Nondimensionalization

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When we start with the equation for the the rate of change of a population:

$$\frac{dP}{dt} = PN(P)$$

Where $N(P)$ is a function of population, and P is the population. If we say $N(P)$ is the following:

$$N(P) = \pm a$$

Where a is constant and the sign depends on whether the population is growing or shrinking.
But this isn't accurate. The larger the population the more competition which will cause the competition; this can be represented more accurately with the following equation:

$$\frac{dP}{dt} = P(a - a'P)$$

Where a and a' are corresponding units to make the entire equation population per time. To nondimensionalize this equation we can turn it into the following form:

$$\frac{d\bar{P}}{d\bar{t}} = \bar{P}(1 - \bar{P})$$

Where $\bar{P} = \frac{P}{P_{ref}}$ and $\bar{t} = \frac{t}{t_{ref}}$. Where P_{ref} and t_{ref} are reference times and populations. We can do this like the following:

$$d\bar{P} = \frac{dP}{P_{ref}} \quad d\bar{t} = \frac{dt}{t_{ref}} \rightarrow d\bar{P}P_{ref} = dP \quad \bar{t}t_{ref} = t$$

If we plug this in to dimensional equation:

$$\frac{d\bar{P}}{d\bar{t}}\frac{P_{ref}}{t_{ref}} = \bar{P}P_{ref}(a - a'\bar{P}P_{ref})$$

$$\frac{d\bar{P}}{d\bar{t}} = \bar{P}t_{ref}a - a'\bar{P}^2P_{ref}t_{ref}$$

So we can conclude:

$$t_{ref}a = 1 \qquad P_{ref}t_{ref}a' = 1$$

and then

$$t_{ref} = \frac{1}{a} \qquad P_{ref} = \frac{a}{a'}$$