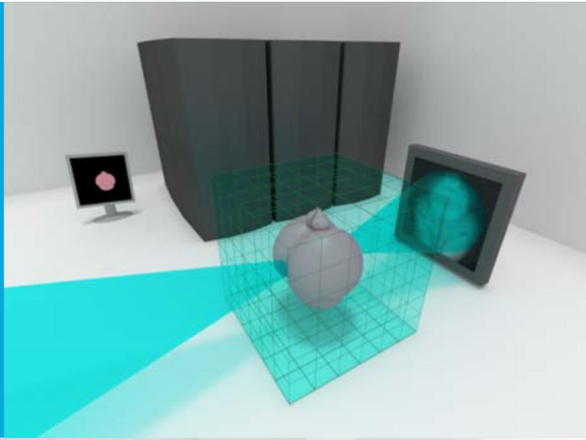
Using automatic differentiation to reconstruct beyond the depth-of-focus
…and more!

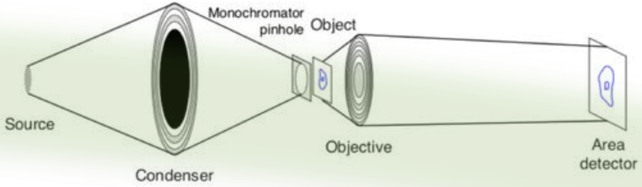


Ming Du
Postdoctoral Appointee
Argonne National Laboratory

November 4, 2019

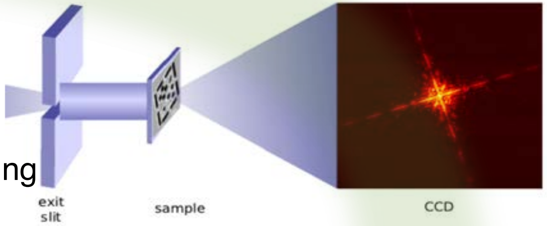
A brief history of x-ray imaging

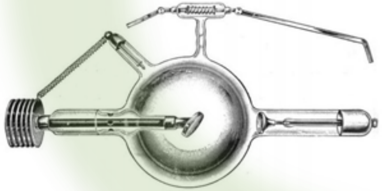
Crookes cathode ray tube



Transmission x-ray microscope

Coherent diffraction imaging





Air ionization x-ray tube

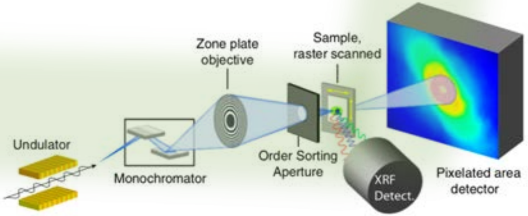
Hot filament x-ray tube

Penetration through several cm

Synchrotron

2D resolution < 20 nm

Ptychography and x-ray fluorescence



Synchrotron with undulator

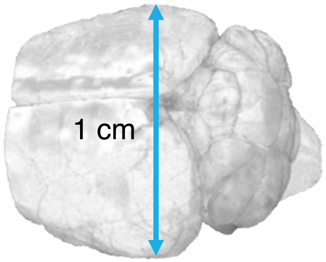
[1] E. Wilson, "Fifty years of synchrotrons," CERN (1996).
[2] J. Deng, D. J. Vine, S. Chen, Y. S. G. Nashed, Q. Jin, N. W. Phillips, T. Peterka, R. Ross, S. Vogt, and C. J. Jacobsen, "Simultaneous cryo X-ray ptychographic and fluorescence microscopy of green algae," Proc Natl Acad Sci USA 112, 2314–2319 (2015).
[3] B. Chen, B. Abbey, R. Dilanian, E. Balaur, G. V. Riessen, M. Junker, C. Q. Tran, M. W. M. Jones, I. McNulty, D. J. Vine, C. T. Putkunz, H. M. Quiney, and K. A. Nugent, "Partial Coherence: a Route to Performing Faster Coherent Diffraction Imaging," J. Phys.: Conf. Ser. 463, 012033–5 (2013).

What are left undone?

Penetration through several cm

+

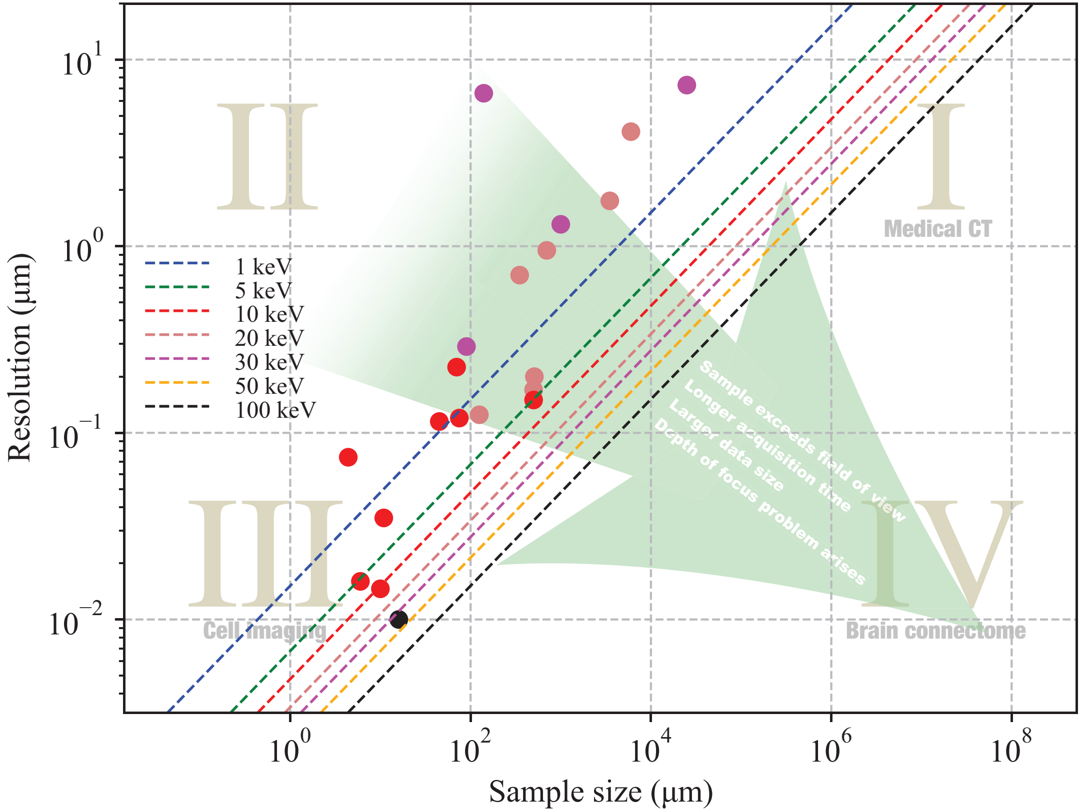
2D resolution < 20 nm



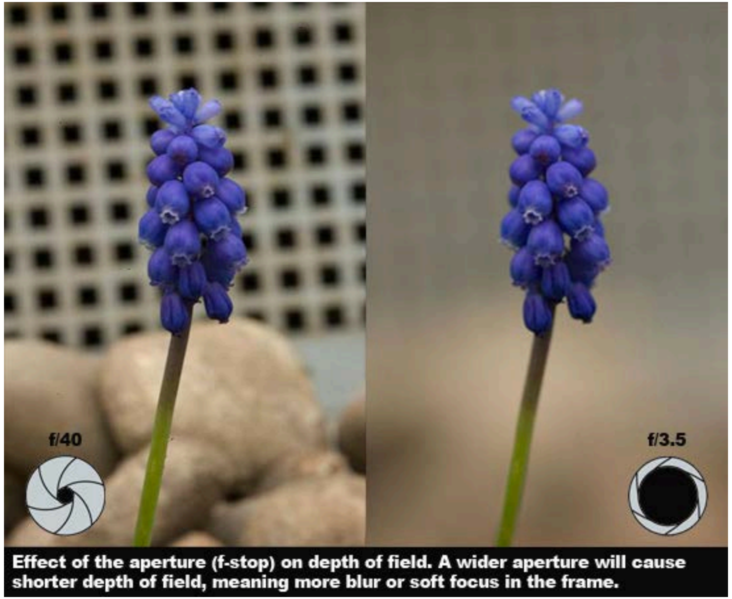
- Better understanding of radiation dose
- Imaging beyond the field of view (FOV)
- Imaging beyond the depth of focus (DOF)

