Cylinder wake model

We explain how the cylinder wake model is implemented, and we go over the related modifications in the BEM equations.

The purpose of this model is to account for radial flow components, and wake skewness. It can be seen as a generalization of other skewed inflow models such as Pitt & Peters, etc. It is based on a cylindrical representation of the wake, as proposed in [Branlard2015] and [Branlard2016]. As argued in [Crawford2006] and [McWilliams2011], the so-derived BEM has improved accuracy for cases with precone and yaw.

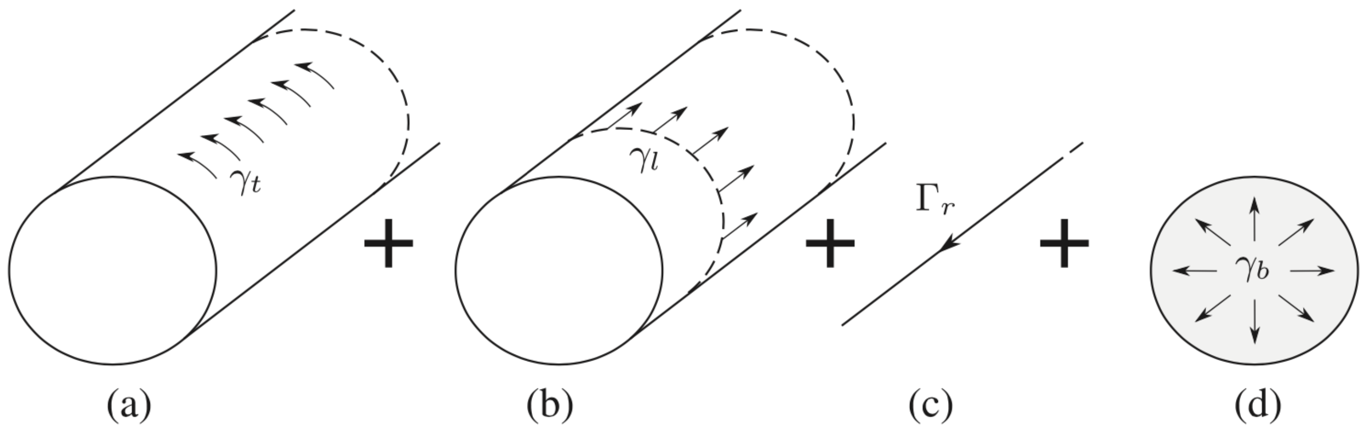
Induced velocities

> **Note**
>
> For details on this section, see [Branlard2016]. Illustrations from this section are also mostly taken from there.

Principle

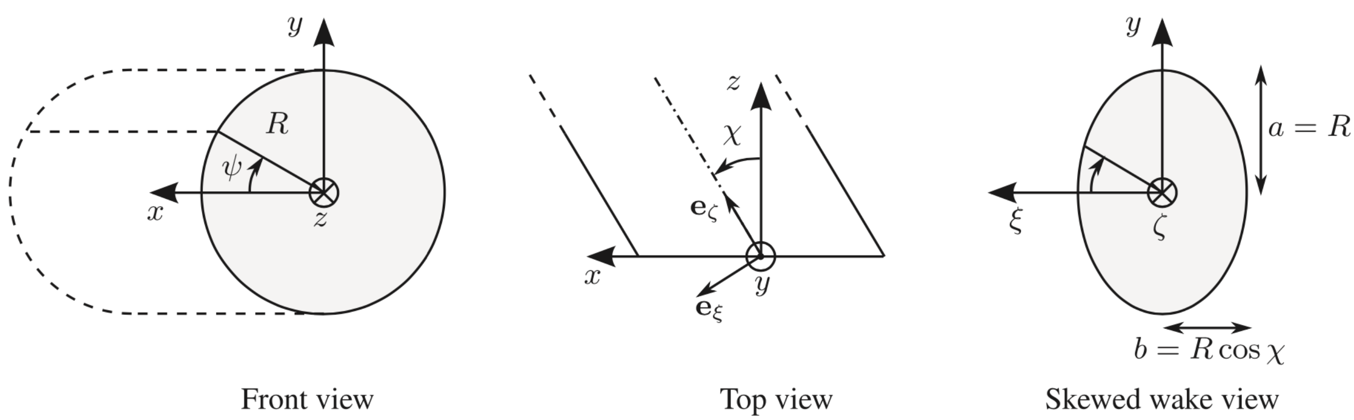
The wake geometry of the wake is assumed fixed. It is composed of a semi-inifinite outer cylindrical vortex sheet of radius `R`, a disk vortex sheet at the location of the rotor and a semi-infinite root vortex filament. The disk has a purely radial vorticity component, and the outer cylindrical sheet has two separate components: axial and tangential. The vorticity/circulation in those elements is assumed uniformly constant.

The indice r refers to the root axial vorticity, b to the bound vorticity on the disk, t to the tangential vorticity in the outer sheet, and l to the axial/longitudinal vorticity in the outer sheet.



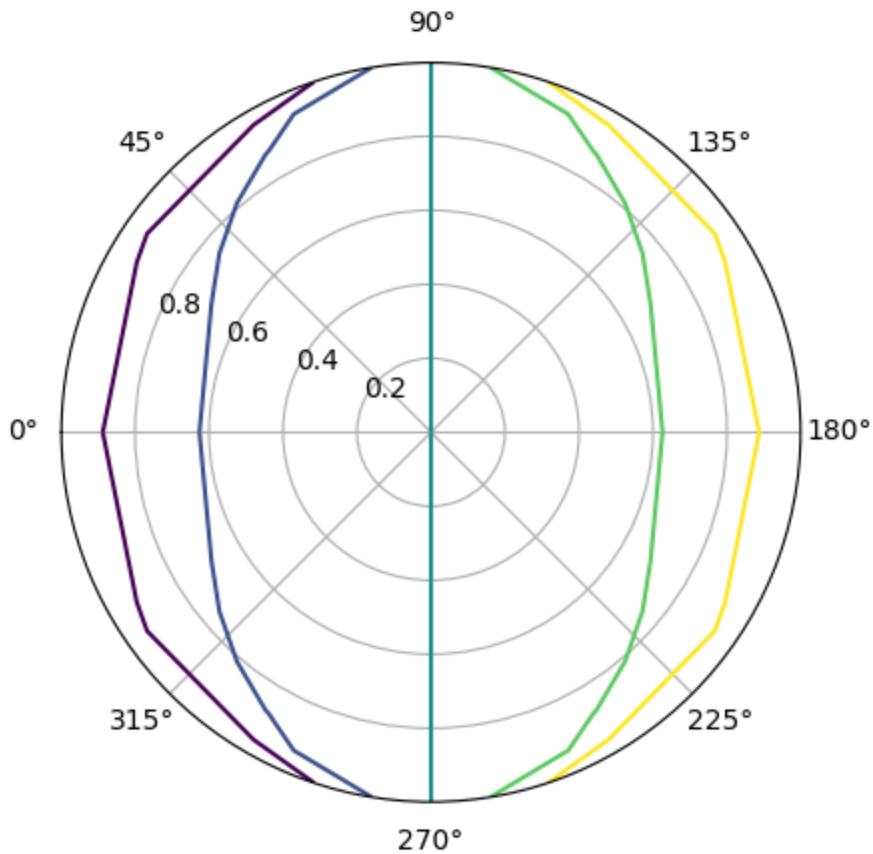
All the vortex instensity components in the different wake objects are related, see next section. For now, let us assume that they are all linearly dependant on γ_t. This will be convenient since the usual result from the Biot-Savart law is $u_{x_0} = \gamma_t/2$, where u_{x_0} is the velocity induced at the edge of a straight circular cylinder with uniform tangential vorticity.

