Stellar Spectra B. LTE Line Formation

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1. Stratification of the solar atmosphere

In this exercise we study the radial stratification of the solar atmosphere by using the FALC model by Fontela et al. (1993)

1.1. FALC temperature stratification

The first thing to do is import the data from the modelfiles, and take a look at how temperature varies with height. This can be seen in figure ??

1.2. FALC density stratification

I start by plotting the total pressure p_{total} against colmn mass m. See figures ?? and ??. We see that they scale linearly. From this we can conclude that we can write

$$p_{total} = Cm \tag{1}$$

where if one finds C for all pressures and column masses and then find the average C, I get $C = g_{surface} = 27398.2 \text{ cm/s}^2$.

Fontena et a. (1993) assumed complete mixing, so we check that this condition holds by plotting the ratio of the hydrogen mass density to the total mass density against height. Next we add the Helium as well and calculate the contribution of helium and hydrogen to the total. From the figure it seems that nearly all of the density is contributed form hydrogen and helium. However if one does the calculation one finds that the average fraction of the remaining elements (the "metals") contributes 0.002 (0.02%) of the total .This can be seen in figure ??

The next quantity to look at is gas density. Gas density is plottes against height in figure ??.

We want to know the pressure scale height H_{ρ} in

$$\rho \approx \rho(0)exp(-h/H_{\rho}) \tag{2}$$

This can be found from the definition

$$H_{\rho} = \frac{kT}{Mg} \tag{3}$$

where k is boltzmanns constant, T is temperature in K, M is the mean molecular weight, and g is surface gravity. If one assumes that most of the molecules in the photosphere is hydrogen we can set $M=m_H$. Inserting the values for the deep photosphere (h=-100) then gives a scale height of $H_{\rho}=196.3$ km. This is not realistic. Assuming that the photosphere only contains helium gives a scale height $H_{\rho}=49.3$ km. The real number is somewhere between these two, but since we don't know the mean molecular weight this method doesn't really work in this case.

Another way to do this is to just mark the point where the density has fallen to 1/e of its original value. In figure ?? this is marked with a line. The point the two lines cross gives $H_{\rho} \approx 150 \mathrm{km}$.

The next step is to compute the gas pressure and plot it against height. We also overplot the product $(n_H + n_e)kT$. See figure ??. There is some difference between the curves. Plotting the ratio between the curves shows this clearly (figure ??). Note however that the sun does not only contain hydrogen. Because of this we must include the hydrogen number density in the ideal gas law. By doing this we obtain figure ?? which shows a curve that seems to be overlapping perfectly. Plotting the ratio between the two curves shows that it deviates only slightly at the fourth decimal place. See figure ??. From this we can conclude that the ideal gas law is a very good approximation.

With that done we want to look at the total hydrogen density. We should also include the electron density, proton density, and the density of the electrons that do not result from hydrogen ionization. The electron density, proton density and electron density can be read out of the FALC model. To find the density of the electrons not from ionized hydrogen we need to assume that all of the protons come from ionized hydrogen. This would indicate that $n_H - n_p$ equals the number of neutral hydrogen atoms. And since each neutral hydrogen has one electron this would be the number of electrons still bound to hydrogen, e.g those not from ionized hydrogen. The density of these should then be

$$n_{be} = (n_H - n_p). (4)$$

This is plotted against height in figure ??.

From the graph we see that the number density of protons is a little above 0 for very low heights. This indicates that the hydrogen gets ionized at this height. As the height increases this goes to zero indicating that the hydrogen remains neutral when height increases. Because of this it is only logical that the electron density goes to zero as well, and that the density of the electrons not from ionized hydrogen approaches the density of the hydrogen (since nearly all of the hydrogen is neutral hydrogen). Figure ?? shows the same with a logarithmic

As the height increases the number density of everything approaches zero, which makes sense since we are looking at the atmosphere of the sun. At some point we should approach vacuum which has almost zero density.

With that done we plot the ionization fraction of hydrogen against height. The ionization fraction of hydrogen fraction is simply

$$n_{HII} = \frac{n_p}{n_H} \tag{5}$$

The next thing one should do is compare the photon density to the particle density. If one assumes thermodynamic equilibrium the photon density N_{phot} in photons per cm³ is given by

$$N_{phot} \approx 20T^3$$
 (6)

| | Sun | Earth |
|--------|---|--|
| ρ | $2.77 \times 10^{-7} \text{g cm}^{-3}$ | $1.23 \times 10^{-3} \text{g cm}^{-3}$ |
| N | $1.30 \times 10^{17} \text{cm}^{-3}$ | $2.57 \times 10^{19} \text{cm}^{-3}$ |
| P | $1.21 \times 10^{5} \text{dyn cm}^{-3}$ | $1.02 \times 10^6 \text{dyn cm}^{-3}$ |
| T | 6520K | 288K |

Table 1. Table with different values for the earth and the sun

$$N_{phot} = 20T_{eff}^3/2\pi \tag{7}$$

1.3. Comparison with the earth's atmosphere

There have been done similar measurements of the earths atmosphere. The values used in this report are from Allen 1976. The first thing to do is to make plots of everything in the table. See figures ??, ??, ??, and ??.