$$\mathrm{DOF} = 5.4\delta_r^2/\lambda$$

For 100 nm resolution, 25 keV, DOF = 1 mm

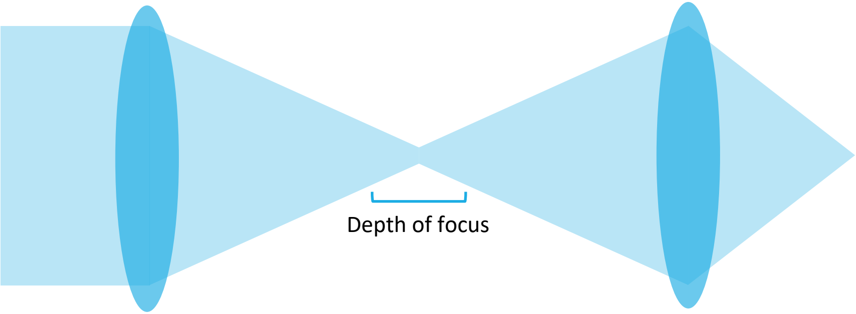


Depth of focus: what is it



http://www.elementsofcinema.com/cinematography/depth-of-field.html

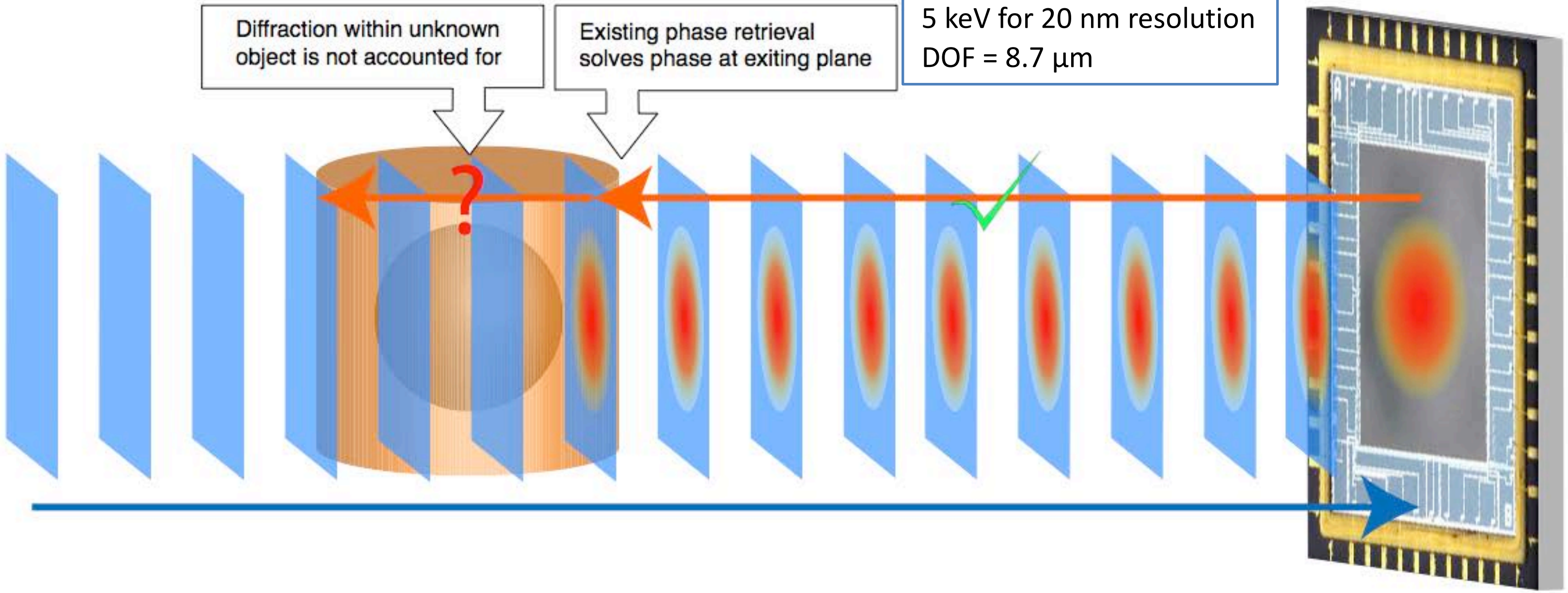
$$\mathrm{DOF} = 5.4\delta_r^2/\lambda$$



In-sample diffraction must be accounted for when t > DOF

$$\mathrm{DOF} = \frac{2}{0.61^2}\frac{\delta_r^2}{\lambda} \simeq 5.4\delta_r\frac{\delta_r}{\lambda}$$ (5.2 prefactor as in Tsai et al., 2016)

5 keV for 20 nm resolution
DOF = 8.7 µm



[1] E. H. R. Tsai, I. Usov, A. Diaz, A. Menzel, and M. Guizar-Sicairos, "X-ray ptychography with extended depth of field," Opt Express 24, 29089–20 (2016).

Maximum likelihood

Realistic imaging is a stochastic process with photon noise following Poisson/Gaussian distribution.

$$p_i(y_i|\boldsymbol{x}) = C \exp\left\{-\frac{[y_i - \bar{y}_i(\boldsymbol{x})]^2}{2\sigma^2}\right\}$$

\# of pixels per view

\# of viewing angles

$$p(\boldsymbol{y}|\boldsymbol{x}) = \prod_{i}^{N_t \times N_\theta} p_i(y_i|\boldsymbol{x}_i)$$

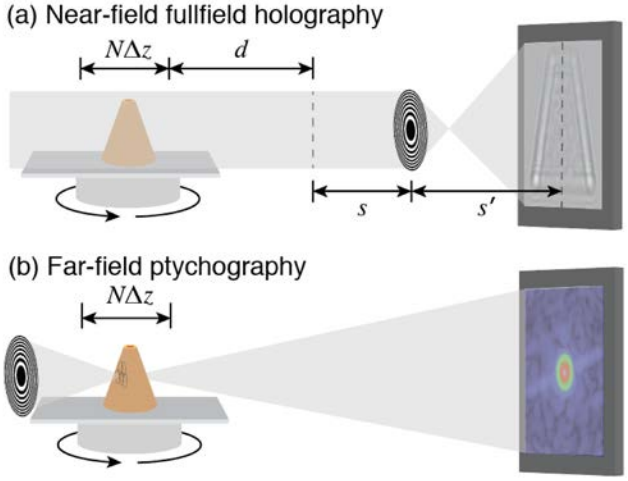
$$\begin{aligned} L(\boldsymbol{x}, \boldsymbol{y}) &= -\ln p(\boldsymbol{y}|\boldsymbol{x}) \\ &= \sum_{i}^{N_t \times N_\theta} -\ln p_i(y_i|\boldsymbol{x}_i) \\ &\propto \sum_{i}^{N_t \times N_\theta} [y_i - \bar{y}_i(\boldsymbol{x})]^2 \end{aligned}$$

The forward model

$$L(\boldsymbol{x},\boldsymbol{y}) = \ln p(\boldsymbol{y}|\boldsymbol{x})$$
$$= \sum_{i}^{N_t \times N_\theta} \ln p_i(y_i|\boldsymbol{x}_i)$$
$$\propto \sum_{i}^{N_t \times N_\theta} [y_i - \boxed{\bar{y}_i(\boldsymbol{x})}]^2$$

Refractive index

$$n = 1 - \delta - i\beta$$



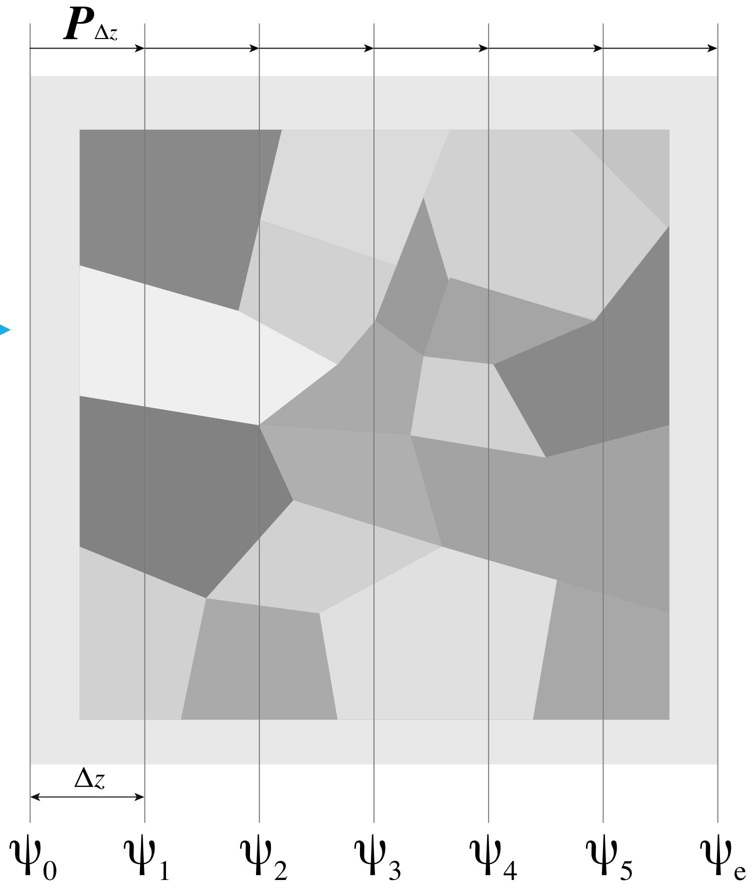
Rotate 0° – 360°

$$f(\boldsymbol{x}) = \bar{\boldsymbol{y}}(\boldsymbol{x}) = \boldsymbol{P}_d \boldsymbol{M}_{\boldsymbol{x}} \psi_0$$

$$f(\boldsymbol{x}) = \bar{\boldsymbol{y}}(\boldsymbol{x}) = \boldsymbol{P}_\infty \boldsymbol{M}_{\boldsymbol{x}} \psi_0$$

Multislice propagation

Fresnel (near-field) propagation



[1] J. M. Cowley and A. F. Moodie, "The scattering of electrons by atoms and crystals. I. A new theoretical approach," Acta Cryst (1957). Q10, 609-619 [doi:10.1107/S0365110X57002194] 10, 1–11 (1957).

