Updated on June 9, 2021

Math 3607: Homework 1

Due: 10:00PM, Wednesday, June 16, 2021

TOTAL: 30 points

1. (Adapted from **LM** [1] 2.2–5.) Write a script which asks for a temperature in Fahrenheit, converts the temperature into Celsius, and prints it.

2. An *oblate spheroid* such as the Earth is obtained by revolving an ellipse about its minor axis as shown in the figure.

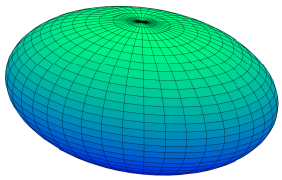


Figure 1: An oblate spheroid. Image created using MATLAB.

The Earth's equatorial radius is about 20km longer than its polar radius. The surface area of an oblate spheroid is given by

$$A(r_1, r_2) = 2\pi \left(r_1^2 + \frac{r_2^2}{\sin(\gamma)} \log\left(\frac{\cos(\gamma)}{1 - \sin(\gamma)} \right) \right),$$

where r_1 is the equatorial radius, r_2 is the polar radius, and

$$\gamma = \arccos\left(\frac{r_2}{r_1} \right).$$

[1] Reference Keys:

- **LM**: *Learning MATLAB, Problem Solving, and Numerical Analysis Through Examples* (Overman)
- **NCM**: *Numerical Computing with MATLB* (Moler)
- **FNC**: *Fundamentals of Numerical Computation* (Driscoll and Braun)

The notation **LM** 2.2–5 indicates Problem 6 at the end of section 2.2 of the textbook by Overman.

We assume $r_2 < r_1$. Write a script that inputs the equatorial and polar radii and displays both $A(r_1, r_2)$ and the approximation $4\pi((r_1 + r_2)/2)^2$. Apply the script to the Earth data $(r_1, r_2) = (6378.137, 6356.752)$. Use `format long g` to display enough digits.

3. (Exercise 2, Relational and Logical Operations) A year is a *leap year* if it is a multiple of 4, except for years divisible by 100 but not by 400. In simpler terms, a non-century year is a leap year if it is divisible by 4; a century year is a leap year if it is divisible by 400.

   For example,

   - Last year (2020) was a leap year. (non-century year; divisible by 4)
   - 1900 was not a leap year. (century year; not divisible by 400)
   - 2000 was a leap year. (century year; divisible by 400)

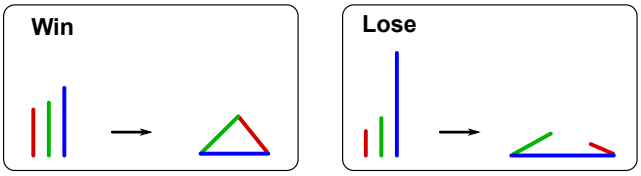
   Write a script which determines whether a given year is a leap year or not.

4. (Exercise 3, Relational and Logical Operations) Recall that Cartesian coordinates (x, y, z) in $\mathbb{R}^3$ are related to spherical coordinates (ρ, ϕ, θ) by

$$x = \rho \sin\phi \cos\theta, \quad y = \rho \sin\phi \sin\theta, \quad z = \rho \cos\phi,$$

   where $\phi \in [0, \pi]$ and $\theta \in [0, 2\pi)$. Write a script which takes Cartesian coordinates as inputs and converts them to spherical ones.

5. (Exercise 2, For-Loop) In the game of 3-Stick, you pick three sticks each having a random length between 0 and 1. You win if you can form a triangle using three sticks; otherwise, you lose.



   Write a script simulating one million games and estimating the probability of winning a game.