We should also se what the density scale height is in the lower atmosphere. Like for the sun we can do this two different ways. Either by using the equation for the scale height

$$H_{\rho} = \frac{kT}{Mg} \tag{8}$$

where everything means the same as before. The difference is that in this case we actually know the mean molecular weight. Inserting values gives $H_{\rho}=8.54~\mathrm{km}$ at the surface of earth. Approximating with the same method as earlier gives $H_{\rho}\approx 9.5~\mathrm{km}$. In this case I would trust the first one more since we know the values we use in the equation to a good precision. Note that the scale height on earth is more than 10 times smaller than on the sun. This is of course caused by the large difference in surface gravity and temperature.

Comparing the rest of the quantities for the sun and earth at the surface shows many differences. Note especially that the ratio between particle density at the surface of earth to the surface of the sun is almost 200.

If we now calculate the column mass for the surface of earth we find that it is $1044 {\rm g~cm^{-2}}$. For the sun the column mass on the surface was $4.404 {\rm g~cm^{-2}}$.

If we now calculate the photon density reaching earth from the sun

$$N_{phot} = \pi \frac{R^2}{D^2} N_{phot}^{top} \tag{9}$$

where N_{phot}^{top} is the photon density we found earlier at the top of the solar atmosphere, we find that $N_{phot} = 4.16 \times 10^7 {\rm cm}^{-3}$ reaching us from the sun. While if we calculate the thermal photon density from the earth atmosphere we get $N_{phot}^{earth} = 4.78 \times 10^8 {\rm cm}^{-3}$. Which means that there are more than 10 times more photons coming from the earth atmosphere on the surface of earth than we receive from the sun. Calculating the ratio of the received photons and the particle density in the air gives a ratio of 1.6×10^{-12} . So the photon density is smaller by 12 orders of magnitude.

2. Continous spectrum from the solar atmosphere

In this section we want to look at the formation of the solar continuum. We concentrate on the visible and near-infrared parts of the spectrum.

2.1. Observed solar continua

We will be using the table provided by Allen (1976) for this part. It specifies the continuum radiation emitted by the sun in the wavelength range $\lambda = 0.2 - 5\mu m$. This table specifies four different quantities as a function of wavelength. The quantites are radially emergent intensity and the astrophysical flux in the solar continuum with, and without, smoothed lines.

Next we convert the distributions into values per frequency bandwitch $\Delta \nu = 1$ Hz, and plot them against wavelength. (See figure ??).

We should try to approximate the intensity of the continuum with a planck fit. Figure 1 has the planck function with 3 different tempetatures in the same plot as the solar continuum intensity. The same can also be done for the ones converted to frequency (See figure 2). Comparing the two it is clear that something in the range 6200-6400 K is as good as we will get. I will go with 6300 K.

Inverting the planck function and inserting the solar continuum produces figure 3 We see that it peaks at $\lambda \approx 1.6 \mu m$. This is in the short-wavelength infrared region of the spectrum. There is also a peak around $0.4 \mu m$. This corresponds to violet in the visual spectrum. So in other words; the light we see receieve from the sun should be slightly more violet in color than the others. Whether increasing the brightness temperature from 6100K to 6500K results in something we can actually see is another question (Note that this could not be seen while inside the earth atmosphere because scattering changes the spectrum one receives considerably). Nevertheless the peak in the infrared region is

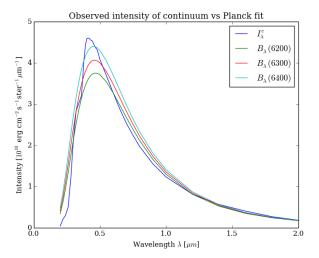


Fig. 1. The solar continuum with three attempts to fit a planck function to it.

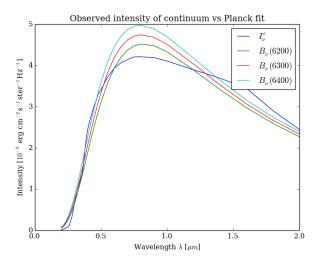


Fig. 2. The solar continuum with three attempts to fit a planck function to it.

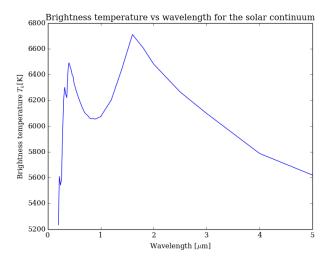


Fig. 3. Brightness temperature of the solar continuum.

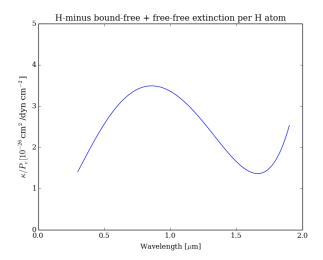


Fig. 4. H⁻ extinction for FALC parameters at surface of solar atmosphere, normalized with the electron pressure $P_e = n_e kT$

even higher than the violet one. That peak is also a wider one which would indicate that more energy is transported through infrared radiation than visible light.

2.2. Continuous extinction

Now we aim to reproduce figure 5 in the project text. We are provided with a function that evaluates polynomial fits for H^- extiction that are given on page 135 ff of Gray (1992). This function takes delivers the total H^- extinction in units of cm² per neutral hydrogen atom. Plotting the values corresponding to the electron density and temperature at the surface of the solar atmosphere results in figure 4

3. References

Rutten, R. J.: 1991, The Generation and Transportation of Radiation, Sterrekundig Instituut Utrecht, The Netherlands