# Project 3 - FYS3150[*]

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An abstract

## I. INTRODUCTION

An introduction.

## II. THEORY AND METHODS

### A. Newton's law of gravitation

Newton's law of gravitation states for two objects of mass $m_1$ and $m_2$, the force on object 1 from object 2 is given by [**?** ],

$$\boldsymbol{F_{1,2}} = \frac{Gm_1m_2}{r^2}\boldsymbol{u_r} = \frac{Gm_1m_2}{r^3}\boldsymbol{r} \tag{1}$$

where $G$ is the gravitational constant and $\boldsymbol{u_r} = \boldsymbol{r}/r$ is a radial unit vector. $\boldsymbol{r}$ is a radial vector pointing at object 2 and $r = |\boldsymbol{r}|$ is the distance. Newton's third law gives us that the force on object 2 from object 1 is $\boldsymbol{F_{2,1}} = -\boldsymbol{F_{1,2}}$. Newton's third law gives us the differential equation governing the motion of object 1

$$\boldsymbol{r}\ddot{(\boldsymbol{t})} = \boldsymbol{a(t)} = \boldsymbol{F_{1,2}}(t, \boldsymbol{r(t)})/m_1, \tag{2}$$

where $\boldsymbol{a}$ is the acceleration, and we can solve this equation to find the motion $\boldsymbol{r(t)}$. For a two-body system this equation will produce closed elliptical orbits around a common center of mass.
If we assume that the orbit of object 2 around object 1 is circular we know that the force obeys the following equation

$$F_{2,1} = \frac{Gm_1m_2}{r^2} = \frac{m_1v_1^2}{r}, \tag{3}$$

which implies that

$$v_1^2 r = Gm_2. \tag{4}$$

Introducing 1 AU $= 1.5 \cdot 10^{11}$m

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### B. The second part

$$1 = 1 \tag{5}$$

### C. Mercury's perihelion precision

In classical mechanics, using Newtons law of gravity, we will get closed elliptical orbits for a planet moving around the sun. The perihelion-position (the point where the planet is closest to the sun) for the planet will appear at the same place every orbit. In real life, this does not happen. What we see is that the perihelion-position will move slightly for every orbit for a planet around the sun. There are different factors in the universe that causes this to happen. One factor is the theory of relativity.
The perihelion-precision is the difference between the postulated classical and the observed movement of the perihelion-position. When subtracting all factors that causes the shift of the perihelion-position except for the theory of relativity, the value of the perihelion precision is 43$''$ per century. [**?** ]

When simulating this in our project, we add a relativistic correction to the classical force, so we get

$$F_{1,2} = \frac{GM_1M_2}{r^2}\left[1 + \frac{3l^2}{r^2c^2}\right] \tag{6}$$

where $l = |\vec{r}\times\vec{v}|$ is the magnitude of the planets orbital angular momentum per unit mass and $c$ is the speed of light in vacuum. We looked at a simulation of Mercury and the sun for a century, and found all the perihelion-positions. We could then calculate the angle $\theta_p$ by

$$\tan\theta_p = \frac{y_p}{x_p} \Rightarrow \theta_p = \arctan(\frac{y_p}{x_p}) \tag{7}$$

where $y_p$ and $x_p$ are the $x$ and $y$ coordinates at the perihelion. The perihelion precision is then given by

$$\theta_{p,classical} - \theta_{p,relativistic} \tag{8}$$

## III. RESULTS AND DISCUSSION

### A. First subpart

An equation reference

### B. Three-body problem

In Figure (1) we have included three dimensional picture of the three-body problem where we have used the Verlet method for integration.

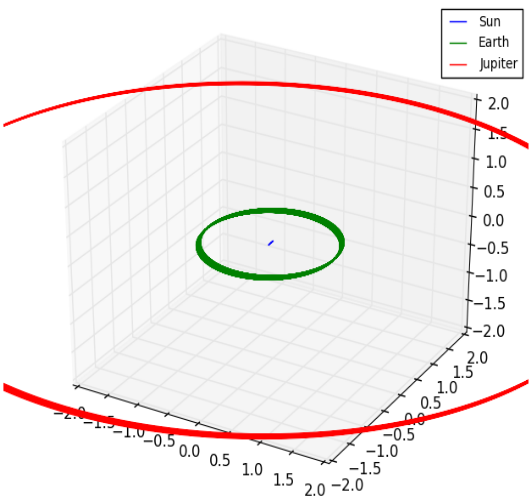
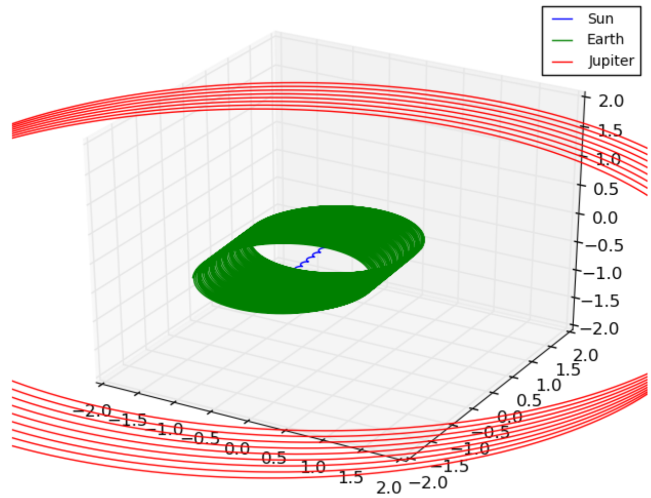


