Fall 2018 Complex Analysis Preliminary Exam

University of Minnesota

Where possible, computations have been also done using SageMath code available on GitHub at github.com/tekaysquared/prelims (feel free to make pull requests!)

2. Write the Laurent expansion of $f(z) = \frac{1}{z^4-1}$ centered at 0 and convergent in $|z| > 1$.

Proof. Factor out z^{-4} to see that

$$f(z) = \frac{1}{z^4(1 - 1/z^4)}$$
$$= \frac{1}{z^4} \sum_{n=0}^{\infty} \frac{1}{z^{4n}}$$

which converges for $|1/z^4| < 1$ which is to say for $|z| > 1$. So then f has a Laurent expansion

$$f(z) = \sum_{n=-\infty}^{\infty} a_n z^n$$

where

$$a_n = \begin{cases} 1 & n = -4k \text{ for nonzero positive integers } k \\ 0 & \text{otherwise} \end{cases}$$

□