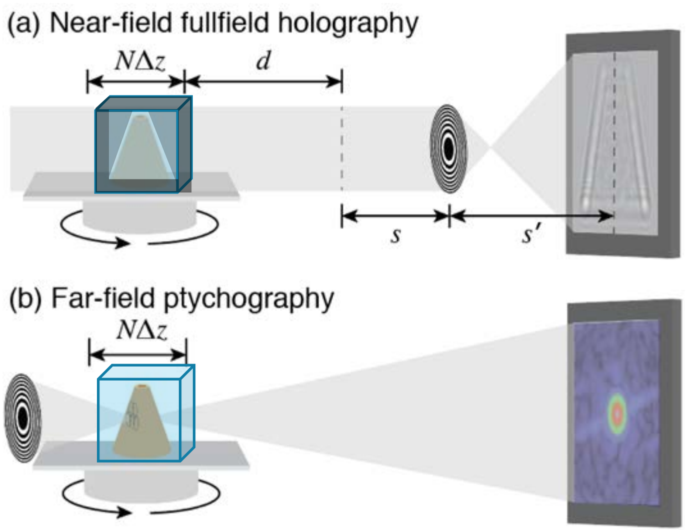
Just something else…

$$n = 1 - \delta - i\beta$$

$$L = \frac{1}{N_\theta N_p N_k} \sum_{\theta,k} \left\| |f(\boldsymbol{x},\theta,k,\Delta z,d)| - \sqrt{\boldsymbol{y}_{\theta,k}} \right\|^2 + \alpha_\delta |\boldsymbol{x}_\delta|_1 + \alpha_\beta |\boldsymbol{x}_\beta|_1 + \gamma \mathrm{TV}(\boldsymbol{x}_\delta)$$

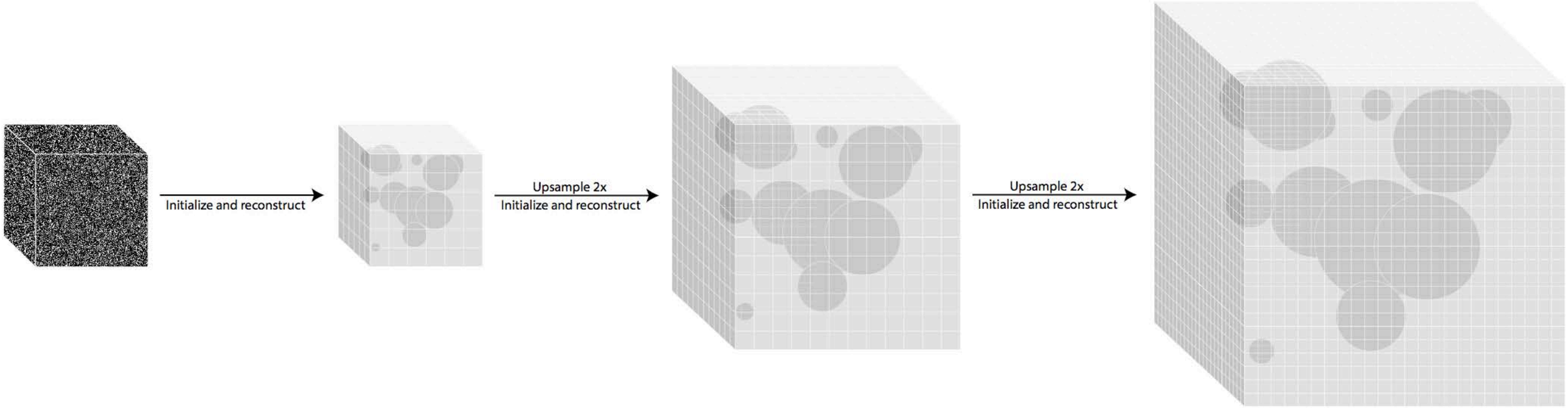
$$\text{subject to } x_w = 0 \text{ for } x_w \notin \Theta \text{ and } x_w \geq 0 \text{ for } x_w \in \Theta.$$

L1 norm	• Object sparsity • Noise and artifact suppression
Total variation	• Object gradient sparsity • Noise and artifact suppression
Non-negativity	• Solution stabilization • Works as long as one avoids anomalous dispersion at an absorption edge
Finite support and shrink-wrap	• For fullfield holography only • Initialized by thresholding conventional reconstruction results • Shrunk by taking out low-value voxels per several iterations



[1] Tibshirani, R. (2011). *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, *73*(3), 273–282.
[2] Horn, R. A. and Johnson, C. R. "Norms for Vectors and Matrices." Ch. 5 in Matrix Analysis. Cambridge, England: Cambridge University Press, 1990.
[3] Sidky, E. Y., & Pan, X. (2008). Physics in Medicine and Biology, 53(17), 4777–4807.
[4] Marchesini, S., He, H., Chapman, H. N., Hau-Riege, S. P., Noy, A., Howells, M. R., et al. (2003). Physical Review B, 68(14), 843–4.

Multiscale reconstruction (frequency advancing)



It’s all about gradient

$$L = \frac{1}{N_\theta N_p N_k} \sum_{\theta,k} \left\| |f(\boldsymbol{x}, \theta, k, \Delta z, d)| - \sqrt{\boldsymbol{y}_{\theta,k}} \right\|^2 + \alpha_\delta |\boldsymbol{x}_\delta|_1 + \alpha_\beta |\boldsymbol{x}_\beta|_1 + \gamma \mathrm{TV}(\boldsymbol{x}_\delta)$$

$$\text{subject to } x_w = 0 \text{ for } x_w \notin \Theta \text{ and } x_w \geq 0 \text{ for } x_w \in \Theta.$$

$$\bar{\boldsymbol{x}} = \underset{\boldsymbol{x}}{\operatorname{argmin}} \{L(\boldsymbol{x})\}$$

