Even and Odd Numbers

Even Numbers and Odd Numbers

Definitions

Definition 1 (Even and Odd Numbers)

- To say that x *is an even number* means that there exists an integer k such that $x = 2k$.

- To say that x *is an odd number* means that there exists an integer k such that $x = 2k + 1$.

Examples

Example 2 (Sum of Odd Numbers)

If x is odd and y is odd, then $x + y$ is even.

Examples (cont')

Example 3 (Product with Even Numbers)

Let x and y be integers. If x is even or y is even, then xy is even.

Fundamental Properties

Even/Odd Dichotomy (I)

Let x be an integer. Then:

1. x is even or x is odd.

2. If x is not even, then x is odd.

3. If x is not odd, then x is even.

Note. If P stands for "x is even" and Q stands for "x is odd", then three statements above are $P \vee Q$, $\neg P \Rightarrow Q$, and $\neg Q \Rightarrow P$, respectively. Note that the three sentences are logically equivalent.

Fundamental Properties (cont')

Even/Odd Dichotomy (II)

Let x be an integer. Then:

① x is not both even and odd.

② If x is even, then x is not odd.

③ If x is odd, then x is not even.

Note. If P stands for "x is even" and Q stands for "x is odd", then three statements above are $\neg(P \wedge Q)$, $P \Rightarrow \neg Q$, and $Q \Rightarrow \neg P$, respectively. Note that the three sentences are logically equivalent.

Examples

Example 4 (When Sum of Two Integers Is Odd)

Let x and y be integers. If $x + y$ is odd, then x is even or y is even.