Math 3607: Homework 9

Wednesday, April 6, 2021

TOTAL: 30 points

- Problems marked with ✎ are to be done by hand; those marked with 💻 are to be solved using a computer.
- **Important note.** Do not use *Symbolic Math Toolbox*. Any work done using `sym` or `syms` will receive NO credit.
- **Another important note.** When asked write a MATLAB function, write one at the end of your live script.

1. (Low-rank approximation using SVD; image compression) 💻 Load `hubble_gray.jpg`, which is a grayscale image taken by the Hubble Space Telescope, convert it to a matrix of floating point pixel intensities, and then display the image in MATLAB by

```
A = imread('hubble_gray.jpg');
imshow(A);
```

   Following the demo in Lecture 28 as a guide,

   (a) Plot the singular values $\sigma_1, \sigma_2, \ldots, \sigma_n$ of A on a log scale (using `semilogy`).

   (b) Plot the accumulation of singular values of A.

   (c) Compute the best approximations of A of rank $2, 20$, and 120 and display the corresponding images using `subplot`.

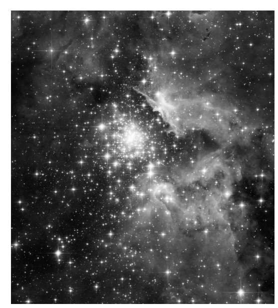

Figure 1: NGC 3603 (Hubble Space Telescope).

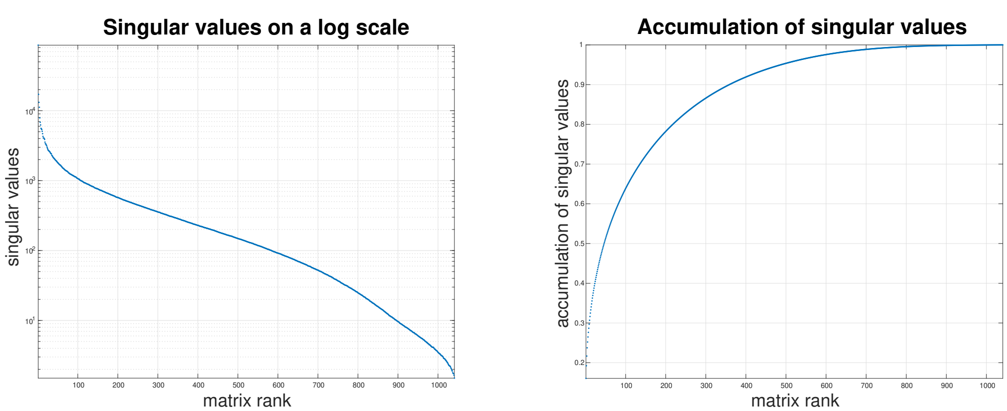

Figure 2: Example outputs for part (a) on the left and part (b) on the right.

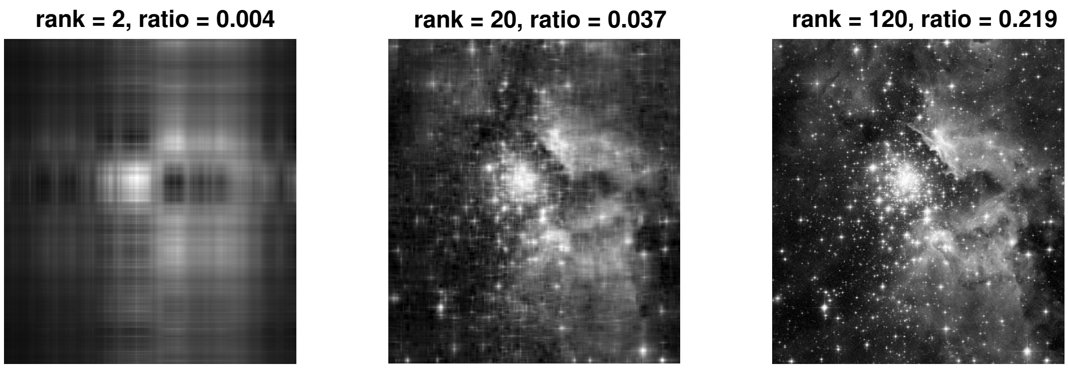

Figure 3: Example output for part (c)

2. (Annuity with `fzero`; **FNC** 4.1.4) A basic type of investment is an annuity: One makes monthly deposits of size P for n months at a fixed annual interest rate r, and at maturity collects the amount
$$\frac{12P}{r}\left(\left(1+\frac{r}{12}\right)^{n}-1\right).$$
Say you want to create an annuity for a term of 300 months and final value of \$1,000,000. Using `fzero`, make a table of the interst rate you will need to get for each of the different contribution values $P = 500, 550, \ldots, 1000$.

3. (Lambert's W function; **FNC** 4.1.6) Lambert's W function is defined as the inverse of xe^x. That is, $y = W(x)$ if and only if $x = ye^y$. Write a function `y = lambertW(x)` that computes W using `fzero`. Make a plot of $W(x)$ for $0 \leqslant x \leqslant 4$.

4. (Fixed-point iteration; adapted from **FNC** 4.2.1 and 4.2.2.) In each case below,

   - $g(x) = \frac{1}{2}\left(x + \frac{9}{x}\right)$, $r = 3$.
   - $g(x) = \pi + \frac{1}{4}\sin(x)$, $r = \pi$.
   - $g(x) = x + 1 - \tan(x/4)$, $r = \pi$.

   (a) Show that the given $g(x)$ has a fixed point at the given r and that fixed point iteration can converge to it.

   (b) Apply fixed point iteration in MATLAB and use a log-linear graph (using `semilogy`) of the error to verify (linear) convergence. Then use numerical values of the error to determine an approximate value for the rate σ.

5. (Convergence of Newton's method) Answer the following questions *by hand*, without using MATLAB.

   (a) Discuss what happens when Newton's method is applied to find a root of
   $$f(x) = \operatorname{sign}(x)\sqrt{|x|},$$
   starting at $x_0 \neq 0$. [1]

[1] $\operatorname{sign}(x)$ is 1 if $x > 0$, -1 if $x < 0$, and 0 if $x = 0$.

(b) In the case of a multiple root, where $f(r) = f'(r) = 0$, the derivation of the quadratic error convergence is invalid. Redo the derivation to show that in this circumstance and with $f''(r) \neq 0$ the error converges only linearly.