

The CMB power spectrum

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Abstract. I compute the CMB power spectrum using all previous parts in the project.

0.1 Introduction

In this project I am following the algorithm presented in Callin (2005)[1] for simulating the cosmic microwave background. This is the final part of the project.

In the first part I set up the background cosmology of the universe, and made a function that could find the conformal time as a function of x . In the second part I computed the electron fraction, electron density, optical depth and visibility function for times around and during recombination. The third part use the two previous to compute the density perturbations, and velocities of dark matter and baryons. This also included the temperature multipoles Θ_l .

This final part combines all of these quantities to compute the final CMB power spectrum.

As previously done I will continue building on the skeleton code provided.

0.2 Equations

In this final part of the project we finally combine everything from the previous projects into the final CMB power spectrum. This is done by a few equations, the first of which is the Transfer function defined

$$\Theta_l(k, x=0) = \int_{-\infty}^0 \tilde{S}(k, x) j_l[k(\eta_0 - \eta)] dx, \quad (1)$$

where j_l is the spherical Bessel functions, and S is the source function defined

$$\tilde{S}(k, x) = \tilde{g} \left[\Theta_0 + \Psi + \frac{1}{4} \Pi \right] + e^{-\tau} [\Psi' - \Phi'] - \frac{1}{ck} \frac{d}{dx} (\mathcal{H} \tilde{g} v_b) + \frac{3}{4c^2 k^2} \frac{d}{dx} \left[\mathcal{H} \frac{d}{dx} (\mathcal{H} \tilde{g} \Pi) \right]. \quad (2)$$

The only variable not explained earlier in this equation is $\Pi = \Theta_2 + \Theta_0^P + \Theta_2^P$, which contain two not previously used variables Θ_0^P , and Θ_2^P . These two are related to the polarization of the temperature multi poles. Since we chose to not include polarization and neutrinos in the last part of the project, these two can be set to zero, which means that for our case $\Pi = \Theta_2$.

The Source function deserves a bit of explaining. Its job is, as expected from its name, to source the spectrum for all k modes at all x values. It consist of four distinct terms each describing different effects.

The first is the local monopole weighted by the visibility function \tilde{g} , this term also includes the fact that the photons climb out of a gravitational potential Ψ and a small correction term Π . This redshifting of the photons is known as the Sachs-Wolfe effect.

The second term is the integrated Sachs-Wolfe effect. This term takes care of the change due to changing gravitational fields the photons encounter on their way from the last scattering surface to us today.

0.4 Results

Underway through the workings of the program I have plotted various quantities to make sure that they return sensible results. An example is the integrand in the Transfer function (figure 1).

