Updated on March 8, 2021

Math 3607: Homework 7

Due: 11:59PM, Monday, March 15, 2021

TOTAL: 20 points

1. (**FNC** 4.1.4) A basic type of investment is an annuity: One makes monthly deposits of size P for n months at a fixed annual interest rate r, and at maturity collects the amount

$$\frac{12P}{r}\left(\left(1+\frac{r}{12}\right)^n - 1\right).$$

Say you want to create an annuity for a term of 300 months and final value of \$1,000,000. Using `fzero`, make a table of the interst rate you will need to get for each of the different contribution values $P = 500, 550, \ldots, 1000$.

2. (**FNC** 4.1.6) Lambert's W function is defined as the inverse of xe^x. That is, $y = W(x)$ if and only if $x = ye^y$. Write a function `y = lambertW(x)` that computes W using `fzero`. Make a plot of $W(x)$ for $0 \leqslant x \leqslant 4$.

3. (Adapted from **FNC** 4.2.1 and 4.2.2.) In each case below,

   - $g(x) = \frac{1}{2}\left(x + \frac{9}{x}\right)$, $r = 3$.
   - $g(x) = \pi + \frac{1}{4}\sin(x)$, $r = \pi$.
   - $g(x) = x + 1 - \tan(x/4)$, $r = \pi$.

   (a) (*by hand*) Show that the given $g(x)$ has a fixed point at the given r and that fixed point iteration can converge to it.

   (b) (*computer*) Apply fixed point iteration in MATLAB and use a log-linear graph (using `semilogy`) of the error to verify linear convergence. Then use numerical values of the error to determine an approximate value for the rate σ (see Lecture 22).

4. Answer the following questions *by hand*, without using MATLAB.

   (a) Discuss what happens when Newton's method is applied to find a root of

   $$f(x) = \operatorname{sign}(x)\sqrt{|x|},$$

   starting at $x_0 \neq 0$. [1]

   (b) In the case of a multiple root, where $f(r) = f'(r) = 0$, the derivation of the quadratic error convergence is invalid. Redo the derivation to show that in this circumstance and with $f''(r) \neq 0$ the error converges only linearly.

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[1] sign(x) is 1 if $x > 0$, -1 if $x < 0$, and 0 if $x = 0$.

5. (**FNC** 4.5.5) Suppose one wants to find the points on the ellipsoid $x^2/25 + y^2/16 + z^2/9 = 1$ that are closest to and farthest from the point $(5, 4, 3)$. The method of Lagrange multipliers implies that any such point satisfies

$$\begin{aligned} x - 5 &= \frac{\lambda x}{25}, \\ y - 4 &= \frac{\lambda y}{16}, \\ z - 3 &= \frac{\lambda z}{9}, \\ 1 &= \frac{1}{25}x^2 + \frac{1}{16}y^2 + \frac{1}{9}z^2 \end{aligned}$$

for an unknown value of λ.

(a) (*by hand*) Write out this system in the form $\mathbf{f}(\mathbf{u}) = \mathbf{0}$.

(b) (*by hand*) Write out the Jacobian matrix of this system.

(c) (*computer*) Use `newtonsys` from class with different initial guesses to find the two roots of this system. Which is the closest point to $(5, 4, 3)$ and which is the farthest?

6. (Optional) Do **LM** 13.1–33 and 34. This is a long problem. The final outcome of the lengthy process is the colorful representation of so-called the *basin of attraction* as shown below.