To include yaw, we allow the wake to be skewed by an angle χ with respect to the rotor frame of reference. Mind the coordinate system attached to the rotor, which does not follow wind turbine conventions!

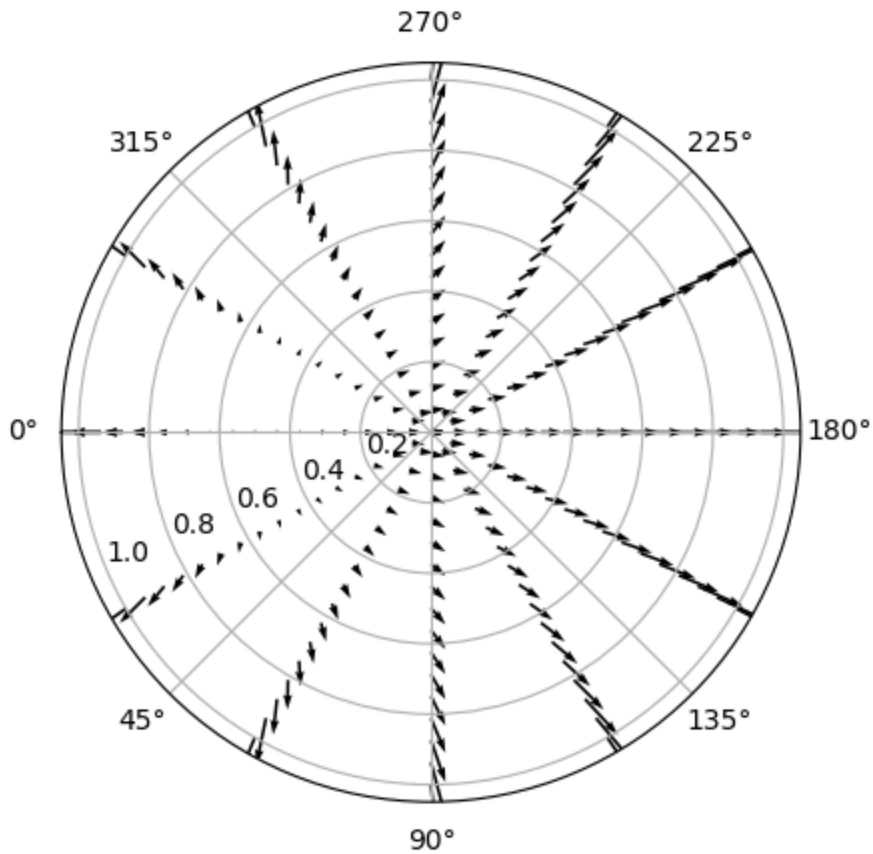


In those conditions, we can compute the velocity induced by each of the vortex component (for a given γ_t) from each object on a point located on the rotor disk. This comes down to numerically integrate [Branlard2016, eq.5-8], which we do with Gauss-Legendre quadrature.

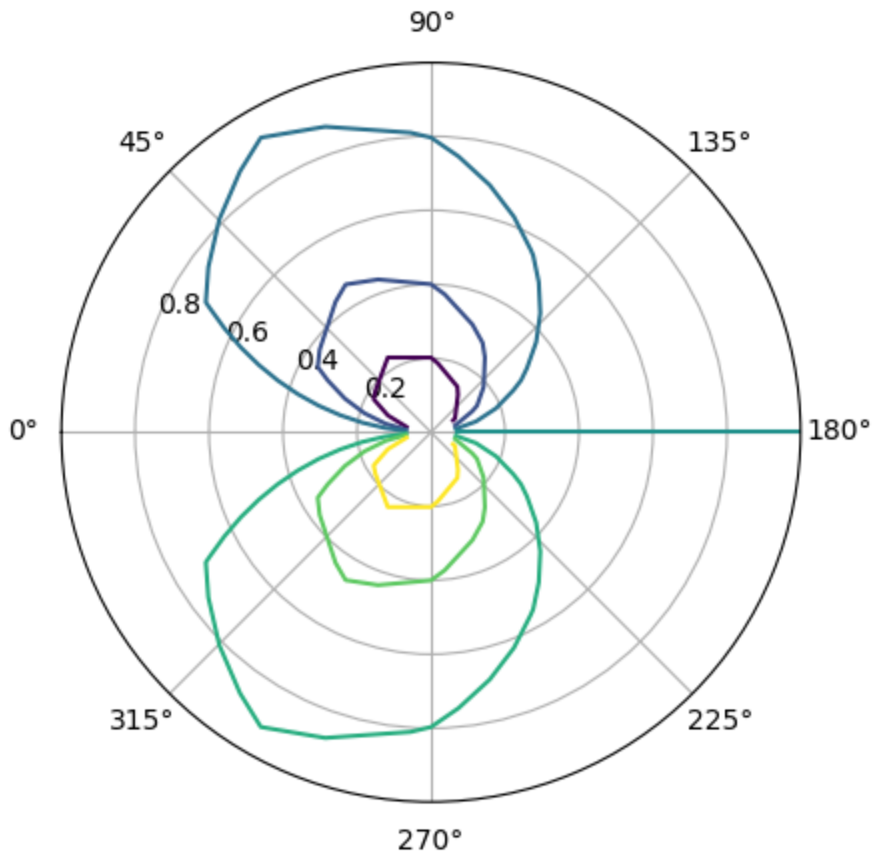
For example, we set $\chi = 30^o$, $\gamma_t = -1$. The axial velocity (z) induced by the tangential component of the outer vorticity cylinder (t) is $u_{z,t}$:



The radial and tangential velocities induced by the tangential component of the outer vorticity cylinder (resp. $u_{r,t}$, $u_{\psi,t}$) are also shown:



Similarly, the axial velocity induced by the root vortex $u_{z,r}$ (assuming $\Gamma_r = -1$):



Note

The dominating wake component on the induced velocities are those from the tangential vorticity in the outer cylinder, and the axial vorticity in the root vortex, see [Branlard2016, sect.4].

Expressions of the induced velocities from the respective wake components

From here on, we resume with standard wind-turbine conventions regarding frame of references.

Crucially, we still assume the independance of annular sections on the rotor (even though it is provably doubtful) we want to keep a simple numerical method. A formal motivation for this is given in [Branlard2015, sect.4.3] for the case of the straight cylinder wake model. In practice this means the following: every anular section has an associated bound vorticity γ_b that corresponds to the circulation of the blades at that location. Neglecting the circulation of all the other anular sections, the vorticity in all other wake components may be expressed as a function of the bound vorticity, by the Kelvin theorem (circulation is conserved). We thus have (see also [Branlard2016, 2.3]):

$$\Gamma_r = -2\pi r \gamma_b$$

$$\gamma_l = -\frac{\Gamma_r}{2\pi R}$$

$$\gamma_t = -\frac{\Gamma_r}{h/cos(\chi)}$$

where h is the pitch of the helix formed by the tangential and axial components in the outer vortex sheet. At high TSR (λ), the latter can be evaluated as $h = \frac{\pi R}{\lambda}(1 + \sqrt{1 - C_T})$ (see [Branlard2015, sect.4] and refs therein).

