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Math 3607: Homework 7

Due: 10:00PM, Wednesday, July 28, 2021

TOTAL: 30 points

You will be writing some MATLAB functions for this assignment. Include all your functions at the end of your live script.

Problems marked with ✎ are to be done by hand; those marked with 💻 are to be solved using a computer.

1. (Annuity with `fzero`; **FNC** 4.1.4) A basic type of investment is an annuity: One makes monthly deposits of size P for n months at a fixed annual interest rate r, and at maturity collects the amount
$$\frac{12P}{r}\left(\left(1+\frac{r}{12}\right)^n - 1\right).$$
Say you want to create an annuity for a term of 300 months and final value of $1,000,000. Using `fzero`, make a table of the interst rate you will need to get for each of the different contribution values $P = 500, 550, \ldots, 1000$.

2. (Lambert's W function; **FNC** 4.1.6) Lambert's W function is defined as the inverse of xe^x. That is, $y = W(x)$ if and only if $x = ye^y$. Write a function `y = lambertW(x)` that computes W using `fzero`. Make a plot of $W(x)$ for $0 \le x \le 4$.

3. (Fixed-point iteration; adapted from **FNC** 4.2.1 and 4.2.2.) In each case below,
   - $g(x) = \frac{1}{2}\left(x + \frac{9}{x}\right)$, $r = 3$.
   - $g(x) = \pi + \frac{1}{4}\sin(x)$, $r = \pi$.
   - $g(x) = x + 1 - \tan(x/4)$, $r = \pi$.

   (a) ✎ Show that the given $g(x)$ has a fixed point at the given r and that fixed point iteration can converge to it.

   (b) 💻 Apply fixed point iteration in MATLAB and use a log-linear graph (using `semilogy`) of the error to verify linear convergence. Then use numerical values of the error to determine an approximate value for the rate σ.

4. (Convergence of Newton's method) ✎ Answer the following questions *by hand*, without using MATLAB.

   (a) Discuss what happens when Newton's method is applied to find a root of
   $$f(x) = \operatorname{sign}(x)\sqrt{|x|},$$
   starting at $x_0 \neq 0$. [1]

[1] $\operatorname{sign}(x)$ is 1 if $x > 0$, -1 if $x < 0$, and 0 if $x = 0$.

(b) In the case of a multiple root, where $f(r) = f'(r) = 0$, the derivation of the quadratic error convergence is invalid. Redo the derivation to show that in this circumstance and with $f''(r) \neq 0$ the error converges only linearly.