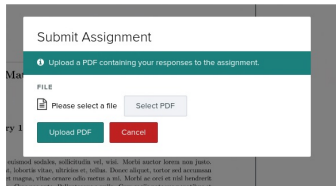
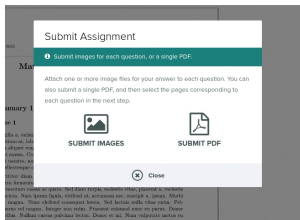


## Proof Techniques

- HW 1 due 11:59 PM.
- Office Hour : 4:30 PM - 6:00 PM (Zoom)
- Quiz 2 on Friday (noon - 11:59 PM)

# Uploading HW to Gradescope



HW01 | Assign Questions and Pages

SUBMITTED AT: JANUARY 15, 9:29 AM

Select questions and pages to indicate where your responses are located. Use **ABC** to deselect all items and **XYZ** to select multiple questions.

QUESTION OUTLINE	
TITLE	POINTS
1 From Wednesday (Section 2: 1, 2, 3, 5, 7)	0.0 pts
2 From Friday (Section 2: 8, 9, 10, 15, 17)	0.0 pts

Select a question or a page.

1 2 3 4

# Select pages for Wed. problems.



## HW01 | Assign Questions and Pages



SUBMITTED AT: JANUARY 19, 9:09 AM

Select questions and pages to indicate where your responses are located. Use **esc** to deselect all items and hold **shift** to select multiple questions.



### Question Outline

Select pages to assign to Question 1.

TITLE

POINTS

1 From Wednesday (Section 2: 1, 2, 3, 5, 7)

0.0 pts

P1 x P2 x

2 From Friday (Section 2: 8, 9, 10, 15, 17)

0.0 pts

**Math 330B: Homework 1**  
No Due Date

**Wednesday, January 12**

**Section 1, Exercise 1**

How did Spinoza, Hume, or Kant understand causality? ...

**Section 2, Exercise 2**

How exactly, Kantianism holds onto its being ...

**Section 3, Exercise 3**

Let us now take up Kant's argument ...

**Thursday, January 13**

**Section 1, Exercise 1**

How exactly, Kantianism holds onto its being ...

**Section 2, Exercise 2**

How exactly, Kantianism holds onto its being ...

**Section 3, Exercise 3**

Let us now take up Kant's argument ...

**Friday, January 14**

**Section 1, Exercise 1**

How exactly, Kantianism holds onto its being ...

**Section 2, Exercise 2**

How exactly, Kantianism holds onto its being ...

**Section 3, Exercise 3**

Let us now take up Kant's argument ...

**Saturday, January 15**

**Section 1, Exercise 1**

How exactly, Kantianism holds onto its being ...

**Section 2, Exercise 2**

How exactly, Kantianism holds onto its being ...

**Section 3, Exercise 3**

Let us now take up Kant's argument ...

Select pages for Fri. problems.



## HW01 | Assign Questions and Pages

SUBMITTED AT: JANUARY 19, 9:09 AM

Select questions and pages to indicate where your responses are located. Use **esc** to deselect all items and hold **shift** to select multiple questions.



### Question Outline

Select pages to assign to Question 2.

TITLE	POINTS
1 From Wednesday (Section 2: 1, 2, 3, 5, 7)	0.0 pts
<b>P1 × P2 ×</b>	
2 From Friday (Section 2: 8, 9, 10, 15, 17)	0.0 pts
<b>P3 × P4 ×</b>	

✕

⋮

⌕

↺

1

Q1 ×

Math 3345: Homework 1

The Five Kites

Wednesday, January 13

Section 2, Exercise 1

How do I begin? I should read the problem carefully and try to understand what is being asked. I should also think about what I know and what I need to find. I should also think about what I can do to solve the problem. I should also think about what I can do to check my answer.

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✕

⋮

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↺

2

Q1 ×

Math 3345: Homework 1

The Five Kites

Wednesday, January 13

Section 2, Exercise 1

How do I begin? I should read the problem carefully and try to understand what is being asked. I should also think about what I know and what I need to find. I should also think about what I can do to solve the problem. I should also think about what I can do to check my answer.

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✕

⋮

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3

Q2 ×

Friday, January 14

Section 3, Exercise 9

How do I begin? I should read the problem carefully and try to understand what is being asked. I should also think about what I know and what I need to find. I should also think about what I can do to solve the problem. I should also think about what I can do to check my answer.

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✕

⋮

⌕

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4

Q2 ×

Friday, January 14

Section 3, Exercise 10

How do I begin? I should read the problem carefully and try to understand what is being asked. I should also think about what I know and what I need to find. I should also think about what I can do to solve the problem. I should also think about what I can do to check my answer.

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# Contents

① Logic of Solving Equations

② Proof by Contradiction

③ Proof by Contraposition

# Logic of Solving Equations

# Solving Equations

Logically speaking, to say that  $x = a$  is a solution of the equation  $f(x) = 0$  is to state

$$f(x) = 0 \iff x = a$$

which usually can be seen by a chain of biconditionals.

