Math 3607: Homework 2

Due: 10:00PM, Wednesday, June 23, 2021

TOTAL: 30 points

1. (Gap of 10) Simulate the tossing of a *biased* coin whose tails is 3 times more likely to be showing than its heads, until the gap between the number of heads and that of tails reaches 10.

2. (Sequences Converging to π) Each of the following sequences converges to π:

$$a_n = \frac{6}{\sqrt{3}} \sum_{k=0}^{n} \frac{(-1)^k}{3^k(2k+1)},$$
$$b_n = 16 \sum_{k=0}^{n} \frac{(-1)^k}{5^{2k+1}(2k+1)} - 4 \sum_{k=0}^{n} \frac{(-1)^k}{239^{2k+1}(2k+1)}.$$

   Write a single script that prints $a_0, \ldots, a_{n_a}$, where n_a is the smallest integer so that $|a_{n_a} - \pi| \leqslant 10^{-6}$ and prints $b_0, \ldots, b_{n_b}$, where n_b is the smallest integer so that $|b_{n_b} - \pi| \leqslant 10^{-6}$.

3. (Birthday Problem) In a group of n randomly chosen people, what is the probability that everyone has a different birthday?

   (a) Find this probability by hand.

   (b) Let $n = 30$. Write a script that generates a group of n people randomly and determines if there are any matches.

   (c) Modify the script above to run a number of simulations and numerically calculate the probability. Try $1000, 10000$, and 100000 simulations. Compare the result with the analytical calculation done in the previous part.

4. (Spiral Triangle to Spiral Polygon; adapted from **LM** [1] 5.9–7, 6.8–34) The following script[2] generates spirals using equilateral triangles as shown in the figure below.

[1] Reference Keys:

- **LM**: *Learning MATLAB, Problem Solving, and Numerical Analysis Through Examples* (Overman)
- **NCM**: *Numerical Computing with MATLB* (Moler)
- **FNC**: *Fundamentals of Numerical Computation* (Driscoll and Braun)

[2] It is slightly modified from the one found on p. 143 of Module 1 lecture slides. Note the introduction of a new variable `d_rot`, which is accountable for the rotation of the innermost triangle.

```
m = 21; d_angle = 4.5; d_rot = 90;
th = linspace(0, 360, 4) + d_rot;
V = [cosd(th);
     sind(th)];
C = colormap(hsv(m));
s = sind(150 - abs(d_angle))/sind(30);
R = [cosd(d_angle) -sind(d_angle);
     sind(d_angle) cosd(d_angle)];
hold off
for i = 1:m
    if i > 1
        V = s*R*V;
    end
    plot(V(1,:), V(2,:), 'Color', C(i,:))
    hold on
end
set(gcf, 'Color', 'w')
axis equal, axis off
```

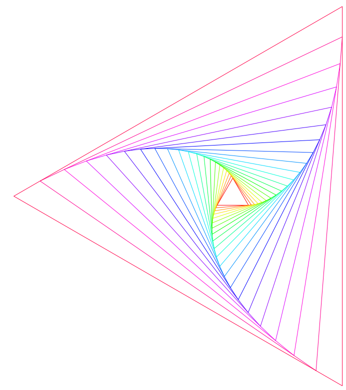

Figure 1: A spiral triangle with $m = 21$ and $\theta = 4.5°$.

(a) Modify the script so that it generates spirals using m regular n-gons for any $n \geqslant 3$. Then turn the script into a function m-file `spiralgon.m`.

```
function V = spiralgon(n, m, d_angle, d_rot)
% SPIRALGON plots spiraling regular n-gons
% input:   n = the number of vertices
%          m = the number of regular n-gons
%          d_angle = the degree angle between successive n-gons
%          (can be positive or negative)
%          d_rot = the degree angle by which the innermost n-gon
%                  is rotated
% output:  V = the vertices of the outermost n-gon
....
```

(b) Run the statements below to generate some aesthetic shapes.

```
clf
subplot(2, 2, 1), spiralgon(3, 41, 4.5, -90);
subplot(2, 2, 2), spiralgon(4, 37, -2.5, 45);
subplot(2, 2, 3), spiralgon(5, 61, 3, -90);
subplot(2, 2, 4), spiralgon(8, 91, -4, 22.5);
```