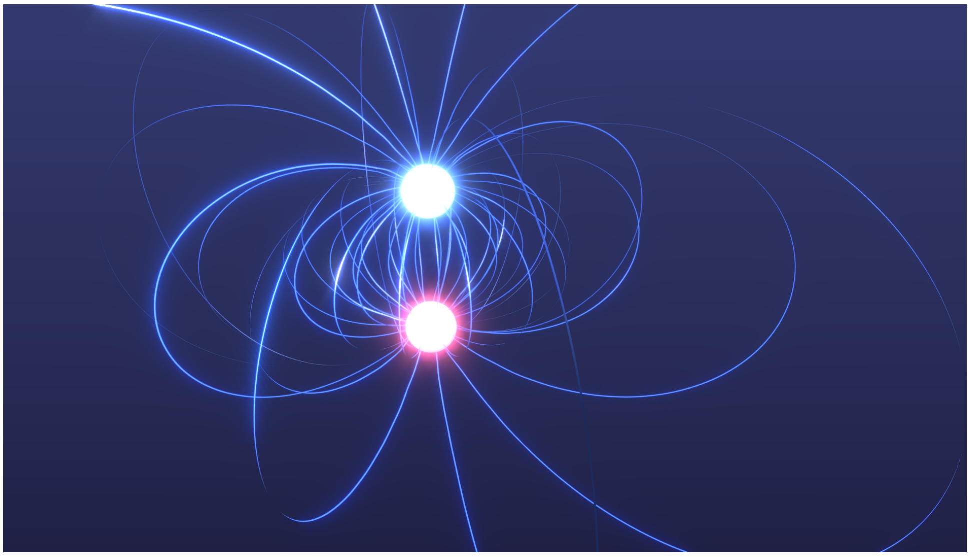
Week 1 – Electric Charges



Nothing is too wonderful to be true if it be consistent with the laws of nature.

Michael Faraday

Exercise 1.1: Coulomb's Law

Coloumb's law is the equation governing the interactions between static charges. But how does the it's *strength* compare to the gravitational attraction? The numbers you need are found in the "Useful constants" section 6 on page 5.

a) Estimate the ratio between the gravitational and electrical attraction between a proton and an electron, F_g/F_e.

Answer: 10^{-41}

b) Suppose you were able to set up an experiment that measured the acceleration of an electron in the presence of a proton. What would the theoretical difference in acceleration be if you did not include the effect of gravitation?

c) Let's say that the electron is orbiting the proton in a radius of $r = 0.53 \times 10^{-10}$ m. What would the velocity of the electron be if it moves in a perfect circle?

Answer: $v = 2.19 \times 10^{6}\,\mathrm{m/s}$

d) If the electrical force is so much stronger then the gravitational force, why isn't every daily observable phenomenon electrical in nature?

Exercise 1.2:

Two positive point charges of charge Q are held fixed at the x-axis at the points $-a$ and a. A third positive charge of charge q and mass m is now placed on the x-axis in such a way that it is constrained to move along this axis. In addition it is placed much closer to the origin than a, i.e. $|x| \ll a$. The charge is now released.

a) Find the frequency of the oscillatory motion of the charge q. *Hint: The binomial expansion is* $(1+z)^n = 1 + nz + \frac{(n-1)n}{2}z^2 + \ldots$ *and if* $|z| \ll 1$ *you can to a very good approximation ignore all higher order terms. Remember also the differential equation for a simple harmonic oscillator* $mx'' = kx$ *with oscillation frequency* $\omega = \sqrt{k/m}$.

   Answer: $\omega = \sqrt{\frac{4kqQ}{ma^3}} = \sqrt{\frac{qQ}{\pi\varepsilon_0 ma^3}}$

b) Suppose now that the charge instead was placed on the y-axis such that the charge is free to move in the xy-plane, now what will the resulting motion be?

Exercise 1.3: Electromagnetic Nuts

a) Imagine two spheres of metal hanging side by side. You observe that the spheres attract. From what you know so far, what can you say about the charges the spheres have? Explain what will happen if the spheres touch. Is it possible that they stick together?

b) A single electron does not hold much charge alone. But how much charge does a visible object contain? Let's study the electrons contained within a glass marble made singly out of SiO_2. We assume that the marble is 1 cm in radius, weighs 1 g and use that SiO_2 has a molar weight of $60.08\,\mathrm{g}\cdot\mathrm{mol}^{-1}$. Each silicon atom contributes 14 electrons, while each oxygen atom contributes with 8 electrons.[1] How much charge would this add up to in total?

   Answer: $Q \approx 49\,200\,\mathrm{C}$.

