

## Applications of LU Factorization

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## ① Applications of PLU Factorization

# Applications of PLU Factorization

- Solving system of eqn.
- computing matrix inverse
- computing determinants

# Solving a Square System

Multiplying  $Ax = b$  on the left by  $P$  we obtain

$$\underbrace{PA}_{=LU} x = \underbrace{Pb}_{=: \beta} \longrightarrow LUx = \beta,$$

which can be solved in two steps:

- Define  $Ux = y$  and solve for  $y$  in the equation

$$Ly = \beta.$$

- Having calculated  $y$ , solve for  $x$  in the equation

$$Ux = y.$$

- $A \vec{x} = \vec{b}$

- $\underline{PA} \vec{x} = \underbrace{P\vec{b}}_{=\vec{\beta}}$

- $L(\underbrace{U \vec{x}}_{\vec{y}}) = \vec{\beta}$

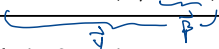
- Solve  $L \vec{y} = \vec{\beta}$  using  
(forward elimination) *forw. elim.*

- Solve  $U \vec{x} = \vec{y}$  using  
(backward substitution) *back. subs.*

## Solving a Square System (cont')

- Using the instructional codes ( `backsub`, `forelim`, `myplu` ):

```
[L,U,P] = myplu(A);  
x = backsub( U, forelim(L, P*b) );
```



- Using MATLAB's built-in functions:

```
[L,U,P] = lu(A);  
x = U \ (L \ (P*b));
```

- The backslash is designed so that triangular systems are solved with the appropriate substitution.
- The most compact way:

```
x = A \ b;
```

- The backslash does partial pivoting and triangular substitutions silently and automatically.

# Computing Inverses

Observe that

$$(PA)^{-1} = (LU)^{-1} \longrightarrow A^{-1}P^{-1} = U^{-1}L^{-1} \longrightarrow LUA^{-1} = P$$

So solve  $LU\mathbf{a}_i = \mathbf{p}_i$  with forward and backward substitution for each column  $\mathbf{p}_i$  of  $P$ . Then

$$A^{-1} = \left[ \begin{array}{c|c|c|c} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n \end{array} \right].$$

# Computing Determinants

Observe that

$$\det(A) = \det(P^{-1}LU) = \det(P^{-1}) \det(L) \det(U) = \frac{\det(L) \det(U)}{\det(P)}.$$

**Useful facts.**

- The determinant of a triangular matrix is the product of its diagonal entries. (What are diagonal entries of  $L$ ?)
- $P$  is a row permutation of the identity matrix (which has determinant 1), and each row swap negates the determinant. So if  $s$  is the number of row swaps, then  $\det(P) = (-1)^s$ .

It follows that

$$\det(A) = (-1)^s \prod_{i=1}^n u_{ii}.$$