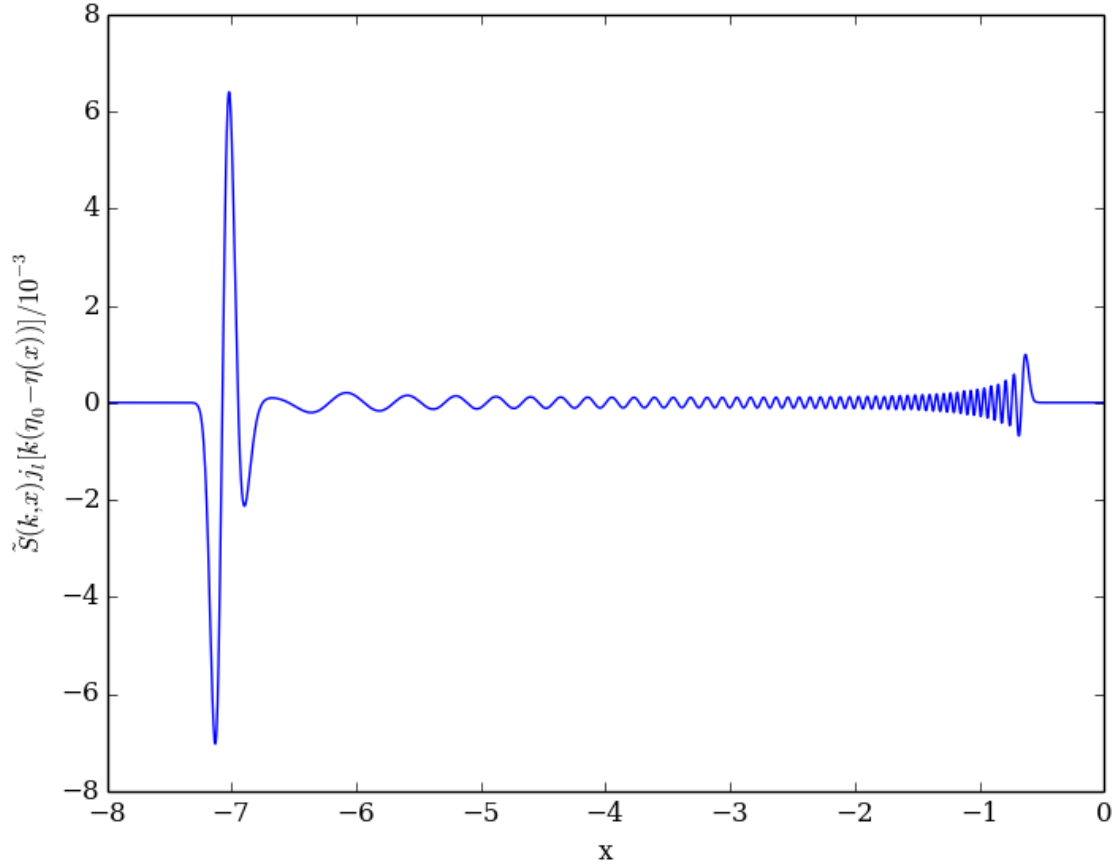


Figure 1: The integrand in the Transfer function integral for $l = 100$, and $k \approx 160H_0/c$. The source function gives a spike around recombination, while the Bessel function makes it oscillate at late times. This is sensible.

Another useful quantity is the Transfer function. This has been plotted for six different k values.

We should also make sure that the integrand in the power spectrum integral behaves

Finally, the CMB power spectrum can be seen in figure 3.

0.5 Conclusions

Unfortunately time did not allow me to use a Metropolis algorithm to estimate the values of the various cosmological parameters. This is something I will have to do on my own time afterwards. This is unfortunate as this would have been the icing on the cake.

