Useful theorems for Manifolds and Topology Preliminary Exams

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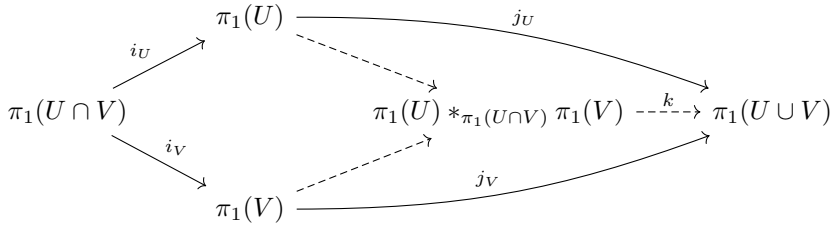
1 Fundamental Group

Theorem (Siefert-van Kampen)**.** *Let U, V be open, path connected topological spaces such that $U \cap V$ is nonempty and path connected. The inclusion maps of $U \hookrightarrow U \cup V$ and $V \hookrightarrow U \cup V$ induce group homomorphisms $j_U : \pi_1(U) \to \pi_1(U \cup V)$ and $j_V : \pi_1(V) \to \pi_1(U \cup V)$. Then $U \cup V$ is path connected, and j_U, j_V form a commutative pushout diagram:*



Since this is a pushout diagram, then k is an isomorphism.

2 Covering Spaces

Theorem. *Let X be path connected, locally path connected, and semilocally simply connected. Then there is a bijection between the set of basepoint-preserving isomorphism classes of path-connected covering spaces $p : (\tilde{X}, \tilde{x}_0) \to (X, x_0)$ and the set of subgroups of $\pi_1(X, x_0)$ obtained by associating the subgroup $p_*(\pi_1(\tilde{X}, \tilde{x}_0))$ to the covering space $(\tilde{X}, \tilde{x}_0)$. If basepoints are ignored, this gives a bijection between isomorphism classes of path-connected covering spaces $p : \tilde{X} \to X$ and conjugacy classes of subgroups of $\pi_1(X, x_0)$.*

Lemma. *If G is an abelian group, then the conjugacy classes of G are all singletons, so if G is finite, then $|G|$ is the number of conjugacy classes.*