Exercise 1.4: Electric Field of the Earth

The earth actually has a net charge which produce an electric field pointing radially toward it's center with a magnitude of about 150 N/C at points near it's surface.

a) What magnitude and sign would the charge on a human being weighing about 80 kg have to be to overcome the force of gravity?

   Answer: $|q| = 5.33$ C

b) Suppose you wanted to kiss your sweetheart while you both went of a levitation trip in the electric field of the earth. Would this be a problem? Calculate the repulsive force between you 1 m apart with the charge you obtained in (a).

c) Assuming that the earth can be considered as a point charge located at the center of the earth[2], what is the total charge of the earth?

[1] The electrons per shell are 2, 8, 4 for Si and 2, 6 for O.
[2] This assumption will be justified later

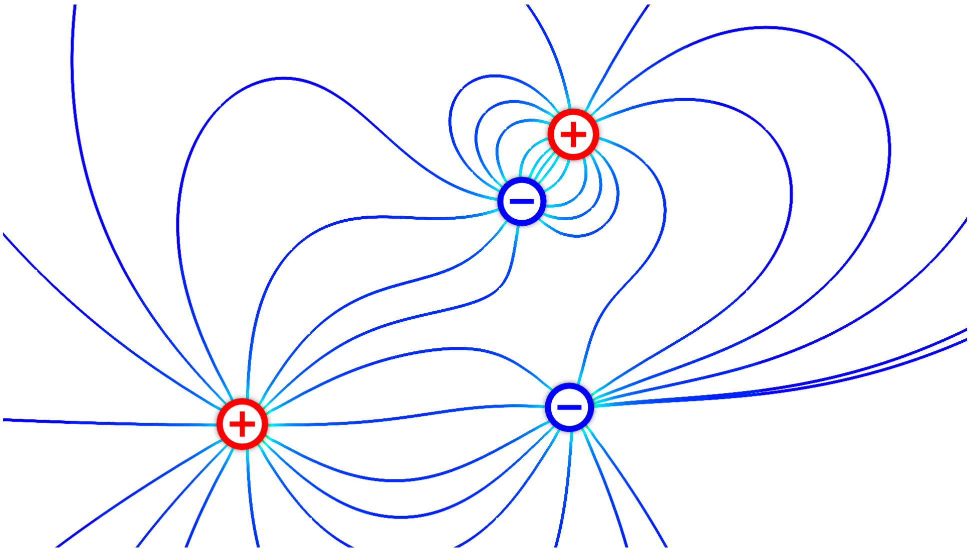


Figure 1: Electric field around four charges in two dimensions.

Answer: E= 600242.4 C

Exercise 1.5: Field from a dipole

Field lines are a great way to visualize the electric field around charges. They convey a lot of information and gives an intuitive way to picture the field. In figure 1 you can see a fairly complicated field from two positive and two negative charges.

In this exercise we will study the field of a dipole. Consider a charge $q_1 = -q$ located at $(-d, 0, 0)$ and a positive charge $q_2 = q$ located at $(d, 0, 0)$. Both charges are held fixed (they are not moving).

a) Make a sketch of the electric field from the dipole in two dimensions.

Let's now dive into some calculations on the dipole. The situation is shown in figure 3. We still have $q_1 = -q$ and $q_2 = q$.

b) Show that the electric field at points on the x-axis where $|x| > d$ can be written as

$$E_x = \frac{1}{2\pi\varepsilon_0}\frac{p|x|}{(x^2-d^2)^2}$$
$$E_y = 0$$
$$E_z = 0,$$

where $\mathbf{p} = 2dq\hat{\mathbf{i}}$ is the dipole moment. Show that when $x \gg d$ E_x will be inversely proportional to x^3.

