Review for Midterm 1

Contents

1. Summations
2. Simulations
3. Data Manipulation

Summations

Different Ways of Forming Sums

To calculate $\sum_{j=1}^{n} a_j b_j$:

Assume that $\vec{a} = [a_1 \; a_2 \; \cdots \; a_n]$, $\vec{b} = [b_1 \; b_2 \; \cdots \; b_n]$ are stored in MATLAB. (Don't know what n is.)

- using a loop

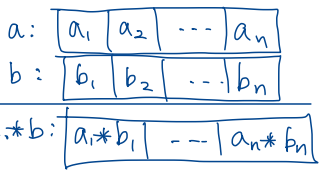
```
S = 0;
for j = 1:length(a)
    S = S + a(j)*b(j);
end
```

"n"

- using `sum`

```
S = sum(a.*b);
```

elementwise multiplication



- **inner product** (linear algebra)

$$\vec{a}\,\vec{b}^{T} = [a_1 \cdots a_n] \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$

```
S = a*b.';
```

Sequence of Partial Sums

→ HW problem (Approx. π)

$a_1 + a_2 + a_3 + \cdots$

To study the convergence of an infinite series $\sum_{j=1}^{\infty} a_j$, form the sequence of partial sums $\{s_n\}$ where

$$s_n = \sum_{j=1}^{n} a_j = a_1 + \cdots + a_n.$$

Assume that $a = [a_1\ a_2\ \cdots\ a_n]$, for some large n, is stored.

$S_1 = a_1$

$S_2 = a_1 + a_2 = S_1 + a_2$

$S_3 = a_1 + a_2 + a_3 = S_2 + a_3$

$S_n = a_1 + \cdots + a_n = S_{n-1} + a_n$

Task: Form a row vector

$S = [S_1\ S_2\ \cdots\ S_n]$

- using a loop

```
n = length(a);
S = zeros(1,n); % preallocation
S(1) = a(1);
for j = 2:n
    S(j) = S(j-1) + a(j);
end
```

- using `cumsum`

```
S = cumsum(a);
```

Simulations

Biased Coin

Question

Simulate the tossing of a biased coin with

$$P(\mathsf{T}) = p, \quad P(\mathsf{H}) = 1 - p.$$

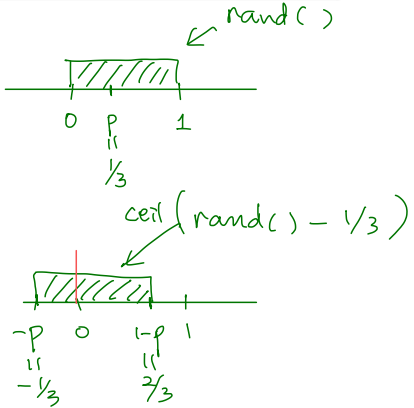
Example $p = 1/3$. $P(T) = 1/3$, $P(H) = 2/3$

Convention: $T \to 0$, $H \to 1$

Method 1 (rand, ceil / floor)

```
p = 1/3;
toss = ceil( rand() - p );
```

Q. `toss = floor( ? );`



Method 2 ("randi" and "find")

Simulate tossing n times.

randi(3, 1, n)


```
tosses = randi(3, 1, n);
tosses(find(tosses == 1)) = 0;
tosses(find(tosses >= 2)) = 1;
```

length(find(tosses == 0))

Biased Coin – Notes

Ideas.

- random number generators
- traditional tools: loops and conditional statements
- the *powerful* `find` function
- one-liner using `ceil` or `floor`

Explore.

- How would you handle similar situations with multiple states with non-uniform probability profile, *e.g.*, a biased dice?

Dice Rolls → Birthday Problem.

Question

Write a script simulating $n = 10,000$ throws of two 6-sided fair dice. What is the probability of obtaining two same numbers? Provide both analytical and numerical answers.

```
throws = randi(6, 2, n);
diff_throws = diff(throws);
length(find(diff_throws == 0))/n
```

Sim 1 Sim 2 ... Sim n

1st dice → | 1 | 2 | 4 | ... | 4 |

2nd dice → | 5 | 3 | 4 | | 6 |

| -4 | -1 | 0 | ... | -2 |

```
bday = randi(365, n, n_sim);
diff_bday = zeros(n-1, n_sim);
diff_bday(find(diff(sort(bday))==0)) = 1;
```

Data Manipulation

Data Manipulation

Download `grades.dat` into your current directory and load it using

```
>> grades = load('grades.dat');
```

To read about how the data are organized, use `type grades.dat`.

Question

1. Determine the number of students.
2. Compute the total grade according to the weights specified in the header. Do this without using a loop.
3. The letter grades are determined by

   - A: $[90, 100]$
   - B: $[80, 90)$
   - C: $[70, 80)$
   - D: $[60, 70)$
   - E: $[0, 60)$

   Find the number of students earning each of the letter grades.