PHY 540 **Statistical Mechanics** Spring 2007

Homework 01 with Solution

Problem 1.1. It is known that a gas has the following properties:

- its entropy $S = aT^b$, and

- the work necessary for its isothermal compression from volume V_2 to volume V_1 is $cT\ln(V_2/V_1)$,

where a, b, and c are positive constants. Find the equation of state, thermodynamic potentials E, W, F, and G, and specific heats C_V and C_p of the gas.

Solution: From the second condition we have:

$$p(T,V_1) = -\frac{dA}{dV_1} = \frac{cT}{V_1},$$

so that the equation of state coincides with that of an ideal gas with $N = c$ (see, e.g., Eq. (42) of the lecture notes). Hence we can use Eqs. (44)-(49) of the notes to accomplish the assignment. In particular, from Eq. (44) we have

$$f'(T) = N\ln V - S = c\ln V - aT^b,$$

so that

$$f(T) = \int \left(c\ln V - aT^b\right)dT = cT\ln V - \frac{a}{b+1}T^{b+1} + d,$$

where d is a constant. Now, Eqs. (43), (45)-(49) of the lecture notes give

$$F = -NT\ln V + f(T) = -cT\ln V + f(T) = -\frac{a}{b+1}T^{b+1} + d,$$

$$E = f - Tf' = (cT\ln V - \frac{a}{b+1}T^{b+1} + d) - T(c\ln V - aT^b) = \frac{ab}{b+1}T^{b+1} + d,$$

$$W = E + pV = E + cT = \frac{ab}{b+1}T^{b+1} + d,$$

$$G = F + pV = F + cT = -\frac{a}{b+1}T^{b+1} + cT + d,$$

$$\Omega = -pV = -cT,$$

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V = (f - Tf')' = abT^b,$$

$$C_p = C_V + N = C_V + c = abT^b + c.$$

Note that all thermodynamic potentials but Ω are still determined up to a constant (d), and that coefficient c is irrelevant for quite a few functions (F, E, W, C_V).