# Existence of Prime Factorization

# Prime Factorization

# Recap

## Principle of Complete Mathematical Induction (PCMI)

Let $P(n)$ be any statement about $n$. Suppose we have proved that

$$P(1) \text{ is true} \tag{1}$$

and that

$$\text{for each } n \in \mathbb{N}, \text{ if } P(1), \ldots, P(n) \text{ are all true, then } P(n+1) \text{ is true.} \tag{2}$$

Then we may conclude that for each natural number $n$, $P(n)$ is true.

# Proof by Complete Induction (Template)

- Declaration:
- BASE CASE:
- INDUCTIVE STEP:
- Conclusion:

# Example: The Existence of Prime Factorization

**Theorem 1 (Existence of Prime Factorization)**

*Each natural number greater than or equal to 2 either is a product of prime numbers or is itself a prime number.*

- We used this result without proof back in Lecture 10; see Remark 4.44.
- It can now be proved using complete induction.
- It is convenient to start from 2.

# Before We Begin …

Recall the definition of a prime number.

- To say that $x$ is prime means that

- (S04E15) $x$ is not a prime number iff

# Proof of Theorem 1

# In Closing

- What would be a challenge had you attempted to prove using induction?