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Chaos Theory in Finance

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Abstract

The theory of chaos is well suited for the understanding of the financial perspectives, because the behavior of the financial market is predetermined whole number of circumstances that are relative to the market can be caused by both internal and external reasons. The theory of chaos for decades was one of the most acute topics in science, but so far it has not been sufficiently used in financial theory and practice. In the course of growing instability and increasing the role of randomness in financial markets, attention to this theory is growing. In this connection, it is important to determine the possibilities and limits of its application in finance, as well as its relation to traditional economic theories. The article attempts to clarify some points related to the possibility of using chaos theory in finance. In particular mechanism of its application to the macro and micro processes, as well as the use of certain methods and instruments such fractal and stochastic processes, prediction.

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Keywords: chaos theory in finance; rational and irrational behavior; spontaneous actions; stochastic processes; fractals; prediction in finance; macroeconomics;

1. Introduction

In recent years, financial institutions were pushed into an unknown territory after unprecedented global trends, such as extremely low and negative interest rates, low commodity prices, and especially hydrocarbons, new technologies and the growing number of cyber-attacks. The traditional banking business model, built on correspondent relations,

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undergoes modernization, while the bank-to-client relationship is passed in an environment where nothing is "ordinary" and a lot of surprises. In general, there have been important disruptions in the payment industry, regulation, macroeconomic risk, capital accumulation and investment, and financial intermediation. As a result, financial markets have changed significantly and most importantly their uncertainty has increased markedly. First of all, innovations growth has led to the constant anticipations of the unexpected. Secondly, there is a transformation of financial markets: on the one hand, numerous nonfinancial services and entities are introduced into the financial sphere, on the other hand, financial activity began to merge with the information, trade and other economic and cultural activities. Thirdly, finance is faced with the problem of big data. As a result, further development of finance is associated with their digitization and further introduction of mathematical techniques and models into the industry. Especially high expectations exist for the application of the theory of chaos in the study of financial markets and the organization of work on them. But this requires a long process of escape from habitual modes of thought, models and expressions.

The theory of chaos for decades has been one of the hot topics in science, but so far has had almost nothing to do with finance. At the present time, timid attempts are made to elucidate the possibilities and outline the limits of its application in finance [10], as well as to conduct a watershed between traditional economic theories and chaos theory [1]. In the article attempts are made to clarify some points connected with the possibility of using the theory of chaos in financial theory and practice.

2. Macroeconomics and microeconomics approaches to the financial chaos

The theory of chaos can be represented in the form of an appropriate section of knowledge suitable for searching order in disorder. Initially, this theory was used to develop tools for trading stocks[7], but now the scope of its application in finance has expanded[5]. The main arguments are: (1) chaos theory is competitive and may well become a "convenient" theory of the financial market, (2) traditional finance does not take into account dynamics, while chaos theory is built on the dynamics of the system, which allows the theory to be brought closer to reality, and (3) Instability is associated not only with the crisis, to which the theory of chaos in the financial market is applied, but also from the hypothesis of Minsk's financial instability, which assumes instability of the financial market as such and in many respects links it to its innovativeness. Overall, these arguments are based on the positive experience of applying the theory of chaos to the analysis of the financial market.

If complexity consists of simpler and smaller systems, a chaotic/complex system can be analyzed through that ones. There are some cases that prove that there is a base system/sequence outside chaos or chaos consists of some simple ordered systems. For example, during the financial crisis of 2008-2009, the overwhelming majority of banks were in order - they were able to organize their activities on a market basis or with the help of the state, but in general the financial markets were in a state of chaos.

An analysis of the reasons for low interest rates can make an interesting and rather simple conclusion: the relative excess of free capital and the relatively prosperous period of the economy (even in view of recent critical surges) affected the interest rate cut. In these conditions, the state has nothing left to do to maintain and push forward this peculiarity of the market. Mathematically, it is quite easy to prove the relationship between the fall in the interest rate and the growth of the free money supply, which seeks active use and growth. And this conclusion follows from both the theory of an effective market and the theory of chaos. So, the market is determined by supply and demand, as the supply of capital grows much faster than the growth of the economy, then the conclusion is quite unambiguous in favor of low interest rates. Nevertheless, this conclusion is not typical for traditionalists.

