Milestone 1: The background evolution of the universe

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Abstract. I set up the background cosmology of the universe.

1. Introduction

In this project I will follow the algorithm presented in Callin (2005) for simulating the cosmic microwave background. This is part one of four for this project. The first part involves computing the expansion history of the universe, as well as looking at the evolution of the density of various matter and energy components. This sets up the background cosmology such that we in the next step can turn to perturbations. I have chosen to make this first part compatible with the inclusion of neutrinos. This can of course be removed by setting the neutrino density to zero and raising the radiation density accordingly.

To ease the development of this code, I have been provided with a skeleton code of the project. This code includes a variety of methods needed to solve the project. There will be a methods section for each part of the project.

2. Methods

3. Equations

For the background cosmology I use the standard Friendmann-Lemaitre-Robertson-Walker metric for flat space. This gives the line element in eq. 1.

$$\begin{aligned} ds^2 &= -dt^2 + a^2(t)(r^2(d\theta^2 + sin^2\theta d\phi^2) \\ &= a^2(\eta)(-d\eta^2 + r^2(d\theta^2 + sin^2\theta d\phi^2)) \end{aligned} \tag{1}$$

where $a(t)$ is the scale factor, and η conformal time. I also introduce a parameter x defined in eq. 2.

$$x = ln(a) \tag{2}$$

We assume that the universe consist of cold dark matter (CDM, m), baryons (b), radiation(r), neutrinos(ν), and a cosmological constant (Λ). With these components, the Hubble parameter H becomes

$$H = \frac{1}{a}\frac{da}{dt} = H_0\sqrt{(\Omega_m + \Omega_b)a^{-3} + (\Omega_r + \Omega_\nu)a^{-4} + \Omega_\Lambda}. \tag{3}$$

I also introduce a scaled H, namely

$$\begin{aligned} \mathcal{H} &= \frac{1}{a}\frac{da}{d\eta} \equiv \frac{\dot{a}}{a} = aH \\ &= H_0\sqrt{(\Omega_m + \Omega_b)a^{-1} + (\Omega_r + \Omega_\nu)a^{-2} + \Omega_\Lambda a^2}. \end{aligned} \tag{4}$$

Note that the dot means derivative with respect to conformal time. Why this is useful will become apparent later. We also want the derivative of this with respect to x.

$$\frac{d\mathcal{H}}{dx} = H_0\frac{\sqrt{-(\Omega_m + \Omega_b)e^{-x} - 2(\Omega_r + \Omega_\nu)e^{-2x} + 2\Omega_\Lambda e^{2x}}}{\sqrt{(\Omega_m + \Omega_b)e^{-x} + (\Omega_r + \Omega_\nu)e^{-2x} + \Omega_\Lambda e^{2x}}}. \tag{5}$$

Ω_x is the relative density of compenent x compared to the critical density ρ_c needed for the universe to be flat.

$$\Omega_x = \frac{\rho_x}{\rho_c} \tag{6}$$

$$\rho_c = \frac{3H^2}{8\pi G} \tag{7}$$

$$\rho_m = \rho_{m,0}a^{-3} \tag{8}$$

$$\rho_b = \rho_{b,0}a^{-3} \tag{9}$$

$$\rho_r = \rho_{r,0}a^{-4} \tag{10}$$

$$\rho_\nu = \rho_{\nu,0}a^{-4} \tag{11}$$

$$\rho_\Lambda = \rho_{\Lambda,0} \tag{12}$$

We will also need to know the distance to the horizon at different times. This can be found by noting that

$$\frac{d\eta}{dt} = \frac{c}{a}$$

which can be rewritten such that

$$\frac{d\eta}{da} = \frac{c}{a\mathcal{H}} \tag{13}$$

There are two ways to solve this differential equation numerically. One can either intergrate directly, or one can use an ordinary differential equation solver(ODE Solver). Since the skeleton code contains an ODE Solver I have chosen the latter. (See the methods section for more information about this.)

4. Results

The Ω_x values indicate the relative density of a given component compared to the critical density of the universe. The critical density being that which corresponds to the universe begin flat. Looking at the figure we see radiation dominating from the start together with neutrinos. This continues for some time until the dark matter component starts to rise. We see that this starts before the baryon component. This is good since that makes it possible for dark matter to form structures that that baryons can later fall into. The radiation and neutrinos die out, and at a later

