Updated on September 15, 2021

Math 3607: Exam 1

Due: 12:00PM (noon), Monday, September 20, 2021

Please read the statements below and sign your name.

Disclaimers and Instructions

- You may use any of the book functions, any of functions provided in lectures, accompanying live scripts, and homework or practice problem solutions, and any of your own helper functions. As stated in the syllabus, you must be able to explain how they are supposed to work. You may be requested to explain your code to me, in which case a proper and satisfactory explanation must be provided to receive any credits on relevant parts.
- You are not allowed to search online forums or even MathWorks website. All problems can be solved with what has been taught in class. If you need to look something up, then you must be doing it incorrectly.
- Unlike for homework, you are not allowed to collaborate with classmates.
- If any code is found to be plagiarized from the internet or another person, you will receive a zero on the *entire* exam and will be reported to the COAM.
- Do not carry out computations using *Symbolic Math Toolbox*. Any work done using `sym`, `syms`, `vpa`, and such will receive NO credit.
- Answers to analytical questions (ones marked with ✎) without supporting work or justification will not receive any credit.

Academic Integrity Statements

- All of the work shown on this exam is my own.
- I will not consult with any resources (MathWorks website, online searches, etc.) other than the textbooks, lecture notes, and supplementary resources provided on the course Carmen pages.
- I will not discuss any part of this exam with anyone, online or offline.
- I understand that academic misconduct during an exam at The Ohio State University is very serious and can result in my failing this class or worse.
- I understand that any suspicious activity on my part will be automatically reported to the OSU Committee on Academic Misconduct (COAM) for their review.

Signature _____

Notation. Problems marked with ✏ are to be done by hand; those marked with 💻 are to be solved using a computer.

1 Golden Ratio by Nested Radical [20 points]

A *nested radical* is an infinite expression of the form

$$\sqrt{a_0 + \sqrt{a_1 + \sqrt{a_2 + \cdots}}}$$

If all the a_k's are equal to 1, the nested radical is equal to ϕ, the golden ratio

$$\phi = \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}} = \frac{1 + \sqrt{5}}{2}.$$

Denote by ϕ_j the truncation of the nested radical representation of the above, that is,

$$\begin{aligned}
\phi_0 &= \sqrt{1} \\
\phi_1 &= \sqrt{1 + \sqrt{1}} = \sqrt{1 + \phi_0} \\
\phi_2 &= \sqrt{1 + \sqrt{1 + \sqrt{1}}} = \sqrt{1 + \phi_1} \\
\phi_3 &= \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1}}}} = \sqrt{1 + \phi_2} \\
&\vdots
\end{aligned}$$

It follows that

$$\phi_j = \sqrt{1 + \phi_{j-1}}, \quad \text{for any } j \geq 1,$$

Furthermore, ϕ_j converges to the golden ratio as $j \to \infty$.

Question. 💻 Use a loop to evaluate ϕ_j for $j = 1, 2, \ldots, n$, where n is the smallest positive integer such that $|\phi_n - \phi| \leq 10^{-12}$. Use `fprintf` with suitable format specifications to tabulate j, ϕ_j, and $|\phi - \phi_j|$ with appropriate headers. Below are first few lines of an example output.

```
j            phi_j     abs. err.
1   1.414213562373   2.0382e-01
2   1.553773974030   6.4260e-02
3   1.598053182479   1.9981e-02
4   1.611847754125   6.1862e-03
.....................................
```

In calculating the errors, use MATLAB's evaluation of

```
phi = (1 + sqrt(5))/2;
```

as the exact value of the golden ratio. Be sure to print out 12 digits of ϕ_j after the decimal point as shown above.

2 Birthday Problem [25 points]

This problem is adapted from LM 3.9–22, which contains a useful hint. It is also a continuation of a recent homework problem.

(a) Write a script which generates a group of n people randomly and determines if there are at least **three** people with the same birthday. This script should take n as an input. Do this without using a loop nor an if-statement. Then print out the content of your script using `type`.

(b) Write another script which runs the previous simulation multiple times and calculates an approximate probability of having at least **three** people with the same birthday. This script should take n and the number of simulations as inputs. Do this without using a loop nor an if-statement. Then print out the content of your script using `type`.

(c) Run the script from part (b) with $n = 30, 40, \ldots, 100$, each with 10 000 simulations.

