Math 3607

Final Exam

- Written: 04/26 (T) 12:00 ~ 04/29 (F) 23:59
  (Gradescope)
- Quiz: Entire Friday (3 attempts ; 1 hour each)
  (Carmen)

04/25/2022

Lecture 38 Review for Final Exam

Eigenvalue Decomposition $\quad A \in \mathbb{R}^{n \times n}$ (square matrix)

$$A\vec{v}_j = \lambda_j \vec{v}_j, \quad j = 1, \cdots, n$$

eigenvalue

eigenvector

- General: $AV = VD$
- EVD: $A = VDV^{-1}$
  (when V is invertible.)
- Useful in studying matrix powers

$$A^k = (VDV^{-1})(VDV^{-1}) \cdots (VDV^{-1})$$
$$= VD^k \boxed{V^{-1}}.$$

MATLAB:

Use "/V".

forward slash.

- HW 8: #2, #3

$$V = \begin{bmatrix} | & | & & | \\ \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_n \\ | & | & & | \end{bmatrix}$$

$$D = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix}$$

Math	MATLAB
$\vec{x} = A^{-1}\vec{y}$	`x = A\y;`
$\vec{x} = \vec{y}^T A^{-1}$	`x = y'/A;`

Singular Value Decomposition $A \in \mathbb{C}^{m\times n}$ $(m \geq n)$

- $A = U\Sigma V^*$ conjugate transpose

  | U, V unitary $\begin{cases} U^*U = UU^* = I_{m\times m} & (U^{-1} = U^*) \\ V^*V = VV^* = I_{n\times n} & (V^{-1} = V^*) \end{cases}$

  | Σ real diagonal

- analytical properties (2-norm)

- Useful in studying A^T, A^H, A^+, ...

  e.g. $A^* = (U\Sigma V^*)^*$
  $= (V^*)^* \Sigma^* U^*$
  $= \underline{V}\,\underline{\Sigma^T}\,\underline{U^*}$

- Low-rank approx. : Image Compression

- HW9: #1

Root finding $\quad f(r) = 0$

- fixed point iteration (linear)
  Newton's method (quad.)
  Secant method (superlinear) $\longrightarrow$ $p = \frac{1+\sqrt{5}}{2} \approx 1.6$, Golden ratio

- Convergence (series analysis)

$$\epsilon_{k+1} \approx C \epsilon_k^p$$

HW 9 : #5 (b)

- Multidimensional Newton's method.

- Lambert's W function

$$y = xe^x \iff x = W(y)$$

$$xe^x - y = 0$$

- HW9 #3
- Week 11 supp.

Piecewise polynomial interpolation and numerical differentiation

- See Supplementary Resources for practice problems

num. diff.

- ☆ Optimal step size h
- ☆ Richardson extrapolation

Airplane Velocity (num. diff.)

The radar stations A and B, separated by the distance $a = 500$ m, track a plane C by recording the angles α and β at one-second intervals. Your goal, back at air traffic control, is to determine the speed of the plane.

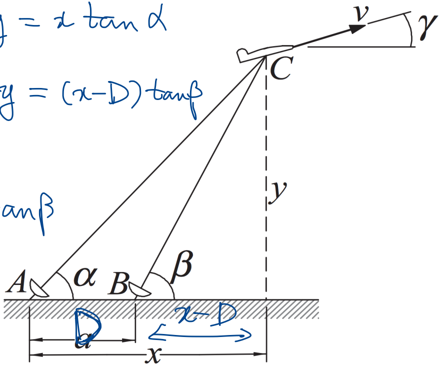
$$\begin{cases} \tan\alpha = \dfrac{y}{x} \implies y = x\tan\alpha \\ \tan\beta = \dfrac{y}{x-D} \implies y = (x-D)\tan\beta \end{cases}$$

$$\Rightarrow x\tan\alpha = (x-D)\tan\beta$$

$$\vdots$$

$$x = \frac{D\tan\beta}{\tan\beta - \tan\alpha}$$

$$y = \frac{D\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}.$$



Data Received from Radar Stations

$\Delta t = 0.1$

time (s)	α (°)	β (°)
0.0	α_1	β_1
0.1	α_2	β_2
0.2	α_3	β_3
⋮	⋮	⋮
1.0	α_{11}	β_{11}

Convert $\longrightarrow$

$x = x(\alpha, \beta)$

$y = y(\alpha, \beta)$

	time (s)	x (m)	y (m)
2^{nd}o. FD →	0.0	x_1	y_1
2^{nd} CD →	0.1	x_2	y_2
	0.2	x_3	y_3
	⋮	⋮	⋮
2^{nd} BD →	1.0	x_{11}	y_{11}

num diff.

t	$x'(t)$	$y'(t)$