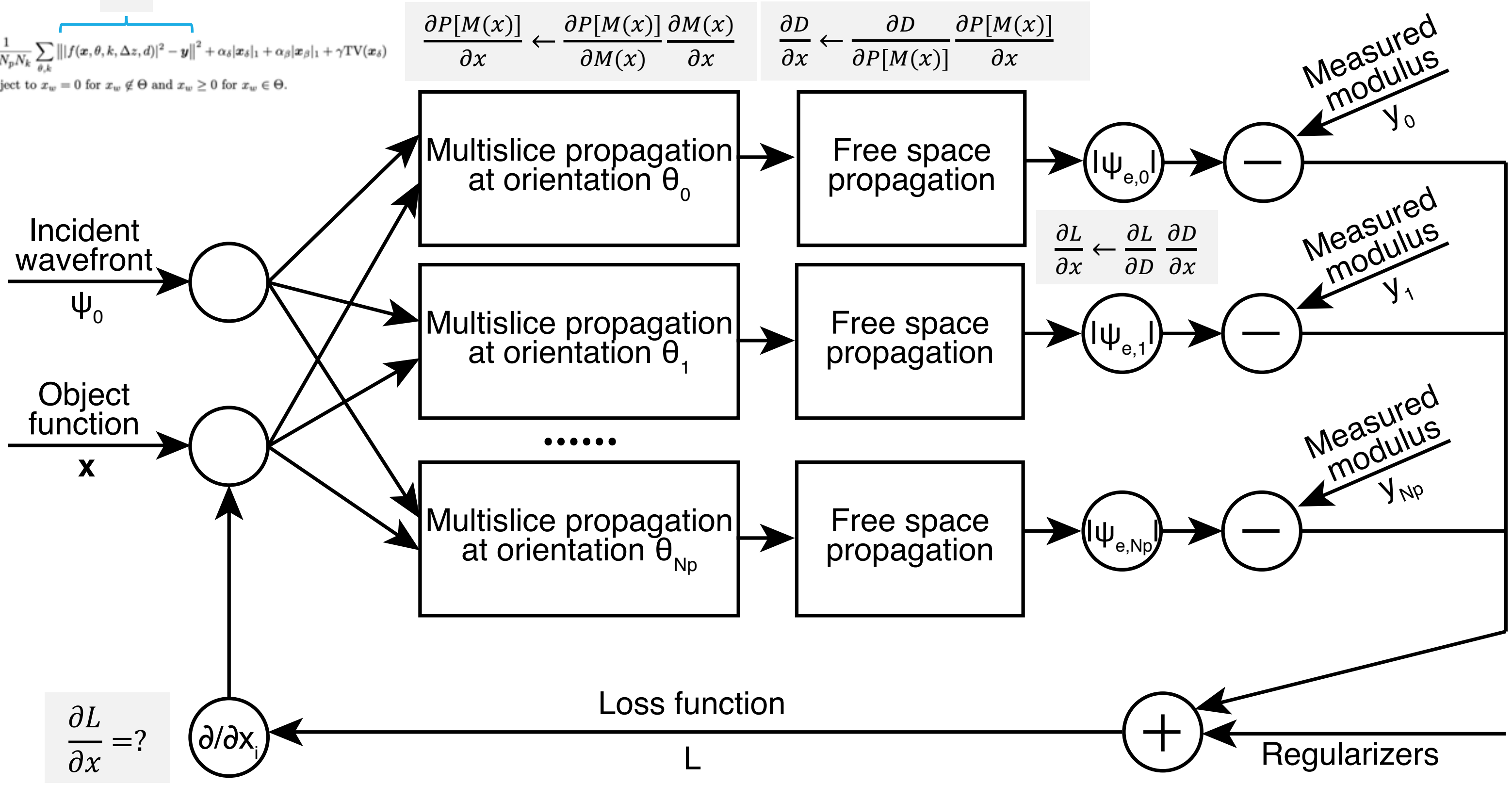
$$\nabla_{\boldsymbol{x}} L = ?$$

$$
\begin{aligned}
L &= c\sum_{\theta,k}\left\| |f(\boldsymbol{x},\theta,k,\Delta z,d)| - \sqrt{y_{\theta,k}}\right\|^2 \\
\nabla_{\boldsymbol{x}} L &= 2c\sum_{\theta,k}\left(|f(\boldsymbol{x},\theta,k,\Delta z,d)| - \sqrt{y_{\theta,k}}\right)^{\mathrm{T}}\nabla_{\boldsymbol{x}} f \\
g_{\theta,k} &= f(\boldsymbol{x},\theta,k,\Delta z,d) = \boldsymbol{P}_d \boldsymbol{M}_{\boldsymbol{x},\theta,\Delta z}\psi_{0,k} \\
\nabla_{\boldsymbol{x}} g_{\theta,k} &= \boldsymbol{P}_d \frac{d\boldsymbol{M}_{\boldsymbol{x},\theta,\Delta z}}{d\boldsymbol{x}}\psi_{0,k} \\
\boldsymbol{M}_{\boldsymbol{x},\theta,\Delta z} &= \prod_j^J \boldsymbol{P}_{\Delta z}\boldsymbol{A}_{\boldsymbol{x},\theta,j} \\
&= \prod_j^J \boldsymbol{P}_{\Delta z}\exp[\mathrm{diag}(\boldsymbol{S}_j\boldsymbol{R}_\theta\boldsymbol{x})] \\
\frac{d\boldsymbol{M}_{\boldsymbol{x},\theta,\Delta z}}{d\boldsymbol{x}} &= \sum_j^J\left\{\prod_{j+1}^J\left\{\boldsymbol{P}_{\Delta z}\exp[\mathrm{diag}(\boldsymbol{S}_j\boldsymbol{R}_\theta\boldsymbol{x})]\right\}\right. \\
&\quad \left\{\boldsymbol{P}_{\Delta z}\exp[\mathrm{diag}(\boldsymbol{S}_j\boldsymbol{R}_\theta\boldsymbol{x})]\,\mathrm{diag}(\mathbf{S}_j\mathbf{R}_\theta)\right\}\prod_0^{j-1}\left.\left\{\mathbf{P}_{\Delta z}\exp[\mathrm{diag}(\mathbf{S}_j\mathbf{R}_\theta\boldsymbol{x})]\right\}\right\}
\end{aligned}
$$

Automatic differentiation: more than machine learning

$$L = \frac{1}{N_\theta N_p N_k} \sum_{\theta,k} \overbrace{\left\| |f(\boldsymbol{x}, \theta, k, \Delta z, d)|^2 - \boldsymbol{y} \right\|^2}^{D} + \alpha_\delta |\boldsymbol{x}_\delta|_1 + \alpha_\beta |\boldsymbol{x}_\beta|_1 + \gamma \mathrm{TV}(\boldsymbol{x}_\delta)$$

subject to $x_w = 0$ for $x_w \notin \Theta$ and $x_w \geq 0$ for $x_w \in \Theta$.



Automatic differentiation: more than machine learning

$$\frac{df}{dx} = \frac{f(x+\epsilon) - f(x-\epsilon)}{2\epsilon}$$

In[110]:= WalkD[x ArcTan[√x], x]

$$\frac{d}{dx} x \tan^{-1}(\sqrt{x})$$

$$= \tan^{-1}(\sqrt{x}) \left(\frac{d}{dx} x\right) + x \left(\frac{d}{dx} \tan^{-1}(\sqrt{x})\right)$$

$$= \tan^{-1}(\sqrt{x}) + x \left(\frac{d}{dx} \tan^{-1}(\sqrt{x})\right)$$

$$= \tan^{-1}(\sqrt{x}) + \frac{x \left(\frac{d}{dx} \sqrt{x}\right)}{1+x}$$

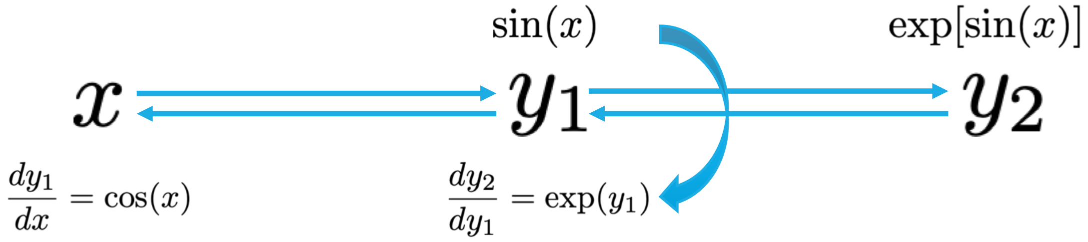
$$= \frac{\sqrt{x}}{2\,(1+x)} + \tan^{-1}(\sqrt{x})$$

Out[110]= $\frac{\sqrt{x}}{2\,(1+x)} + \mathrm{ArcTan}[\sqrt{x}]$

$$f(x) = y_2[y_1(x)]$$

$$y_1(w) = \sin(w)$$

$$y_2(w) = \exp(w)$$



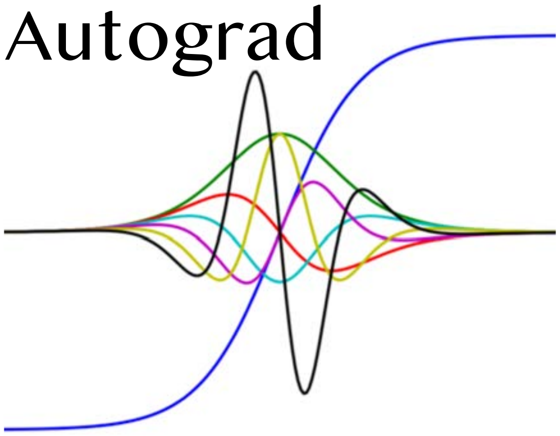
$$\frac{df}{dx} = \frac{dy_2}{dy_1}\frac{dy_1}{dx}$$

Automatic differentiation: more than machine learning

TensorFlow

(Has been applied for ptychography by Nashed *et al.*)

HOROVOD

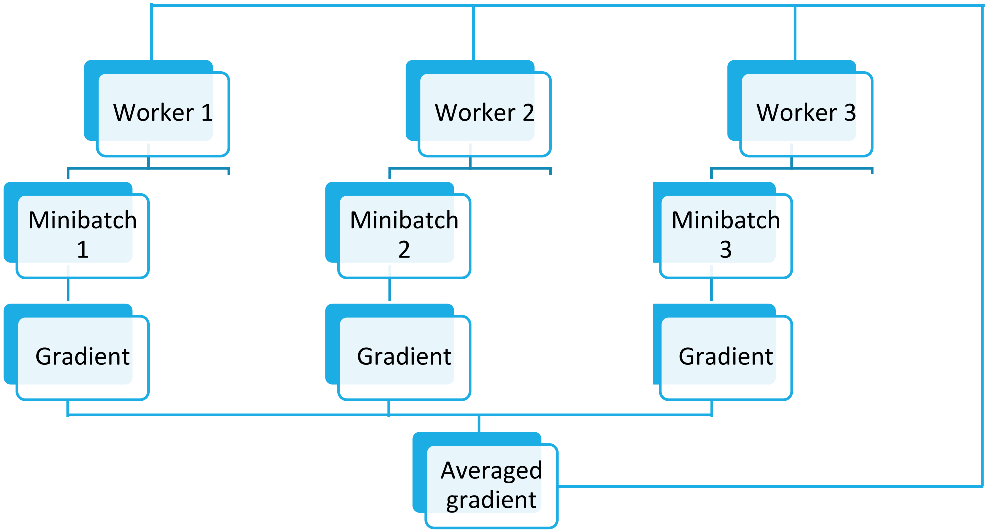


[1] M. Abadi *et al.*, "Tensorflow: Large-scale machine learning on heterogeneous distributed systems," arXiv cs.DC, arXiv:1603.04467 (2016).
[2] D. Maclaurin, "Modeling, Inference and Optimization with Composable Differentiable Procedures," (PhD thesis), Harvard University (2014).
[3] Nashed, Y. S. G., Peterka, T., Deng, J., & Jacobsen, C. (2017). Procedia Computer Science, 108, 404–414.

