Rational Numbers

Contents

Rational Numbers

Definition

Definition 1 (Rational Numbers)

To say that x *is a rational number* means that there exist integers m and n such that $n \neq 0$ and $x = m/n$.

Examples

Example 2 (Sum of Rational Numbers)

Let u and v be rational numbers. Then $u + v$ is a rational number.

Special Forms of Rational Numbers

A given rational number x can be expressed in many different ways. For example,

$$\frac{7}{3} = \frac{-7}{-3} = \frac{14}{6} = \frac{-14}{-6} = \cdots = \frac{350}{150} = \cdots .$$

The fact that each rational number can be written in lowest terms such as $7/3$ can be proved later once we learn complete induction. For now, we can prove the following:

Rational Number as An Integer Divided by A Natural Number

Let x be a rational number. Then there exists an integer a and a natural number b such that $x = a/b$.

Irrational Numbers

Definition

Definition 3 (Irrational Numbers)

To say that x *is an irrational number* means that x is a real number and x is not a rational number.

Note

Remember that each irrational number is a real number!

$$\text{“}x \text{ is an irrational number.”} \not\equiv \text{“}x \text{ is not a rational number.”}$$

Consider the following question.

Question. Determine whether each of the following is true or false. Explain your answers.

1. For each $x \in \mathbb{C}$, if x is an irrational number, then x is not a rational number.

2. For each $x \in \mathbb{C}$, if x is not a rational number, then x is an irrational number.

Example 4 (Sum of Rational and Irrational Numbers)

Let x be a rational number and let y be an irrational number. Then $x + y$ is an irrational number.

Examples (cont')

Question. Let x and y be real numbers. Determine whether each of the following is true or false. Explain.

1. If xy is rational, then x and y are rational.

2. If $x + y$ is rational, then x and y are rational.

Irrationality of $\sqrt{2}$

Theorem 5

1. *Let x be a rational number. Then $x^2 \neq 2$.*
2. *$\sqrt{2}$ is irrational.*