Home work

Sec 1

$o\text{-}t\text{-}o$ = one-to-one

12

a) add is not o-t-o since, for example, add (1, 1) = add (0, 2) while $(1, 1) \neq (0, 2)$.

It is onto, however

$y \in \mathbb{R}$ $y = add(x)$ solution $x = (y, 0)$

b) is not o-t-o

mult (1, 4) = mult (2, 2)

while

$(1, 4) \neq (2, 2)$

$y \in \mathbb{R}$ $y = mult(x)$ $x = (y, 1)$

~~[illegible]~~

~~a) [illegible]~~

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a) h isn't o-t-o : $h(1) = 3 = h(-1)$ $1 \neq -1$

b) h is o-t-o : $h(x_1) = h(x_2)$

$x_1^2 + 2 = x_2^2 + 2$

$x_1^2 = x_2^2$ $x_1 = \pm x_2$

18

a) $f(x) = (x+7)^2 - 100$

$f(x_1) = f(x_2)$

$(x_1+7)^2 - 100 = (x_2+7)^2 - 100$

$(x_1+7)^2 = (x_2+7)^2$

$|x_1+7| = |x_2+7|$

$x_1, x_2 \in N$

$x_1 + 7 > 0$

$x_2 + 7 > 0$

$x_1 + 7 = x_2 + 7$

$x_1 = x_2$

it is o-t-o

b) is not o-t-o

$f(0) = f(-14) = (-51$

it is not onto

c) It is not o-t-o

$f(0) = f(-14)$ $y \geq -100$

$x = \sqrt{\ldots}$

Sec 2

4. let $y = f^{-1}(x)$

$x = f(y) = -\sqrt{y}$

$y = x^2 = f^{-1}(x)$

10.

a) $f \circ g = \{(1,1), (3,8), (2,1), (4,9), (5,1)\}$

$g \circ f$ is not $f = \{1, 2, 3, 8, 9\} \not\subset$ dom $g = \{1, 2, 3, 4, 5\}$

$f \circ f$ is not $f = \{1, 2, 3, 8, 9\} \not\subset$ dom $f = \{1, 2, 3, 4, 5\}$

$g \circ g = \{(1,2) (2,2), (3,2) (4,1) (5,2)\}$

c) Since f is ~~onst~~ o-t-o and onto, it has an inverse $f^{-1} = \{(8,1) (9,3) (3,4) (1,2) (2,5)\}$

& g is not o-t-o g does not have inverse

13.

$$g \circ f(x) = \frac{\frac{1}{x}}{x+1} = \frac{x+1}{x},$$

$$f \circ g(x) = \frac{\frac{1}{x}}{\frac{1}{x}+1} = \frac{1}{x+1}$$

$$h \circ g \circ f(x) = h\left(\frac{x+1}{x}\right) = \frac{x+1}{x} + 1 = \frac{2x+1}{x}$$

$$h \circ g \circ h(x) = f\left(\frac{1}{x+1}\right) = \frac{\frac{1}{x+1}}{\frac{1}{x+1}+1} = \frac{1}{x+2}$$

5 (a)

$g(m_1, n_1) = g(m_2, n_2)$

$(m_1, f(n_1)) = (m_2, f(n_2))$

$m_1 = m_2$ $\quad f(n_1) = f(n_2)$

f is one-to-one $\quad n_1 = n_2$

g is one-to-one

$(m_1, n_1) = (m_2, n_2)$

g is onto let $(a, b) \in N \times Z$

f is onto $\quad n \in N$

$f(n) = b$

$g(a, n) = (a, f(n)) = (a, b)$

g is onto

B) let $f: N \to Z$ be any o-t-o onto function (~~for example~~, the function defined in ~~…~~)

$g: N \times N \to N \times Z$

$g(m, n) = (m, f(n))$

is o-t-o

$N \times N \to N \times Z$

12. a) $f(x) = kx + l$

$f(x) = \cancel{x} + 1$

b) $f(x) = 2x + 4$

is o-t-o

$(0, 1)$ and $(4, 6$

c) $f: (0, 1) \to (a, b)$

$f(x) = a + (b - a)x$ is o-t-o

$g: (0, 1) \to (c, d)$

$g(x) = c + (d - c)$

f^{-1} provides a o-t-o

$(a, b) \to (c, d$

$g \circ f^{-1}(x) = c + \frac{d - c}{b - a}(x - a)$