Financial chaos is often perceived as a mess that often causes prostration and reckless actions. As a result, it increases the helplessness that prevents correct decisions. At the same time, financial chaos is composed of a complex system in which there are various somehow-ordered subsystems that are in some way structured and there are interactions between them of some sort. In general, in financial chaos, the order-to-disorder ratio is quite problematic and largely depends on the specific situation on the market. The fact is that the extreme points of the cycle – crisis and boom – can not be characterized as a market disorder, but also they do not fit the definition of order. Rather, it is characterized by the notion of "financial chaos." In this case, they must be analyzed from the standpoint of the theory of financial chaos. At the same time in the financial science efficient market theory is more commonly used for analysis of the financial market as a whole, according to which any deviation from the balance of the market are considered to be market anomalies. In this regard, it is very important to clarify the place and role of both theories in the study of the financial market.

There are bull and bear markets on the stock market. In bull markets, good news is greeted with euphoria and strong stock buying, while bad news is discounted or ignored. In bear markets, the opposite is true: good news is often ignored, and bad news leads to repeated bouts of sale. The fuel underlying this attitude is motivation, which determines the behavior of people, private companies and government in the market. The totality of expected and spontaneous actions, based on both rational and irrational behavior in aggregate, is a chaotic order of the market. In simplification, the theory of chaos for financial markets is a set of rules for the behavior of markets in general, and some parts and their structures. Big data technologies make it possible to approximate the understanding of chaos as a financial market system with the corresponding laws of behavior and an estimate of the probabilities of random events on it.

According to some estimates, there is little evidence of low-dimensional chaos in financial markets [10]. It's so hard to imagine that any well-known bank has chaos, unless it's the Lehman Brothers. If this were so, then it would be connected with non-payments and would mean bankruptcy. Nevertheless, in certain periods of time may the chaotic state of some individual structures (microeconomic financial chaos) that settled by crisis management or risk-management. The theory and the mechanism of microeconomic financial chaos regulation are sufficiently developed. Another thing when it comes to large, complex market structures – financial macroeconomic. In fact, the theory of financial chaos (macroeconomic financial chaos) is applied not only when there are problems of non-payments, debt is growing rapidly and massive bankruptcies are observed, but indeed at any point in time when financial control is lost [1]. The chaos can be associated not only with bad, but with good events, in particular with progress in the financial sphere. Financial markets are subject to various shocks. Among them, for example, an important place is occupied by technological and product innovations, alternative payment systems, the transition to direct monetary relations between buyers and sellers, creditors and debtors, investors and entrepreneurs, bypassing financial intermediaries (based on B2B, C2B, P2P systems, and crowdfunding). As a result, market uncertainty rises sharply. For the analysis of such processes, the theory of chaos is quite applicable. Financial chaos arises when deterministic dynamic systems (financial markets) are sensitive to initial conditions and generate effective unpredictable long-term behaviors motivated by spontaneous actions and shock effects.

In general, high-level chaos (macroeconomic financial chaos) is indistinguishable from a long-running stochastic process, while low-dimensional chaos (microeconomic financial chaos) with high entropy at the micro level, which is characterized by a temporary nature of the action – the system either would be normalized (for example, by securitizing debt, solvency is restored) or destroyed (liquidation as a result of bankruptcy)[8].

3. Fractal waves of financial chaos

The idea that markets exhibit fractal behavior can be exploited for prediction[4]. Since prices and return are not historically lognormal, recently new methods are being used to assess the future. In particular, fractal analysis is used, which differs from the usual one, and the results obtained with it can also be different. New approaches to forecasting are based not on the extrapolation of current trends, nor on the account of the cyclical occurrence of events, but on the principle of fractal analysis, which considers the resumption and wave propagation of events[3].

The theory of chaos teaches us to expect the unexpected. While the traditional efficient market hypothesis states that asset prices fully reflect all available information, and their movement is entirely expected. The theory of chaos deals with nonlinear things that cannot be predicted, controlled or managed. Therefore, it seems that it is useless for financial management. Nevertheless, through a whole set of different tools, one can deal with chaotic things and events in the financial market and thereby use them in management[6].

