Review for Midterm 1

Contents

Summations

Different Ways of Forming Sums

To calculate $\sum_{j=1}^{n} a_j b_j$:

Assume $\vec{a} = [a_1 \; a_2 \; \cdots \; a_n]$ and $\vec{b} = [b_1 \; b_2 \; \cdots \; b_n]$ are saved in MATLAB.

- using a loop

```
s = 0;
for j = 1 : length(a)        % "n"
    s = s + a(j) * b(j);
end
```

- using `sum`

```
s = sum(a .* b);
```

elementwise multiplication

a = | a_1 | a_2 | $\cdots$ | a_n |

b = | b_1 | b_2 | $\cdots$ | b_n |

$a .* b$ = | $a_1 * b_1$ | $a_2 * b_2$ | $\cdots$ | $a_n * b_n$ |

- **inner product**

Note (linear algebra)

$$\vec{a}\,\vec{b}^{T} = [a_1 \cdots a_n] \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix} = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$

```
s = a * b.';
```

Sequence of Partial Sums — HW (Approximating π)

To study the convergence of an infinite series $\sum_{j=1}^{\infty} a_j$, form the sequence of partial sums $\{s_n\}$ where

Calc 2

$a_1 + a_2 + a_3 + \cdots$

$$s_n = \sum_{j=1}^{n} a_j = a_1 + \cdots + a_n.$$

Assume $\vec{a} = [a_1\ a_2\ \cdots\ a_n]$ is stored.

- using a loop

```
n = length(a);
S = zeros(1, n); % preallocation
S(1) = a(1);
for j = 2:n
    S(j) = S(j-1) + a(j);
end
```

- using `cumsum`

```
S = cumsum(a);
```

Vectorized code.

$$\begin{aligned} S_1 &= a_1 \\ S_2 &= a_1 + a_2 \\ S_3 &= \underbrace{a_1 + a_2}_{S_2} + a_3 \\ &\vdots \\ S_n &= a_1 + a_2 + \cdots + a_n \end{aligned}$$

Want: $[S_1, S_2, \cdots, S_n]$

Simulations

Biased Coin

Question

Simulate the tossing of a biased coin with

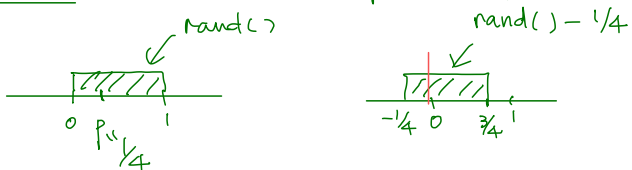
$$P(\mathsf{T}) = p, \quad P(\mathsf{H}) = 1 - p.$$

Example $p = 1/4$

T : 0 , H : 1

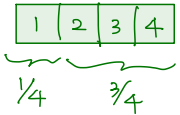
toss = ceil(rand( ) − 1/4);

Method 1 (rand, ceil, floor, if-statement)



Method 2 (randi)

```
toss = randi(4,1,1);
if toss == 1
    toss = 0;
else
    toss = 1;
end
```



Alternately:

```
toss = randi(4,1,1);
toss( find(toss==1) ) = 0;
toss( find(toss~=1) ) = 1;
```

n

more suitable when "toss" is an array.

Biased Coin – Notes

Ideas.

- random number generators
- traditional tools: loops and conditional statements
- the *powerful* `find` function
- one-liner using `ceil` or `floor`

Explore.

- How would you handle similar situations with multiple states with non-uniform probability profile, *e.g.*, a biased dice?

Dice Rolls

Question

Write a script simulating $n = 10,000$ throws of two 6-sided fair dice. What is the probability of obtaining two same numbers? Provide both analytical and numerical answers.

Data Manipulation

Data Manipulation

Download `grades.dat` into your current directory and load it using

```
>> grades = load('grades.dat');
```

To read about how the data are organized, use `type grades.dat`.

Question

1. Determine the number of students.
2. Compute the total grade according to the weights specified in the header. Do this without using a loop.
3. The letter grades are determined by

   - A: $[90, 100]$
   - B: $[80, 90)$
   - C: $[70, 80)$
   - D: $[60, 70)$
   - E: $[0, 60)$

   Find the number of students earning each of the letter grades.