More Notes on Set Operations and Venn Diagrams

More Notes on Set Operations

Office Hours (This week only)

- TW 4:45 ~ 6:15

Relative Complement $A \setminus B = \{x : x \in A \text{ and } x \notin B\}$

If it is understood/agreed that all sets in a discussion are subsets of a fixed set T, one often uses the short-hand notation A^c (read as "A complement") in place of $T \setminus A$.

Example. Let A and B be subsets of a fixed set T. Then

- $(A^c)^c = A$
- $A \setminus B = A \cap B^c$
- De Morgan's laws (with S replaced by T) can be written succinctly as
  1. $(A \cup B)^c = A^c \cap B^c$
  2. $(A \cap B)^c = A^c \cup B^c$

$$\begin{cases} S \setminus (A \cup B) = (S \setminus A) \cap (S \setminus B) \\ S \setminus (A \cap B) = (S \setminus A) \cup (S \setminus B) \end{cases}$$

$T \setminus (A \cup B) = (A \cup B)^c$

Relative Complement (cont')

Recall: $X = Y$ means $(\forall x)(x \in X \text{ iff } x \in Y)$

Revisiting S10E15(a)

Let S, A, and B be sets. Then

$$S \setminus (A \setminus B) = (S \setminus A) \cup (S \cap B).$$

Previously (Do this way for HW this week.)

Proof: For each object x,

$x \in S \setminus (A \setminus B)$

iff $x \in S$ and $x \notin A \setminus B$

iff $x \in S$ and $\neg(x \in A \setminus B)$

iff $x \in S$ and $\neg(x \in A \text{ and } x \notin B) \ldots$

Now with the complement notation:

Proof Let T be a set containing $S, A,$ and B. Then

$$\begin{aligned} S \setminus (A \setminus B) &= S \cap (A \setminus B)^c \\ &= S \cap (A \cap B^c)^c \quad \text{] by 2nd bullet pt.} \\ &= S \cap (A^c \cup (B^c)^c) \quad \text{by DeMorgan's} \\ &= S \cap (A^c \cup B) \quad \text{] by 1st bullet pt.} \end{aligned}$$

$$= (S \cap A^c) \cup (S \cap B) \quad \text{by dist. law}$$

$$= (S \setminus A) \cup (S \cap B)$$

$\square$

Disjointness

i.e., A and B do not share any element in common.

Definition 1 (Disjointness)

- To say that two sets A and B are *disjoint* means that $A \cap B = \varnothing$.
- To say that several sets $A, B, C, \dots$ are *pairwise disjoint* means that each two of them are disjoint.
- To say that a set of sets $\mathcal{M}$ is *pairwise disjoint* means that each two distinct element of $\mathcal{M}$ are disjoint. $(\forall A \in \mathcal{M})(\forall B \in \mathcal{M})[A \neq B \Rightarrow A \cap B = \emptyset]$

E.g., To say that A, B, and C are pairwise disjoint means $A \cap B = \emptyset$, $A \cap C = \emptyset$, and $B \cap C = \emptyset$.

Example.

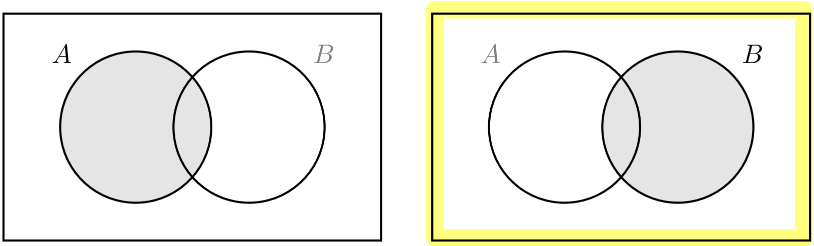
- The sets $A = \{2k : k \in \mathbb{Z}\}$ and $B = \{2k + 1 : k \in \mathbb{Z}\}$ are disjoint.
- The set $\mathcal{M} = \{\{1, 2, 3\}, \{4, 5, 6\}, \{3, 6, 9\}\}$ is not pairwise disjoint, because $\{1, 2, 3\} \cap \{3, 6, 9\} \neq \varnothing$.

Venn Diagrams

Venn Diagrams

Venn diagrams provide a graphical means to confirm set identities.

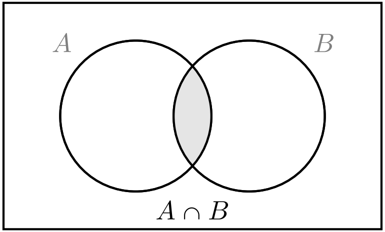
- The universe of discourse is represented by a rectangle;
- Subsets of the universe of discourse are represented by regions within the rectangle.

