Updated on June 23, 2021

Math 3607: Exam 1

Due: 11:59PM, Friday, June 25, 2021

Please read the statements below and sign your name.

Disclaimers and Instructions

- You may use any of the book functions (quote), any of functions provided in lectures, accompanying live scripts, and homework or practice problem solutions, and any of your own helper functions. As stated in the syllabus, you must be able to explain how they are supposed to work. You may be requested to explain your code to me, in which case a proper and satisfactory explanation must be provided to receive any credits on relevant parts.
- You are not allowed to search online forums or even MathWorks website. All problems can be solved with what has been taught in class. If you need to look something up, then you must be doing it incorrectly.
- If any code is found to be plagiarized from the internet or another person, you will receive a zero on the *entire* exam (both parts).

Academic Integrity Statements

- All of the work shown on this exam is my own.
- I will not consult with any resources (MATLAB documentation, online searches, etc.) other than the textbooks, lecture notes, and supplementary resources provided on the course Carmen pages.
- I will not discuss any part of this exam with anyone, online or offline.
- I understand that academic misconduct during an exam at The Ohio State University is very serious and can result in my failing this class or worse.
- I understand that any suspicious activity on my part will be automatically reported to the OSU Committee on Academic Misconduct for their review.

Signature ____________________________

1 Surface Plot [16 points]

Plot the surface represented by

$$\begin{aligned} x &= u(3 + \cos(v)) \cos(2u), \\ y &= u(3 + \cos(v)) \sin(2u), \\ z &= u \sin(v) - 3u, \end{aligned}$$

for $u \in [0, 2\pi]$, $v \in [0, 2\pi]$.

2 Continued Fraction [20 points]

A *continued fraction* is an infinite expression of the form

$$a_0 + \cfrac{1}{a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \cdots}}}.$$

If all the a_k's are equal to 1, the continued fraction is equal to ϕ, the golden ratio:

$$\phi = 1 + \cfrac{1}{1 + \frac{1}{1 + \frac{1}{1 + \cdots}}} = \frac{1 + \sqrt{5}}{2}.$$

Denote by ϕ_n the n-term truncation of the continued fraction representation of the above, that is,

$$\begin{aligned} \phi_0 &= 1, \\ \phi_1 &= 1 + \frac{1}{1} = 1 + \frac{1}{\phi_0}, \\ \phi_2 &= 1 + \cfrac{1}{1 + \frac{1}{1}} = 1 + \frac{1}{\phi_1}, \\ \phi_3 &= 1 + \cfrac{1}{1 + \frac{1}{1 + \frac{1}{1}}} = 1 + \frac{1}{\phi_2}, \\ &\vdots \\ \phi_n &= 1 + \frac{1}{\phi_{n-1}}, \quad \text{for any } n \geqslant 1. \end{aligned}$$

They can be used to approximate the golden ratio, *i.e.*, $\phi_n \to \phi$ as $n \to \infty$.

Use a loop to evaluate ϕ_j for $j = 1, 2, \ldots$ until you get 16 correct digits after the decimal place. Use `fprintf` in each iteration to show that the iterates are converging to the correct value. Your first few iterates and errors should look like this:

```
n   phi_n                 error
1   2.0000000000000000   3.8197e-01
2   1.5000000000000000   1.1803e-01
3   1.6666666666666665   4.8633e-02
4   1.6000000000000001   1.8034e-02
5   1.6250000000000000   6.9660e-03
6   1.6153846153846154   2.6494e-03
.................................
```

3 Array Operations [24 points]

You are planning a bicycle trip along a 400 mile stretch of a very straight midwestern rural highway and plan to stop each night at a different town. The towns are irregularly spaced, but you have mileage markers for each town, given in the array

$$\texttt{Miles} = [0, 27, 69, 101, 120, 154, 178, 211, 235, 278, 306, 327, 356, 391, 400]$$

You would like to compute the distances you have to travel each day, as well as other statistics about your trip.

To start, create an array `Miles` containing the mile markers above, and a second array `Dist` containing the distances between each of the mile markers and answer the following questions.

(a) What is the shortest distance you will have to bicycle on any day?

(b) What is the longest distance?

(c) What is your average daily distance?

(d) How far will you have to go on day 7?