As mentioned before, we rather express the various vorticity/circulations as a function of γ_t:

$$\Gamma_r = -\gamma_t \frac{h}{\cos\chi}$$

$$\gamma_l = \frac{\gamma_t}{2\pi R} \frac{h}{\cos\chi}$$

$$\gamma_b = \frac{\gamma_t}{2\pi r} \frac{h}{\cos\chi}$$

Note that all are linear in γ_t.

Warning

In the following, we neglect the influence of the bound vorticity on the velocity measured in the rotor plane. This is mainly to decouple the BEM computation from each blade.

The axial, tangential and radial induced velocities at the rotor are

$$u_x = u_{x,t} + u_{x,r} + u_{x,l} \triangleq I_x \gamma_t$$

$$u_\psi = u_{\psi,t} + u_{\psi,r} + u_{\psi,l} \triangleq I_\psi \gamma_t$$

$$u_r = u_{r,t} + u_{r,l} \triangleq I_r \gamma_t$$

Notice that the root vortex never induces radial velocity.

The I factors are the previously-mentioned integrals that only depend on geometric factors: (r, R, ψ, x, χ). In practice, in spite of the blade bending and/or (pre)cone, we always evaluate I at $x = 0$. The reason is that the head of the cylinder model is flat and passes through the rotor hub. Hence, it makes more sense to evaluate induced velocity in the rotor plane. We also neglect the influence of the tilt angle on I.

Danger

TODO: for completeness, write the expressions for $I_.$, and be careful with the change of convention between Branlart and standard WT.

In the far wake, the induced velocities are

$$u_{x_2} = \gamma_t$$

$$u_{\psi_2} = u_{\psi,r}\big|_{x\to\infty} = \frac{\Gamma_r}{2\pi r \cos\chi \cos\Theta} \triangleq I_{\psi_2}\gamma_t$$

$$u_{r_2} = 0$$

since $u_{\psi,l}$ is uniformly zero inside the cylinder wake, and there is no tangential velocity induced by the tangential vorticity component.

Usage in the BEM

We showed that the induced veloctities at the rotor can be expressed as

$$\begin{aligned} u_x &= I_x\gamma_t \\ u_\psi &= I_\psi\gamma_t u_r &= I_r\gamma_t \end{aligned}$$

where, again, the I factors are computed numerically and do not depend on γ

For the sequel, it will be convenient to define `epsilon factors`. The conventional result generally used in BEM corresponds to $\epsilon_x = \epsilon_\psi = 1$. Conventional BEM also neglects u_r (i.e., $\epsilon_r = 0$).

$$\begin{aligned} u_x \epsilon_x &= \gamma_t / 2 \\ u_\psi \epsilon_\psi &= u_{\psi_2} / 2 \\ u_r &= \epsilon_r u_x \end{aligned}$$

such that

$$\begin{aligned} \epsilon_x &= 1/(2 I_x) \\ \epsilon_\psi &= \frac{1}{2} \frac{I_{\psi_2}}{I_\psi} \\ \epsilon_r &= I_r / I_x \end{aligned}$$

Note

Relation to other skewed wake models

As noted in [Branlard2016], the cylinder wake model can be seen as a generalization of other models that have been proposed to treat yaw. The relation between these models and the current one is established by noticing that the axial induced velocity has the form of the model proposed by Glauert:

$$u_x = \frac{\gamma_t}{2}(1 + K \sin\psi)$$

where K has an expression that depends on the model.

For the `Coleman et al.` model for instance,

$$K_{\mathrm{PittPeters}} = \frac{r}{R} \tan(\chi/2)$$

The `Pitt & Peters` model gives,

$$K_{\mathrm{Coleman}} = \frac{r}{R} \frac{15\pi}{32} \tan(\chi/2)$$

although some authors proposed to use a factor $15\pi/64$ instead (see [Ning2015]). Other models can be found in the litterature (see e.g. [Micallef2016])

In the present model, u_x is influenced by the various component of vorticity in the wake (see the above equation $u_x = u_{x,t} + u_{x,r} + u_{x,l}$). In that sense it is more general. However, we can establish a link with the above simple models if we we consider only the velocity induced by the tangential component of the tip vorticity (neglecting $u_{x,r}$ and $u_{x,l}$). Indeed, the velocity induced by the tangential vorticity in the cylinder also has the form $u_{x,t} = \frac{\gamma_t}{2}(1 + K_{x,t} \sin\psi)$, where $K_{x,t}$ takes the form of an integral that needs to be computed numerically. Furthermore, we can show that the linearization of $K_{x,t}$ leads us to recover the exact same expression for K as the `Coleman et al.` model.

Adapted BEM equations

We re-develop the BEM equations following the same logic as in [Ning2021], but for the general case of a turbine with yaw and non-straight/preconed blades. We make use of the definitions of the epsilon factors, and the standard definition of the induction factors:

$$a = \frac{u_x}{V_x} \quad a' = \frac{u_\psi}{V_y}$$

We refer the interested reader to Branlard2015, sect.4.2 for a formal explanation of the relation between the BEM theory and the vortex-induced velocity.

Warning

The sign conventions used here may depend on the type of operation (turbine/propeller). We follow the same definitions as in [Ning2021].

We account for a yaw angle χ_0. At a given radial station r measured along the blade, the local coning and sweep angles are β and s, and the distance to the rotor shaft is $r_a \approx z_a$ (see figures below).

The main idea behind this development is to relate the local forces on the blades (which depend on the 2D aerodynamics expressed in a plane normal to the deflected blade) to the updated momentum equations. **The axial momentum (both axial and angular) is expressed in the direction normal to the rotor plane** (see reference frames hereafter).

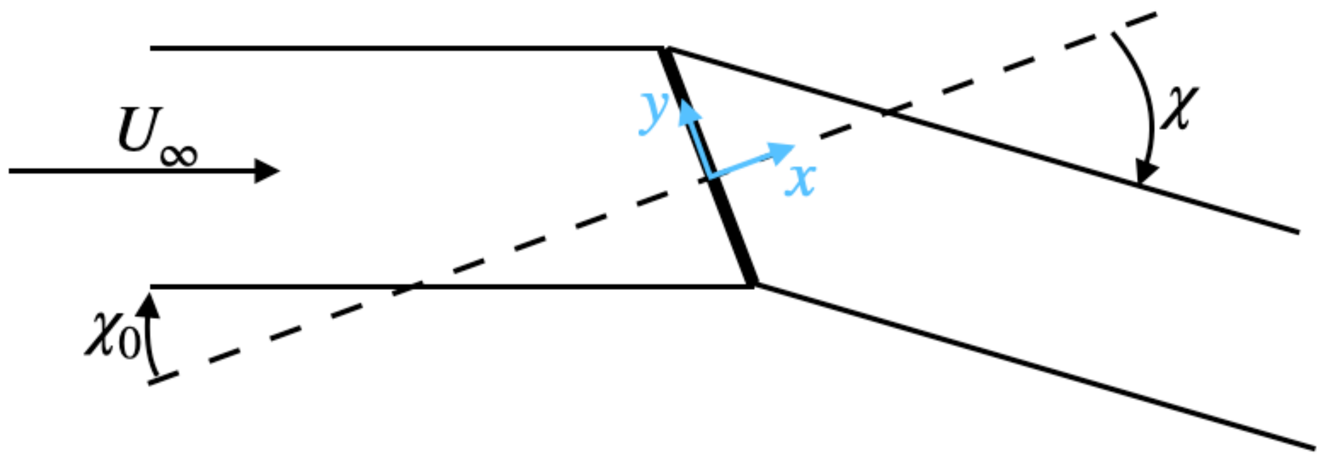
Note

Generally, the yaw angle χ_0 is smaller than the skew angle χ. See references in [Branlard2016, sect.2.1] for relations between them. If we neglect the tilt angle, we can simply use the relation from Burton's Wind Energy handbook [Ning2015, eq.31]

$$\chi = (0.6a + 1)\chi_0$$

Danger

:warning: should clarify if $a = u_x/V_\infty$ or $a = u_{x_2}/V_\infty$ in that formula