3 Approximation of π [25 points]

Each of the following sequences converges to π:

$$a_n = \frac{6}{\sqrt{3}} \sum_{k=0}^{n} \frac{(-1)^k}{3^k(2k+1)},$$
$$b_n = 16 \sum_{k=0}^{n} \frac{(-1)^k}{5^{2k+1}(2k+1)} - 4 \sum_{k=0}^{n} \frac{(-1)^k}{239^{2k+1}(2k+1)}.$$

(a) Generate two row vectors `a` $= (a_0, a_1, a_2, \ldots, a_{30})$ and `b` $= (b_0, b_1, b_2, \ldots, b_{30})$, without using a loop.

(b) Using `a` and `b` from the previous part, plot a_n and b_n against n for $n = 0, \ldots, 30$ on a single graph. Circle the data points and connect them with lines. Give the plot a title, label axes, and create legends as shown below.

(c) Using `semilogy` instead of `plot`, plot $|a_n - \pi|$ and $|b_n - \pi|$ against n for $n = 0, \ldots, 30$ on a single log-linear graph. Circle the data points and connect them with lines. Give the plot a title, label axes, and create legends as shown below.

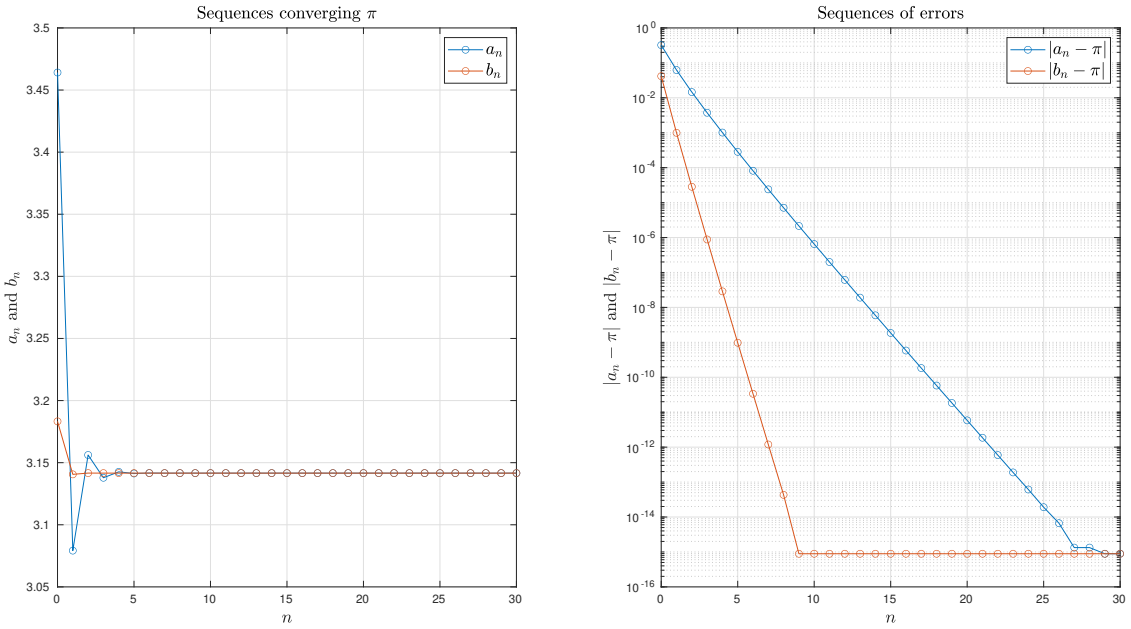

Figure 1: Example outputs for part (b) on the left and part (c) on the right.

Note. The code for each part must be put within a single code block. For part (a), no output needs to be printed, i.e., do not print out the elements of `a` nor `b`.

4 Array Operations [30 points]

Load the stock price data for Company A from January 2000 through May 2018 by

```
T = load('stocks.dat');
```

The data file contains opening price, daily high/low, closing price, etc. In this problem, we will be working with the *adjusted close price* which is found on the 5-th column of `T`. Note that the stock data are ordered from most recent to oldest.

For parts (a) through (d), answer each question with ONE MATLAB statement, without using a loop.

(a) Extract all adjusted close price into a single column vector with the oldest price appearing first and the most recent price appearing last. Name the column vector as `adjclose`.

