Spring 2019 Complex Analysis Preliminary Exam

University of Minnesota

Where possible, computations have been also done using SageMath code available on GitHub at github.com/tekaysquared/p
(feel free to make pull requests!)

2. Write the first three terms of the Laurent expansion of $f(z) = \dfrac{1}{z^5 - 1}$ centered at 0 and convergent in $|z| < 1$.

   Proof. Observe that

   $$\frac{1}{z^5 - 1} = \frac{-1}{1 - z^5} = -\sum_{n=0}^{\infty} z^{5n}$$

   which converges for $|z^5| < 1$ which is $|z|^5 < 1$ or $|z| < 1$. Thus, the first three nonzero terms of the expansion of f are $a_0 = -1$, $a_5 = -1$, and $a_{10} = -1$. □