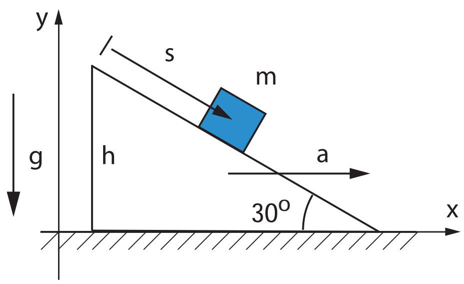
FYS 3120 Classical Mechanics and Electrodynamics
Spring semester 2014

Problem Set 3

Problem 3.1

An object with mass m slides without friction on an inclined plane. The plane is forced to to move horizontally with a constant acceleration a. In this case a natural choice of generalized coordinate will be the displacement s of the object along the tilted surface.



a) Assume first the inclined plane to be at rest, with vanishing velocity and acceleration. Express the Cartesian coordinates x, y of the sliding object as functions of s. Find the Lagrangian expressed in terms of s and $\dot{s}$.

b) Assume next the acceleration a to be constant and non-vanishing. Again find the Cartesian coordinates, and their time derivatives, expressed in terms of s and $\dot{s}$ and determine the Lagrangian.

c) Find Lagrange's equation for the system, and solve the equation under the assumption that the body starts at time $t = 0$ from the top of the inclined plane ($s = 0$) with zero velocity relative to the plane.

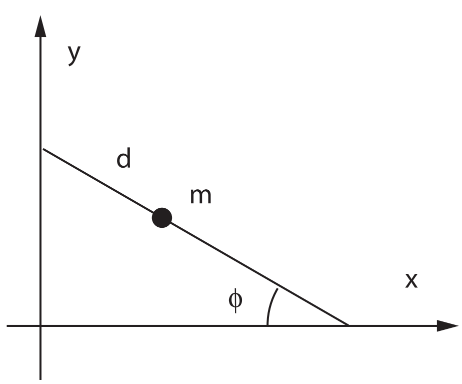


Figure 1: Problem 3.2

Problem 3.2

A rigid rod of length d is forced to move, in the horizontal plane, in such a way that the end points are in contact with the coordinate axes. The angle ϕ increases linearly with time, $\phi = \omega t$. A small body with mass m slides without friction along the rod.

a) Explain why the system has one degree of freedom and make a convenient choice of generalized coordinate. Find the corresponding expression for the Lagrangian.

b) Derive Lagrange's equation for the system.

Problem 3.3

Two bodies with the same mass, m, are connected with a massless rope through a small hole in a smooth horizontal plane. One body is moving on the plane, the other one is hanging at the end of the rope and can move vertically. At all instances the rope is tight. The acceleration due to gravity is g.

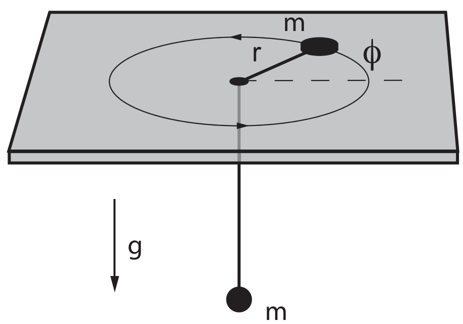


Figure 2: Problem 3.2

a) Find the Lagrange's equations of motion in polar coordinates (r, θ).

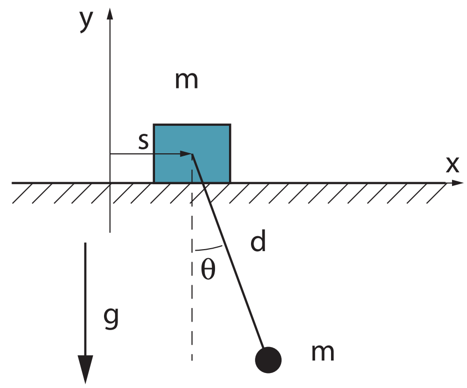
b) Reduce the equations of motion to a one dimensional problem in r and make a qualitative description of the motion.

Problem 3.4

A pendulum is connected to a box, which can slide without friction in a horizontal direction. Assume that all motion takes place in the vertical x, y-plane. The box and the pendulum bob have equal masses m. The pendulum rod, which has length d is considered to be massless. As generalized coordinates use in this problem s as the x-coordinate of the center of mass of the box, and θ as the angle of the pendulum rod relative to the vertical direction. As initial condition choose at time $t = 0$ both the box and the pendulum to be at zero velocity, with the pendulum angle being θ_0

a) Find the Lagrangian of the system expressed in terms of the generalized coordinates and their velocities.

b) Explain what is meant by s being a cyclic coordinate. Since s is cyclic $\dot{s}$ can be expressed in terms of θ and its derivative. Use this in order to re-express the Lagrangian as a function only of θ and $\dot{\theta}$.



c) Show that Lagrange's equation, for the new Lagrangian, gets the form

$$(1 - \frac{1}{2}\cos^2\theta)\ddot{\theta} + \frac{1}{2}\sin\theta\cos\theta\,\dot{\theta}^2 + \frac{g}{d}\sin\theta = 0 \tag{1}$$

d) With θ_0 sufficiently small, the equation of motion can be simplified by making a "small oscillation" approximation. This means here to make an expansion of the θ dependent functions in (1), and keeping in the equation only the first order (linear) terms in θ or its derivatives.

Show that the equation then gets the form of a harmonic oscillator equation, and determine the angular frequency of the pendulum oscillations.