For example, we see that  $x^2 = 5x - 6$  if and only if  $x = 2$  or  $x = 3$  by:

$$\begin{aligned} x^2 = 5x - 6 &\iff x^2 - 5x + 6 = 0 \\ &\iff (x - 2)(x - 3) = 0 \\ &\iff x - 2 = 0 \text{ or } x - 3 = 0 \\ &\iff x = 2 \text{ or } x = 3. \end{aligned}$$

One needs to be careful to confirm that all steps are true biconditional sentences.

# Examples

## Rational Equation

Solve the equation

$$\frac{x-2}{x^2+2x-8} = \frac{1}{8}.$$

**Erroneous solution.**

$$\Rightarrow x-2 = (1/8)(x^2+2x-8)$$

$$\Leftrightarrow 8x-16 = x^2+2x-8$$

$$\Leftrightarrow 0 = x^2 - 6x + 8 = \underbrace{(x-2)}_{\substack{|| \\ 0}} \underbrace{(x-4)}_{\substack{|| \\ 0}}$$

$$\Leftrightarrow x = 2, 4$$

When  $x=2$ ,

$$\begin{aligned} x^2 + 2x - 8 &= 2^2 + 2 \cdot 2 - 8 \\ &= 4 + 4 - 8 = 0. \end{aligned}$$

✓ Which step is not a true biconditional sentence?



## Examples (cont')

intermediate steps omitted

Correct solution.

$$\bullet \quad \frac{x-2}{x^2+2x-8} = \frac{1}{8} \Rightarrow x=2 \text{ or } \underline{x=4}$$

• Now if  $x=2$ , then  $x^2+2x-8 = 4+4-8=0$ ,

so  $\frac{x-2}{x^2+2x-8}$  is undefined. So  $x=2$  is not a solution.

• If  $x=4$ , then  $x^2+2x-8 = 16+8-8=16$  and  $x-2 = 4-2=2$ ,

so  $\frac{x-2}{x^2+2x-8} = \frac{2}{16} = \frac{1}{8}$ . So  $x=4$  is a solution.

# Examples

## Equation Involving Radicals

Solve the equation

$$x = -\sqrt{x+6}$$

An erroneous solution:

$$\begin{aligned} \Rightarrow x^2 &= x + 6 && \text{squaring both sides} \\ \Leftrightarrow x^2 - x - 6 &= 0 \\ \Leftrightarrow (x+2)(x-3) &= 0 \\ \Leftrightarrow x &= -2, 3 \end{aligned}$$

$$\begin{aligned} x^2 &= 4 \\ \Leftrightarrow x &= \pm\sqrt{4} = \pm 2 \end{aligned}$$

Is  $x = 3$  a solution of the original equation?

$$\text{No, because } -\sqrt{3+6} = -\sqrt{9} = -3 \neq 3.$$

## Examples (cont')

**Correct solution.**

If  $x = -\sqrt{x+6}$ , then  $x = -2$  or  $x = 3$ .

Now if  $x = -2$ , then  $x = -\sqrt{x+6}$ .

If  $x = 3$ , then  $x \neq -\sqrt{x+6}$ .

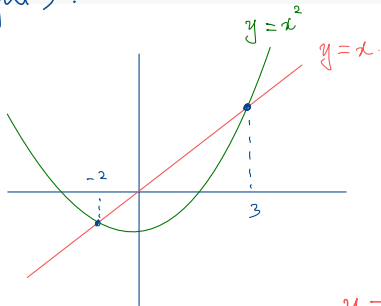
Therefore,  $x = -\sqrt{x+6}$  iff  $x = -2$ . ( $x = -2$  is the soln of eqn.)

# Practical approach (for insight).

One view

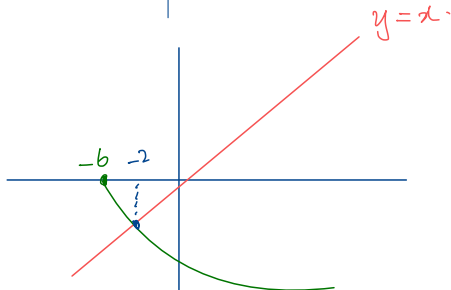
$$x^2 = x + b$$

$$x^2 - b = x$$



Orig.

$$x = -\sqrt{x+b}$$



# Proof by Contradiction

# Overview: Proof Techniques

The follow is the list of proof techniques discussed in Section 2 of the textbook:

☒ Conditional proof      Last frz.

☐ Proof by contradiction

☐ Proof by contraposition

• used to prove  $P \Rightarrow Q$ .

• Template

A1: Assume  $P$  is true.

| Work to show  $Q$  is true.

Discharging A1,  $P \Rightarrow Q$  is true under no assumption.

$P$	$Q$	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

# Contradictions

A *contradiction* is a sentence of the form  $Q \wedge \neg Q$ , which is false regardless of the truth value of  $Q$ .

Why?

$Q$	$\neg Q$	$Q \wedge \neg Q$
T	F	F
F	T	F

Uses in proofs.

