# Rational and Irrational Numbers

# Contents

# Rational Numbers

# Definition

**Definition 1 (Rational Numbers)**

To say that $x$ *is a rational number* means that there exist integers $m$ and $n$ such that $n \neq 0$ and $x = m/n$.

### Example 2 (Sum of Rational Numbers)

Let $u$ and $v$ be rational numbers. Then $u + v$ is a rational number.

# Special Forms of Rational Numbers

A given rational number $x$ can be expressed in many different ways. For example,

$$\frac{7}{3} = \frac{-7}{-3} = \frac{14}{6} = \frac{-14}{-6} = \cdots = \frac{350}{150} = \cdots .$$

The fact that each rational number can be written in lowest terms such as $7/3$ can be proved later once we learn complete induction. For now, we can prove the following:

## Rational Number as An Integer Divided by A Natural Number

Let $x$ be a rational number. Then there exists an integer $a$ and a natural number $b$ such that $x = a/b$.

# Irrational Numbers

# Definition

**Definition 3 (Irrational Numbers)**

To say that $x$ *is an irrational number* means that $x$ is a real number and $x$ is not a rational number.

# Note

Remember that each irrational number is a real number!

$$\text{“}x \text{ is an irrational number.”} \neq \text{“}x \text{ is not a rational number.”}$$

Consider the following question.

---

**Question.** Determine whether each of the following is true or false. Explain your answers.

1. For each $x \in \mathbb{C}$, if $x$ is an irrational number, then $x$ is not a rational number.

2. For each $x \in \mathbb{C}$, if $x$ is not a rational number, then $x$ is an irrational number.

## Example 4 (Sum of Rational and Irrational Numbers)

Let $x$ be a rational number and let $y$ be an irrational number. Then $x + y$ is an irrational number.

# Examples (cont')

**Question.** Let $x$ and $y$ be real numbers. Determine whether each of the following is true or false. Explain.

1. If $xy$ is rational, then $x$ and $y$ are rational.

2. If $x + y$ is rational, then $x$ and $y$ are rational.

# Irrationality of $\sqrt{2}$

**Theorem 5**

1. *Let $x$ be a rational number. Then $x^2 \neq 2$.*
2. *$\sqrt{2}$ is irrational.*