The best way to learn about chaos in financial market is to explore it. First of all, let's make two assumptions: the motion goes from chaos to order and then again to chaos – so the cycle is over (completed) and is coming a new one; although Chaos is passing through a cycle, situations are rarely exactly replicated and repeated. In the plotting of many financial markets on simple graphs it can be revealed clear that, at least in theory, there is often a desire to achieve equilibrium of some sort. For instance: imagine a stock exchange of 100 shares of different entities. In order to trade these shares, the exchange will spawn one Clearing House with 10 employee, a Depository, an Information agency, and three churches. And for argument's sake, we will assume that this setup pleases everybody and equilibrium is achieved. But then the Cat & Cat's company decides to make an IPO for collect capital $ 100 million for open a new ice cream plant, that increase the number of shares to 101. The stock exchange expands rapidly to accommodate one new share and brokers start to trade with it. Then there issue 30 additional shares and the market capitalization increases by $ 3 billion, or 50 percent. At least two variants of market reaction are possible: the same number of brokers, services companies and employees engaged in the market and with the same capital continue to serve the new volumes and the equilibrium is maintained; issuance of new shares attracted additional brokers and investors also need to expand the clearing operations, depository and information services with the involvement of additional employees. This situation is some kind of equilibrium (static type-of-equilibrium)[3].

After a while, investors and speculators begin to lose interest in this stock exchange and a number of brokers stopped to operate, the turnover of the Clearing House and the Depositary decreased and employees lost their jobs. Demand fell, but another stock exchange beginning to expand its activities and include brokers from the first one and move to the quotation of shares withdrawn from the first stock exchange. And so it is repeated over and over again. Much is already in the process of transition, but the first stock exchange does not change its plans yet. The stock exchange keeps the trade running for a year and then comes to the conclusion that there are not enough business (listing shares, brokers, and investors) and shut it down. But the demand for capital is growing and a new stock exchange is needed to organize its movement. And a new stock exchange meets this demand. This situation is also some kind of equilibrium, but a dynamic one (dynamic type-of-equilibrium). The difference between a Simple Dynamic and a Complex Dynamic Equilibrium is that a Simple represents a state to which a system finally settles, while a Complex Dynamic Equilibrium represents some kind of trajectory upon which a system runs from situation to situation without ever settling down. The discovery of Simple Equilibrium (it was the basis of the effective market theory) was exciting and explained a lot, but the most awesome phenomenon Chaos Theory discovered was so called Self-Similarity. Unveiling Self-Similarity allowed everyone a glimpse of the magical mechanisms that shape, for example, financial markets. How does market direct shares and manage the capital flows, or do stock exchanges expand or contract? Chaos Theory has an answer: Self Similarity, a fundamental principle that allows forming blocks to mimic their own shape in the market they make. An image or a copy that displays Self-Similarity is usually called a fractal in mathematics. If in the 1980s there was a reduction in exchanges, and then there was a sharp expansion and the emergence of new types of exchanges, which can be described using a fractal process. Usually, fractals arise in the study of nonlinear dynamical systems. Financial markets are such systems of some sort. That is why the modern process of the emergence of crypto currency exchanges is a model of fractal self-creation.

Let F (z) is a polynomial z0 – is a complex number. Consider the following sequence: z0, z1 = F (z0), z2 = F (F (z0)) = F (z1), z3 = F (F(F(z0))) = F(z2),… We are interested in the behavior of this sequence with the aspiration n to infinity. However, in life there can not be an infinite number of exchanges - some exchanges appear, others disappear and so a certain balance remains - the cycles of expansion of exchanges alternate with cycles of their compression. However, in theory it is possible to build such structures. So this sequence can: strive for infinity, strive for the final limit, and demonstrate in the limit the cyclic behavior, for example: z1, z2, z3, … Behave chaotically, that is, do not demonstrate any of the three mentioned types of behavior. Set of values z0, for which the sequence demonstrates one particular type of behavior, as well as the set of bifurcation points between different types, often has fractal properties. For example, the stock exchange simulates the appearance of an exchange of derivatives from shares of securities. Stochastic (random) fractals can be used for their modeling. Examples of stochastic fractals: trajectory of Brownian motion. Through the process of "copy" of exchanges, a network of exchanges is being created – process of Self-Similarities, which shape the financial market. The same process occurs with securities (shares, bonds, notes, etc.) - self-reproduction (Self-Similarities) in a new form, in a new quality, and new companies. This process of self-reproduction is very chaotic. The system of self-reproduction of the financial market is complicated by the fact that each new repeating structure is unlike the previous one, although formally it is a "copy", which significantly increases the chaotic development of the financial system. Self-similarity is a really big deal. It occurs all over finance and that's why self-similarity is one of the key principles that shape financial system the way it is.

