Math 3607: Homework 8

Due: 10:00PM, Tuesday, November 2, 2021

TOTAL: 30 points

- Problems marked with ✏ are to be done by hand; those marked with 💻 are to be solved using a computer.

- **Important note.** Do not use *Symbolic Math Toolbox*. Any work done using `sym` or `syms` will receive NO credit.

- **Another important note.** When asked write a MATLAB function, write one at the end of your live script.

1. (Low-rank approximation using SVD; image compression) 💻 Load `hubble_gray.jpg`, which is a grayscale image taken by the Hubble Space Telescope, convert it to a matrix of floating point pixel intensities, and then display the image in MATLAB by

   ```
   A = imread('hubble_gray.jpg');
   imshow(A);
   ```

   Following the demo in Lecture 23 as a guide,

   (a) Plot the singular values $\sigma_1, \sigma_2, \ldots, \sigma_n$ of A on a log scale (using `semilogy`).

   (b) Plot the accumulation of singular values of A.

   (c) Compute the best approximations of A of rank $2, 20$, and 120 and display the corresponding images using `subplot`.
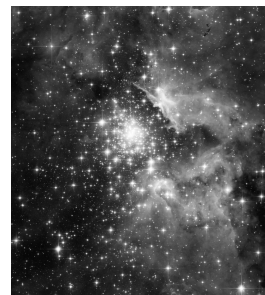
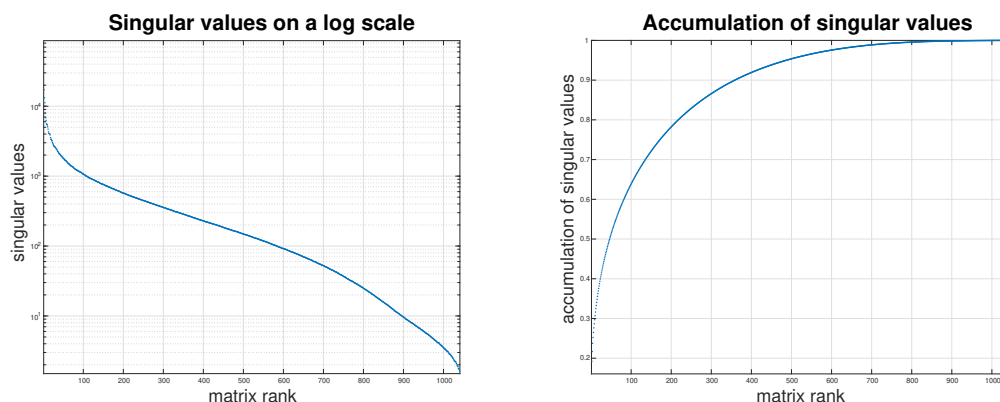

Figure 1: NGC 3603 (Hubble Space Telescope).

Figure 2: Example outputs for part (a) on the left and part (b) on the right.
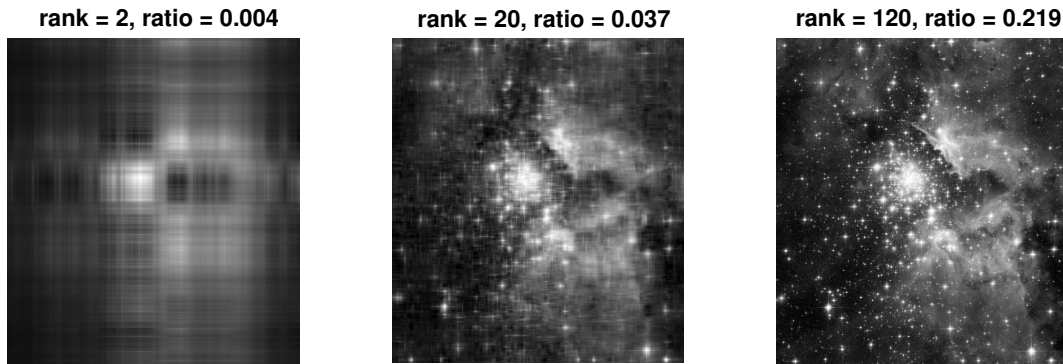
| rank = 2, ratio = 0.004 | rank = 20, ratio = 0.037 | rank = 120, ratio = 0.219 |

Figure 3: Example output for part (c)

2. (Annuity with `fzero`; **FNC** 4.1.4) 🖥 A basic type of investment is an annuity: One makes monthly deposits of size P for n months at a fixed annual interest rate r, and at maturity collects the amount

$$\frac{12P}{r}\left(\left(1 + \frac{r}{12}\right)^n - 1\right).$$

Say you want to create an annuity for a term of 300 months and final value of $1,000,000. Using `fzero`, make a table of the interst rate you will need to get for each of the different contribution values $P = 500, 550, \ldots, 1000$.

3. (Lambert's W function; **FNC** 4.1.6) 🖥 Lambert's W function is defined as the inverse of xe^x. That is, $y = W(x)$ if and only if $x = ye^y$. Write a function `y = lambertW(x)` that computes W using `fzero`. Make a plot of $W(x)$ for $0 \le x \le 4$.

4. (Fixed-point iteration; adapted from **FNC** 4.2.1 and 4.2.2.) In each case below,

   - $g(x) = \frac{1}{2}\left(x + \frac{9}{x}\right)$, $r = 3$.
   - $g(x) = \pi + \frac{1}{4}\sin(x)$, $r = \pi$.
   - $g(x) = x + 1 - \tan(x/4)$, $r = \pi$.

   (a) ✏ Show that the given $g(x)$ has a fixed point at the given r and that fixed point iteration can converge to it.

   (b) 🖥 Apply fixed point iteration in MATLAB and use a log-linear graph (using `semilogy`) of the error to verify linear convergence. Then use numerical values of the error to determine an approximate value for the rate σ.

5. (Convergence of Newton's method) ✏ Answer the following questions *by hand*, without using MATLAB.

   (a) Discuss what happens when Newton's method is applied to find a root of

   $$f(x) = \text{sign}(x)\sqrt{|x|},$$

   starting at $x_0 \ne 0$. [1]

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[1] $\text{sign}(x)$ is 1 if $x > 0$, -1 if $x < 0$, and 0 if $x = 0$.

(b) In the case of a multiple root, where $f(r) = f'(r) = 0$, the derivation of the quadratic error convergence is invalid. Redo the derivation to show that in this circumstance and with $f''(r) \neq 0$ the error converges only linearly.