Automatic differentiation: more than machine learning

HOROVOD

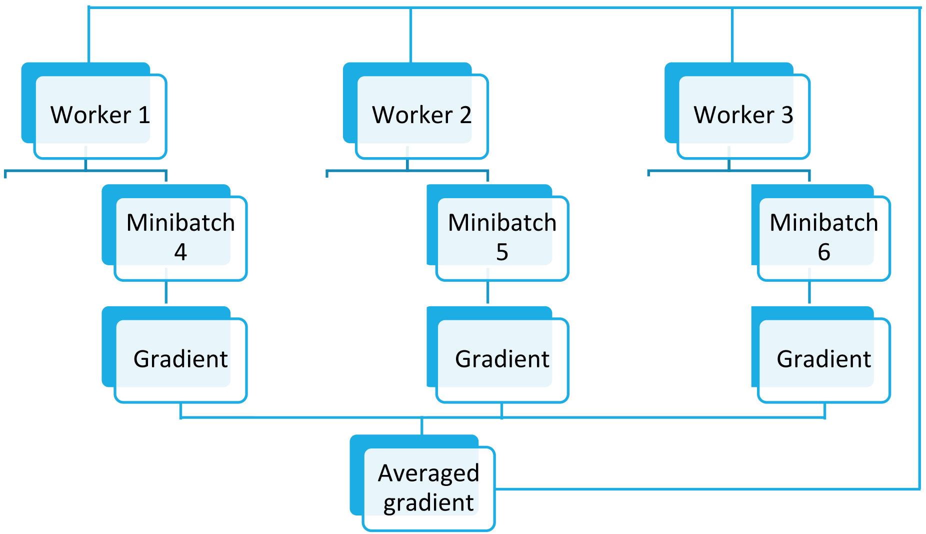
Divide projection data into minibatches to fit in memory



Automatic differentiation: more than machine learning

HOROVOD

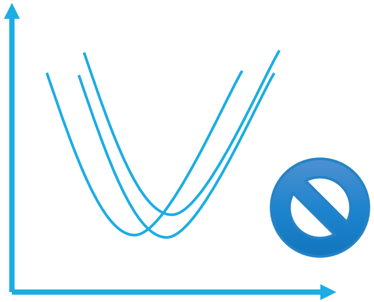
Divide projection data into minibatches to fit in memory

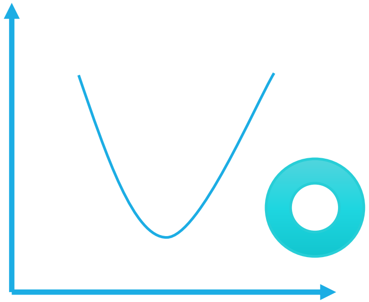


Gradient accumulation: fighting uncertainty

```
# build model
while stopping_criterion_not_met:
    for i_update_chunk = 1 ... n_chunks:
        for i_minibatch = 1 ... n_minibatches:
            accum_gradient += optimizer.compute_gradient
        optimizer.apply_gradient(accum_gradient)
```

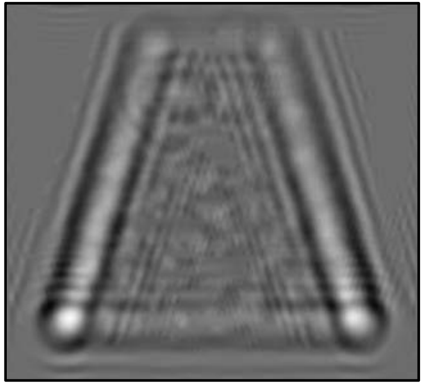
Minibatch: subset of data
Chunk: batch of minibatches

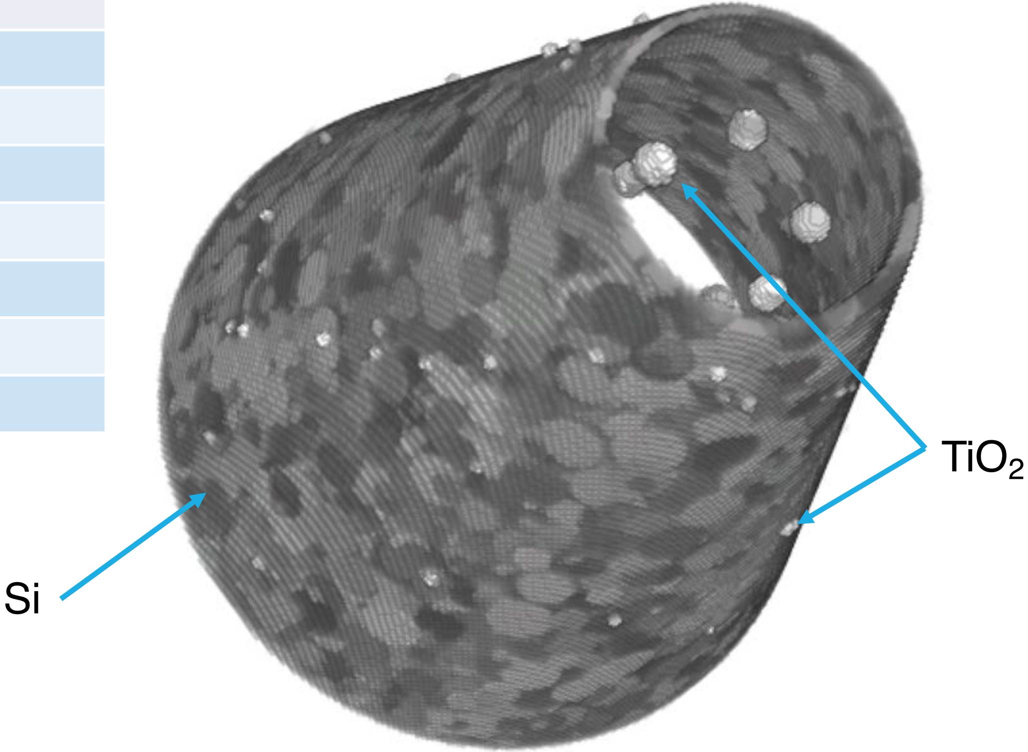




Test case 1: a designed sample

Pixel size (nm)	1
Energy (eV)	5000
Depth of focus (nm)	21.77
Largest sample thickness (nm)	200
Sample-detector distance (nm)	1000
Object grid size	256^3
# of projections	500
# of diffraction spots in ptychography	23×23

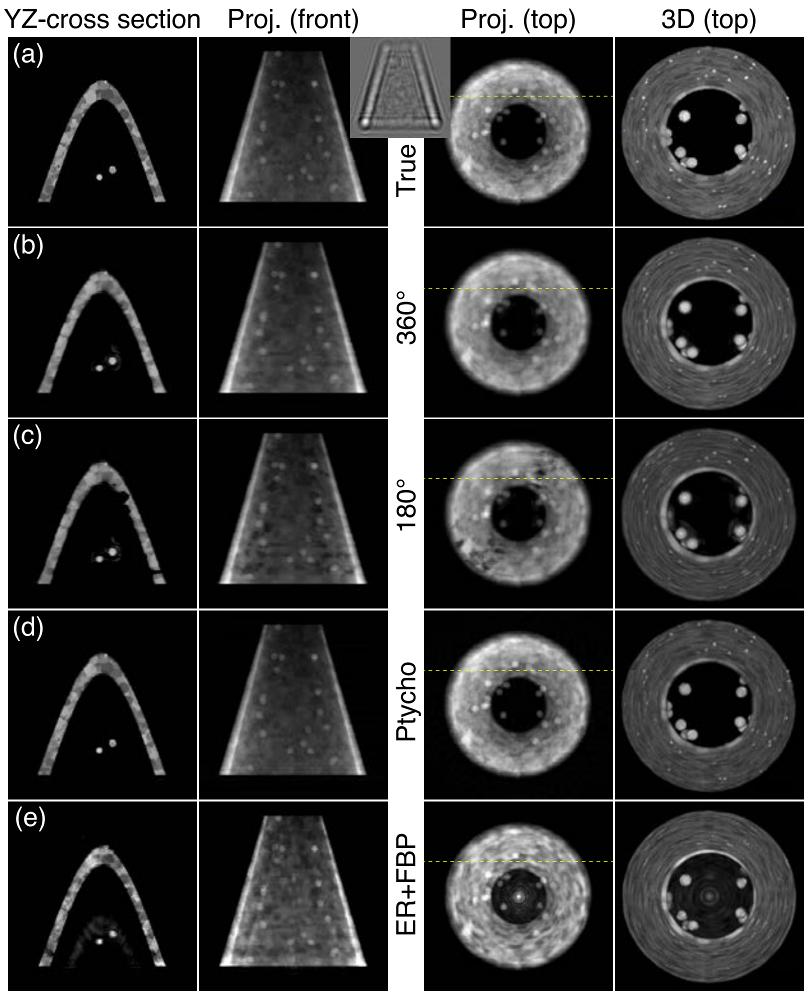




Test case 1: a designed sample

Pixel size (nm)	1
Energy (eV)	5000
Depth of focus (nm)	21.77
Largest sample thickness (nm)	200
Sample-detector distance (nm)	1000
Object grid size	256^3
# of projections	500
# of diffraction spots in ptychography	23×23
Platform	Cooley
Fullfield # of threads	4
Fullfield time	5 h
Ptychography # of threads	20
Ptychography time	45.9 h

