Lec 10: Review of Topic 1

Tips

Loops

#5, HW02

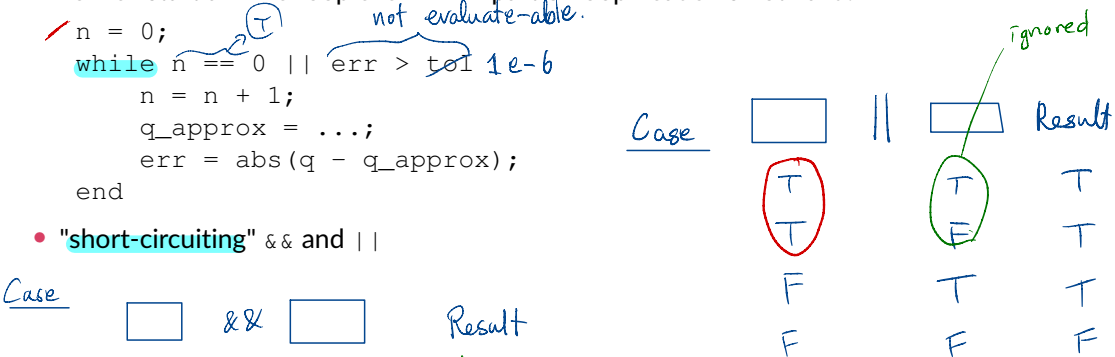
- To kick start a while-loop even when part of loop header is not valid:

```
n = 0;
while n == 0 || err > tol
    n = n + 1;
    q_approx = ...;
    err = abs(q - q_approx);
end
```

not evaluate-able.

1e-6

- "short-circuiting" `&&` and `||`



Forming Sums

Assume:

$\vec{a} = (a_1, a_2, \cdots, a_n)$

$\vec{b} = (b_1, b_2, \cdots, b_n)$

} row vectors stored in MATLAB

To calculate $\sum_{j=1}^{n} a_j b_j$: $= s$

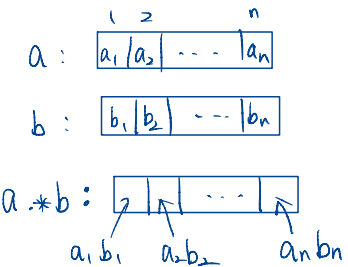
- using a loop

```
s = 0;
for j = 1:length(a)
    s = a(j) * b(j) + s;
end
```

(traditional)

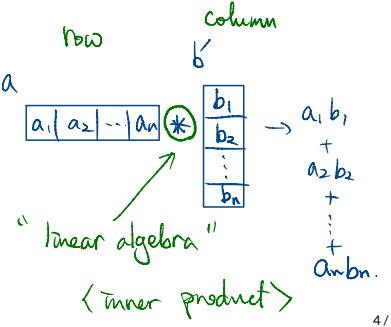
- using `sum`

```
s = sum( a .* b )
```



- **inner product**

```
s = a * b'
```



Sequence of Partial Sums

→ #5, HW02 (convergence to π)

To study the convergence of an infinite series $S = \sum_{j=0}^{\infty} a_j$, form the sequence of partial sums $\{s_n\}$ where

$$s_n = \sum_{j=0}^{n} a_j = a_0 + a_1 + \cdots + a_n.$$

$s_0 = a_0$

$s_1 = a_0 + a_1$

$s_2 = a_0 + a_1 + a_2$

⋮

Assume n is fixed (& is stored).

Assume $(a_0, a_1, a_2, \cdots, a_n)$ stored.

cumulative sum.

- using a loop

```
for k = 0 : n
    S_k = 0
    for j = 0 : k
        S_k = S_k + a(j);
    end
    disp(S_k)
end
```

s_0
s_1
s_2
⋮
s_n

- using `cumsum`

```
S = cumsum(a)
```

a : | a_0 | a_1 | - - - | a_n | (1, 2, n+1)

cumsum(a) : | s_0 | s_1 | s_2 | . . . | s_n |

a_0 $\quad a_0+a_1$ $\quad a_0+a_1+a_2$ $\quad \sum_{j=0}^{n} a_j$

Simple Examples

Biased Coin

Question

Simulate the tossing of a biased coin with

$$P(\mathsf{T}) = p, \quad P(\mathsf{H}) = 1 - p.$$

$0 \le p \le 1$

$= q$

Randomly generate

$p = P(T) = 3P(H)$

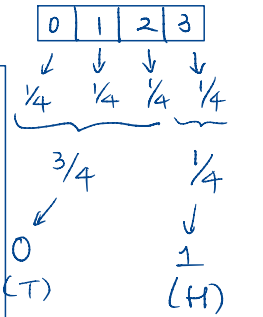
$\Rightarrow P(H) = \frac{1}{3}p$

$\therefore P(T) + P(H) = p + \frac{1}{3}p = \frac{4}{3}p = 1$

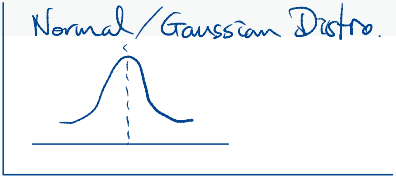
$\Rightarrow p = \frac{3}{4}, \; q = \frac{1}{4}$

$p = 3/4, \; q = 1/4$

```
toss = randi([0 3]);
if toss < 3
    toss = 0;
else
    toss = 1;
end
```



Biased Coin – Notes



Ideas.

