Stellar Spectra B. LTE Line Formation

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1. Stratification of the solar atmosphere

In this exercise we study the radial stratification of the solar atmosphere by using the FALC model by Fontela et al. (1993)

1.1. FALC temperature stratification

The first thing to do is import the data from the modelfiles, and take a look at how temperature varies with height. This can be seen in figure **??**

1.2. FALC density stratification

I start by plotting the total pressure p_{total} against colmn mass m. See figures **??** and **??**. We see that they scale linearly. From this we can conclude that we can write

$$p_{total} = Cm \tag{1}$$

where if one finds C for all pressures and column masses and then find the average C, I get $C = g_{surface} = 27398.2$ cm/s^2.

Fontena et a. (1993) assumed complete mixing, so we check that this condition holds by plotting the ratio of the hydrogen mass density to the total mass density against height. Next we add the Helium as well and calculate the contribution of helium and hydrogen to the total. From the figure it seems that nearly all of the density is contributed form hydrogen and helium. However if one does the calculation one finds that the average fraction of the remaining elements (the "metals") contributes 0.002 (0.02%) of the total .This can be seen in figure **??**

Next we plot the column mass against height. See figure **??**. Note that the curve becomes nearly straight if we make the y-axis logarithmic in figure **??**. This is caused by XXXXXXXXXXXXXXXXXXXXXXXXXX.

The next quantity to look at is gas density. Gas density is plottes against height in figure **??**.

We want to know the pressure scale height H_ρ in

$$\rho \approx \rho(0)exp(-h/H_\rho) \tag{2}$$

This can be found from the definition

$$H_\rho = \frac{kT}{Mg} \tag{3}$$

where k is boltzmanns constant, T is temperature in K, M is the mean molecular weight, and g is surface gravity. If one assumes that most of the molecules in the photosphere is hydrogen we can set $M = m_H$. Inserting the values for the deep photosphere ($h = -100$) then gives a scale height of $H_\rho = 196.3$ km. This is not realistic. Assuming that the photosphere only contains helium gives a scale height $H_\rho = 49.3km$. The real number is somewhere between these two, but since we don't know the mean molecular weight this method doesn't really work in this case.

Another way to do this is to just mark the point where the density has fallen to $1/e$ of its original value. In figure **??** this is marked with a line. The point the two lines cross gives $H_\rho \approx$ 150km.

The next step is to compute the gas pressure and plot it against height. We also overplot the product $(n_H + n_e)kT$. See figure **??**. There is some difference between the curves. Plotting the ratio between the curves shows this clearly (figure **??**). Note however that the sun does not only contain hydrogen. Because of this we must include the hydrogen number density in the ideal gas law. By doing this we obtain figure **??** which shows a curve that seems to be overlapping perfectly. Plotting the ratio between the two curves shows that it deviates only slightly at the fourth decimal place. See figure **??**. From this we can conclude that the ideal gas law is a very good approximation.

With that done we want to look at the total hydrogen density. We should also include the electron density, proton density, and the density of the electrons that do not result from hydrogen ionization. The electron density, proton density and electron density can be read out of the FALC model. To find the density of the electrons not from ionized hydrogen we need to assume that all of the protons come from ionized hydrogen. This would indicate that $n_H - n_p$ equals the number of neutral hydrogen atoms. And since each neutral hydrogen has one electron this would be the number of electrons still bound to hydrogen, e.g those not from ionized hydrogen. The density of these should then be

$$n_{be} = (n_H - n_p). \tag{4}$$

This is plotted against height in figure **??**.

From the graph we see that the number density of protons is a little above 0 for very low heights. This indicates that the hydrogen gets ionized at this height. As the height increases this goes to zero indicating that the hydrogen remains neutral when height increases. Because of this it is only logical that the electron density goes to zero as well, and that the density of the elctrons not from ionized hydrogen approaches the density of the hydrogen (since nearly all of the hydrogen is neutral hydrogen). Figure **??** shows the same with a logarithmic

As the height increases the number density of everything approaches zero, which makes sense since we are looking at the atmosphere of the sun. At some point we should approach vacuum which has almost zero density.

