

Logical Connectives (I)

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Introduction

Sentences: Building Blocks in Logic

In logic, one seeks to determine which sentences are true and which are false.

Examples:

- Seoul is the capital of South Korea.
- $2 + 5 = 7$.
- $5 < 9$.
- 2022 is an even number.
- Cleveland is the capital of Illinois.
- $7 + 8 = 13$.
- $9 < 5$.
- 2021 is an even number.

Terminology. (*Truth value* of a sentence)

- When a sentence is true, its truth value is “true”.
- When a sentence is false, its truth value is “false”.

Logical Connectives

Simple sentences are put together using *logical connectives* such as

“not”, “and”, “or”, “implies”, and “if and only if”

to build compound sentences.

Propositional calculus (or *sentential calculus*) studies the truth values of compound sentences in terms of the truth values of their constituent sentences.

Convention. Denote sentences by letters such as P , Q , R , and so on, which are often referred to as *propositional variables*.

Symbols for Logical Connectives

| Logical Connectives | Symbols | Big Words |
|---------------------|-------------------|---------------|
| "not" | \neg | negation |
| "and" | \wedge | conjunction |
| "or" | \vee | disjunction |
| "implies" | \Rightarrow | conditional |
| "if and only if" | \Leftrightarrow | biconditional |

Negation, Conjunction, and Disjunction

Negation

Negation

Given a sentence P , the sentence $\neg P$ is called *the negation of P* .

- If P is a true sentence, then the sentence $\neg P$ is considered to be false.
- If P is a false sentence, then the sentence $\neg P$ is considered to be true.

| P | $\neg P$ |
|-----|----------|
| T | F |
| F | T |

Terminology. A sentence Q is said to be a *negative sentence* when Q is the negation of some other sentence.

Side: Logical Equivalence

If two sentences A and B always have the same truth values, we say that

A is logically equivalent to B .

and write

$$A \equiv B.$$

Example. Note that

- If P is true, then $\neg P$ is false, so $\neg\neg P$ is true.
- If P is false, then $\neg P$ is true, so $\neg\neg P$ is false.

Thus, $P \equiv \neg\neg P$.

Question. If you want to show two sentences are logically equivalent, what should you do?

Conjunction

Conjunction

Given sentences P and Q , the sentence $P \wedge Q$ is called *the conjunction of P and Q* .

- $P \wedge Q$ is considered to be true just when both of P and Q are true.
- If at least one of them is false, then $P \wedge Q$ is considered to be false.

| P | Q | $P \wedge Q$ |
|-----|-----|--------------|
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

Terminology. A sentence R is said to be a *conjunctive sentence* when R is of the form $P \wedge Q$, where P and Q are some other sentences. In this case, P and Q are called the *conjunctands* in R .

Conjunction (cont')

Some properties of \wedge .

- \wedge is *commutative*, that is, $Q \wedge P$ is logically equivalent to $P \wedge Q$.

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- \wedge is *associative*, that is, $P \wedge (Q \wedge R)$ is logically equivalent to $(P \wedge Q) \wedge R$.

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Disjunction

Disjunction

Given sentences P and Q , the sentence $P \vee Q$ is called *the disjunction of P and Q* .

- $P \vee Q$ is considered to be true just when at least one of P and Q is true.
- If both P and Q are false, then $P \vee Q$ is considered to be false.

| P | Q | $P \vee Q$ |
|-----|-----|------------|
| T | T | T |
| T | F | T |
| F | T | T |
| F | F | F |

Terminology. A sentence R is said to be a *disjunctive sentence* when R is of the form $P \vee Q$, where P and Q are some other sentences. In this case, P and Q are called the *disjunctands* in R .

Disjunction (cont')

Some properties of \vee .

- \vee is *commutative*, that is, $Q \vee P$ is logically equivalent to $P \vee Q$.

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- \vee is *associative*, that is, $P \vee (Q \vee R)$ is logically equivalent to $(P \vee Q) \vee R$.

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