- random number generators
- traditional tools: loops and conditional statements
- the *powerful* `find` function
- one-liner using `ceil` or `floor`

Explore.

- How would you handle similar situations with multiple states with non-uniform probability profile, *e.g.*, a biased dice?

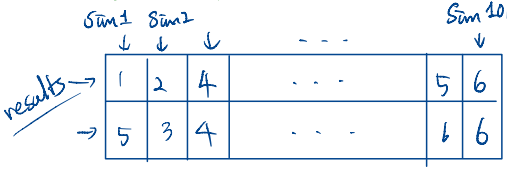
Dice Rolls

Birthday Problem

Question

Write a script simulating $n = 10,000$ throws of two 6-sided fair dice. What is the probability of obtaining two same numbers? Provide both analytical and numerical answers.

- randi ([1 6] , 2 , 1e4)
  - [1 6]: 6-sided
  - 2: two dice
  - 1e4: # of throws



- diff : difference along columns
- find : detect when the diff = 0

Finding Factors

Question

Given a positive integer n, finds all factors. Do it using a single MATLAB statement.

- If $n = 12$,

$f(12) = 6$ [1, 2, 3, 4, 6, 12]

- p is a factor of n if $n = pk$, $k \in \mathbb{N}$.

- "mod" is going to be useful.
  ("rem")
- "find" function

Ans.

```
find( mod(n, 1:n) == 0 )
```

Finding Factors – Notes

a factor of 4

mod(4, 2) : remainder of 4 ÷ 2 (0)

mod(4, 3) : " 4 ÷ 3 (1)

not a factor of 4

Ideas.

- the `mod` function: detecting a factor
- the `find` function: do it in one scoop

w/o using a loop.

Explore.

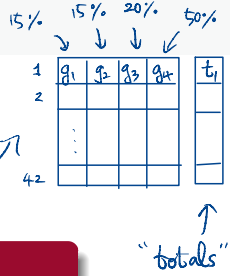
- The built-in function `factor` finds all prime factors. Use it to write a prime factorization of an integer.

Data Manipulation

Download `grades.dat` into your current directory and load it using

```
>> grades = load('grades.dat');
```

To read about how the data are organized, use `type grades.dat`.



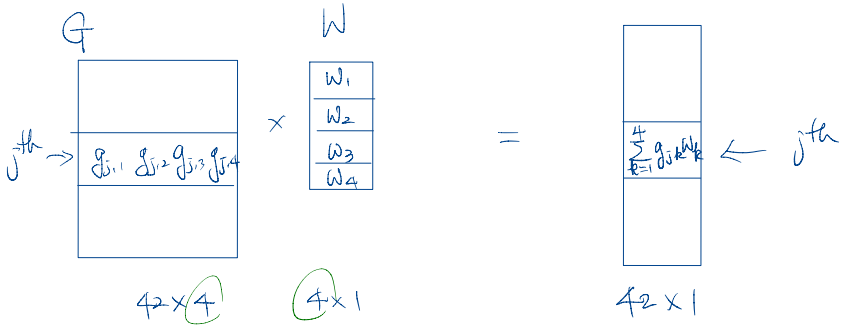
Question

1. Determine the number of students.
2. Compute the total grade according to the weights specified in the header. Do this without using a loop.
3. The letter grades are determined by
   - A: $[90, 100]$
   - B: $[80, 90)$
   - C: $[70, 80)$
   - D: $[60, 70)$
   - E: $[0, 60)$

   Find the number of students earning each of the letter grades.