Warning. For this part, put a semicolon at the end to suppress the output, `adjclose`, since it is very long.

From this point onward, *adjusted close price* will be referred to simply as *stock price*.

(b) Calculate the absolute gain by taking the difference between the oldest and the most recent stock prices.

(c) Calculate the relative gain *in percentage* by dividing the absolute gain by the initial stock price and multiplying it by 100.

(d) Using `adjclose`, construct another column vector `monthlyAvg` whose elements are 30-day average stock prices, that is,

$$\texttt{monthlyAvg} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_{154} \end{bmatrix},$$

where a_1 is the average stock prices of the first 30 days, a_2 is the average stock prices of the second 30 days, etc.

Warning. For this part, put a semicolon at the end to suppress the output, `monthlyAvg`, since it is very long.

For the remaining parts, you may use loops.

(e) Using `adjclose`, construct the column vector `monthlyMovingAvg` whose elements are 30-day *moving average* stock prices, that is,

$$\texttt{monthlyMovingAvg} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{4620} \end{bmatrix}, \quad \text{where} \quad b_j = \begin{cases} \dfrac{1}{j}\displaystyle\sum_{k=1}^{j} p_k, & \text{if } 1 \le j \le 30 \\ \dfrac{1}{30}\displaystyle\sum_{k=j-29}^{j} p_k, & \text{if } j > 30 \end{cases}$$

in which p_k is the kth element of `adjclose`. You may use a loop, but to earn the full mark for this part, do it without using an if-statement. Correct answers with if-statements will still earn partial credits.

(f) Using `adjclose` and `monthlyMovingAvg`, plot the stock prices and their 30-day moving averages in a single graph. Then use

```
xlim([length(adjclose)-729, length(adjclose)])
```

to show only the last two years of the data. Create a title, label the graph, and create a legend as shown below.

Hint. You may use `1:length(adjclose)` as x-data, but it is not necessary.

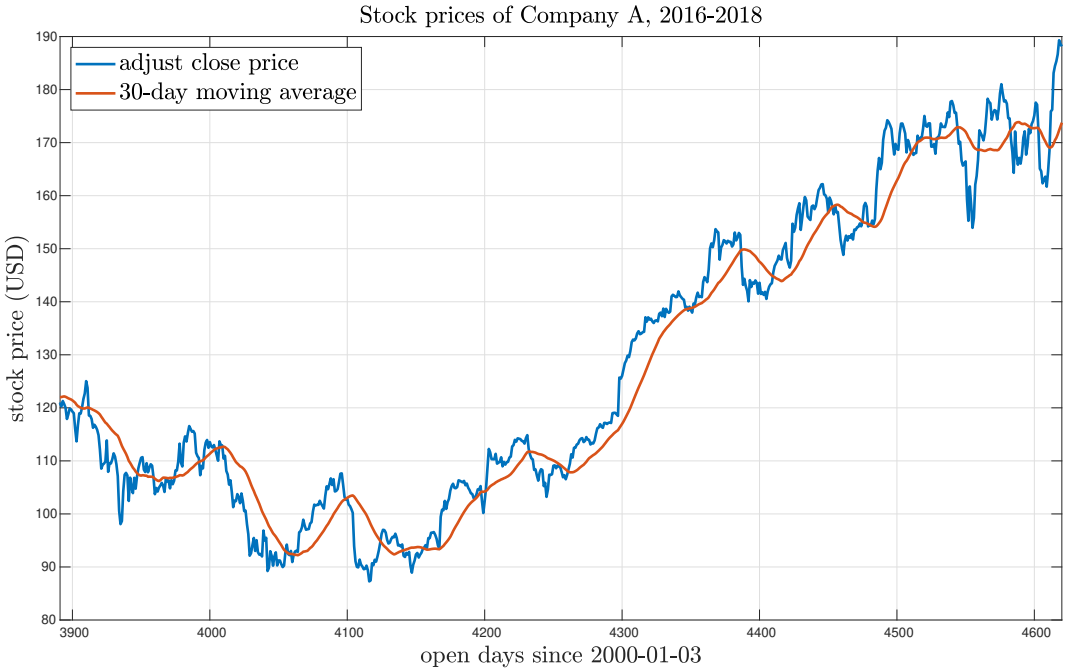

Figure 2: Example output for part (f)