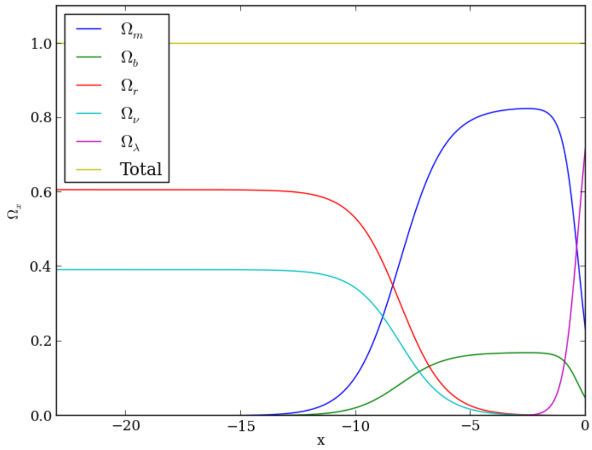

Fig. 1. The figure shows the evolution of the relative densities. They behave as expected from

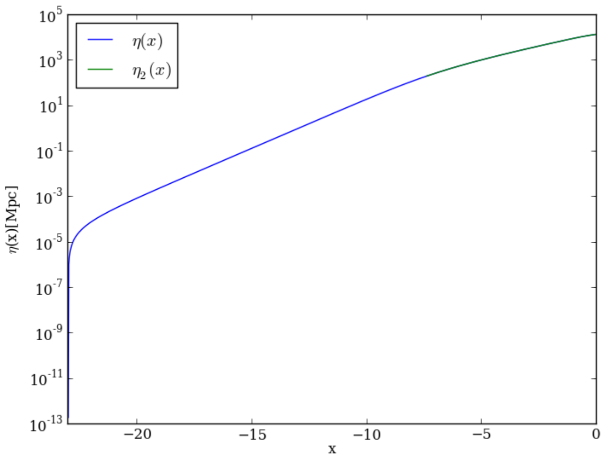

Fig. 2.

point vacuum energy shoots up. At the end we end up with approximately 70% vacuum energy(dark energy?), 25% dark matter, and 5% baryons. This is exactly as it should be since that is what they were set to be at the present.

Conformal time, denoted $\eta(x)$ measures the distance to the particle horizon for a given x value. A nice test of this is to insert the x-value of today The answer should then be the radius of the observable universe. At the present this is measured to be approximately 14 billion parsecs (14Gpc). The graph hits this value fairly well, indicating that everything is working properly so far. Note also that there are in fact two graphs in this figure, one which is calculated from the differential equation for η. And another one made by splining the first one and finding η values at arbitrary values between those of the first function.

The Hubble parameter H is depicted both as a function of x, and z. Whether this is good or not is hard to say for the first part. We can at least put some faith in its correctness by the fact that it ends at $H_0 \approx 70\text{km s}^{-1}\text{Mpc}^{-1}$, which is what H_0 was set to be.

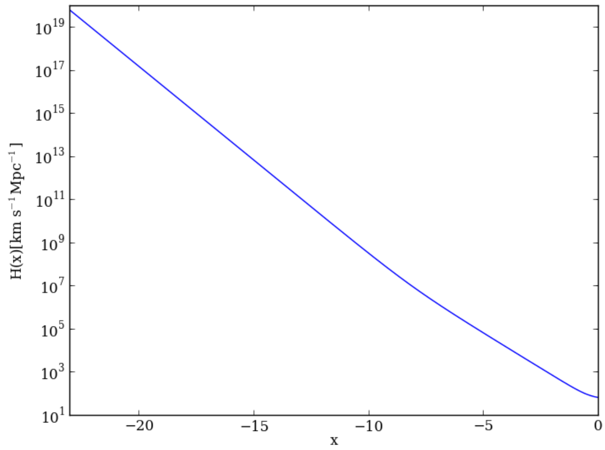

Fig. 3.

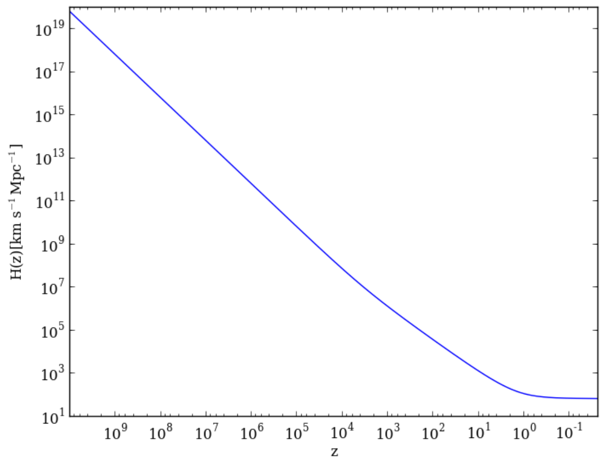

Fig. 4.

5. Conclusions

6. Source code

7. References

Itzhak Bars; John Terning (November 2009). Extra Dimensions in Space and Time. Springer. pp. 27 Callin (2005)