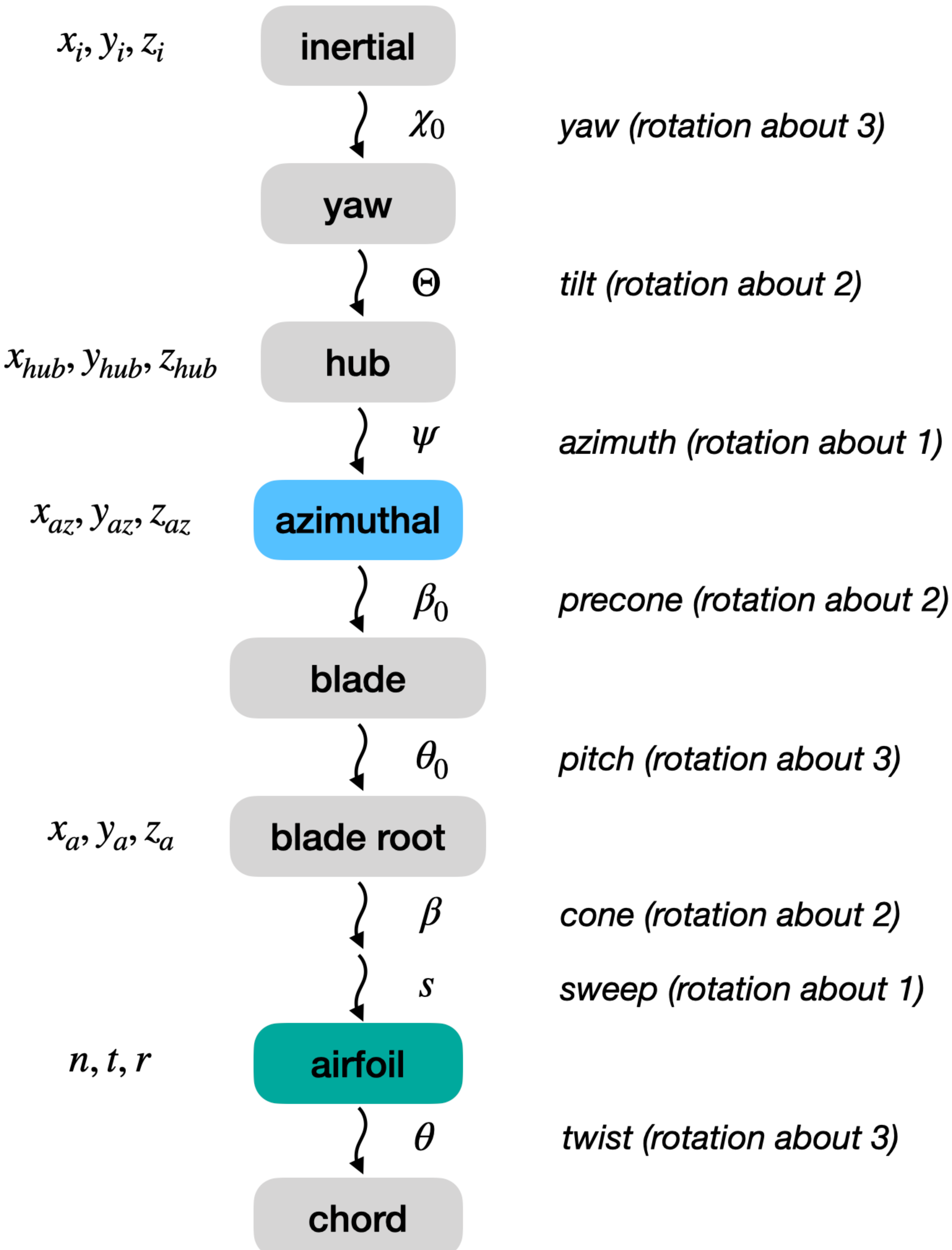
Note

Notations: V_i are external velocities (from the inflow and the rotation); u_i are wake-induced velocities; U_i are the sum of V_i and u_i.

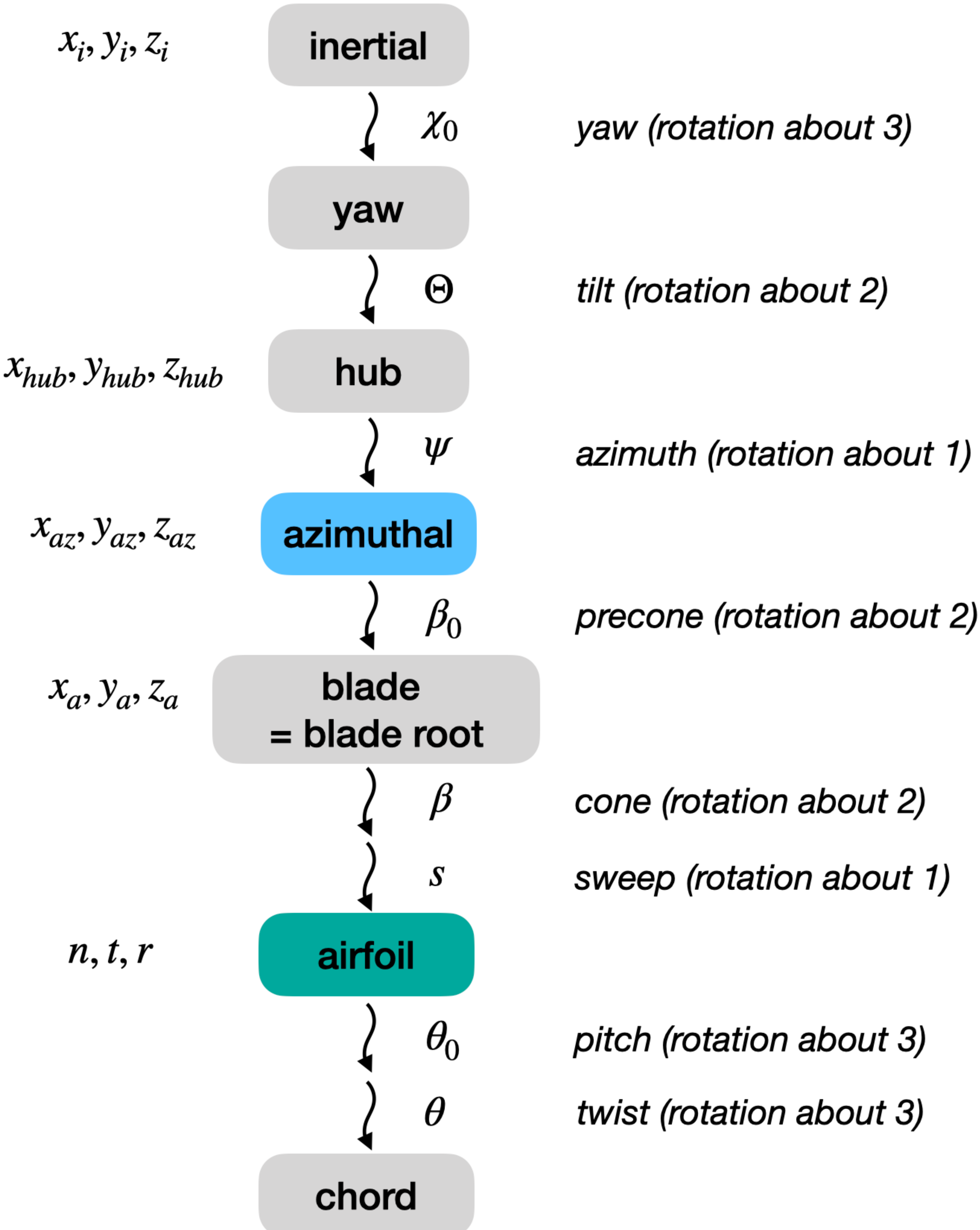
Reference frames

We present some figures modified from [Ning2015].