The theory of chaos requires a new way of thinking. Chaos is a product of uncertainty and unexpectedness and at the same time engine of them. The entire financial market is a chaotic unpredictable complex system that is inherently chaotic and constantly on the move from order to disorder and back. In this movement there is the very essence of chaos. Uncertainty as well unexpectedness appears to be ones of the most characteristic qualities of contemporary financial market.

For example, a large bank has, at any point in time, a certain capital, assets and liabilities and a certain position in the market. Both the size and position in the market can be accurately calculated or measured. But the real state and all parameters of the impact on the market cannot be accurately known. Both the position of the bank is known and the capital. If the size of the bank can be accurately known, its impact on the market can be either expected or unexpected. In turn, depending on the market situation, the bank can dramatically change its priorities and activities. Consequently, we do not know all the parameters of the system called "bank". But the bank is not alone in the market. He interacts with other banks, with clients and other financial institutions. In addition, control over banks is changing, which also has an impact on the bank. Therefore, it is impossible to fully describe all the parameters of even one bank, and not because we are not smart enough or our tools are not precise enough. It's just impossible. So there is the principle of uncertainty in the market. It is impossible to fully describe all the characteristics of the bank and its impact on the market. Thus, in the market, financial institutions interact, in which all the characteristics and all the directions of the impact on the market are uncertain. As a result, the foundation of financial chaos is formed. These arguments differ sharply from traditional opinions about financial institutions and financial markets as fully cognizable objects that are in a very definite and controlled state. To study such objects follows from the position of the theory of chaos. Financial-uncertainty has a very peculiar consequence. It cannot be predicted when and where insolvency, bankruptcy, and default will come up. Each financial institute is sovereign and cannot be coerced and controlled fully. And this is the basis for a variety of interactions, each of which can represent a certain order and focus on the stability of each participant, but in general, represent a chaotic complex set – chaos.

Fractal analysis of the financial market allows making a fairly unambiguous conclusion - the acceleration of the process of self-reproduction (Self-Similarities) of capital provokes a fall in the interest rate. In the same direction, there are a number of other factors that directly follow from the theory of chaos. From formal positions and proceeding from traditional assumptions, the growth of market uncertainty causes an increase in the price of money. But the opposite has happened in the real world – it seems that uncertainty causes a fall in the growth rate of interest, which can be explained in terms of the greater risk tolerance. However, the theory of chaos puts everything in its place. Infinite patterns created by repeating a simple process again and again in a continuous feedback loop. Self-reproduction of capital has accelerated significantly during the financializtion of the economy (constant repeating self-similarity)[9].

Another example: to distinguish the theory of chaos from the inherited instability in the analysis of the consequences of the crisis[11]. If build an inverted pyramid, it will eventually fall over not because the theory of chaos that predict this, but because of inherent instability. The repackaged mortgages leading up to 2008 to the crisis were inherently unstable because they relied on positive feedback loops that went negative (for a long time, the annual increase in the price of houses was higher than the payments due for credit, which made it easy to repay mortgages) when house values collapsed. It was, however, largely random, when the failure occurred, but that wasn't theory of chaos in itself; it was what could theoretically be predicted by the theory of chaos.

