{d}{dx}\left(e^{4x}\sqrt{2+\tan^{-1}(3x^2)}\right)$$

(b) Consider the curve given by

$$\sin(xy) = y(x-3).$$

i. Verify that the point $(3, \pi)$ lies on the curve.

ii. Write an equation of the line tangent to the curve at the point $(3, \pi)$.

Problem 2. (L'Hôpital's rule)

For each limit below

- state the **form** of the limit;
- indicate whether the form is **indeterminate** or not;
- evaluate the limit, if it exists or if it is $+\infty$ or $-\infty$. Otherwise, write "does not exist". If the form is **indeterminate**, show your work. You may use L'Hôpital's rule.

(a) $\lim_{x\to 1^+} \Big[\ln(x)\Big]^x$

(b) $\lim_{x\to e} \dfrac{\ln(x)-1}{x-e}$

(c) $\lim_{x\to 0^+} \Big(\sin(x)\ln(x)\Big)$

Problem 3. (Various topics)

Answer the following questions.

(a) Solve the initial value problem (IVP),

$$\begin{cases} y' = \sec^2(x) + 10\sin(5x) & \text{(DE)} \\ y(0) = 4 & \text{(IC)} \end{cases}.$$

(b) Let g be the function given by

$$g(x) = \begin{cases} 3e^{x-2} & \text{if } x \leq 2, \\ 6\cos\left(\dfrac{\pi}{6}x\right) & \text{if } x > 2. \end{cases}$$

State the **definition of continuity**. Use the definition of continuity to determine whether the function g is continuous at $x = 2$. Show your work.

Problem 4. (Integral exercises)

Evaluate the following integrals. Show your work.

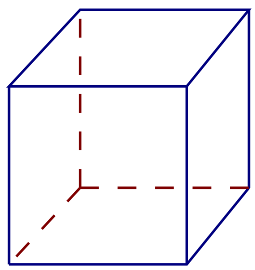
(a) $\displaystyle\int_0^{\pi} \frac{\sin x}{(2\cos x)^2}\, dx$

(b) $\displaystyle\int \frac{x-2}{\sqrt{x+3}}\, dx$

(c) $\displaystyle\int_{-\pi}^{\pi} \left(x^3 \cos x + 5 + \frac{2x}{3+x^2} \right) dx$

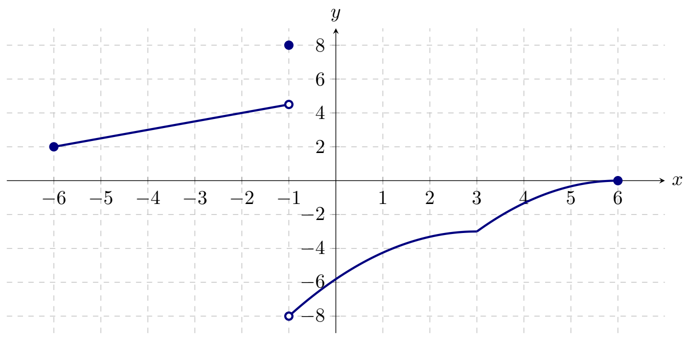
Problem 5. (Optimization)

Suppose an airline policy states that all baggage must be box-shaped with a sum of length, width, and height not exceeding 64 in. What are the dimensions and volume of a square-based box with the greatest volume under these conditions?



Problem 6. (Understanding functions from graphs)

The graph of the function f on its domain $[-6, 6]$ is shown in the figure below.



Use the graph of f to answer the questions below.

(a) Determine the **range** of f. Write your answer in interval notation.

(b) Let f^{-1} be the inverse of f. Determine the **range** of f^{-1}. Write your answer in interval notation.

(c) Find the following values or say "does not exist".

i. $f^{-1}(0)$

ii. $f^{-1}(8)$

iii. $f'(3)$

iv. $f'(-2)$

v. $f^{-1}(4)$

vi. $\dfrac{df^{-1}}{dx}(4)$

(d) In the figure above, sketch the graph of f^{-1}.

