

Hints for Homework 6

1. (FLOP counting; **FNC** 2.5.5) Watch the video “Counting Flops.”
2. (Matrix norms)
3. (Understanding matrix multiplication) This problem is to be done purely by hand and most of your answers will involve some sort of summation. Below are some similar questions and answers.

Example 1. What is the j^{th} column of $\mathbf{r}\mathbf{w}^T$, where $\mathbf{r} \in \mathbb{R}^m$ and $\mathbf{w} \in \mathbb{R}^n$? What is the size of the resulting matrix?

Answer. The expression $\mathbf{r}\mathbf{w}^T$, as an outer product, produces an $m \times n$ matrix

$$\begin{aligned} \mathbf{r}\mathbf{w}^T &= \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \end{bmatrix} \begin{bmatrix} w_1 & w_2 & \cdots & w_n \end{bmatrix} \\ &= \begin{bmatrix} r_1 w_1 & r_1 w_2 & \cdots & r_1 w_j & \cdots & r_1 w_n \\ r_2 w_1 & r_2 w_2 & \cdots & r_2 w_j & \cdots & r_2 w_n \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ r_m w_1 & r_m w_2 & \cdots & r_m w_j & \cdots & r_m w_n \end{bmatrix}, \end{aligned}$$

whose j^{th} column is

$$\begin{bmatrix} r_1 w_j \\ r_2 w_j \\ \vdots \\ r_m w_j \end{bmatrix}.$$

Notation. For a matrix M , denote by $[M]_{ij} = m_{ij}$ the (i, j) -element of M .

Observation. Since rows of M becomes columns of M^T , and vice versa, the (i, j) -element of M^T is the same as the (j, i) -element of M , *i.e.*,

$$[M^T]_{ij} = [M]_{ji} = m_{ji}.$$

This is going to be useful in the following example.

Example 2. What is the j^{th} element of the column vector $A^T \mathbf{x}$, where $A \in \mathbb{R}^{m \times n}$ and $\mathbf{x} \in \mathbb{R}^m$? What is the size of the resulting vector?

Answer. Since $A \in \mathbb{R}^{m \times n}$, $A^T \in \mathbb{R}^{n \times m}$ and so $A^T \mathbf{x} \in \mathbb{R}^n$, a column vector with n elements. The j^{th} element of $A^T \mathbf{x}$ is

$$\sum_{i=1}^m [A^T]_{ji} x_i = \sum_{i=1}^m [A]_{ij} x_i = \sum_{i=1}^m a_{ij} x_i.$$

4. (Periodic fit; **FNC** 3.1.3)

See the video demo provided for this week.

5. (Visualizing matrix norms; adapted from **LM** 9.4–26.)

The key modification must take place in the lines where \mathbf{X} is defined and `norm_Y` is calculated. Even for p other than 2, I suggest you keep the line where \mathbf{X} is calculated using the cosine and sine functions, but modify it immediately by dividing each column by its own vector p -norm. Think about how to compactly compute the p -norms of all columns at once, using a single line of MATLAB statement. The rest can be simply re-used in your function with minimal modification.