Logical Connectives (I)

Contents

Introduction

Sentences: Building Blocks in Logic

In logic, one seeks to determine which sentences are true and which are false.

Examples:

- Seoul is the capital of South Korea.
- $2 + 5 = 7$.
- $5 < 9$.
- 2022 is an even number.
- Cleveland is the capital of Illinois.
- $7 + 8 = 13$.
- $9 < 5$.
- 2021 is an even number.

Terminology. (*Truth value* of a sentence)

- When a sentence is true, its truth value is “true”.
- When a sentence is false, its truth value is “false”.

Logical Connectives

Simple sentences are put together using *logical connectives* such as

“not”, “and”, “or”, “implies”, and “if and only if”

to build compound sentences.

Propositional calculus (or *sentential calculus*) studies the truth values of compound sentences in terms of the truth values of their constituent sentences.

Convention. Denote sentences by letters such as P, Q, R, and so on, which are often referred to as *propositional variables*.

Symbols for Logical Connectives

Logical Connectives	Symbols	Big Words
“not”	$\neg$	negation
“and”	$\wedge$	conjunction
“or”	$\vee$	disjunction
“implies”	$\Rightarrow$	conditional
“if and only if”	$\Leftrightarrow$	biconditional

Negation, Conjunction, and Disjunction

Negation

Negation

Given a sentence P, the sentence $\neg P$ is called *the negation of* P.

- If P is a true sentence, then the sentence $\neg P$ is considered to be false.
- If P is a false sentence, then the sentence $\neg P$ is considered to be true.

P	$\neg P$
T	F
F	T

Terminology. A sentence Q is said to be a *negative sentence* when Q is the negation of some other sentence.

Logical Equivalence

If two sentences A and B always have the same truth values, we say that

$$A \textit{ is logically equivalent to } B.$$

and write

$$A \equiv B.$$

Example. Note that

- If P is true, then $\neg P$ is false, so $\neg\neg P$ is true.
- If P is false, then $\neg P$ is true, so $\neg\neg P$ is false.

Thus, $P \equiv \neg\neg P$.

Question. If you want to show two sentences are logically equivalent, what should you do?

Conjunction

Conjunction

Given sentences P and Q, the sentence $P \wedge Q$ is called *the conjunction of P and Q*.

- $P \wedge Q$ is considered to be true just when both of P and Q are true.
- If at least one of them is false, then $P \wedge Q$ is considered to be false.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Terminology. A sentence R is said to be a *conjunctive sentence* when R is of the form $P \wedge Q$, where P and Q are some other sentences. In this case, P and Q are called the *conjunctands* in R.

Conjunction (cont')

Some properties of $\wedge$.

- $\wedge$ is *commutative*, that is, $Q \wedge P$ is logically equivalent to $P \wedge Q$.

- $\wedge$ is *associative*, that is, $P \wedge (Q \wedge R)$ is logically equivalent to $(P \wedge Q) \wedge R$.

Disjunction

Disjunction

Given sentences P and Q, the sentence $P \vee Q$ is called *the disjunction of P and Q*.

- $P \vee Q$ is considered to be true just when at least one of P and Q is true.
- If both P and Q are false, then $P \vee Q$ is considered to be false.

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Terminology. A sentence R is said to be a *disjunctive sentence* when R is of the form $P \vee Q$, where P and Q are some other sentences. In this case, P and Q are called the *disjunctands* in R.

Disjunction (cont')

Some properties of $\vee$.

- $\vee$ is *commutative*, that is, $Q \vee P$ is logically equivalent to $P \vee Q$.

- $\vee$ is *associative*, that is, $P \vee (Q \vee R)$ is logically equivalent to $(P \vee Q) \vee R$.