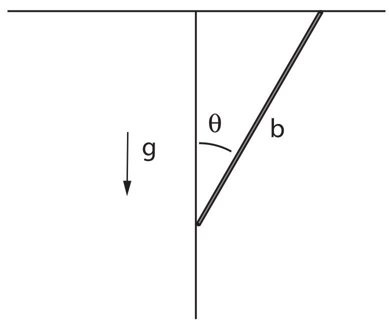
FYS 3120 Classical Mechanics and Electrodynamics
Spring semester 2014

Problem Set 4

Note: This is a compulsory hand-in exercise (”oblig”). Written/printed solutions should be returned to Ekspedisjonskontoret before closing time (15:00) Monday, February 10. Please use name rather than student number on the solution. You may use Norwegian or English according to your preference. If you have questions concerning the problems, please contact Jon Magne Leinaas (Ø471).

Problem 4.1

The figure shows a rod of length b and mass m, with the mass evenly distributed along the rod. One endpoint of the rod is constrained to move along a horisontal line and the other endpoint along a vertical line. The two lines are in the same plane. There is no friction and the acceleration due to gravity is g.



a) Find the Lagrangian L with the angle θ as coordinate, and show that Lagrange's equation gives

$$\ddot{\theta} + \frac{3g}{2b}\sin\theta = 0 \tag{1}$$

(Concerning the moment of inertia of the rod, see Problem 2.2.)

b) What is the stable equilibrium position of the rod? Find the period T_0 for small oscillations about equilibrium.

c) Since L has no explicit time dependence, there is a corresponding constant of motion. What is the expression for this constant and what is the interpretation. Comment on how the expression is related to the equation of motion.

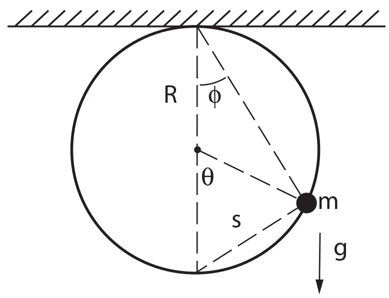
d) Assume the rod oscillates about the equilibrium position with a maximum angle θ_0, with $0 < \theta_0 \leq \pi/2$. Show that the period T of the oscillations is generally expressed by the integral

$$T = T_0\frac{\sqrt{2}}{\pi}\int_0^{\theta_0}\frac{d\theta}{\sqrt{\cos\theta - \cos\theta_0}} \tag{2}$$

Determine the ratio T/T_0 for the maximum amplitude $\theta_0 = \pi/2$. (In Rottman you will find a general formula, which can be used to express the integral in terms of Euler gamma-functions.)

Problem 4.2

A particle of mass m is attached to the circumference of a rigid circular hoop of radius R. The the hoop rolls on the underside of a horizontal line. We assume the hoop to be massless and the motion to take place in a vertical plane.



a) What is the relation between the two angles θ and ϕ? With ϕ as generalized coordinate show that the Lagrangian has the form

$$L = 2mR^2 \cos^2 \phi \, (4 \, \dot{\phi}^2 + \frac{g}{R}) \tag{3}$$

b) Find the corresponding Lagrange's equation.

c) Show next that the Lagrangian simplifies to that of a one-dimensional harmonic oscillator when s is used as generalized coordinate. What is the period of oscillations?

d) Why is there a maximal allowed amplitude for the oscillations in s? Give a qualitative description of what happens when the total energy is larger than the energy that corresponds to the maximum amplitude.

Problem 4.3 (Exam 2008)

A particle moves on a parabolic surface given by the equation $z = (\lambda/2)(x^2 + y^2)$ where z is the Cartesian coordinate in the vertical direction, x and y are orthogonal coordinates in the horizontal plane and λ is a constant. The particle has mass m and moves without friction on the surface under influence of gravitation. The gravitational acceleration g acts in the negative z-direction. The particle's position is given by the polar coordinates (r, θ) of the *projection* of the position vector into the x, y plane.

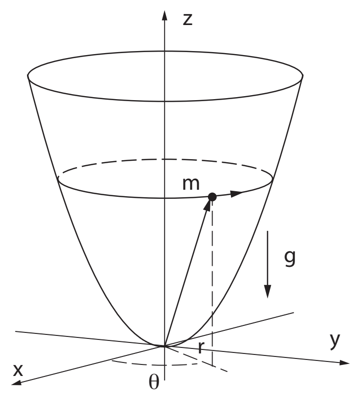
a) Show that the Lagrangian for this system is

$$L = \frac{1}{2}m[(1 + \lambda^2 r^2)\dot{r}^2 + r^2\dot{\theta}^2 - g\lambda r^2] \tag{4}$$

and find Lagrange's equations for the particle.

b) Use the fact that there is a cyclic coordinate to show that the equations can be reduced to a single equation in the radial variable r. What is the condition for the particle to move in a circle with radius $r = r_0$?

c) Assume that the path of the particle deviates little from the circular motion so that $r = r_0 + \rho$, where ρ is small. Show that under this condition the radial equation can be reduced to a harmonic



oscillator equation for the small variable ρ and determine the corresponding angular frequency. Give a qualitative description of the motion of the particle.