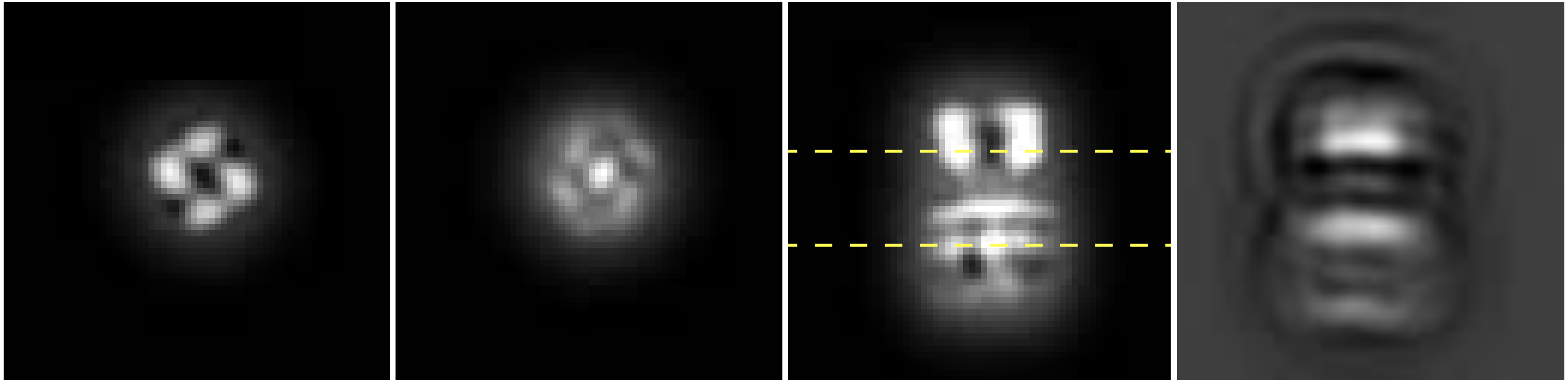
True: simulated object
360°: full-filed reconstruction with 360° projection data
180°: full-filed reconstruction with 180° projection data
Ptycho: ptychography reconstruction with 360° projection data
ER+FBP: reconstruction with conventional CDI and tomography



Test case 2: a semi-experimental protein molecule

Human adhesin complex originally acquired using EM; data retrieved from EM databank.

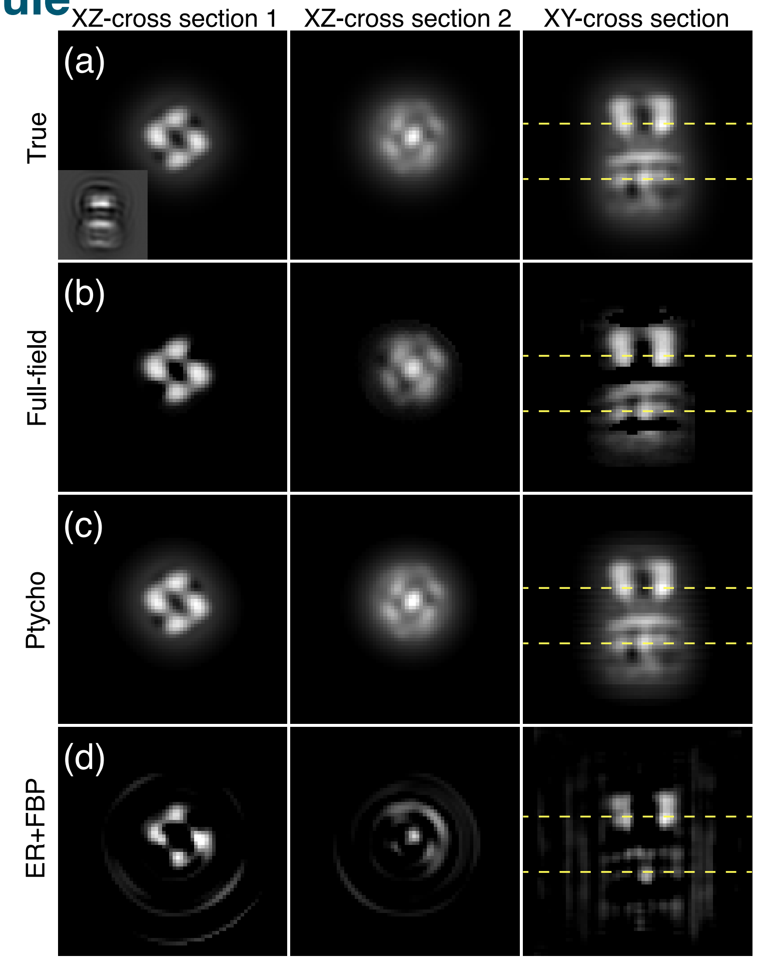
Pixel size (nm)	0.67
Energy (eV)	800
Depth of focus (nm)	1.56
Largest sample thickness (nm)	30
Object grid size	64^3
Num. of projections	500
# of diffraction spots in ptychography	23×23



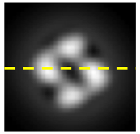
Test case 2: a semi-experimental protein molecule

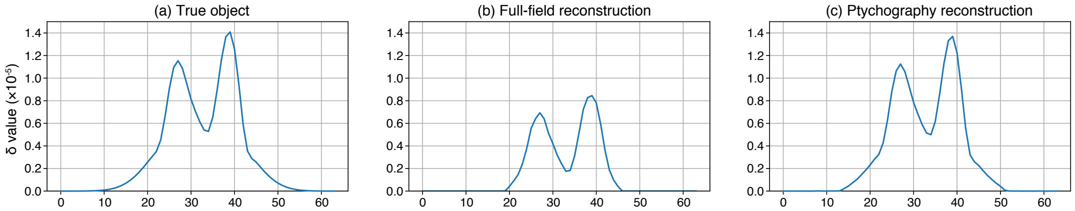
Pixel size (nm)	0.67
Energy (eV)	800
Depth of focus (nm)	1.56
Largest sample thickness (nm)	30
Object grid size	64^3
Num. of projections	500
# of diffraction spots in ptycho.	23×23

Fullfield platform	Workstation (GPU)
Fullfield # of threads	4
Fullfield time	0.15 h
Ptychography platform	Cooley
Ptycho. # of threads	20
Ptychography time	1.45 h

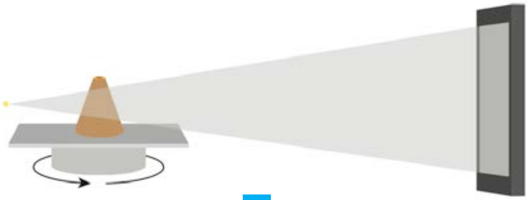


Full-field reconstruction throws away halos, while ptychography preserves it.

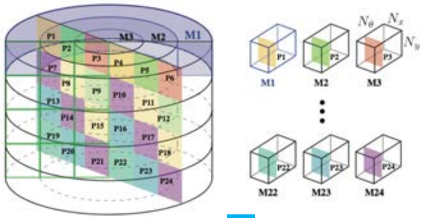




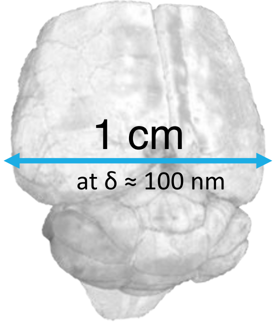
Future perspectives

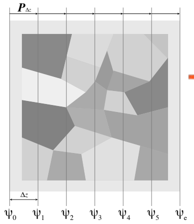


Point projection



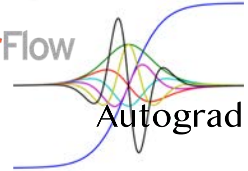
Tomosaic

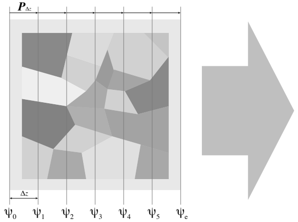


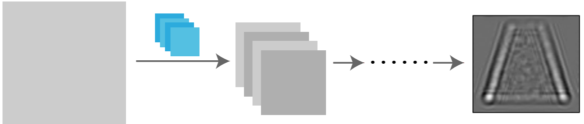


TensorFlow

Beyond DOF

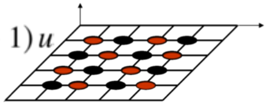






Convolutional neural network

$$\frac{\partial u}{\partial z} = \frac{1}{2ikn}\Delta_{\perp}u - ik(n-1)u$$



O(N) Finite difference propagation

One code, two or three dimensions, and… many more things doable!

