Stability of Critical Points

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$$\frac{dP}{dt} = P\left(1 - P - \frac{AV}{P+C}\right) = f$$
$$\frac{dV}{dt} = BV(1 - \frac{V}{P}) = g$$

$$J = \begin{bmatrix} \frac{\partial f}{\partial P} & \frac{\partial f}{\partial V} \\ \frac{\partial g}{\partial P} & \frac{\partial g}{\partial V} \end{bmatrix}$$

$$J = \begin{bmatrix} 1 - 2P - \frac{AV}{P+C} + \frac{APV}{(P+C)^2} & \frac{-AP}{P+C} \\ \frac{BV^2}{P^2} & B - \frac{2BV}{P} \end{bmatrix}$$

For critical the point $(P, V) = (1, 0)$ the Jacobian matrix is the following:

$$J = \begin{bmatrix} -1 & \frac{-A}{1+C} \\ 0 & B \end{bmatrix}$$

The eigenvalues for this matrix can be calculated as follows:

$$0 = \begin{vmatrix} -1 - \lambda & \frac{-A}{1+C} \\ 0 & B - \lambda \end{vmatrix}$$

$$0 = (-1 - \lambda)(B - \lambda)$$

This says the eigenvalues are $\lambda = -1$ and $\lambda = B$. For the critical point $(P, V) = (1, 0)$ to not be a saddle B would need to be negative.