With that done we plot the ionization fraction of hydrogen against height. The ionization fraction of hydrogen fraction is simply

$$n_{HII} = \frac{n_p}{n_H} \tag{5}$$

See figure **??**. Note the logarithmic scale. There is a clear resemblence to the temperature plot (see figure **??**). XXXXXXXXXXXXXXXXXXXXXXXXXX EXPLAIN WHY THAT IS AND WHY IT IS TILTED XXXXXXXXXXXXXXXXXXXXXXXXXX

The next thing one should do is compare the photon density to the particle density. If one assumes thermodynamic equilibrium the photon density N_{phot} in photons per cm^3 is given by

$$N_{phot} \approx 20T^3 \tag{6}$$

	Sun	Earth
ρ	2.77×10^{-7}g cm^{-3}	1.23×10^{-3}g cm^{-3}
N	1.30×10^{17}cm^{-3}	2.57×10^{19}cm^{-3}
P	1.21×10^{5}dyn cm^{-3}	1.02×10^{6}dyn cm^{-3}
T	6520K	288K

Table 1. Table with different values for the earth and the sun

XXXXXXXXXXXXXXXXXXXXXXXXX EXPLAIN WHY THIS IS XXXXXXXXXXXXXXXXXXXXXXXXX. Calculating this value for the deep photosphere ($h = -100$) gives a photon density $N_{phot} = 1.66 \times 10^{13}$ cm^{-3}. In comparison the hydrogen density at the same location is $N_H = 1.35 \times 10^{17}$ cm^{-3}. Unfortunately we can not assume thermodynamic equilibrium higher up in the atmosphere. The equation for the photon density there instead becomes

$$N_{phot} = 20T_{eff}^3/2\pi \tag{7}$$

where $T_{eff} = 5770$K is the effective solar temperature. Calculating this for the highest point in our data gives $N_{phot} = 6.11 \times 10^{11}$ cm^{-3}. The density of hydrogen a t the same point is $N_H = 5.58 \times 10^9$. At this height there are more photons than hydrogen atoms. Note that the hydrogen density dropped by 8 orders of magnitude, while the photon density dropped only 2. The medium at this high altitude is insensitive to the photons because XXXXXXXXXXXXXXXXXXXXXXXXX EXPLAIN WHY XXXXXXXXXXXXXXXXXXXXXXXXX.

1.3. Comparison with the earth's atmosphere

There have been done similar measurements of the earths atmosphere. The values used in this report are from Allen 1976. The first thing to do is to make plots of everything in the table. See figures **??**, **??**, **??**, and **??**.

Taking a closer look at pressure and density reveals that if one normalizes each to their respective maximums and then plot them together we get a graph that is very close to identical. XXXXXXXXXXXXXXXXXXXXXXXXX COMMENT WHY THEY LOOK THE SAME XXXXXXXXXXXXXXXXXXXXXXXXX

Next we plot the mean molecular weight $\mu_E \equiv \overline{m}/m_H$ against height. See figure **??**. We see that it decreases at higher altitudes. This is caused by XXXXXXXXXXXXXXXXXXXXXXXXX EXPLAIN WHY XXXXXXXXXXXXXXXXXXXXXXXXX.

We should also se what the density scale height is in the lower atmosphere. Like for the sun we can do this two different ways. Either by using the equation for the scale height

$$H_\rho = \frac{kT}{Mg} \tag{8}$$

where everything means the same as before. The difference is that in this case we actually know the mean molecular weight. Inserting values gives $H_\rho = 8.54$ km at the surface of earth. Approximating with the same method as earlier gives $H_\rho \approx 9.5$ km. In this case I would trust the first one more since we know the values we use in the equation to a good precision. Note that the scale height on earth is more than 10 times smaller than on the sun. This is of course caused by the large difference in surface gravity and temperature.

Comparing the rest of the quantities for the sun and earth at the surface shows many differences. Note especially that the ratio between particle density at the surface of earth to the surface of the sun is almost 200.

If we now calculate the column mass for the surface of earth we find that it is 1044g cm^{-2}. For the sun the column mass on the surface was 4.404g cm^{-2}.

If we now calculate the photon density reaching earth from the sun

$$N_{phot} = \pi \frac{R^2}{D^2} N_{phot}^{top} \tag{9}$$

where N_{phot}^{top} is the photon density we found earlier at the top of the solar atmosphere, we find that $N_{phot} = 4.16 \times 10^7$cm^{-3} reaching us from the sun. While if we calculate the thermal photon density from the earth atmosphere we get $N_{phot}^{earth} = 4.78 \times 10^8$cm^{-3}. Which means that there are more than 10 times more photons coming from the earth atmosphere on the surface of earth than we receive from the sun. Calculating the ratio of the received photons and the particle density in the air gives a ratio of 1.6×10^{-12}. So the photon density is smaller by 12 orders of magnitude.

XXXXXXXXXXXXXXXXXXXXXXXXX COMMENT XXXXXXXXXXXXXXXXXXXXXXXXX.

2. Continous spectrum from the solar atmosphere

In this section we want to look at the formation of the solar continuum. We concentrate on the visible and near-infrared parts of the spectrum.

