Set Operations

Set Operations

Unions, Intersections, and Relative Complements

Definition 1 (Set Operations)

Let A and B be sets.

- The *union of A and B* (denoted $A \cup B$) is the set of all things that belong to at least one of the sets A and B; in other words,

$$A \cup B = \{x : x \in A \text{ or } x \in B\}.$$

- The *intersection of A and B* (denoted $A \cap B$) is the set of all things that belong to both of the sets A and B; in other words,

$$A \cap B = \{x : x \in A \text{ and } x \in B\}.$$

- The *relative complement of B in A* (denoted $A \backslash B$) is the set of all things that belong to A but not to B; in other words,

$$A \backslash B = \{x : x \in A \text{ and } x \notin B\}.$$

Notes on Set Operations

- Short ways to read $A \cup B$, $A \cap B$, and $A \backslash B$ are "A union B," "A intersect B," and "A less B" respectively.
- $A \cup B$ should not be read "A or B." $A \cap B$ should not be read "A and B." We use the connectives "and" and "or" to connect sentences, not nouns.
- The results of set operations are another sets, so they are nouns. Hence, one must not write something like "$A \cup B$ iff $x \in A$ or $x \in B$." Instead, write "$x \in A \cup B$ iff $x \in A$ or $x \in B$."

Set Inclusion and Set Operations

Example 2 (:B$_{\text{example}}$:)

Let A and B be sets. Then:

1. $A \subseteq A \cup B$ and $B \subseteq A \cup B$.
2. $A \cap B \subseteq A$ and $A \cap B \subseteq B$.

Set Inclusion and Set Operations (cont')

Example 3 (:B$_{\text{example}}$:)

Let A, B, and C be sets. Then:

1. If $A \subseteq C$ and $B \subseteq C$, then $A \cup B \subseteq C$.
2. If $C \subseteq A$ and $C \subseteq B$, then $C \subseteq A \cap B$.

Set Inclusion and Set Operations (cont')

Example 4 (Equivalence to Set Inclusion)

Let A and B be sets. Then:

1. $A \subseteq B$ iff $A \cup B = B$.
2. $A \subseteq B$ iff $A \cap B = A$.
3. $A \subseteq B$ iff $A \backslash B = \varnothing$.