Answer: When $x \gg d$

$$E_x \approx \frac{1}{2\pi\varepsilon_0}\frac{p}{x^3}$$

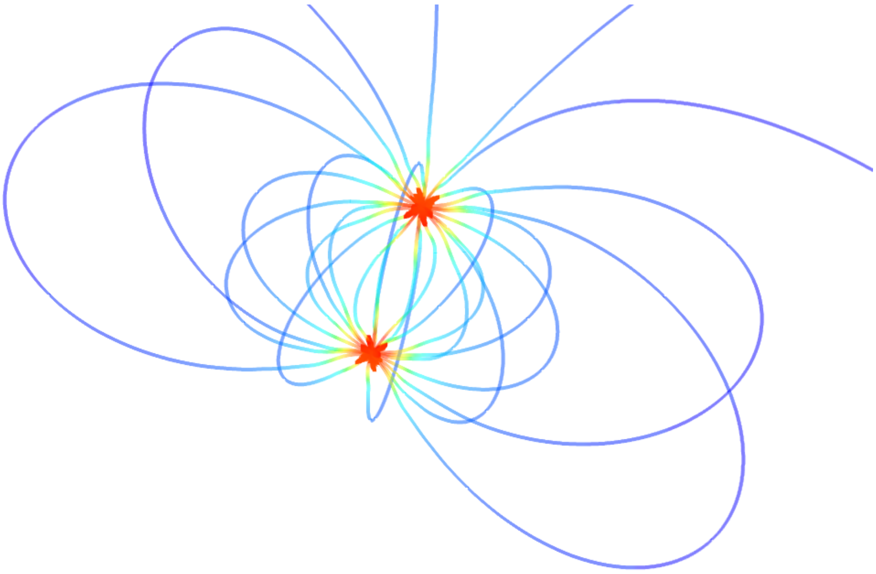


Figure 2: Electric field around a dipole in 3D.

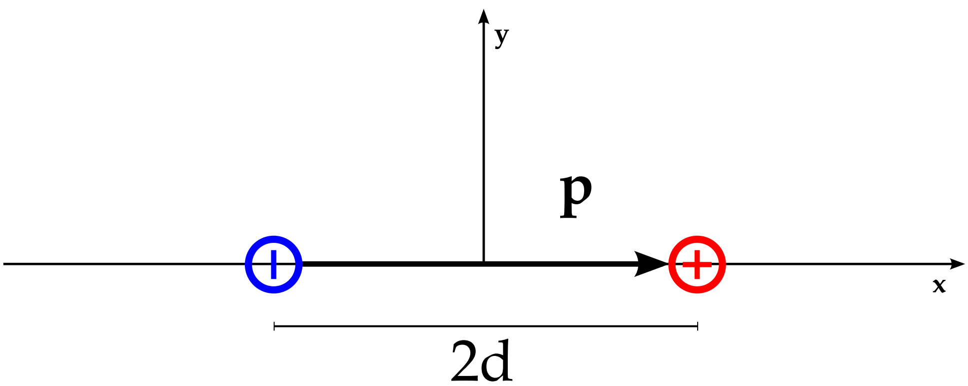


Figure 3: A dipole centered at the origin with **p** along the x-axis.

c) Can you find any points along the x-axis where the field is exactly zero?

d) Now suppose the charges weren't exactly equal in magnitude. That is $|q_1| \neq |q_2|$. Will there now be any points on the x-axis where the field is zero?

Hint: Analyze the field in three regions, $x > d$, $x < -d$ and $-d < x < d$. You don't need to do any calculations.

e) Let the charges have equal magnitude such that our configuration again is identical to the dipole configuration of figure 3. Find the field along the y-axis and show that for $y \gg d$ it will proportional to the inverse of y^3.

Hint: How will the vectors add along the y-axis? Make a sketch.

Answer:

The field will only point in the negative x-direction and

$$E_x = -\frac{1}{4\pi\varepsilon_0}\frac{p}{(y^2+d^2)^{3/2}} \approx -\frac{1}{4\pi\varepsilon_0}\frac{p}{y^3}$$

for $y \gg d$.

Exercise 1.6: The water molecule

In water molecules (H_2O) the oxygen atom is more electronegative and tends to attract electrons more strongly, resulting in a non-uniformly distribution of charges. This in additional to the molecular shape of water, give the water molecules the nature of electric dipoles with a dipole moment at $6.17 \times 10^{-30}\,\mathrm{C \cdot m}$ Consider now that the water molecules are placed in a uniform electric field $\mathbf{E}$ with magnitude of $1.6 \times 10^{6}\,\mathrm{N/C}$ in positive $x-$direction.

a) What is the change in electric potential energy when the dipole moment of a molecule changes its orientation with respect to $\mathbf{E}$ from parallel to perpendicular?

Answer: $\Delta U = 9.87 \times 10^{-24} J$

b) Assume now that the angle between the the electric field and the dipole moment of the dipole is 145 °, when going clockwise from $\mathbf{E}$ to $\mathbf{p}$. Find the magnitude and the direction of the torque exerted on the dipole.

Answer: $\tau = 5.66 \cdot 10^{-24} N \cdot m$, out of the page.

Useful constants

These constants might be useful in some of this week's exercises.

Electrical permittivity in vacuum:	$\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2N^{-1}m^{-2}}$
Gravitational constant:	$G = 6.67 \times 10^{-11}\,\mathrm{m^2kg^{-1}s^{-2}}$
Proton mass:	$m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$
Proton charge:	$q_p = 1e = 1.602 \times 10^{-19}\,\mathrm{C}$
Electron mass:	$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$
Electron charge:	$q_e = -1e = -1.602 \times 10^{-19}\,\mathrm{C}$