Updated on January 20, 2021

Math 3607: Homework 2

Due: 11:59PM, Monday, January 25, 2021

1. A year is a *leap year* if it is a multiple of 4, except for years divisible by 100 but not by 400. In simpler terms, a non-century year is a leap year if it is divisible by 4; a century year is a leap year if it is divisible by 400.

   For example,

   - Last year (2020) was a leap year. (non-century year; divisible by 4)
   - 1900 was not a leap year. (century year; not divisible by 400)
   - 2000 was a leap year. (century year; divisible by 400)

   Write a script which determines whether a given year is a leap year or not.

2. Recall that Cartesian coordinates (x, y, z) in $\mathbb{R}^3$ are related to spherical coordinates (ρ, ϕ, θ) by

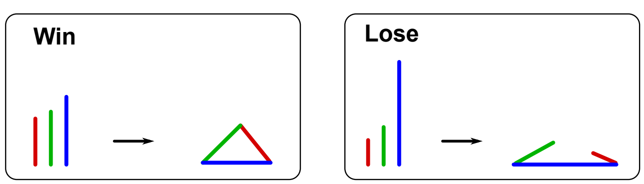
   $$x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta, \quad z = \rho \cos\phi,$$

   where $\phi \in [0, \pi]$ and $\theta \in [0, 2\pi)$. Write a script which takes Cartesian coordinates as inputs and converts them to spherical ones.

3. Do **LM** [1] 5.4–1(b,d,f,h,j).

   Use *Code Example* in Live Script to format answers as non-executable code. Follow the instruction found in **Note** below the problem.

4. (Exercise 2, Lecture 4) In the game of 3-Stick, you pick three sticks each having a random length between 0 and 1. You win if you can form a triangle using three sticks; otherwise, you lose.



   Write a script simulating one million games and estimating the probability of winning a game.

[1] Reference Keys:

- **LM**: *Learning MATLAB, Problem Solving, and Numerical Analysis Through Examples* (Overman)
- **NCM**: *Numerical Computing with MATLB* (Moler)
- **FNC**: *Fundamentals of Numerical Computation* (Driscoll and Braun)

5. Each of the following sequences converges to π:

$$a_n = \frac{6}{\sqrt{3}} \sum_{k=0}^{n} \frac{(-1)^k}{3^k(2k+1)},$$

$$b_n = 16 \sum_{k=0}^{n} \frac{(-1)^k}{5^{2k+1}(2k+1)} - 4 \sum_{k=0}^{n} \frac{(-1)^k}{239^{2k+1}(2k+1)}.$$

Write a single script that prints $a_0, \ldots, a_{n_a}$, where n_a is the smallest integer so that $|a_{n_a} - \pi| \leq 10^{-6}$ and prints $b_0, \ldots, b_{n_b}$, where n_b is the smallest integer so that $|b_{n_b} - \pi| \leq 10^{-6}$.