Looking at the turbine from a multi-body perspective, the chaining between frames and corresponding rotations and angle is the following:

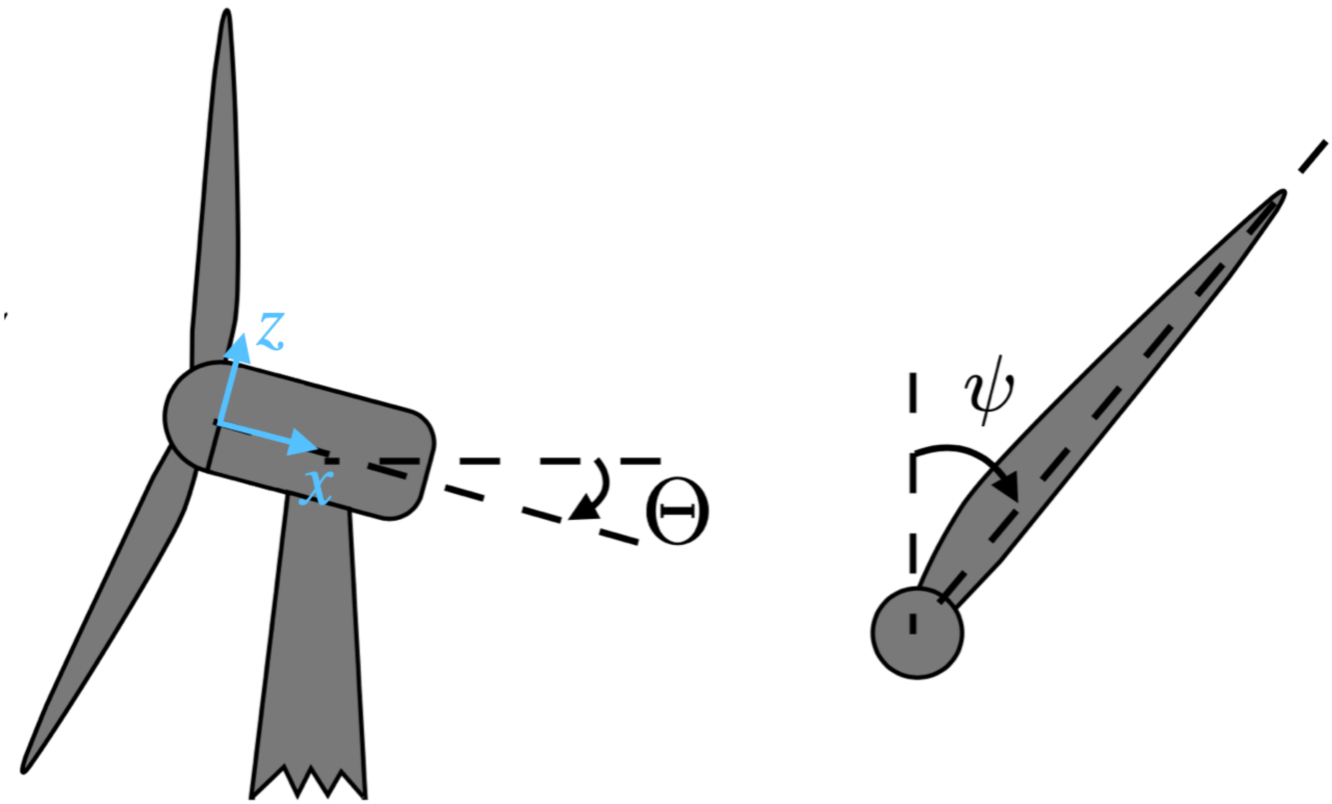

x_i, y_i, z_i
inertial
χ_0
yaw (rotation about 3)
yaw
Θ
tilt (rotation about 2)
$x_{hub}, y_{hub}, z_{hub}$
hub
ψ
azimuth (rotation about 1)
x_{az}, y_{az}, z_{az}
azimuthal
β_0
precone (rotation about 2)
blade
θ_0
pitch (rotation about 3)
x_a, y_a, z_a
blade root
β
cone (rotation about 2)
s
sweep (rotation about 1)
n, t, r
airfoil
θ
twist (rotation about 3)
chord

For the purpose of the present BEM implementation however, we introduce an approximation that corresponds to slightly altering that order, as follows:



What this approximation does is that it considers that the pitch actuation results in uniformly modifying the airfoil pitch angle along the blade span, as if it was locally twisted instead of being 'rigidly' rotated at the root. This essentially affects deformed/non-straight blades, and does not change anything for straight blades. We will see hereafter that this assumption his crucial in decoupling tangential and axial inductions, resulting in equations that can be solved sequentially.

Rotor-related frame:



We will express the momentum in the direction normal to the rotor plane (after yaw and tilt, **before coning and sweep**). That is where our $x, y, z \equiv x_{az}, y_{az}, z_{az}$ coordinate system is defined.

Note

Definitions (caution: tilt and precone are inverted w.r.t. AeroDyn!):

- positive tilt results in rotor facing upwards (upwind turbines have $\Theta > 0$)
- positive precone results in blade leaning forward (upwind turbines have $\beta_0 > 0$)
- positive yaw results in the wind coming from your right if you sit on top of the hub facing the wind

