Lec 34: Problem Solving Session (2/2)

Exercise Problems

Optimal h

Question. Suppose that a function $f(x)$ is numerically calculated by the following procedure.

```
function y = f(x)
    a = 1; b = cos(x);
    for i = 1:5
        c = b;
        b = sqrt(a.*b);
        a = (a + c)/2;
    end
    y = (pi/2)./a;
end
```

Compute $f'(\pi/4)$ as accurately as possible using a method of numerical differentiation.

Logarithmic Integral

The **logarithmic integral** is a special mathematical function defined by the equation

$$\operatorname{li}(x) = \int_2^x \frac{dt}{\ln t}.$$

Find $\operatorname{li}(200)$ by means of the composite trapezoid method.

Quadrature Exercise

Compute

$$\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$$

by using small and large values for the limits of integration and applying a numerical method. Then compute it by making the change of variable

$$x = -\ln t.$$

Quadrature Exercise

Find the area of the ellipse $y^2 + 4x^2 = 1$.

Airplane Velocity

The radar stations A and B, separated by the distance $a = 500$ m, track a plane C by recording the angles α and β at one-second intervals. Your goal, back at air traffic control, is to determine the speed of the plane.