

Lec 28: Problem Solving Session

Rootfinding

FZERO to Solve Complex Problem

- **FNC 4.1.5** (Kepler's Law)

Lambert W-Function

- **FNC 4.1.6**

More With Lambert W-Function

Question. Show that solutions of the equation $2^x = 5x$

$$r = -\frac{W(-\log(2)/5)}{\log 2}.$$

(Here, as usual in this class, $\log(\cdot) = \ln(\cdot)$ is the natural logarithmic function.)
Then numerically verify the result using `fzero`¹

¹Two real-valued solutions, $r_1 \approx 0.2355$ and $r_2 \approx 4.488$.

FPI: When Convergence Is Faster Than Expected

- **FNC 4.2.6**

FPI: Conditions for Convergence

- **FNC 4.2.7**

Stopping Criteria

- **FNC 4.3.8**

Linear Convergence of Newton's Method

Newton's Method for Multiple Roots

Assume that $f \in C^{m+1}[a, b]$ has a root r of multiplicity m . Then Newton's method is locally convergent to r , and the error ϵ_k at step k satisfies

$$\lim_{k \rightarrow \infty} \frac{\epsilon_{k+1}}{\epsilon_k} = \frac{m-1}{m} \quad (\text{linear convergence})$$

- See Problem 4 of HW07 (**FNC 4.3.7**)
- Remedy: Modify the iteration formula

$$x_{k+1} = x_k - \frac{mf(x_k)}{f'(x_k)}$$