CSC317 Fall 2022 Myles Greene-Beaupre
Instructor: Odelia Schwartz
Assignment 4

Please complete your own homework and do not copy solutions from the web.

Due Sept 27 2022 by midnight, on Blackboard. Note: Some of the questions are based on the Cormen textbook. Points listed below are for relative weighting of the questions in this assignment. Each assignment will in the end be weighted equally.

1. Show that the solution to the recurrence T(n) = T(n/4)+T(3n/4) + cn, where c is a constant, is Omega of $\Omega(nlogn)$ by using a recursion tree. Draw the recursion tree and show how you obtain this answer. To do so, you will need to consider both a lower bound to the height of the tree and the amount of work at each level of the tree. Hint: In class we examined a similar but slightly different recurrence and showed that the solution is $O(n\log n)$. You can follow similar logic for this recurrence, but for Ω. (10 points)

Handwritten work is on last page (page 3)

2. **In many of the earlier algorithms in class, such as Insertion Sort, we analyzed worst case performance. (a) What are we typically analyzing with randomized algorithms? (b) Why do we randomize the candidates in the hiring problem of the textbook before running the algorithm and analysis? (10 points)**

(a) With randomized algorithms we are typically analyzing the average run time rather than the worst case runtime.
(b) Because the cost to hire and fire depends on the order of the candidates, and it is expensive. By randomizing our list of candidates, we can be sure there is no bias that might affect our runtime.

3. **In the probability review in class, we showed an easier way to compute the Expected value (average) of the sum of two dice, by using the Linearity of Expectations. (a) Show how you would use the Linearity of Expectations property to compute the Expected value (average) of the sum of 10 fair dice. (b) Do the same, but now compute the Expected value (average) of the sum of 10 biased dice in which the probability of obtaining value 6 is 1 and the probability of all other values are 0. Show all your steps of using Expectations and Linearity of Expectations. (10 points)**

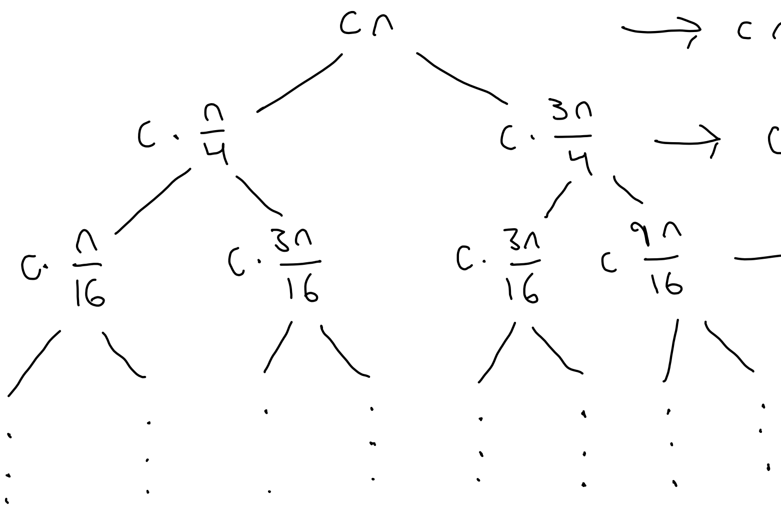
(a) We can say X represents the sum of 10 dice, and X1, X2, …, Xi represents the value of each dice. Because of the linearity of expectations, we know that the expected value of the sum of each dice value is equal to the sum of the expected values of each dice value. The expected value of Xi for any given i is equal to 1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(1/6) + 6(1/6) = 21/6 = 3.5. Therefore, for i = 10 dice rolls, and following the linearity of expectations, E[X1] + E[X2] + E[X3] + E[X4] + E[X5] + E[X6] + E[X7] + E[X8] + E[X9] + E[X10] = 3.5 + 3.5 + 3.5 + 3.5 + 3.5 + 3.5 + 3.5 + 3.5 + 3.5 + 3.5 = 35. So the expected value of the sum of 10

fair dice is 35, and in general we can say the expected value of the sum of i fair dice is equal to 3.5i. This gives us a quick and easy formula to plug into for any number of dice rolls.
(b) Since now the probability of getting value 6 is 1 and the probability of getting any other value is 0, we know E[Xi] for any i = 1(0) + 2(0) + 3(0) + 4(0) + 5(0) + 6(1) + 7(0) + 8(0) + 9(0) + 10(0) = 6. Following the linearity of expectations, E[X1] + E[X2] + E[X3] + E[X4] + E[X5] + E[X6] + E[X7] + E[X8] + E[X9] + E[X10] = 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 = 60. This makes sense, since if we roll a dice 10 times and we get a value of 6 every time, we know the sum of the values of our 10 rolls must be 6 * 10 = 60, and this will be the case every time, so the average will be 60 no matter how many times we do our 10 rolls.

4. **In the randomized hiring problem, (a) What is the probability that the first candidate is hired? (b) what is the probability that the fifth candidate is hired? Explain your answers. (10 points)**

(a) There is a 100% chance that the first candidate is hired, or a probability of 1. Intuitively, this is because there are no other candidates to compare it against, so the first candidate is automatically the best we've seen so far. You could also plug the number of candidates n into the general formula 1 / n since the probability that the nth candidate is hired is 1 out of the total number of candidates seen so far, so for example the probability that candidate 1 is hired is 1 / 1 = 1 (100% chance).
(b) Using the formula described in part a, we can see that the probability that candidate 5 is hired is 1 / 5 = 0.2. This means there is a 20% chance that the fifth candidate is hired, assuming a fair world.

$T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{3n}{4}\right) + cn$



$\rightarrow cn$

$\rightarrow c\left(\frac{n}{4} + \frac{3n}{4}\right) = c\frac{4n}{4} = cn$

$\rightarrow c\left(\frac{n}{16} + \frac{3n}{16} + \frac{3n}{16} + \frac{9n}{16}\right) = c\frac{16n}{16} = cn$

$\rightarrow$ every level will have cost cn

- We will consider shorter path since we want lower bound (Ω)
- Left branch ($\frac{n}{4}$) is shorter path (get's to bottom faster)
- Height of left branch is $\log_4(n)$, but we ignore base

Since constant factor $\left(\log_b(a) = \frac{\log_c(a)}{\log_c(b)}\right)$

- With a height of $\log(n)$ and a cost at each level of

cn, we can multiply these and get $cn\log(n)$

- Ignoring constants we can say we have found our lower bound

to be $\Omega(n\log(n))$ ✓