2.1. Observed solar continua

We will be using the table provided by Allen (1976) for this part. It specifies the continuum radiation emitted by the sun in the wavelength range $\lambda = 0.2 - 5\mu$m. This table specifies four different quantities as a function of wavelength. The quantites are radially emergent intensity and the astrophysical flux in the solar continuum with, and without, smoothed lines.

The four quantities are plotted in figure **??**. XXXXXXXXXXXXXXXXXXXXXXXXX EXPLAIN WHY THEY SHARE THE SAME UNITS AND THEIR DIFFERENCES XXXXXXXXXXXXXXXXXXXXXXXXX

Next we convert the distributions into values per frequency bandwitch $\Delta\nu = 1$ Hz, and plot them against wavelength. (See figure **??**).

We should try to approximate the intensity of the continuum with a planck fit. Figure 1 has the planck function with 3 different tempetatures in the same plot as the solar continuum intensity. The same can also be done for the ones converted to frequency (See figure 2). Comparing the two it is clear that something in the range 6200-6400 K is as good as we will get. I will go with 6300 K.

Inverting the planck function and inserting the solar continuum produces figure 3 We see that it peaks at $\lambda \approx 1.6\mu$m

3. References

Rutten, R. J.: 1991, The Generation and Transportation of Radiation, Sterrekundig Instituut Utrecht, The Netherlands

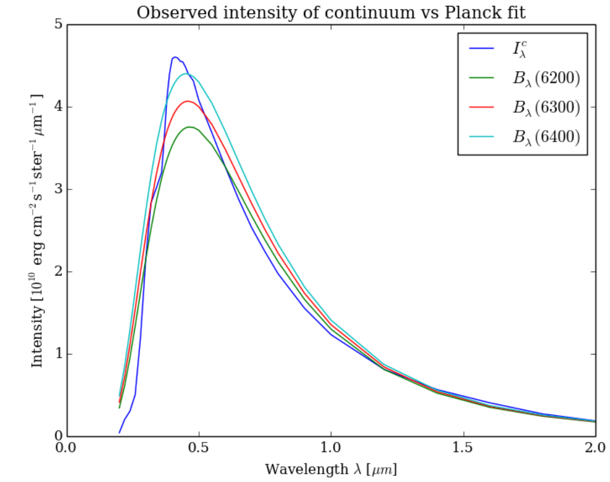

Fig. 1. The solar continuum with three attempts to fit a planck function to it.

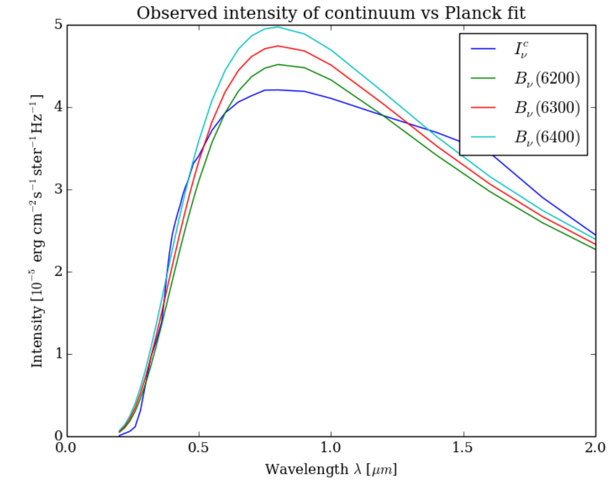

Fig. 2. The solar continuum with three attempts to fit a planck function to it.

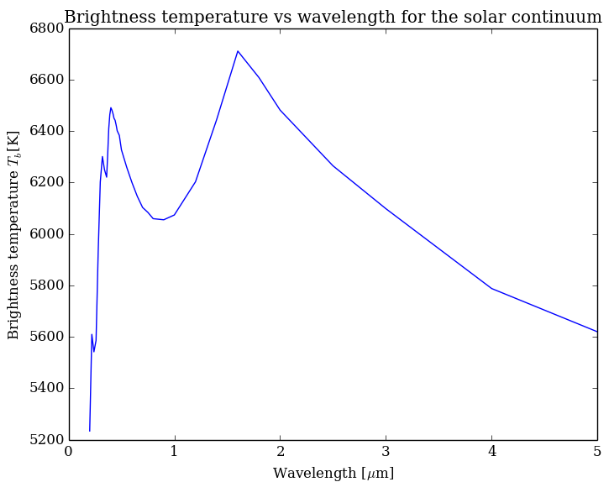

Fig. 3. Brightness temperature of the solar continuum.