# Proof by Contradiction

## Proof by Contradiction

To prove a sentence  $P$ , assume  $\neg P$  and deduce a contradiction. This approach is known as the method of *proof by contradiction*.

**Template.** To prove  $P$ :

- Begin with “Assume  $\neg P$  is true.”
- Deduce a contradiction.
- Conclude that  $P$  is true.

Why does it work?

$P$	$\neg P$	$\wedge$	$\neg P \Rightarrow \wedge$
T	F	F	T
F	T	F	F

*a contradiction* (with an arrow pointing to the  $\wedge$  column)



## Proof by Contradiction (cont')

### Example

Let  $n$  be an integer. Using the method of proof by contradiction, prove that

If  $n^2$  is an odd number, then  $n$  is an odd number.

A1: Assume  $n^2$  is an odd number.

We wish to show  $n$  is an odd number.

Assume (towards a contradiction)  $n$  is not an odd number.

Since  $n$  is an integer,  $n$  must be an even number.

So  $n^2$  is an even number, that is,  $n^2$  is not an odd number.

This leads to a contradiction, so we must reject the assumption that  $n$  is not an odd number. So  $n$  is an odd number.

Discharging A1, we see that if  $n^2$  is an odd number, then  $n$  is an odd number under no assumption.



# Proof of a Negative Sentence

The usual way to prove a negative sentence  $\neg P$  is to prove by contradiction, that is, assume  $P$  and deduce a contradiction.

**Why does it work?**

Proof by Contradiction on  $\neg P$ .

# Proof of a Negative Sentence (cont')

## Section 2, Exercise 23

Use the method of conditional proof to explain in words why

$$\underbrace{[(P \Rightarrow Q) \wedge \neg Q]}_{A_1} \Rightarrow \underbrace{\neg P}_{C_1}$$

is a tautology.

Suggestion: Conditional proof.

A1: Assume  $A_1$  is true.

NTS  $C_1$  is true.

$\vdots$  } proof of a negative sentence.

## Proof of a Negative Sentence (cont')

# Proof by Contraposition

# Overview: Proof Techniques

The follow is the list of proof techniques discussed in Section 2 of the textbook:

- ☒ Conditional proof
  - ☒ Proof by contradiction
  - ☐ Proof by contraposition
- 

To prove  $P \Rightarrow Q$ ,  
it suffices to assume  $P$  is true  
and show  $Q$  is true.

To prove  $P$ ,  
assume  $\neg P$  is true  
and deduce a contradiction ( $Q \wedge \neg Q$ ).

# Contrapositive

Given  $P \Rightarrow Q$ , the related conditional sentence  $\neg Q \Rightarrow \neg P$  is called the **contrapositive of  $P \Rightarrow Q$** . Note that  $P \Rightarrow Q$  is logically equivalent to  $\neg Q \Rightarrow \neg P$ . (Confirm this using a truth table.)

**Example.** Given the conditional sentence

A: If today is Sunday, then I do not have to go to work today.

- Converse of A:  $Q \Rightarrow P$

If I do not have to go to work today, then today is Sunday.

- Contrapositive of A:  $\neg Q \Rightarrow \neg P$

If I have to go to work today, then today is not Sunday.

No work  
Sunday



# Proof by Contraposition

## Proof by Contraposition

To prove  $P \Rightarrow Q$ , it suffices to prove  $\neg Q \Rightarrow \neg P$ .

Confirm using a truth table that

$$(P \Rightarrow Q) \equiv (\neg Q \Rightarrow \neg P).$$

## Proof by Contraposition (cont')

### Example (revisited)

Let  $n$  be an integer. Using the method of proof by contraposition, prove that

If  $n^2$  is an odd number, then  $n$  is an odd number.

*Solution.* The given sentence is logically equivalent to the sentence

If  $n$  is not an odd number, then  $n^2$  is not an odd number.

which we will prove. A C

A1: Assume that  $n$  is not an odd number.

(We wish to show that  $n^2$  is not an odd number.)

Since  $n$  is an integer but  $n$  is not an odd number,  $n$  is an even number.

Hence  $n^2$  is an even number, so  $n^2$  is not an odd number.

We have shown this under A1.

Discharging A1, we conclude that the conditional sentence (\*) is true under no assumptions. This completes the proof by contraposition that the original conditional sentence is true. □

(\*) Contrapositive  
of the given.

# Proof by Contradiction vs Proof by Contraposition

Let's examine the two proof techniques in proving  $P \Rightarrow Q$ .

**Proof by contradiction.**

Assume  $P$ . (NTS  $Q$  is true)

Assume  $\neg Q$ .

Show  $\neg P$ .

Contradiction,  $P \wedge \neg P$ !

So  $Q$  must be true.

Therefore,  $P \Rightarrow Q$ .

**Proof by contraposition.**

(show  $\neg Q \Rightarrow \neg P$ )

Assume  $\neg Q$ .

Show  $\neg P$ . (if this can be done w/o  $P$ .)

So  $\neg Q \Rightarrow \neg P$ .

Therefore,  $P \Rightarrow Q$ , by contraposition.

Homework (1/19 ; due Wed 1/26)

Section 2 : # 19, 20, 24