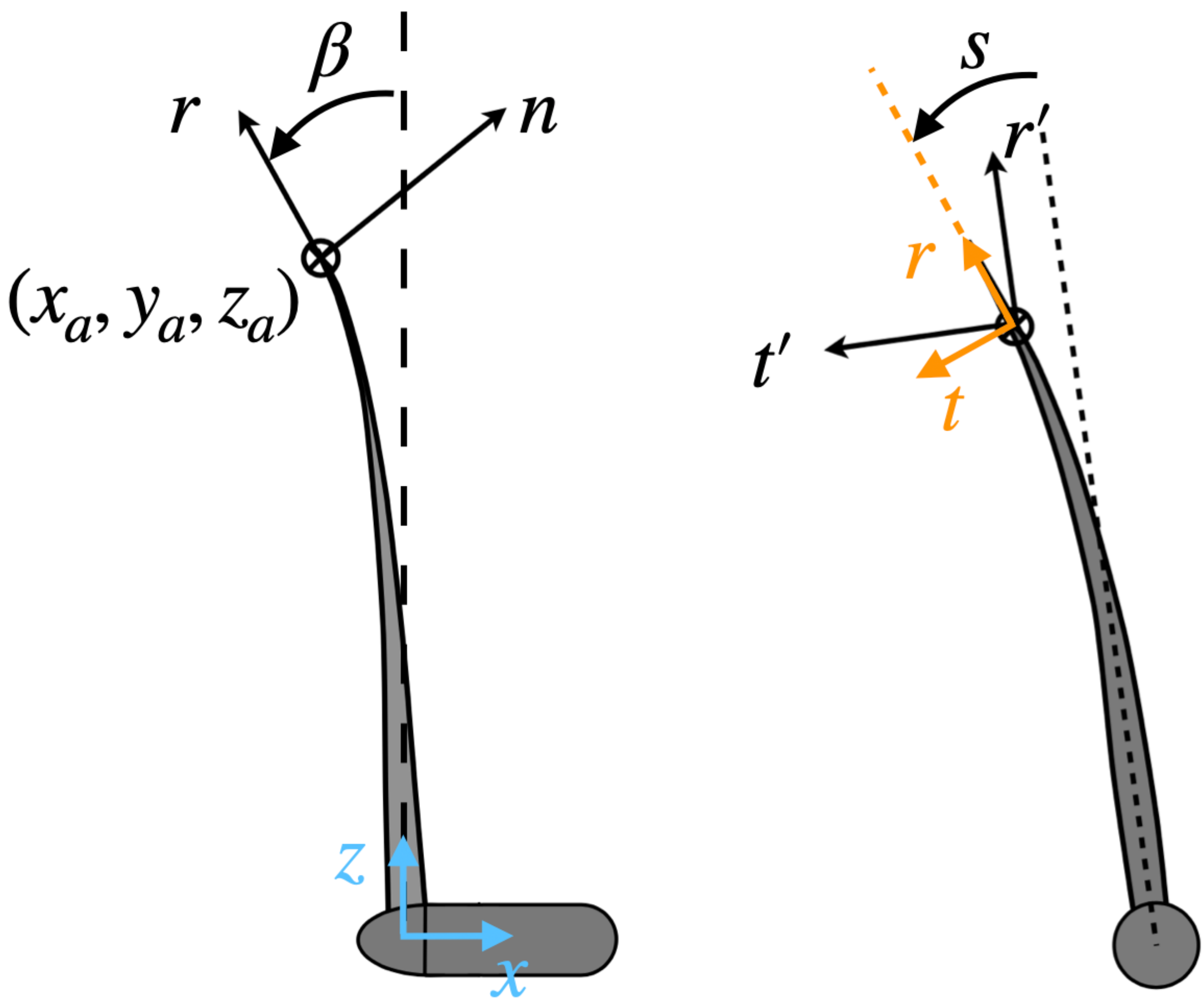
Danger

TODO: check yaw convention throughout the code

(Hence, for $\psi = 0$, $\beta = -\beta_0$ gives the same results $\forall \beta$)

Blade related-frames:

The blade root frame is handy because we expect that any structural code will output the blade deformation in those coordinates. For a straight blade, we will have $z_a = r$ and $x_a = y_a = 0$.



Note that β is the local coning angle (that accounts only for flapwise bending).

Note

Another potentially useful assumption is *sweep through shearing* that replaces r, t with r', t'. This consists in neglecting the change of orientation of the blade related to sweep, leaving r' parallel to z_a.

Danger

Need to assess the necessity of this assumption.

Note

Definitions

- positive conicity is a positive rotation about `y`, that is downwards (i.e., opposed to the precone!)
- positive sweep is a positive rotation about `x`

External velocity at the blade

Given the position of a blade segment in the rotor plane parametrized by $y_{az} = y_a, z_{az} = z_a \cos(\beta_0)$, and rotor info ψ, Θ, χ_0, one can obtain the componnents of the upstream velocity and rotational velocity in the rotor ($x = x_{az}, y = y_{az}, z = z_{az}$) frame.

$$
\begin{aligned}
V_x &= V_\infty \cos(\chi_0) \cos(\Theta) \\
V_y &= V_\infty (\cos(\chi_0) \sin(\Theta) \sin(\psi) - \sin(\chi_0) \cos(\psi)) + \Omega \cos(\beta_0) z_a \\
V_z &= V_\infty (\cos(\chi_0) \sin(\Theta) \cos(\psi) + \sin(\chi_0) \sin(\psi)) - \Omega y_a
\end{aligned}
$$

This is essentially Eq.28 in [Ning2015], but expressing the velocities in the frame *before* precone (and not after), since we are interested in velocities in the rotor plane. V_∞ may include a dependency on z_i to account for shear.

Warning

Theoretically, y_a should be accounted for in the above expression since it may introduce components of rotational velocity in both V_y, V_z. We choose to neglect that effect since the sweep deflection is likely small.

Danger

TODO: CHECK THAT WE NEED THIS, because currently it is coded in the op routine

Note that in the basic BEM, induction factors are defined with respect to V_infty and $R\Omega$ respectively. Since all unsteady effects are here neglected, we express the BEM equations by assuming we can replace these with the instantaneous velocities V_x, V_y.

Rotor mass flow

For an anular section $A_a = 2\pi r_a dr$, assuming local conditions (velocities) apply to the entire annulus:

$$\dot{m} = \rho A_a (V_x + u_x) = \rho V_x A_a (1 + a)$$

Note

Some authors keep the definition of $a = \frac{u_x}{V_\infty}$, which leads to

$$\dot{m} = \rho A_a V_\infty (\cos(\chi_0) \cos(\Theta) + a)$$

and similar expressions in the sequel.

Warning

Is there really a reason to favor one or the other approach?

Thrust coefficient

With the Prandtl loss function F:

$$T = \frac{1}{2} \rho V_x^2 A_a C_T = \dot{m} \Delta V F$$

where the Δ is taken between far-field velocities measured normally to the rotor plane

$$\Delta V = (V_\infty \cos(\chi_0) - (V_\infty \cos(\chi_0) - \gamma_t \cos(\chi))) \cos(\Theta) = \gamma_t \cos(\chi) \cos(\Theta) = 2 u_x \epsilon_x \cos(\chi$$

Thus,

$$C_T = 4a(1+a)\epsilon_x \cos(\chi)\cos(\Theta)F$$

> **Warning**
>
> The latter expression is different from that proposed in [McWilliams2011]: $C_T = 4a(\cos(\chi_0) + \epsilon_x a)\epsilon_x$. It is also different from Glauert's theory, see in [Ning2015]: $C_T = 4sqrt1 + a(2\cos(\chi_0) + a)aF$. All of them collapse to the the propeller-brake/momentum region formula with no yaw $C_T = 4(1+a)aF$ [Ning2021].

