Georgia Institute of Technology

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Computational Investing, Part I

141: CAPM For Portfolios

Find out how modern electronic markets work, why stock prices change in the ways they do, and how computation can help our understanding of them. Learn to build algorithms and visualizations to inform investing practice.

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Module Objectives

- Recap and add to CAPM toolset
- Understand CAPM for portfolios
- Understand portfolio beta and portfolio alpha
- Learn how to manage market risk

Recommended Readings

- *Grinold & Kahn chapters 5 & 6*

Recall CAPM for Individual Stock

$$r_i(t) = \underbrace{\beta_i \cdot r_m(t)}_{\text{market}} + \underbrace{\alpha_i}_{\text{"Skill"}}$$

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$\alpha(t)$ Skill

$\epsilon(t)$ random

Now We Extend to *Portfolios*

h_i = % holdings in i

$$r_p(t) = \sum_i h_i \, r_i(t)$$

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$$r_i(t) = \beta_i r_m(t) + \alpha_i$$

Example: Portfolio with 2 Holdings

$h_1 = .25$, $h_2 = .75$

$\beta_1 = 3.0$ $\beta_2 = 1.0$

$$r_p(t) = \sum_i h_i r_i(t)$$

Example: Portfolio with 2 Holdings

$h_1 = .25$, $h_2 = .75$

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$$r_p = h_1(\beta_1 r_m + \alpha_1) + h_2(\beta_2 r_m + \alpha_2)$$

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$h_1 = .25, \quad h_2 = .75$

$\beta_1 = 3.0 \quad \beta_2 = 1.0$

$$r_p = h_1(\beta_1 r_m + \alpha_1) + h_2(\beta_2 r_m + \alpha_2)$$

$$= h_1\beta_1 r_m + h_2\beta_2 r_m + h_1\alpha_1 + h_2\alpha_2$$

return due to market

return due to "skill"

Example: Portfolio with 2 Holdings

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$\beta_1 = 3.0 \quad \beta_2 = 1.0$

$$r_p = h_1(\beta_1 r_m + \alpha_1) + h_2(\beta_2 r_m + \alpha_2)$$

$$= h_1\beta_1 r_m + h_2\beta_2 r_m + h_1\alpha_1 + h_2\alpha_2$$

$$.75\, r_m + .75\, r_m + .25\alpha_1 + .75\alpha_2$$

$$= 1.5\, r_m + .25\alpha_1 + .75\alpha_2$$

Summary

- Return for a portfolio is just a weighted sum of the individual returns of the holdings.
- We can separate out the component of returns related to:
  - Market (Beta)
  - Skill (Alpha)

Next: Example: How to Remove Market Risk