Catalog of Definitions

Section 4: First Examples of Mathematical Proofs.

Definition (p. 40)**.** To say that x *is an even number* means that there exists an integer k such that $x = 2k$.

Definition (p. 40)**.** To say that x *is an odd number* means that there exists an integer k such that $x = 2k+1$.

Definition (p. 43)**.** To say that x *is a rational number* means that there exist integers m and n such that $n \neq 0$ and $x = m/n$.

Definition (p. 44)**.** To say that x *is an irrational number* means that x is a real number and x is not a rational number.

Definition (p. 45)**.** Let d and x be integers. To say that d *divides* x means that there exists an integer k such that $x = kd$.

Definition (p. 47)**.** To say that x *is a prime number* means that $x \in \mathbb{N}$ and $x \neq 1$ and for each $a \in \mathbb{N}$, for each $b \in \mathbb{N}$, if $x = ab$, then $a = 1$ or $b = 1$.

Definition (p. 51)**.** Let a, b, and m be integers. To say that a *is congruent to* b *modulo* m (written $a \equiv b \mod m$) means that m divides $b - a$.

Section 10: Sets.

Definition (p. 106)**.** The *empty set* is the set that has no elements, usually denoted by $\varnothing$.

Definition (p. 106)**.** Let A and B be sets.

- To say that A *is a subset of* B (denoted $A \subseteq B$) means that for each x, if $x \in A$, then $x \in B$.
- To say that A *is a proper subset of* B (denoted $A \subseteq B$) means that $A \subseteq B$ and $A \neq B$.

Definition (p. 108)**.** Let A and B be sets.

- The *union of* A *and* B (denoted $A \cup B$) is the set of all things that belong to at least one of the sets A and B; in other words,
$$A \cup B = \{x : x \in A \text{ or } x \in B\}.$$

- The *intersection of A and B* (denoted $A \cap B$) is the set of all things that belong to both of the sets A and B; in other words,
$$A \cap B = \{x : x \in A \text{ and } x \in B\}.$$
- The *relative complement of B in A* (denoted $A \setminus B$) is the set of all things that belong to A but not to B; in other words,
$$A \setminus B = \{x : x \in A \text{ and } x \notin B\}.$$

Definition (p. 111)**.** To say that two sets A and B are *disjoint* means that $A \cap B = \varnothing$.

Definition (p. 115)**.** Let $\mathcal{A}$ be a set of sets. Then *the union of $\mathcal{A}$* (denoted $\bigcup \mathcal{A}$) is the set of all things that belong to at least one of the sets in $\mathcal{A}$; in other words,

$$\bigcup \mathcal{A} = \{x : x \in A \text{ for some } A \in \mathcal{A}\}.$$

Definition (p. 115)**.** Let $\mathcal{A}$ be a nonempty set of sets. Then *the intersection of $\mathcal{A}$* (denoted $\bigcap \mathcal{A}$) is the set of all things that belong to all of the sets in $\mathcal{A}$; in other words,

$$\bigcap \mathcal{A} = \{x : x \in A \text{ for each } A \in \mathcal{A}\}.$$

Definition (p. 116)**.** Let A be a set. The *power set of A* (denoted $\mathcal{P}(A)$) is the set of all subsets of A; in other words, $\mathcal{P}(A) = \{S : S \subseteq A\}$.

Definition (p. 117)**.** Let a and b be any objects. The *ordered pair* (a, b) is the set $\big\{\{a\}, \{a, b\}\big\}$.

Definition (p. 118)**.** Let A and B be sets. Then the *Cartesian product of A and B* (denoted $A \times B$) is the set of all ordered pairs (x, y) such that $x \in A$ and $y \in B$; in other words,

$$A \times B = \{(x, y) : x \in A \text{ and } y \in B.$$

Section 11: Functions.

Definition (p. 121)**.** Let A and B be sets.

- To say that f *is a function on A* means that f is a function and $\mathrm{Dom}(f) = A$.
- To say that f *is a function from A to B* (denoted $f : A \to B$) means that f is a function, $\mathrm{Dom}(f) = A$, and for each x, if $x \in A$, then $f(x) \in B$.

Definition (p. 126)**.** Let f be a function. The *range of f* (denoted $\mathrm{Rng}(f)$) is the set of all values of f; in other words,

$$\begin{aligned}\mathrm{Rng}(f) &= \{f(x) : x \in \mathrm{Dom}(f)\}\\ &= \{y : y = f(x) \text{ for some } x \in \mathrm{Dom}(f)\}.\end{aligned}$$

Definition (p. 127)**.** Let f and g be functions. Then *the composition of* g *with* f is the function, denoted $g \circ f$, that is defined by

$$(g \circ f)(x) = g(f(x))$$

for all $x \in \mathrm{Dom}(f)$ such that $f(x) \in \mathrm{Dom}(g)$.

Definition (p. 128)**.** Let A and B be sets. To say that f *is a surjection from* A *to* B means that f is a function from A to B and for each $y \in B$, there exists $x \in A$ such that $f(x) = y$.

Definition (p. 128)**.** To say that f *is an injection* means that f is a function and for all $x_1, x_2 \in \mathrm{Dom}(f)$, if $f(x_1) = f(x_2)$, then $x_1 = x_2$.

Definition (p. 129)**.** Let f be an injection. Then for each $y \in \mathrm{Rng}(f)$, we shall write $f^{-1}(y)$ for the unique $x \in \mathrm{Dom}(f)$ such that $f(x) = y$. This defines a function f^{-1} from $\mathrm{Rng}(f)$ to $\mathrm{Dom}(f)$. The function f^{-1} is called the *inverse of the function* f.

Definition (p. 130)**.** Let A and B be sets. To say that f *is a bijection from* A *to* B means that f is both a surjection from A to B and an injection.

Section 13: The Fundamental Principles of Counting.

Definition (p. 147)**.** Let A and B be sets. To say that A *is equinumerous to* B (denoted $A \approx B$) means that there exists a bijection from A to B.

Definition (p. 148)**.** Let A be a set and let $n \in \boldsymbol{\omega}$. To say that A *has* n *elements* means that A is equinumerous to $\{1, \ldots, n\}$.

Definition (p. 148)**.** Let A be a set.

- To say that A *is finite* means that there exists $n \in \boldsymbol{\omega}$ such that A has n elements.
- To say that A *is infinite* means that A is not finite.

Section 15: Infinite Sets.

Definition (p. 167)**.** Let A be a set.

- To say that A *is denumerable* means that A is equinumerous to $\mathbb{N}$.
- To say that A *is countable* means that A is finite or denumerable.
- To say that A *is uncountable* means that A is not countable.