# Subsets

# Algebra of Set Operations

# Not an Element

**Proposition 1**

*Let $A$ and $B$ be sets and let $x$ be any object. Then:*

1. $x \notin A \cup B$ iff $x \notin A$ and $x \notin B$.
2. $x \notin A \cap B$ iff $x \notin A$ or $x \notin B$.
3. $x \notin A \backslash B$ iff $x \notin A$ or $x \in B$.

# De Morgan's Laws for Sets

**Theorem 1 (De Morgan's Laws for Sets)**

*Let $S$, $A$, and $B$ be sets. Then:*

1. $S \backslash (A \cup B) = (S \backslash A) \cap (S \backslash B)$.
2. $S \backslash (A \cap B) = (S \backslash A) \cup (S \backslash B)$.

# Distributive Laws for Unions and Intersections

**Theorem 2 (Distributive Laws for Unions and Intersections)**

*Let $S$, $A$, and $B$ be sets. Then:*

1. $S \cap (A \cup B) = (S \cap A) \cup (S \cap B)$.
2. $S \cup (A \cap B) = (S \cup A) \cap (S \cup B)$.

# Associative Laws for Unions and Intersections

**Proposition 2 (Associative Laws for Unions and Intersections)**

*Let $A$, $B$, and $C$ be sets. Then:*

1. $(A \cup B) \cup C = A \cup (B \cup C)$
2. $(A \cap B) \cap C = A \cap (B \cap C)$

# Commutative Laws for Unions and Intersections

**Proposition 3 (Commutative Laws for Unions and Intersections)**

*Let $A$ and $B$ be sets. Then:*

1. $A \cup B = B \cup A$
2. $A \cap B = B \cap A$