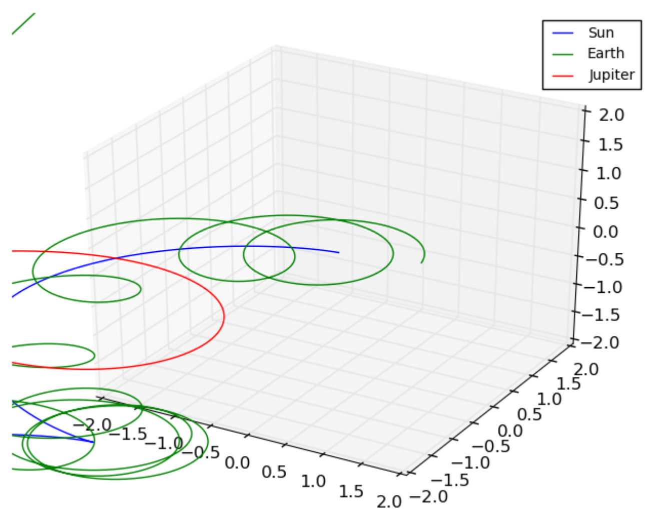

FIG. (1) The three-body problem where Jupiter is the red graph and the earth is the green. The motion of the sun can barely be seen as the blue graph.

Analysing this plot by looking at it tells us that the system is stable. The Sun seems to be moving slightly and the planets move in what appears to be closed and elliptical orbits.

In Figure (2) the motion of the planets are clearly different. In (a) Jupiter's mass is multiplied by 10 and in (b) the mass is multiplied by 1000 (which is close to the mass of the sun).




(a) $M_{Jupiter} \times 10$




(b) $M_{Jupiter} \times 1000$

FIG. (2) Earth-Sun-Jupiter three-body system for bigger values of $M_{jupiter}$

### C. Mercury's perihelion precision

In this project we looked at Mercury's perihelion-precision. We then added a relativistic correction to the force (equation 6) and compared the angle at perihelion (equation 7) to the situation with classical Newtonian gravitational force.

To get the right precision in this part of the project it was crucial to have enough steps for integration. In Table 1 we have shown the time and the perihelion precision for two different numbers of integration-steps. The perihelion precision is calculated by Equation 8 for the $\theta_p$ values after a century.

We expected the perihelion-precision to be 43″. For $n = 10^8$ integration-steps we get an answer that is close enough to show that this is caused by relativistic correction. Also, if we were to to more integration-steps, our computers would spend too much time doing it. In Figure (?) we have included a plot for Mercury's perihelion precision. From this we can see that the precision between the classical case and the relativistic case gets

TABLE (I) Perihelion precision for different timesteps

| **Integration steps $n$** | **Time $[s]$** | **Perihelion precision ["]** |
|---|---|---|
| $10^8$ | 0.3 | 47.72 |
| $10^5$ | 234.3 | 165.0 |

bigger as time goes by.

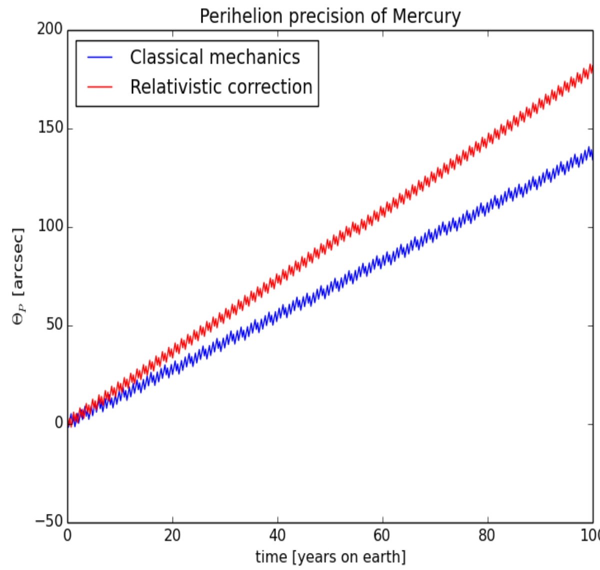



FIG. (3) Perihelion precision for Mercury around the sun for a century. The blue graph represent the classical case and the red graph represent the relativistic case. This is done with $n = 10^8$ integrationsteps

## IV. CONCLUSION

Do stuff.

## V. APPENDIX

All code used is available at: The programs used in this project are listed in this section:

**main.cpp:** Program1

**plot.py:** Program2