4. From Bayesian thought process to Brownian motion in finance

For example, with a very large simplification (ideal-abstract state of the economy), there are only two economic variables - interest rates and inflation. They can either grow or fall in a particular way. The regulator – the Central Bank – solves the following question: what is the probability that interest rate and inflation will decline. Selection frames (the first scenario): 1) Up interest rates, Downward inflation; 2) Down, Up; 3) Down, Down. From this sample, one can see that there is a quarter (25%) probability that interest rates, as well as inflation, will decrease. However, if the Central Bank has an incomplete information about the economy (second scenario), which shows that one of the two variables (interest rates or inflation) is certainly down, then what will be the probability that both interest rates and inflation are down too? With this new information, the sample shows shrinking (Up, Down), (Down, Up) and (Down, Down). Such case shows the probability for both these variables will go down would be 1/3 (33.33%).

The above sample is a highly stylized and a highly simplistic scenario. But it shows where such methods of reasoning will take place – the Bayesian process. There is a prior probability and there is this new information and based on that information is changing our view of the financial world. It is a Bayesian way of thought in the finance. But the system is much more complicated and the variables are incomparable more and some are moving in one direction, others in the opposite direction, and all are moving at different rates. At this level of computer technology, the probability of each of them is almost impossible to calculate. This state of the system is completely determined as chaotic. In this case, Bayesian thinking is given to Brownian motion.

Radical transformations in the finance – from uncertainty to certainty and from randomness to predictability – can be estimated from the position of determining the level of probability of events associated with the digitization of the industry as well as Big Data adoption. The main issues are reduced to the allocation of random events that can affect the entire system. The subsequent classification and transition to parameterization, including the calculation of the probability of occurrence of events and the degree of its impact on the entire system, will allow identifying and taking into account current events. With this approach, it is possible to assess their impact on the succession or preservation of current trends, as well as to identify the emergence of new trends and determine the likelihood of their development and impact on finance.

Quantitatively, all the changes are distributed between 0 and 1, where the probability of a reliable event is 1, the impossible is 0, and the random one is from 0 to 1. First of all, the problem of the possibility of introducing innovation into the financial sphere is solved. In the future, the reliability of this event is estimated, through the calculation of the level of its certainty. The probability of an event A, while the function P should not take a value less than 0 (negative value) and greater than 1, that is, is in the interval between two values; In this case, the probability level is determined for the given characteristics: P (A) = 0 and P (B) = 1. The sensitivity level of calculations and their reliability depends on the time frame, the state of the events under consideration.

The stochastic process is used to predict the equity market. For example, all the current conditions of the equity market (E), everywhere within it, to predict what will be doing in the distant future one must know all the current context throughout the economy (set of subsystems – σ) and government regulation as well, since the market and Central Bank (as well the Securities and exchange committee) through interest rate (I) and regulation control the equity prices (R) are interacted. In addition, stock prices are affected by the currency exchange rate (C). In view of the inevitable inaccuracy and incompleteness of equity market observations, precise very-long-range forecasting would seem to be non-existent. So even if the stocks in the market all interacted with buyers and sellers non-randomly, but a totally cause-and-effect (deterministic) manner, this still couldn't predict with certainty what they would do or what the market would be. This market behavior can be studied through analysis of a chaotic mathematical model. There are several traffic models, which were developed to show that the system dynamics could pass under certain conditions to chaos.

Thus, the future conditions of the equity market can be represented as a kind of stochastic process consisting of a number of random variables I, R, C, that are indexed by some mathematical set, which means that each random variable of a stochastic process is somehow connected with an element in the set. The set used to index random variables is determined by the state regulation of the interest rate, the equity market and the exchange rate. A set of indices is some subset of a real line, for example, a temporary statistical series of interest rates, which gave the index the interpretation of time. Each random variable takes values from the same space – equity market. This space can be, for example, an integer n-dimensional so-called Euclidean space. Using a stochastic process, one can obtain many results because of its randomness. A special model is compiled for each calculation. Therefore, multivariate modeling is used. In each case of Brownian motion, time is defined as follows: $0 \leqslant t \leqslant 1$. The set of indices of random processes is a nonnegative number (ie, all values are positive). Given the time factor, the state space is a three-dimensional (Euclidean space), that is, future changes in individual indicators of the development of the stock market economy are estimated.