Torque coefficient

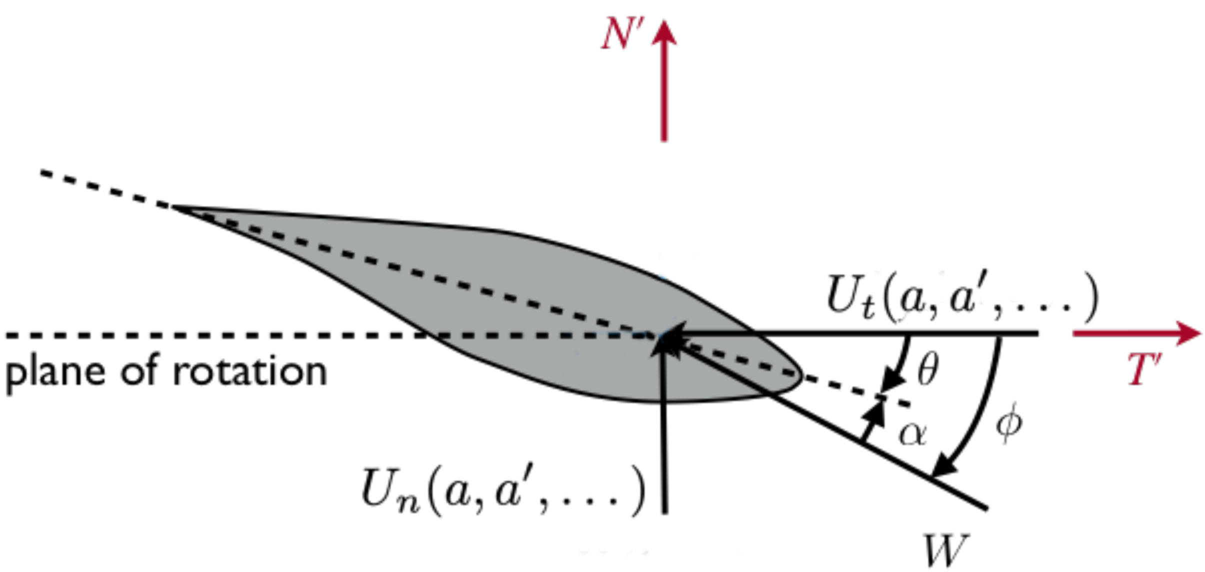
Recalling that u_{ψ_2} is the far field tangential velocity in the cylinder,

$$Q = \frac{1}{2}\rho V_x^2 A_a r_a C_Q = \dot{m} r_a u_{\psi_2} F = \dot{m} r_a 2 u_\psi \epsilon_\psi F$$

$$C_Q = 4a'(1+a)\epsilon_\psi V_y / V_x F$$

Airfoil aerodynamics

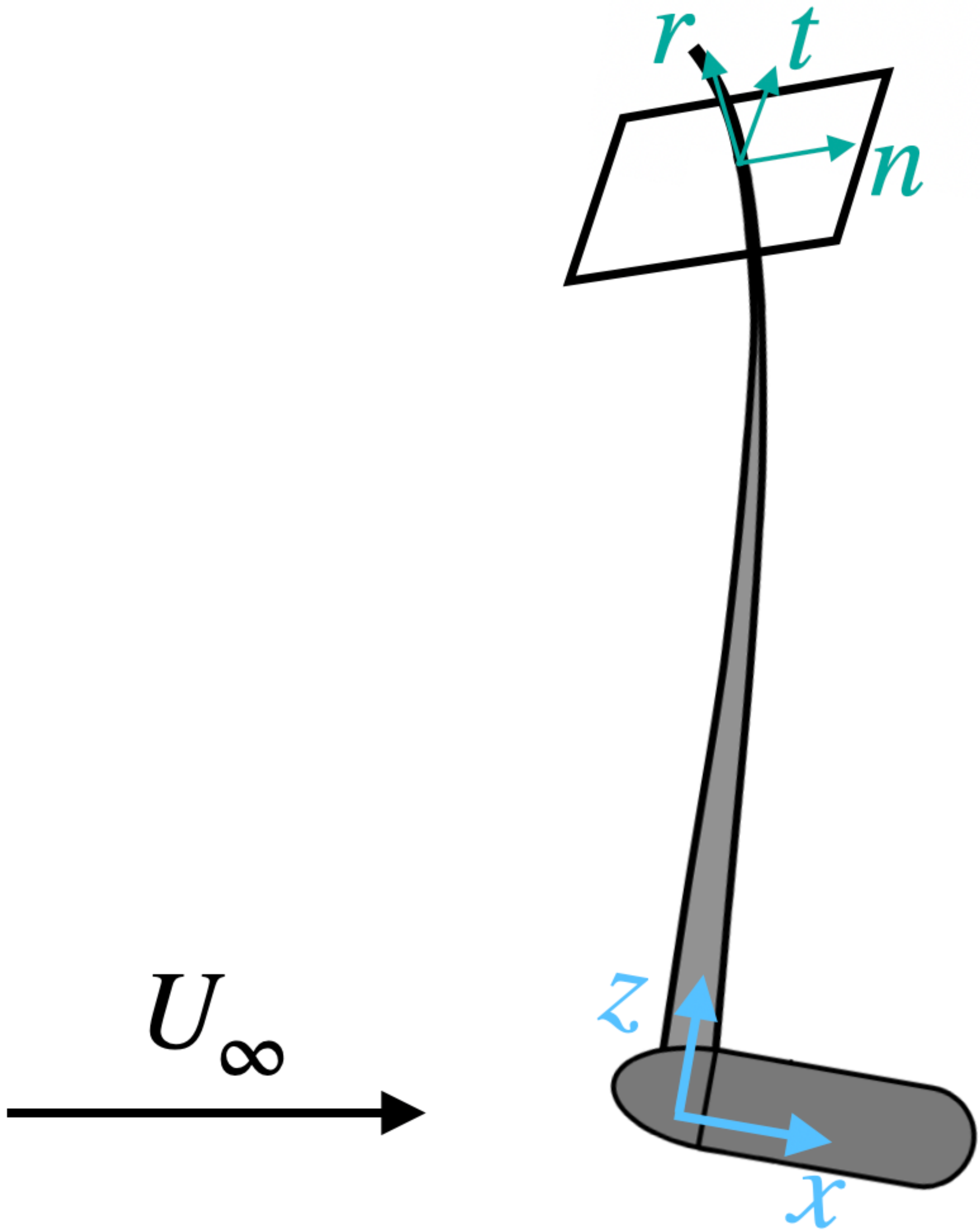
The 2-D aerodynamics has to be expressed in a plane normal to the blade axis, rotated by the angle β and s with respect to the rotor reference plane. In fact, the blade is allowed to deflect, and may have a local coning and sweep angle. The forces on the airfoil are transferred to the rotor reference plane so as to express the axial and angular momentum in that reference plane.



By definition, $\tan(\phi) = \frac{U_n}{U_t}$ that we can rewrite

$$\frac{\sin(\phi)}{U_n} - \frac{\cos(\phi)}{U_t} = 0$$

We need two things to happen: obtain the velocities U_n, U_t in the local blade frame, and transform the forces expressed in the local blade frame to the rotor frame.



The local 2-D aerodynamics yields forces parallel to the normal and tangential direction:

$$f_n = \frac{1}{2}\rho W^2 c c_n(\phi) dr$$
$$f_t = \frac{1}{2}\rho W^2 c c_t(\phi) dr$$

and we need to express these forces in the coordinate system associated with the rotor disk

$$f_x = \frac{1}{2}\rho W^2 c c_x(c_n, c_t, \beta_0, \theta_0, \beta, s) dr$$
$$f_y = \frac{1}{2}\rho W^2 c c_y(c_n, c_t, \beta_0, \theta_0, \beta, s) dr$$

where c_x, c_y are coordinate transformations (blade to rotor).

Similarly, we need to express W, U_n, U_t as a function of the velocities in the rotor c.s.:

$$[U_n, U_t, U_r]^T = A[U_x, U_y, U_z]^T = A[V_x(1+a), V_y(1-a'), V_z + V_x a \epsilon_r]^T$$

where A is the rotation matrix between the rotor frame and the local blade frame (that includes precone β_0, pitchθ_0, coning β, sweep s):

$$A = \begin{pmatrix} 1 & 0 & \\ 0 & \cos s & \sin s \\ 0 & -\sin s & \cos s \end{pmatrix} \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix} \begin{pmatrix} \cos\theta_0 & \sin\theta_0 & 0 \\ -\sin\theta_0 & \cos\theta_0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\beta_0 & 0 & - \\ 0 & 1 & \\ \sin\beta_0 & 0 & \end{pmatrix}$$

Warning

In theory, a' is related to the tangential velocity component V_ψ which may slightly differ from V_y when the blade center line has a non-zero y in the azimuthal frame (i.e., the blade is not purely radial). However, we here neglect this effect, assuming that $V_\psi = V_y$ and that the rotational induced velocity is similarly obtained $u_y = u_\psi = V_y a'$. This is the main reason for invoking the previously mentioned hypothesis of sweep though shear.

