HW02, Problem 2 (Hint)

Problem

Each of the following sequences converges to π:

$$a_n = \frac{6}{\sqrt{3}} \sum_{k=0}^{n} \frac{(-1)^k}{3^k(2k+1)},$$
$$b_n = 16 \sum_{k=0}^{n} \frac{(-1)^k}{5^{2k+1}(2k+1)} - 4 \sum_{k=0}^{n} \frac{(-1)^k}{239^{2k+1}(2k+1)}.$$

Write a single script that prints $a_0, \ldots, a_{n_a}$, where n_a is the smallest integer so that $|a_{n_a} - \pi| \leqslant 10^{-6}$ and prints $b_0, \ldots, b_{n_b}$, where n_b is the smallest integer so that $|b_{n_b} - \pi| \leqslant 10^{-6}$.

Hint using a similar example

Consider the following series which is famously known to be convergent.

$$\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}.$$

In other words, the sequence of partial sums $\{s_n\}$ is convergent with $\lim_{n\to\infty} s_n = \pi^2/6$ where

$$s_n = \sum_{k=1}^{n} \frac{1}{k^2}.$$

Let's see how many terms are needed so that the partial sum is close enough to the true value. More concretely, let's find the smallest integer n so that $e_n := |s_n - \pi^2/6| \leqslant 10^{-2}$.

- To calculate a partial sum s_n, use a for-loop:

```
% assume n is given
s_n = 0;
for k = 1:n
    s_n = s_n + 1/k^2;
end
```

  This will be a central building block in our program.

- To design a loop which continues to repeat while $e_n > 10^{-2}$:

```
s = pi^2/6;                % exact sum
errBound = 1e-2;           % error bound
err_n = 1;                 % so that loop below can be initiated
n = 1;
while err_n > errBound     % the loop will close when err_n <= errBound
    s_n = 0;
    for k=1:n
        s_n = s_n + 1/k^2;
    end
    err_n = abs(s_n - s);
    n = n + 1;
end
```

The two snippets above should you a good starting point.