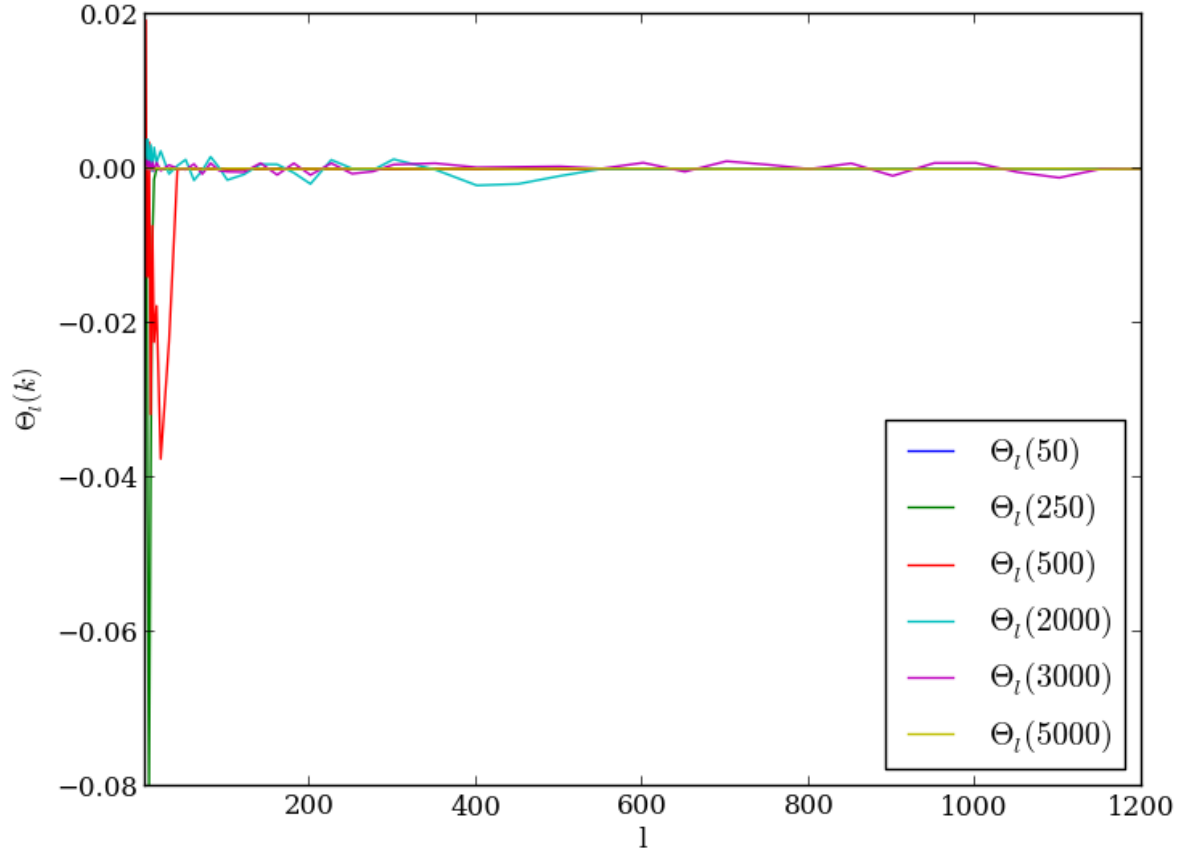


Figure 2: The Transfer function for k number 50,250,500,2000,3000, and 5000. Note that this does not correspond to the actual value of k. If the reader is interested in the actual k value he/she will have to use equation 14 presented in part 3 of the project for each of the k values.

0.6 References

- [1] P. Callin, astro-ph/0606683
- [2] P. A. R. Ade et al. [Planck Collaboration], arXiv:1502.02114 [astro-ph.CO].

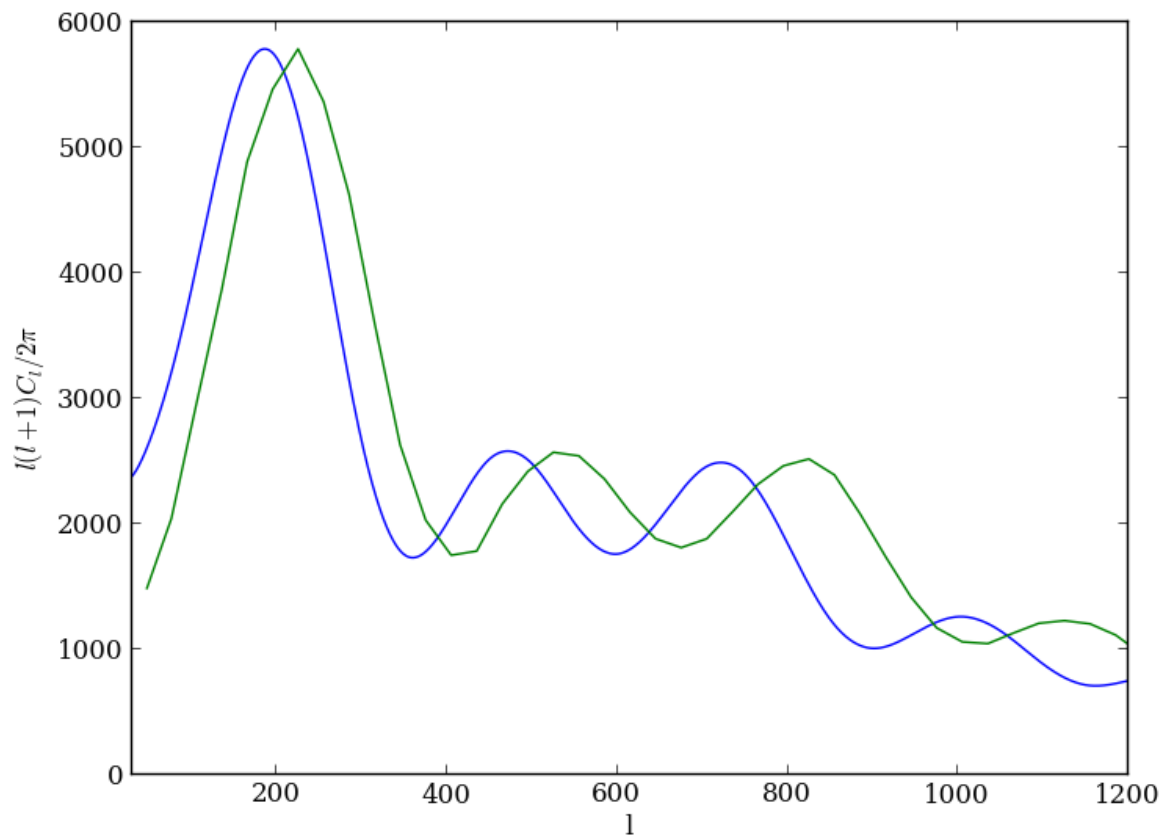


Figure 3: The CMB power spectrum. Computed in blue, and observed data in green.

0.7 Source code

The source code for the function made for computing the high resolution source function in `evolution.mod.f90` is included as well as the file `cl_mod.f90` file used for computing the final power spectrum is included for inspection. This file depends on all files previously used in the three earlier parts of the project.