




Math 3607: Homework 9

Wednesday, April 6, 2021

TOTAL: 30 points

- Problems marked with  are to be done by hand; those marked with  are to be solved using a computer.
 - **Important note.** Do not use *Symbolic Math Toolbox*. Any work done using `sym` or `syms` will receive NO credit.
 - **Another important note.** When asked write a MATLAB function, write one at the end of your live script.
1. (Low-rank approximation using SVD; image compression)  Load `hubble_gray.jpg`, which is a grayscale image taken by the Hubble Space Telescope, convert it to a matrix of floating point pixel intensities, and then display the image in MATLAB by

```
A = imread('hubble_gray.jpg');  
imshow(A);
```

Following the demo in Lecture 23 as a guide,

- (a) Plot the singular values $\sigma_1, \sigma_2, \dots, \sigma_n$ of A on a log scale (using `semilogy`).
- (b) Plot the accumulation of singular values of A .
- (c) Compute the best approximations of A of rank 2, 20, and 120 and display the corresponding images using `subplot`.



Figure 1: NGC 3603
(Hubble Space Telescope).

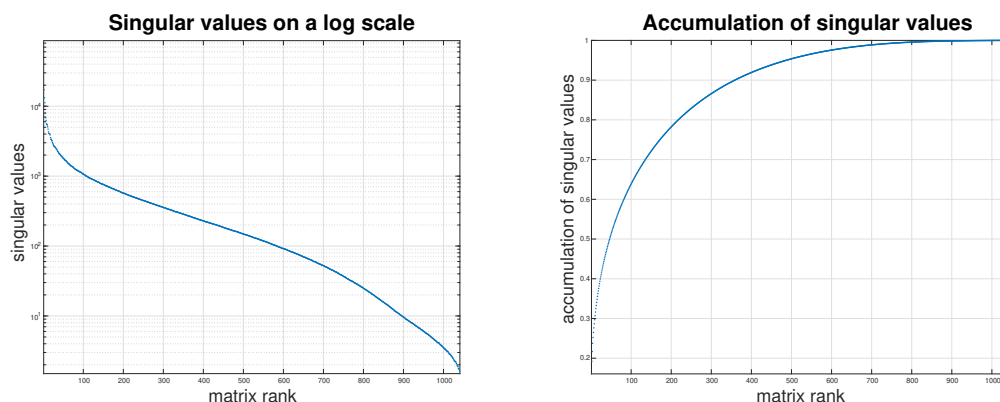


Figure 2: Example outputs for part (a) on the left and part (b) on the right.

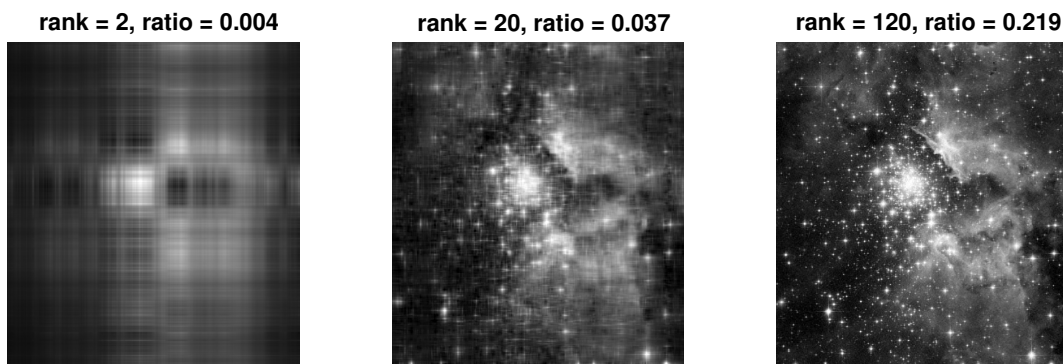


Figure 3: Example output for part (c)

2. (Annuity with `fzero`; **FNC** 4.1.4) A basic type of investment is an annuity: One makes monthly deposits of size P for n months at a fixed annual interest rate r , and at maturity collects the amount

$$\frac{12P}{r} \left(\left(1 + \frac{r}{12} \right)^n - 1 \right).$$

Say you want to create an annuity for a term of 300 months and final value of \$1,000,000. Using `fzero`, make a table of the interest rate you will need to get for each of the different contribution values $P = 500, 550, \dots, 1000$.

3. (Lambert's W function; **FNC** 4.1.6) Lambert's W function is defined as the inverse of xe^x . That is, $y = W(x)$ if and only if $x = ye^y$. Write a function `y = lambertW(x)` that computes W using `fzero`. Make a plot of $W(x)$ for $0 \leq x \leq 4$.

4. (Fixed-point iteration; adapted from **FNC** 4.2.1 and 4.2.2.) In each case below,

- $g(x) = \frac{1}{2} \left(x + \frac{9}{x} \right)$, $r = 3$.
- $g(x) = \pi + \frac{1}{4} \sin(x)$, $r = \pi$.
- $g(x) = x + 1 - \tan(x/4)$, $r = \pi$.

- (a) Show that the given $g(x)$ has a fixed point at the given r and that fixed point iteration can converge to it.

- (b) Apply fixed point iteration in MATLAB and use a log-linear graph (using `semilogy`) of the error to verify (linear) convergence. Then use numerical values of the error to determine an approximate value for the rate σ .

5. (Convergence of Newton's method) Answer the following questions *by hand*, without using MATLAB.

- (a) Discuss what happens when Newton's method is applied to find a root of

$$f(x) = \text{sign}(x)\sqrt{|x|},$$

starting at $x_0 \neq 0$. ¹

¹ $\text{sign}(x)$ is 1 if $x > 0$, -1 if $x < 0$, and 0 if $x = 0$.

- (b) In the case of a multiple root, where $f(r) = f'(r) = 0$, the derivation of the quadratic error convergence is invalid. Redo the derivation to show that in this circumstance and with $f''(r) \neq 0$ the error converges only linearly.