Gaussian noise model

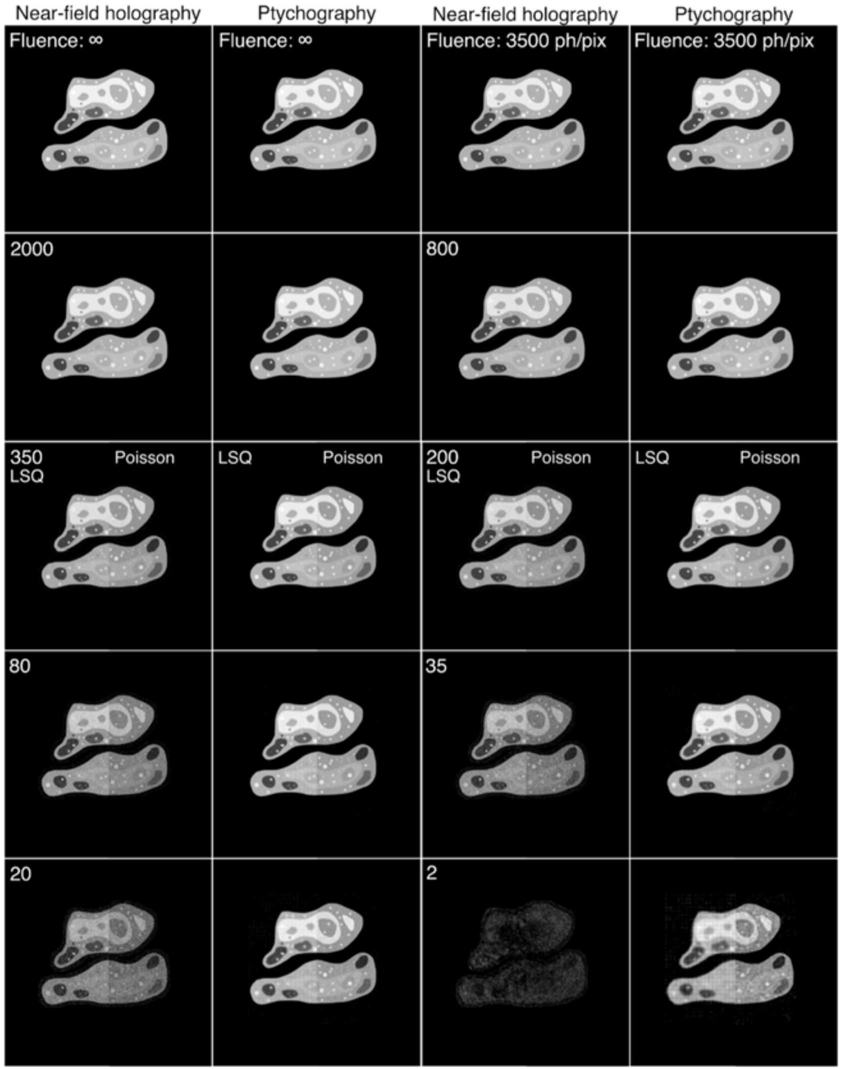
$$L(\boldsymbol{x}, \boldsymbol{y}) \propto \sum_i \|y_i - \bar{y_i}\|^2$$

Poisson noise model

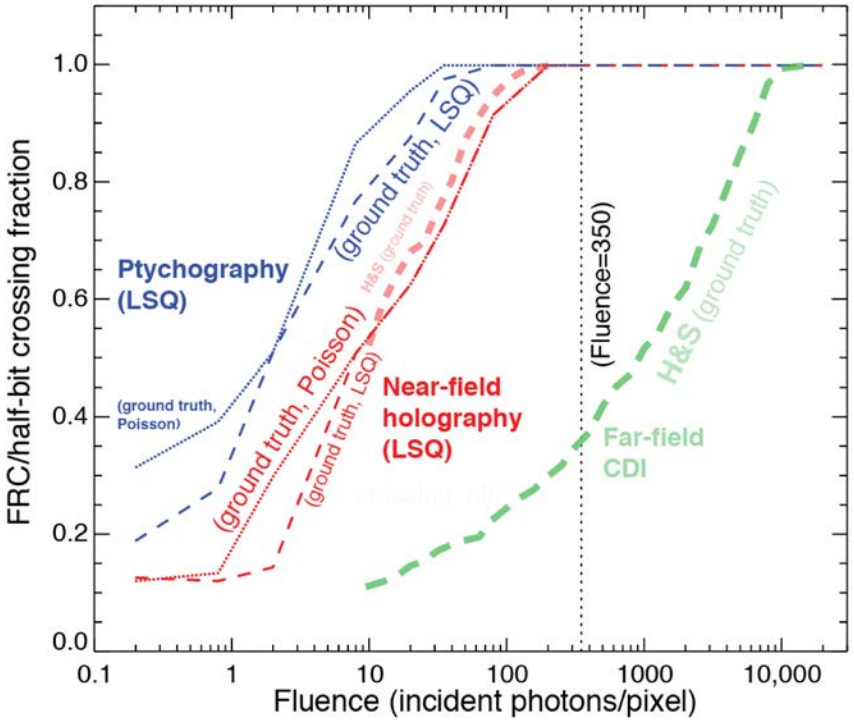
$$L(\boldsymbol{x}, \boldsymbol{y}) \propto \sum_i [\bar{y_i} - y_i \log(\bar{y_i})]$$

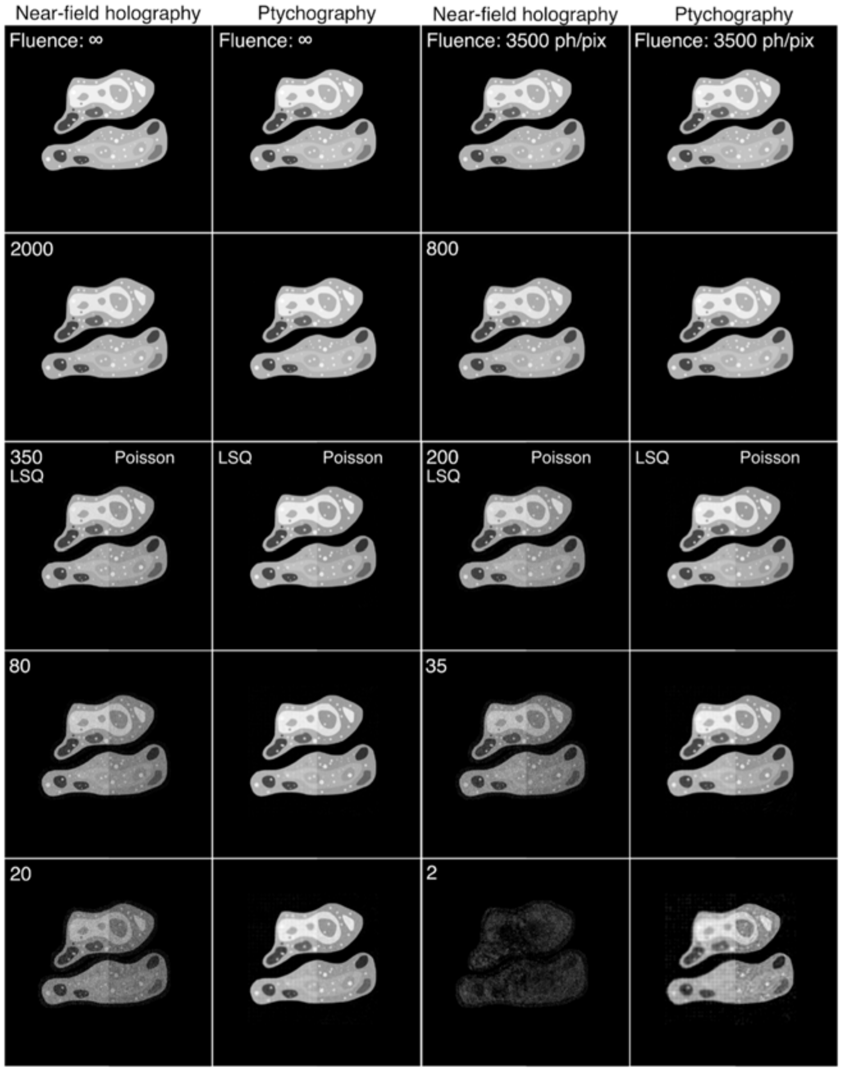
```
loss = tf.reduce_mean(tf.squared_difference(tf.abs(exiting_ls),
    tf.abs(this_prj_batch[i])))
```

```
loss = tf.reduce_mean(tf.abs(exiting_ls) ** 2 * poisson_multiplier) –
    tf.abs(this_prj_batch[i]) ** 2 * poisson_multiplier *
    tf.log(tf.abs(exiting_ls) ** 2 * poisson_multiplier)
```

