Updated on June 22, 2021

Math 3607: Homework 3

Due: 10:00PM, Wednesday, June 30, 2021

TOTAL: 30 points

1. Do **LM** 9.3–3(a).

2. Do **LM** 9.3–11.

3. (Inverting hyperbolic cosine; **FNC** 1.3.6) The function

$$x = \cosh(t) = \frac{e^t + e^{-t}}{2}$$

can be inverted to yield a formula for $\operatorname{acosh}(x)$:

$$t = \log\left(x - \sqrt{x^2 - 1}\right) \qquad (\star)$$

where $\log(\cdot)$ denotes the natural logarithmic function $\ln(\cdot)$. In MATLAB, let `t=-4:-4:-16` and `x=cosh(t)`.

   (a) Find the condition number of the problem $f(x) = \operatorname{acosh}(x)$. (You may use Equation $(\star)$, or look up a formula for f' in a calculus book.) Evaluate κ_f at the entries of `x` in MATLAB.

   (b) Use Equation $(\star)$ on `x` to approximate `t`. Record the accuracy of the answers (by displaying absolute and/or relative errors), and explain. (Warning: Use `format long` to get enough digits or use `fprintf` with a suitable format.)

   (c) An alternate formula for $\operatorname{acosh}(x)$ is

$$t = -2\log\left(\sqrt{\frac{x+1}{2}} + \sqrt{\frac{x-1}{2}}\right). \qquad (\dagger)$$

   Apply Equation $(\dagger)$ to `x` and record the accuracy as in part (b). Comment on your observation.

   (d) Based on your experiments, which of the formulas $(\star)$ and $(\dagger)$ is unstable? What is the problem with that formula?