(e) Find the x-coordinates of all **critical points** of f on the interval $(-6, 6)$ or say "no critical points".

(f) Find the x-coordinates of all **local maxima** of f on the interval $(-6, 6)$ or say "no local maxima".

(g) Order the following four numbers from smallest to largest:

$$f'(2),\ f'(2.6),\ f'(3.2),\ f'(3.3)$$

(h) Find the limit if it exists. Otherwise, write "does not exist".

i. $\lim_{x\to 0} f(x)$

ii. $\lim_{x\to 6^-} f(x)$

iii. $\lim_{x\to -1^+} f(x)$

iv. $\lim_{x\to -1} f(x)$

(i) Find the **average rate of change** of the function f on the interval $[0,3]$. Show work.

(j) Circle the interval on which the function f satisfies the conditions of the mean value theorem.

i. $[-6,4]$
ii. $[-1,4]$
iii. $[0,4]$
iv. $[1,3]$
v. $[2,4]$
vi. No previous answer is correct.

Problem 7. (Understanding functions from tables)

Let f be a function that is **differentiable** on the interval $(0, 6)$. Particular values of f and f' are given in the table below. We also know that the function f', the derivative of f, is continuous on the interval $[1, 5]$. Use the table below to answer the following parts. Show your work.

x	1	2	3	4	5
$f(x)$	-2	1	-1	2	3
$f'(x)$	-4	3	4	-2	-1

(a) Find the limit below or say "does not exist".

$$\lim_{h\to 0} \frac{f(3+h) - f(3)}{h}$$

(b) Find $L(x)$, the linear approximation to f at $a = 3$.

(c) Use the linearization from part (b) to estimate $f(2.7)$.

(d) Compute the **right** Riemann sum of f', the derivative of f, on $[1, 5]$ for $n = 2$. Show your work.

(e) Express the **limit of right Riemann sums** on $[1, 5]$ as a definite integral, then evaluate the definite integral.

(f) Compute the derivative below. Explain.

$$\frac{d}{dx}\int_1^4 f(t)\,dt$$

(g) Compute the value of the derivative

$$\left[\frac{d}{dx}\int_1^x \sqrt{f(t)+7}\,dt\right]_{x=4}$$

(h) Compute the value of the derivative

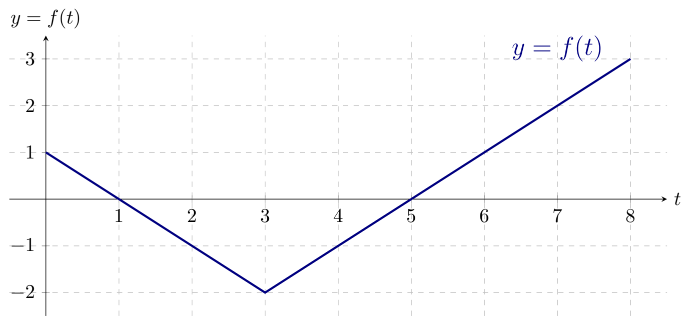
$$\left[\frac{d}{dx}\int_1^{\sqrt{x}} f'(t)\,dt\right]_{x=4}$$

(i) Compute the value of the derivative

$$\left[\frac{d}{dx}\left(\frac{xf(x)}{7}\right)\right]_{x=2}$$

Problem 8. (Accumulation function and the fundamental theorem of calculus)

The function f is continuous on $[0, 8]$. The graph of f is shown below.



Let $A(x) = \int_0^x f(t)\,dt$ for $0 \leq x \leq 8$.

(a) Find the value.

i. $A(0)$

ii. $A(4)$

iii. $A'(4)$

(b) Complete the following sentence.

The function A attains its minimum value on $[0, 8]$ at $x =$ ________.

(c) Complete the following sentence.

The function A is both **decreasing** and **concave up** on the interval ____________.

(d) Sketch the graph of A in the figure below.