




Math 3607: Homework 10


Due: 10:00PM, Tuesday, November 23, 2021

TOTAL: 30 points

- Problems marked with  are to be done by hand; those marked with  are to be solved using a computer.
- **Important note.** Do not use *Symbolic Math Toolbox*. Any work done using `sym` or `syms` will receive NO credit.
- **Another important note.** When asked write a MATLAB function, write one at the end of your live script.

1. (Polynomial vs. piecewise polynomial interpolation; **FNC 5.1.2**)  The following table gives the life expectancy in the U.S. by year of birth:

year	1980	1985	1990	1995	2000	2005	2010
expectancy	73.7	74.7	75.4	75.8	77.0	77.8	78.7

- (a) Defining “year since 1980” as the independent variable, use `polyfit` to construct and plot the polynomial interpolant of the data.
 - (b) Use `interp1` to construct and plot a piecewise cubic interpolant (use ‘`spline`’ option) of the data.
 - (c) Use both methods to estimate the life expectancy for a person born in 2007. Which value is more believable?
2. (Piecewise cubic interpolation; **FNC 5.1.3**)  The following two point sets define the top and bottom of a flying saucer shape:


Top:

x	0	0.51	0.96	1.06	1.29	1.55	1.73	2.13	2.61
y	0	0.16	0.16	0.43	0.62	0.48	0.19	0.18	0




Bottom:

x	0	0.58	1.04	1.25	1.56	1.76	2.19	2.61
y	0	-0.16	-0.15	-0.30	-0.29	-0.12	-0.12	0

Use piecewise cubic interpolation to make a picture of the flying saucer.

3. (Quadratic interpolant by hand; **FNC 5.1.4**)  Define

$$q(x) = \frac{a}{2}x(x-1) - b(x-1)(x+1) + \frac{c}{2}x(x+1).$$

- (a) Show that q is a polynomial interpolant of the points $(-1, a)$, $(0, b)$, $(1, c)$.
 - (b) Use a change of variable to find a quadratic polynomial interpolant p for the points $(x_0 - h, a)$, (x_0, b) , $(x_0 + h, c)$.
4. (Cardinal cubic splines; **FNC** 5.3.5)  Although the cardinal cubic splines are intractable in closed form, they can be found numerically. Each cardinal spline interpolates the data from one column of an identity matrix. Define the nodes $\mathbf{t} = [0, 0.075, 0.25, 0.55, 1]^T$. Plot over $[0, 1]$ the five cardinal functions for this node set over the interval $[0, 1]$.
 5. (Piecewise quadratic interpolation; adapted from **FNC** 5.3.6.)  Suppose you were to define a piecewise quadratic spline that interpolates n given values and has a continuous first derivative. Follow the derivation presented in lecture to express all of the interpolation and continuity conditions. How many additional conditions are required to make a square system for the coefficients? Justify your answer.
 6. (Cubic splines in 2-D)  At the top of p. 1569 of **LM**, the term *pseudo-arc length* is introduced with an example script `an_ant.m`. Read it. Then do **LM** 12.2–15.