(e) If you would like to stop each day for lunch at exactly the halfway point for each day's journey, what mileage values should you plug into your GPS to assure that you do not miss lunch?

(f) How can you recover your array `Miles` from your array `Dist`?

Answer these questions without using a loop[1].

[1] A correct answer obtained using a loop will earn half of the allotted points

4 Tiling with Spiral Polygons [40 points]

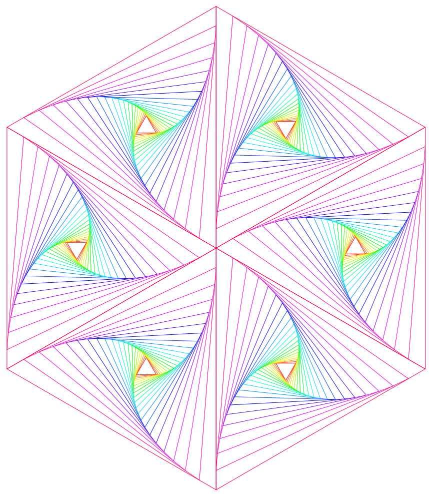

Figure 1: Tiling with spiral triangles.

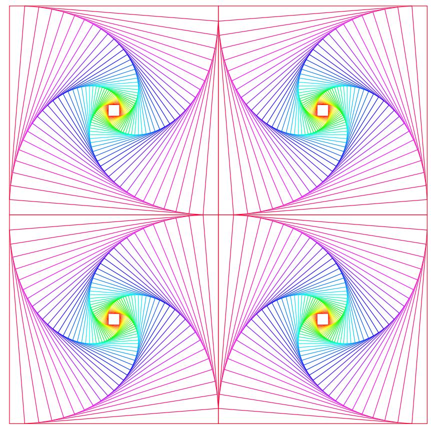

Figure 2: Tiling with spiral squares.

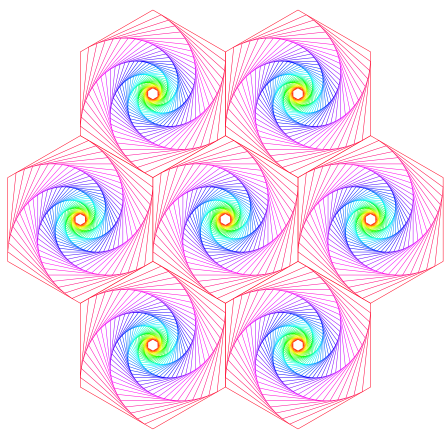

Figure 3: Tiling with spiral hexagons.

(a) Modify the function[2] `spiralgon.m` from Homework 2 so that it now takes `n`, `m`, `d_angle`, `d_rot`, and `shift`. The additional input argument `shift` is a two-vector (a vector with two elements) and it shifts the center of all the polygons to the point (`shift(1)`, `shift(2)`). Consequently, the plotting statement must now be changed to

```
plot(V(1,:)+shift(1), V(2,:)+shift(2), 'Color', C(i,:))
```

In addition, comment out the “ `hold off` ” statement before the `for` loop.

(b) Generate Figure 1 by following the steps below.

- Run `spiralgon`, letting `n=3`, `m=21`, `d_angle=4.5`, `d_rot=90`, and `shift=[0 0]'` and save `V`. Its first column is the location of the leftmost vertex. Follow the call with “ `hold off` ” so that the image disappears. It was only executed to return `V`.

[2] Include the function at the end of the live script.

- Modify `shift` so that the next time you run this function, the leftmost vertex will be at the origin. Run `spiralgon` again and you have 1/6th of your figure.
- Now use a `for` loop and plot the next five spiral triangles. You have to add 60° to `d_rot` for each new spiral triangle and also shift `shift` by 60°.

(c) Generate Figure 2 by patching four spiral squares as in the previous part. Each of the four spiral squares is generated with `m=41` squares. Systematically determine other input arguments so as to reproduce the shape. Explain your choice of inputs. (*Hint.* Pay attention to the direction of each spiral. Study the statements in Problem 4(b) of Homework 2. They were carefully chosen to ensure that all shapes are aesthetically configured. Or read the instructions found in **LM** 6.8–34, 35.)

(d) Generate Figure 3. Each of the spiral hexagons is generated with `m=51`. Determine other input arguments so as to reproduce the shape. Justify your choice of inputs.