The full expression of the total velocities in the airfoil frame reads

$$\begin{bmatrix} U_n \\ U_t \end{bmatrix} \begin{pmatrix} \cos\beta\cos\theta_0\cos\beta_0 - \sin\beta\sin\beta_0 & \cos\beta\sin\theta_0 & -\cos\beta\cos\theta_0\sin\beta_0 - \sin\beta\cos\beta_0 \\ \dots & \dots & \dots \end{pmatrix}$$

Note

What we obtain at this stage is a system where we can hardly obtain an expression of U_n or U_t as a function of only a or a'. This will prevent the expression of the residual under the form of a single equation, as a function of ϕ. However, we notice that if we neglect A_{12}, the expression of U_n does not depend on a' which effectively decouples the system. Similarly, we may want to have that U_t only depends on V_y. We will thus arbitrarily set $\sin\theta_0 = 0$, which achieves both objectives.

We further neglect the sweep angle in this coordinate transformation, to simplify the expressions and because sweep angle is generally small.

Warning

For consistency, we also neglect the sweep in c_x, c_y..

With these assumtions, we obtain

$$
\begin{aligned}
U_n &= (\cos\beta\cos\theta_0\cos\beta_0 - \sin\beta\sin\beta_0)V_x(1+a) - (\cos\beta\cos\theta_0\sin\beta_0 + \sin\beta\cos\beta_0)(V_z \\
U_t &= \cos\theta_0 V_y(1-a')
\end{aligned}
$$

Thus,

$$
\begin{aligned}
U_n &= p_1 V_x(1+a) + p_2(V_z + V_x a\epsilon_r) \\
U_t &= p_3 V_y(1-a')
\end{aligned}
$$

Note

With unsteady flexible blades, the displacement velocity of the blades expressed in the airfoil frame, v_n, v_t, may be added directly here:

$$
\begin{aligned}
U_n &= \ldots - v_n \\
U_t &= \ldots - v_t
\end{aligned}
$$

This only slightly modifies the BEM equations hereunder.

Danger

Maybe more than slightly... Consider doing $a(V_x + v_x) = u_x$, i.e. add deflection velocity to the external velocities?

BEM equations

We equate the momentum equations and the local 2D aerodynamics, in order to obtain a expression for a, a' as a function of ϕ. Then, we can use the 1-residual equation

$$R(\phi) = \frac{\sin(\phi)}{U_n} - \frac{\cos(\phi)}{U_t} = 0$$

Also, $W = \frac{U_n}{\sin\phi} = \frac{U_t}{\cos\phi}$.

We equate the thrust coefficient deduced from axial momentum and from the airfoil aerodynamics:

$$\frac{a(1+a)}{((p_1 V_x + p_2 V_z) + (p_1 + p_2 \epsilon_r) V_x a)^2} = \frac{1}{\epsilon_x \cos(\chi) \cos(\Theta) F V_x^2 \sin^2\phi} \frac{c c_x}{8\pi z_a} \triangleq \kappa$$

We can then solve for a:

$$\frac{a(1+a)}{(b_1 + b_2 a)^2} = \kappa$$

$$a = \frac{-2b_1 b_2 \kappa + 1 \pm \quad 4b_1^2\kappa - b_1 b_2 \kappa + 1}{2(b_2^2\kappa - 1)} \quad \text{with} \pm b_2 \quad 4b_1^2\kappa - b_1 b_2\kappa + 1 - 2b_1 + b_2 \neq 0$$

Warning

There are two roots to this equation. Hopefully, one can be ruled out from physics considerations. This has to be verified, though.

Knowing a, the tangential equilibrium yields ...

$$\frac{a'}{1-a'} = \frac{b_1 + b_2 a}{V_x(1+a)} \frac{p_3}{4\epsilon_\psi F \sin\phi\cos\phi} \frac{c c_y}{2\pi z_a}$$

$$\triangleq \kappa'$$

that we can easily invert for a' "as usual".

> **Note**
>
> **Relation with the original BEM** The main difference to remember is that, with the present expression of the BEM equation, everything is written in the blade azimuthal frame whereas the original implementation considered the thrust in the blade root frame (i.e. normal to the blade and not normal to the rotor). We
>
> However, by considering the following changes, the present implementation reverts to the original CCBlade implementation:
>
> - the wake epsilon factor take their original value: $\epsilon_x = \epsilon_\psi = 1, \epsilon_r = 0$
> - tilt and yaw do not enter the in κ, κ': $\cos(\Theta) = \cos(\chi) = 1$,
> - `sigma_p = B*chord/(2.0*pi*r)` where `r` does not account for precone.
> - the velocity normal to the blade is `Vx = (cos(precone) * Vx_az - sin(precone) * Vz_az)`. Everywhere else, we set `p1 = 1, p2 = 0` such that `cn = cx, ct = cy, b2 = b1 = Vx`
> - the local thrust (`Np`) is rotated by the precone angle before the integration of the rotor thrust.
>
> For backward compatibility, this remains the default behavior of the code. The new implementation is turned on only when passing `wakeCyl = true` to the rotor object.

Summary of the assumptions

- we neglect the influence of the bound vortices of the other blades on the current blade. This is valid if all the blade have the same circulation, which is not exactly the case in yaw or with shear.
- we neglect the influence of the longitudinal vorticity components of the tip cylinder on the velocity measured normal to the disk
- we still assume the independance of each annular section (theoretically only valid for no yaw, no cone, and high TSR [Branlard2015]). Otherwise, the determination of the axial induction at a given radial station would depend on all the other sations, requiring to solve a large system of equations (see also [Branlard2015, sect.4.3]).
- the wake geometry is assumed as explained above: no wake expansion, plus the wake vorticity is only shed at the blade root and the blade tip. The wake expansion could be taken into account by a discretization in the axial direction [Crawford2006] and iterations... But anyway, the wake expansion results in an increase of induction near the tip, which is overtaken by the Prandtl tip correction.
- the evaluation of the pitch of the helix h assumes high TSR
- we neglect the blade flap/coning angle when evaluating the `epsilon` factors. The reason is that

the cylider wake model is centered on the rotor hub, and does not come forward when the blades bend.

- we neglect the wake redirection by the tilt angle (i.e., the wake is parallel to the ground). Even more, we consider that the integrals I in the computation of the induced velocities do not depend on Θ.
- we neglect the sweep angle in the computation of the normal and tangential velocities (this angle should be small anyway) *This assumption might be unnecessary.*

TODO

- [ ] write the expressions for I
- [ ] checks sign consistency between wake model and BEM: psi, yaw, tilt
- [ ] check sign consistency of a, etc. with Ning2021
- [ ] shall we define a=ux/Uinf or a=ux/Ux ?
- [ ] smoothly connect to the high induction model. Treat the particular cases ofsome missing velocities
- [ ] would it be better to pre-evaluate epsilon and look it up during BEM computation, or can we just keep re-evaluating it on the run?
- [ ] update `openmdao/python/ccblade`
- [ ] test the whole method with con/swp.

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Wake model examples »