Subsets

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Definition 1 (Subsets)

Let A and B be sets.

- To say that A *is a subset of* B (denoted $A \subseteq B$) means that for each x, if $x \in A$, then $x \in B$.
- To say that A *is a proper subset of* B (denoted $A \subsetneq B$) means that $A \subseteq B$ and $A \neq B$.

Notes.

- The relation $\subseteq$ is called *set inclusion*.
- The notation $B \supseteq A$ means the same as $A \subseteq B$ and is read "B is a superset of A."

Set Inclusion

Proposition 1 (Set Inclusion as Relation)

Set inclusion is reflexive, antisymmetric, and transitive. In other words

1. *For each set A, we have $A \subseteq A$. (Reflexivity.)*
2. *For all sets A and B, if $A \subseteq B$ and $B \subseteq A$, then $A = B$. (Antisymmetry.)*
3. *For all sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$. (Transitivity.)*

Empty Set

Proposition 2

For each set A, we have $\varnothing \subseteq A$.

- The proof involves a *vacuously true* statement.
- Conversely, if a set is a subset of any set, then it must be the empty set. In other words,

  (S10E05) Let A be a set such that for each set B, we have $A \subseteq B$. Then
  $$A = \varnothing.$$

Exercise 1 (Subsets)

Answer the following questions.

1. Is $\{3, 5\}$ a subset of $\{2, 3, 5\}$?
2. Is $\{2, \{3, 5\}\}$ a subset of $\{2, 3, 5\}$?
3. Write down all subsets of $\{1, 2, 3\}$.

Exercise 2 ($\in$ vs. $\subseteq$)

Find two sets A and B such that:

1. $A \in B$ and $A \subseteq B$.
2. $A \in B$ and $A \nsubseteq B$.
3. $A \notin B$ and $A \subseteq B$.
4. $A \notin B$ and $A \nsubseteq B$.