

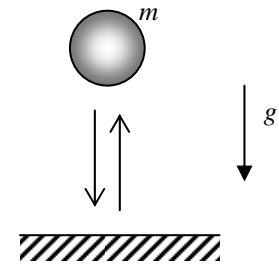
Problem 2.1. (Graded of 10 points.) Use the general result derived in class,

$$T = (2m_{\text{ef}})^{1/2} \int_B^A \frac{dq}{[H - U_{\text{ef}}(q)]^{1/2}}, \quad (1)$$

to find the functional dependence of the period T of oscillations of a 1D particle of mass m in the potential $U(x) = \alpha x^{2n}$ ($\alpha > 0$, n is a positive integer) on energy E . Explore the limit $n \rightarrow \infty$.

Problem 2.2. (10 points.) A small, very stiff ball is bouncing of the floor. Neglecting energy loss,

- integrate the equation of motion directly to find the ball height as a function of time (sketch the result);
- explain how the problem (including bouncing) may be described as a particle motion in a 1D field of potential forces; sketch the corresponding potential energy $U(x)$;
- check that the relation between the bouncing period T and the ball energy E , following from your first result, does satisfy general Eq. (1).



Problem 2.3. (10 points.) Use the time-domain approach (i.e. the Green's function method) to find dynamics in a linear oscillator with damping, $x(t)$ induced by a resonant force suddenly turned on:

$$F(t) = \begin{cases} 0, & \text{for } t < 0, \\ F_0 \cos \omega_0 t, & \text{for } t > 0. \end{cases} \quad (2)$$

Sketch (or plot) the resulting function $x(t)$, and give its physical interpretation. Explore the trend at $\delta \rightarrow 0$.

Notice: The frequency in Eq. (2) is the real own frequency ω_0 of the oscillator, rather than its re-normalized value $\omega'_0 = \sqrt{\omega_0^2 - \delta^2}$.