Updated on February 9, 2021

Math 3607: Homework 4

Due: 11:59PM, Monday, February 15, 2021

1. (Handling large numbers and scientific notation; adapted from **LM** 5.6) A product of terms can grow or decay much faster than a sum of terms, leading to an *overflow* or an *uderflow* in a floating-point architecture. This difficulty can usually be avoided by replacing

$$P_n = \prod_{i=1}^{n} a_i \quad \text{by} \quad \log|P_n| = \sum_{i=1}^{n} \log|a_i| \tag{$\spadesuit$}$$

as long as none of the terms are 0; if one or more terms are 0 the product is immediate. The result is then $P_n = f \times 10^m$ in (base-10) scientific notation where $|f| \in [1, 10)$ and f can be positive or negative, and where m is an integer.

Write a MATLAB function which calculates the product of all the elements of an input vector `a` by using ($\spadesuit$).

- The name of the function should be `logprod.m` and must output f and m.
- $f = 0$ if one of the elements of `a` is 0.
- The code must check each element to determine if it is positive, negative, or zero, and also keep track of the overall sign of the product.
- If a zero element is found, the function must exit immediately with $f = m = 0$.

2. (Tiling with spiral polygons; adapted from **LM** 6.8–34,35) Modify the function `spiralgon.m` you wrote for Exam 1 so that it now takes `n`, `m`, `d_angle`, `d_rot`, and `shift`. The additional input argument `shift` is a two-vector and it shifts the center of all the polygons to the point (`shift(1)`, `shift(2)`). Consequently, `plot` must now be changed to

```
plot(V(1,:)+shift(1), V(2,:)+shift(2), 'Color', C(i,:))
```

In addition, comment out the " `hold off` " statement before the `for` loop.

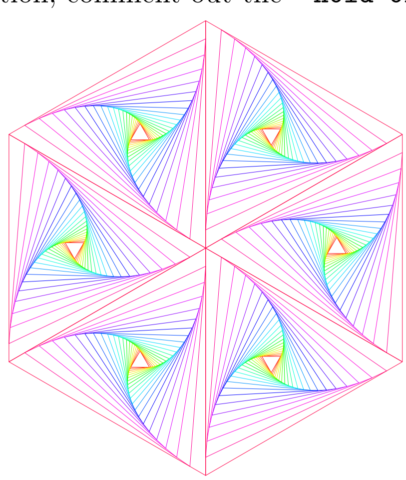

Figure 1: Tiling with spiral triangles.

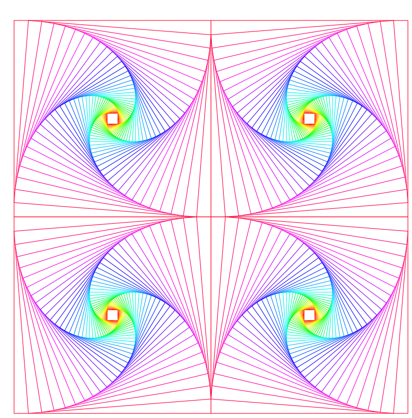

Figure 2: Tiling with spiral squares.

(a) Generate Figure 1.

- Run `spiralgon`, letting `n=3`, `m=21`, `d_angle=4.5`, `d_rot=90`, and `shift=[0 0]'` and save `V`. Its first column is the location of the leftmost vertex. Follow the call with " `hold off` " so that the image disappears. It was only executed to return `V`.
- Modify `shift` so that the next time you run this function, the leftmost vertex will be at the origin. Run `spiralgon` again and you have 1/6th of your figure.
- Now use a `for` loop and plot the next five spiral triangles. You have to add 60° to `d_rot` for each new spiral triangle and also shift `shift` by 60°.

(b) Generate Figure 2 by patching four spiral squares as in the previous part. Each of the four spiral squares is generated with `m=41` squares. Systematically determine other input arguments so as to reproduce the shape. (*Hint.* Pay attention to the direction of each spiral. Study the parameters provided in Problem 1(b) of Exam 1. They were carefully chosen to ensure that all shapes are nicely configured. Or read the instructions found in **LM** 6.8–34, 35.)

(c) Generate Figure 3. Each of the spiral hexagons is generated with `m=51`. Determine other input arguments so as to reproduce the shape.

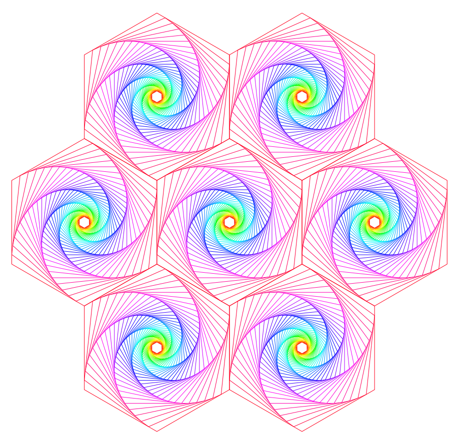

Figure 3: Tiling with spiral hexagons.

3. Do **LM** 9.3–3(a).

4. Do **LM** 9.3–10.

5. (Inverting hyperbolic cosine; **FNC** 1.3.6) The function

$$x = \cosh(t) = \frac{e^t + e^{-t}}{2}$$

can be inverted to yield a formula for acosh(x):

$$t = \log\left(x - \sqrt{x^2 - 1}\right) \qquad (\star)$$

where $\log(\cdot)$ denotes the natural logarithmic function $\ln(\cdot)$. In MATLAB, let `t=-4:-4:-16` and `x=cosh(t)`.

(a) Find the condition number of the problem $f(x) = \operatorname{acosh}(x)$. (You may use Equation ($\star$), or look up a formula for f' in a calculus book.) Evaluate κ_f at the entries of `x` in MATLAB.

(b) Use Equation ($\star$) on `x` to approximate `t`. Record the accuracy of the answers (by displaying absolute and/or relative errors), and explain. (Warning: Use `format long` to get enough digits or use `fprintf` with a suitable format.)

(c) An alternate formula for $\operatorname{acosh}(x)$ is

$$t = -2\log\left(\sqrt{\frac{x+1}{2}} + \sqrt{\frac{x-1}{2}}\right). \tag{$\dagger$}$$

Apply Equation ($\dagger$) to `x` and record the accuracy as in part (b). Comment on your observation.

(d) Based on your experiments, which of the formulas ($\star$) and ($\dagger$) is unstable? What is the problem with that formula?