$t_1 = 0.15\, g_1 + 0.15\, g_2 + 0.2\, g_3 + 0.5\, g_4$

"weighted average"


G
W
j^{th}
$g_{j,1}$ $g_{j,2}$ $g_{j,3}$ $g_{j,4}$
w_1
w_2
w_3
w_4
$\sum_{k=1}^{4} g_{j,k} w_k$
j^{th}
42 × 4
4 × 1
42 × 1

Spiral Triangle: Tying Up Loose Ends

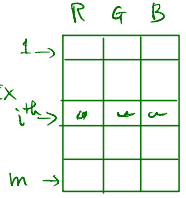
Recall: Entire Code

jet, parula, winter, ···, hot, pink, ··· (default)

```
1  m = 21; d_angle = 4.5;
2  th = linspace(0, 360, 4) + 90;
3  V = [cosd(th);
4       sind(th)];
5  C = colormap(hsv(m));
6  s = sind(150 - abs(d_angle))/sind(30);
7  R = [cosd(d_angle) -sind(d_angle);
8       sind(d_angle) cosd(d_angle)];
9  hold off
10 for i = 1:m
11     if i > 1
12         V = s*R*V;
13     end
14     plot(V(1,:), V(2,:), 'Color', C(i,:))
15     hold on
16 end
17 set(gcf, 'Color', 'w')
18 axis equal, axis off
```

to take into account both clockwise & counterclockwise rotation.

Color matrix



background.

Understanding Line 6

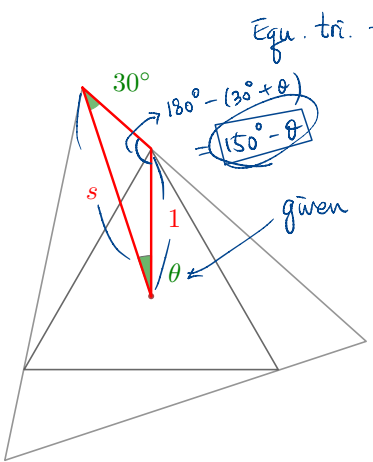
To create the desired spiraling effect, the scaling factor must be calculated carefully.

- Useful: Law of Sine

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

- Compute the scaling factor s:

$$\frac{\sin 30^\circ}{1} = \frac{\sin(150^\circ - \theta)}{s}$$

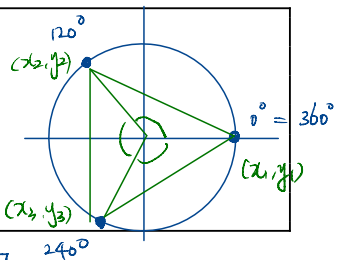


Understanding Line 12

0, 120, 240, 360

```
th = linspace(0, 360, 4) + 90;
V = [cosd(th);
     sind(th)];
s = sind(150 - abs(d_angle))/sind(30);
R = [cosd(d_angle) -sind(d_angle);
     sind(d_angle) cosd(d_angle)];
V = s*R*V;      % <----
```

rot. mat.



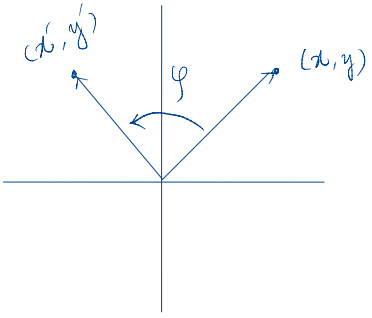
$$V = \begin{bmatrix} \cos(0^\circ) & \cos(120^\circ) & \cos(240^\circ) & \cos(360^\circ) \\ \sin(0^\circ) & \sin(120^\circ) & \sin(240^\circ) & \sin(360^\circ) \end{bmatrix} \in \mathbb{R}^{2\times 4}$$

the same

$$\begin{cases} x_1 = \cos(0^\circ) \\ y_1 = \sin(0^\circ) \end{cases}$$

⋮

2-D rotation



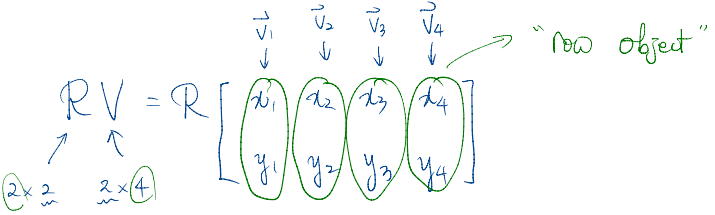
$$R_\varphi = \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix}$$

(φ : angle of rotation)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = R_\varphi \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} x\cos\varphi - y\sin\varphi \\ x\sin\varphi + y\cos\varphi \end{bmatrix}$$



$$= \begin{bmatrix} R\vec{v}_1 & R\vec{v}_2 & R\vec{v}_3 & R\vec{v}_4 \end{bmatrix}$$

2×1 2×1 2×1 2×1

2×4

rotated coor. of 1st vertex 2nd 3rd 4th

Understanding Line 5 (More on Coloring)

- Using RGB colors in plots
- `colormap`

Simple

```
plot(x, y, 'r')
```

g
b
m
c
w
k
y

Alternate (RGB) ed veer ive

	R	G	B
Red :	1	0	0
Green :	0	1	0
Blue :	0	0	1.
	⋮		

```
plot(x, y, 'Color', [0.6, 0.6, 0.6])
```