Multivarianceness of the results requires a special technique of choice of the best results. In practice, often resort to peer review, which is due both to insufficient application of mathematical methods and information technology, and lack of long-rage data that affect the stock market. The stochastic process of determining the future state of the equity market can be represented as a set of random variables (I, R, C) defined on a common probability space (Ω, F, P), where Ω is the sample space, F is a sigma-algebra (a subsystem set – σ), and P is a probability measure – random variable indexed by some set T, all variables have values in the same space S that is measurable with respect to a sigma-algebra Σ. In other words, there is a given probability space (Ω, F, P) – the state of the equity market in the predicted period – and a measurable space (S, Σ), then the stochastic process is a collection of S-valued random variables, which can be written in the form: $\{X(t) : t \in T\}$. In many financial problems a point $t \in T$ had the meaning of time, so X(t) is random variable representing a value observed at time t. A random process is actually a function of two variables, $t \in T$ and $w \in \Omega$ – time and possible state of the equity market. There are also other options for using stochastic processes to forecast the equity market, but in each case, the logic and tools would comply with Brownian motion.

Financial markets are predictable for a while, and then “appear” to become random. First of all, a lot depends on the timeframe of prediction. The amount of time during which the behavior of the financial market can be effectively predicted is determined by three factors: willingness to endure the uncertainty in the forecast, accurate measurement of the current status of the financial market, and the time scale, depending on the dynamics of the system. The dynamics of the system and the overall rate of change largely depends on the business cycle phase, as well as on the cycles of various markets (exchange market cycles of stocks, bonds, derivatives, swaps, futures, currencies, oil financial contracts, etc.) for which forecasting is being performed. On stock market and over-the-counter market cycles proceed according to their own characteristics and their own speed. In addition, general business cycles are superimposed on sub-cycles, which modify the course and duration of sub-cycles.

In financial markets, the uncertainty in a forecast increases with exponential time. For chaotic systems, the following rules apply: the doubling of the predicted time is greater than the squares of the proportional uncertainty in the forecast. In practice, this means that a meaningful prediction can not be made over an interval of more than two or three times the time of Lyapunov (Alexander Lyapunov, 1857-1918). Thus, it is possible to determine the time of financial market withdrawal from the normal to the chaotic state. But if meaningful predictions cannot be made, the system will be random. In this case, the assessment of the financial markets is difficult. Nevertheless, the theory of chaos gives a general view of the prospects for the development of finance in the face of increasing uncertainty and chaos. Suppose that financial market stability asymptotically approaches. If we use Liapunov's stability theorem, we can determine the time necessary for both the transition of the financial market to equilibrium and the entrance from it. Consider the system: x = f (x), assume that x = 0 is the equilibrium point, and x = f(x(t) □ D □ Ŕ is the domain characterizing deviations from equilibrium.

Since the theory of chaos concerns deterministic systems, their behavior can in principle be predicted. From this position, the theory is well suited for the understanding of the financial perspectives, because the behavior of the financial market is predetermined whole number of circumstances that are relative to the market can be caused by both internal and external reasons.

5. Conclusions

Thus, the application of chaos theory to finance opens additional opportunities for the understanding, evaluation and vision of financial markets development prospects. On the other hand, attempts to assess the prospects of the financial market from the perspective of chaos theory expand the borders of its application and stimulate the development of new tools and methods of mathematical analysis.

A financial market is highly complex, and the only prediction what can make is that it is unpredictable. The unpredictability of the financial market is due to the randomness of many events on it. Through the theory of chaos, one can try to catch the structure of unpredictability and display it in a variety of templates. The chaos theory is a revolutionary approach to understanding and forecasting the behavior of financial markets. The beginning of its application coincided with the transition of finance to the use of big data. As computers and knowledge became more powerful chaos theory could become a larger part of evaluating and predicting in financial markets. Externally, actions in the financial markets may seem random, but the theory of chaos sometimes allows us to identify patterns that are at the basis of randomness, stability or chaotic overload of a complex financial system. But there is no possibility of accurate prediction, because the system is so sensitive to the initial conditions or small amounts of various noises and friction, internal and external influences. Nevertheless, it provides benchmarks for the development of markets and allows assessing the probability levels of certain events and actions.

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