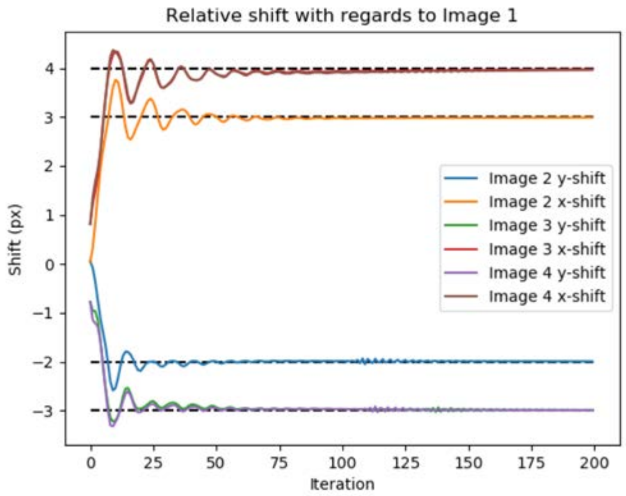


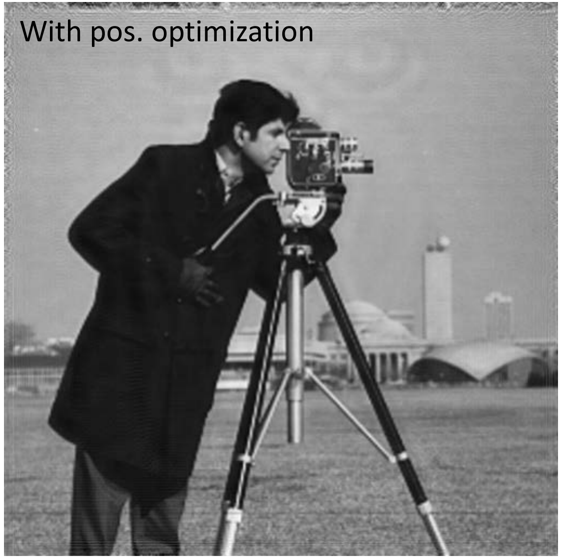
One code, two or three dimensions, and… many more things doable!

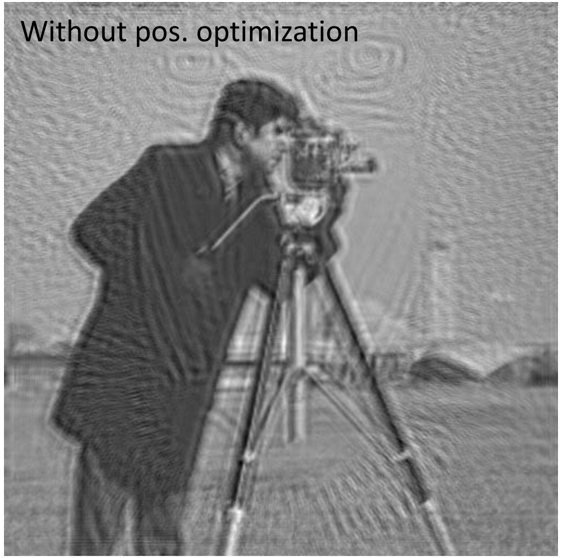


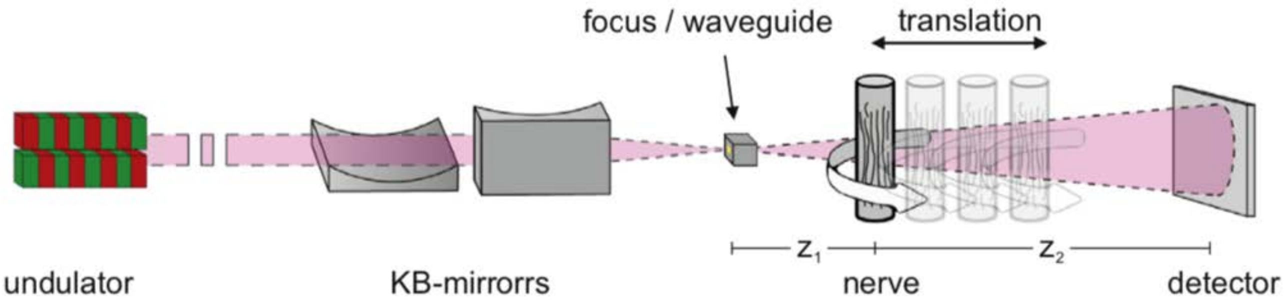


Jointed reconstruction with projection alignment for multi-distance holography









Bartels, M., Krenkel, M., Cloetens, P., Möbius, W. & Salditt, T. Myelinated mouse nerves studied by X-ray phase contrast zoom tomography. *Journal of Structural Biology* **192,** 561–568 (2015).

Acknowledgement

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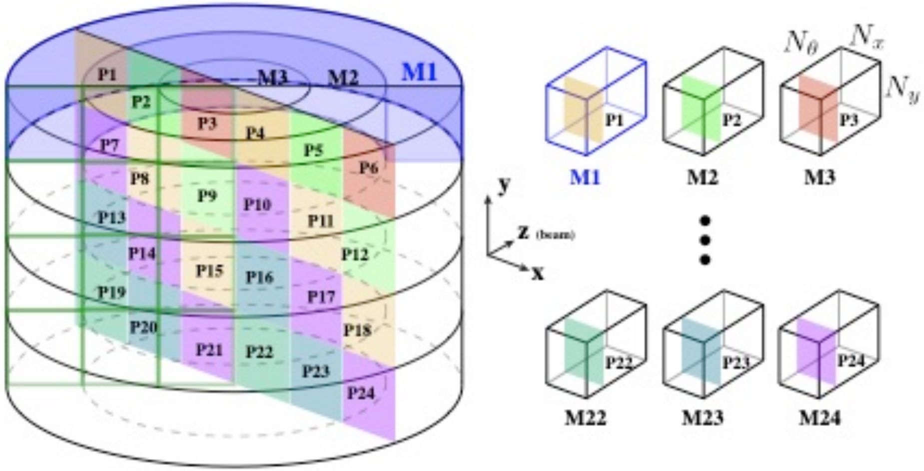
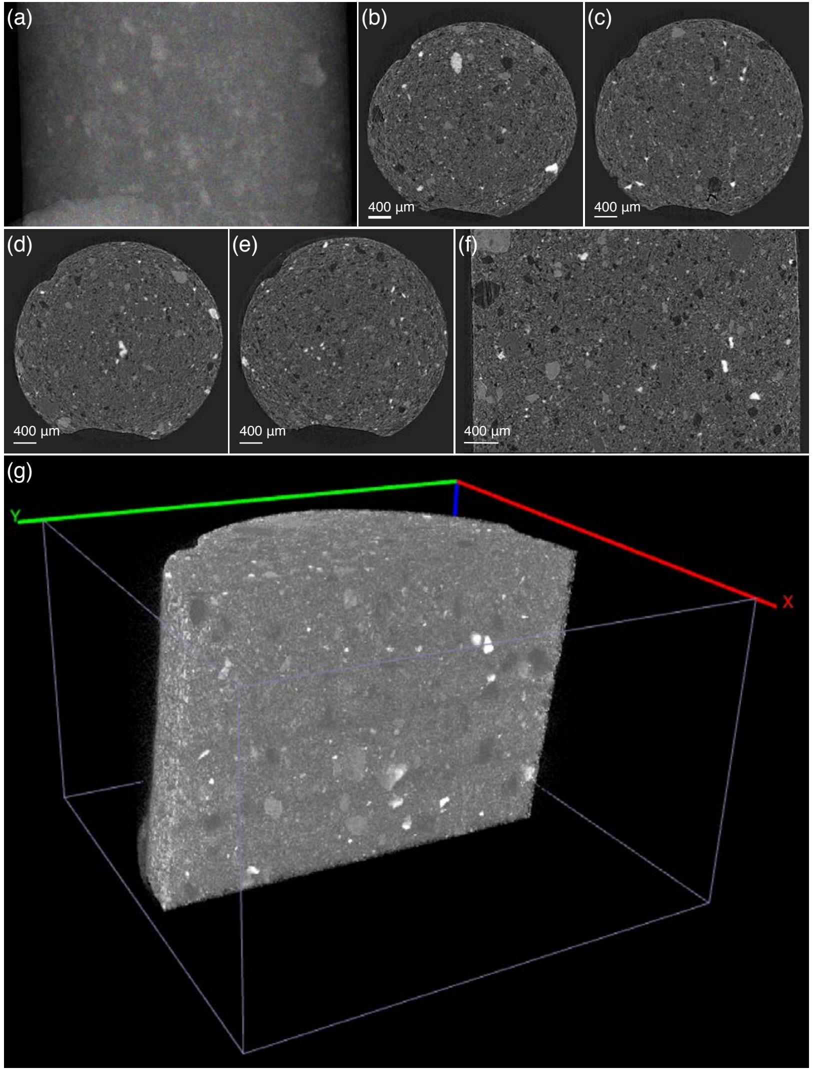
Tomosaic at 2-BM

- Large-scale tomography acquisition, stitching and reconstruction
- Supercomputer-friendly





Thank you

Q & A