Hints for Homework 7

1. (Using `eig`; **FNC** 7.2.3) Some useful facts from linear algebra to remember:
   - The rank of a matrix is the number of linearly independent rows (or columns).
   - If a square matrix $A \in \mathbb{R}^{n\times n}$ has rank less than n, it is singular.
   - An eigenvalue λ of a matrix A is a scalar for which $A - \lambda I$ is singular (Why?).

2. (Polynomial evaluation of matrices; **FNC** 7.2.5 and Su20 final exam) This problem showcases a situation in which EVD enables an economical computation of A^k. Recall from lecture that if A has an EVD $A = VDV^{-1}$, then

$$\begin{aligned} A^2 &= (VDV^{-1})(VDV^{-1}) = VD^2V^{-1}, \\ A^3 &= (VDV^{-1})(VDV^{-1})(VDV^{-1}) = VD^3V^{-1}, \\ &\vdots \end{aligned}$$

   This will be useful.

   As for Horner's methods, see Problem 12(b) of Module 2 practice problem set. With a simple modification to the code, it can be used for both scalar and vector inputs.

   Note. If you want to test your code (the problem does not require any testing), use `polyval` for cases where `x` is a scalar or a vector and `polyvalm` for cases where `x` is a square matrix. Do recall that MATLAB uses a different convention in arranging polynomial coefficients, so you need to use `flip` accordingly.

3. (Singular values by hand) Look for the theorem[1] which reveals a connection between SVD and EVD. Since A in the problem is a real matrix, $A^* = A^{\mathrm{T}}$. It is your job to determine which one of $A^{\mathrm{T}}A$ or AA^{T} to use. *Hint.* The problem demands a 2×2 eigenvalue problem.

4. (SVD and the 2-norm)
   (a) Let $A = U\Sigma V^{\mathrm{T}}$ be the SVD of A. Then

$$A^{\mathrm{T}} = \left(U\Sigma V^{\mathrm{T}}\right)^{\mathrm{T}} = V\Sigma^{\mathrm{T}}U^{\mathrm{T}}.$$

   What kind of matrix is Σ^{T} and what does it equal?

   (b) Look for the theorem which describes how SVD is related to the matrix 2-norm. The result of part (a) is also useful.

[1] From Lecture 22 (10/22/21, F): Properties of SVD.

5. (Vandermonde matrix, SVD, and rank)

   I hope by now that everyone is comfortable creating a Vandermonde-type matrix; see also Problem 3 of HW5. The semi-log plot for part (b) should plot singular values (vertical axis) against integers $1, \ldots, 25$ (horizontal axis). The vertical axis need to be in log scale, so use `semilogy`, *e.g.*,

```
semilogy( <indices>, <singular values> )
```