Updated on August 25, 2021

Math 3607: Homework 1

Due: 10:00PM, Tuesday, August 31, 2021

- Problems marked with are to be done by hand; those marked with are to be solved using a computer. In addition, see below [1] for reference keys and notations.
- **Important note.** This is a course on numerical computations, not on symbolic ones[2]. Though MATLAB is capable of carrying out symbolic calculations using *Symbolic Math Toolbox*, we will never use it. Any work done using `sym` or `syms` will receive NO credit.

TOTAL: 30 points

1. (**LM** 2.1–15(a): Continued square roots) Let r be defined by

$$r = \sqrt{k + \sqrt{k + \sqrt{k + \sqrt{k + \sqrt{k + \cdots}}}}}$$

   where k is any positive integer.

   (a) Calculate r by hand and write down the result as an analytical formula (in terms of k).

   (b) Calculate the value of r numerically for $k = 2, 3$, and 4.

2. (**LM** 2.1–39(b): Roots of unity)

   (a) Use the approach explained in the problem (the one using Euler's formula) to find the five distinct solutions of $x^5 = -1$ in MATLAB.

   (b) (Optional) If you want more challenge, by factoring $x^5 + 1$ and solving quadratic equations, find the analytical expressions for the solutions not in terms of trigonometric functions, but in terms of radicals. Then use MATLAB to confirm your results (against the ones obtained using Euler's formula).

3. (Temperature conversion; adapted from **LM** 2.2–5.) Write a script which asks for a temperature in Fahrenheit. It should then output the temperature in degrees Celsius, kelvins, and degree Rankine[3]. Then run your code with a temperature of 134°F.

4. (Oblate spheroid) An *oblate spheroid* such as the Earth is obtained by revolving an ellipse about its minor axis as shown in the figure.

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[1] References:

- **LM**: *Learning MATLAB, Problem Solving, and Numerical Analysis Through Examples* (Overman)
- **NCM**: *Numerical Computing with MATLB* (Moler)
- **FNC**: *Fundamentals of Numerical Computation* (Driscoll and Braun)

The notation **LM** 2.2–5 indicates Problem 5 at the end of section 2.2 of the textbook by Overman.

[2] To learn about the difference between numerical and symbolic computations, please read the prologue of **FNC**.

[3] William Rankine was a Scottish engineer and physicist who proposed this scale in 1859, 11 years after the Kelvin scale. It also is zero at absolute zero, an increase of 1°Ra is exactly the same as an increase of 1°F.

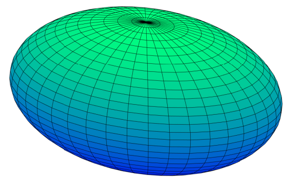

Figure 1: An oblate spheroid. Image created using MATLAB.

The Earth's equatorial radius is about 20km longer than its polar radius. The surface area of an oblate spheroid is given by

$$A(r_1, r_2) = 2\pi \left(r_1^2 + \frac{r_2^2}{\sin(\gamma)} \log \left(\frac{\cos(\gamma)}{1 - \sin(\gamma)} \right) \right),$$

where r_1 is the equatorial radius, r_2 is the polar radius, and

$$\gamma = \arccos \left(\frac{r_2}{r_1} \right).$$

We assume $r_2 < r_1$. Write a script that inputs the equatorial and polar radii and displays both $A(r_1, r_2)$ and the approximation $4\pi((r_1 + r_2)/2)^2$. Apply the script to the Earth data $(r_1, r_2) = (6378.137, 6356.752)$. Use `format long g` to display enough digits.