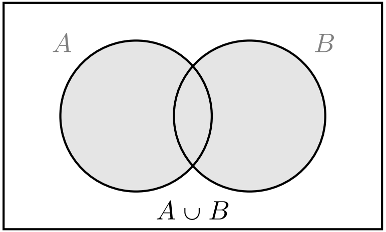


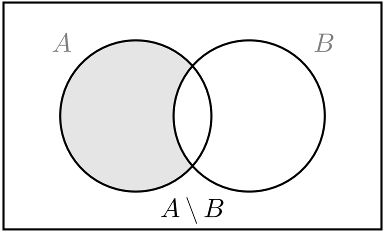
Venn Diagrams: Set Operations on Two Sets

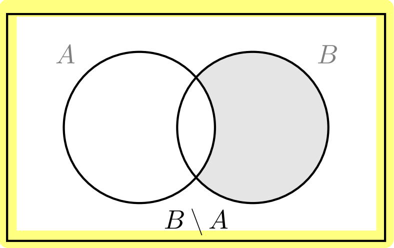
$A \cap B = \{x : x \in A \text{ \underline{and} } x \in B\}$

$A \cup B = \{x : x \in A \text{ \underline{or} } x \in B\}$







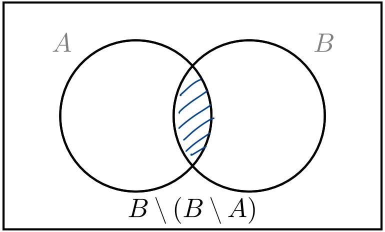


Question. In the diagram below, shade the region representing the set $B \setminus (B \setminus A)$. Make an observation.

S10 E15 (b):

$B \setminus (B \setminus A) = B \cap A$

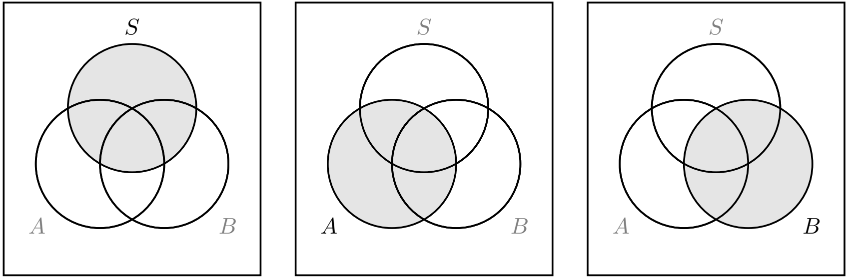
$= A \cap B.$



Obs. This region is the same as the one representing $A \cap B$.

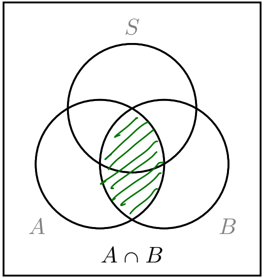
That is, this confirms that $B \setminus (B \setminus A) = A \cap B$ pictorially.

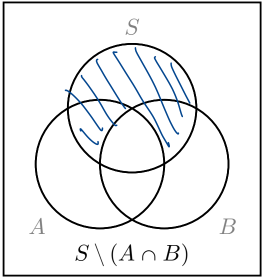
Venn Diagrams: Set Operations on Three Sets



Venn Diagrams: Set Operations on Three Sets (cont')

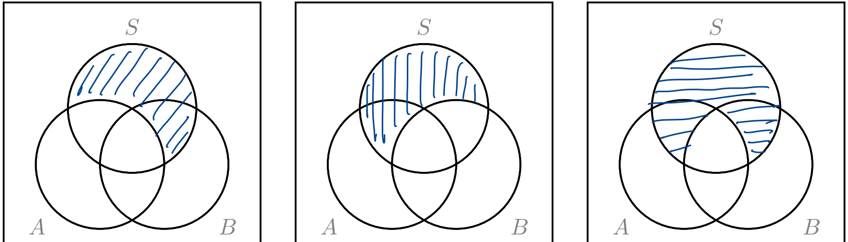
Question. In the diagrams below, shade the regions representing the sets $A \cap B$ and $S \setminus (A \cap B)$.





Venn Diagrams: Set Operations on Three Sets (cont')

Question. In the diagrams below, shade the regions representing the sets $S \setminus A$, $S \setminus B$, and $(S \setminus A) \cup (S \setminus B)$.

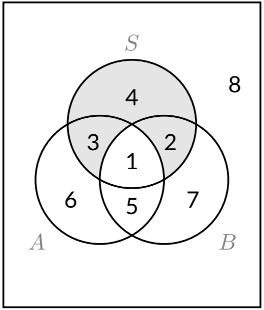


$S \setminus A$ $\quad$ $S \setminus B$ $\quad$ $(S \setminus A) \cup (S \setminus B)$

Observation? $S \setminus (A \cap B) = (S \setminus A) \cup (S \setminus B)$ (De Morgan's law)

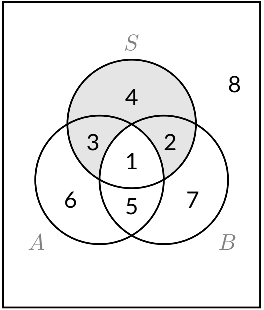
Venn Diagram and Truth Table

Understanding $S \setminus (A \cap B)$



	$x \in S$	$x \in A$	$x \in B$	$x \in A \wedge x \in B$	$x \in S \wedge \neg(x \in A \wedge x \in B)$
1.	T	T	T	T	F
2.	T	T	F	F	T
3.	T	F	T	F	T
4.	T	F	F	F	T
5.	F	T	T	T	F
6.	F	T	F	F	F
7.	F	F	T	F	F
8.	F	F	F	F	F

Understanding $(S \setminus A) \cup (S \setminus B)$



	$x \in S \wedge x \notin A$	$x \in S \wedge x \notin B$	$(x \in S \wedge x \notin A) \vee (x \in S \wedge x \notin B)$
1.	F	F	F
2.	T	F	T
3.	F	T	T
4.	T	T	T
5.	F	F	F
6.	F	F	F
7.	F	F	F
8.	F	F	F