Updated on July 12, 2021

Math 3607: Homework 6

Due: 10:00PM, Wednesday, July 21, 2021

TOTAL: 30 points

You will be writing some MATLAB functions for this assignment. Include all your functions at the end of your live script.

Problems marked with are to be done by hand; those marked with are to be solved using a computer.

1. (Polynomial evaluation of matrices) Let $p(z) = c_1 + c_2 z + \cdots + c_n z^{n-1}$. The value of p for a square matrix input is defined as

$$p(X) = c_1 I + c_2 X + \cdots + c_n X^{n-1}.$$

   (a) Show that if $X \in \mathbb{R}^{k\times k}$ has an EVD, then $p(X)$ can be found using only evaluations of p at the eigenvalues and two matrix multiplications.

   (b) Complete the following program which, given coefficients $\mathbf{c} = (c_1, c_2, \ldots, c_n)^{\mathrm{T}}$, evaluates the corresponding polynomial at $\mathbf{x}$, which can be a number, a vector, or a square matrix. If $\mathbf{x}$ is a scalar or a vector, use *Horner's method*[1]; if $\mathbf{x}$ is a square matrix, use the result from the previous part.

```
function y = mypolyval(c, x)
%MYPOLYVAL evaluates a polynomial at points  x  given its coeffs.
% Input:
%   c   coefficient vector (c_1, c_2, ..., c_n)^T
%   x   points of evaluation
%       - if x is a scalar or a vector, use Horner's method
%       - if x is a square matrix, use the result from (a)
%       - otherwise, produce an error message.
```

2. (Singular values by hand) Calculate the singular values of

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix}$$

   by solving a 2×2 eigenvalue problem.

3. (SVD and the 2-norm) Let $A \in \mathbb{R}^{n\times n}$. Show that

   (a) A and A^{T} have the same singular values.

[1] See Problem 12 of Module 2 practice problem set.

(b) $\|A\|_2 = \|A^{\mathrm{T}}\|_2$.

4. (Vandermonde matrix, SVD, and rank) Let $\mathbf{x}$ be a vector of 1000 equally spaced points between 0 and 1, and let A_n be the $1000 \times n$ Vandermonde-type matrix whose (i, j) entry is x_i^{j-1} for $j = 1, \ldots, n$.

   (a) Print out the singular values of A_1, A_2, and A_3.

   (b) Make a semi-log plot of the singular values of A_{25}.

   (c) Use `rank` to find the rank of A_{25}. How does this relate to the graph from part (b)? You may want to use the help document for the `rank` command to understand what it does.