FYS 3120 Classical Mechanics and Electrodynamics
Spring semester 2014

Problem set 1

Problem 1.1

An Atwood's machine consists of three parts, with masses $m_1 = 4m$, $m_2 = 2m$ and $m_3 = m$, that move vertically, and two rotating pulleys, which we treat as massless. The lengths of the ropes, which we also consider as massless, have fixed lengths l_1 and l_2.

Explain why the number of degrees of the system is 2 and and choose a corresponding set of generalized coordinates. Find the potential and kinetic energies of the system expressed as functions of the generalized coordinates and their time derivatives.

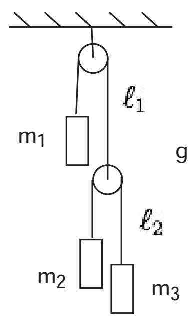

Figure 1: A fall machine

Problem 1.2

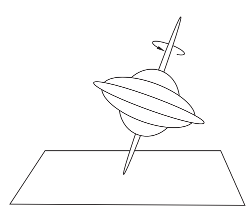

Figure 2: A rotating top

A rotating top is set into motion on a horizontal floor. Count the number of degrees of freedom of the top.

Problem 1.3

Three identicals rods of mass m and length l are connected by frictionless joints, as shown in the figure, with the distance between the points of suspension being equal to the length of the rods. The rods move in the plane. We remind about the expression for the moment of of inertia of one of the rods about its endpoint, $I = \frac{1}{3}ml^2$.

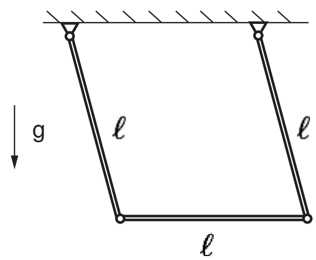

Figure 3: System of moving rods

Show that there is one degree of freedom in this system and choose a suitable generalized coordinate. Find the Lagrangian $L = T - V$ expressed as a function of the generalized coordinate and its time derivative.

Problem 1.4

A rotating pendulum is illustrated in Fig. 4. It is composed of several parts; a fixed vertical rod VR, on the top of this a horizontal rod HR, which rotates about its midpoint O with a constant angular velocity ω, finally a pendulum with a rod, which is at on end fixed at point A of the horizontal rod HR and at the other end connected to a pendulum bob B. The pendulum rod we consider as massless and rigid, while the mass of the pendulum bob is m. The pendulum moves without friction, but is at all times constrained so that the pendulum rod lies in the plane orthogonal to the the direction of HR. The distance SA is a and the length of the pendulum rod is b. The angle between the pendulum rod and the vertical direction is denoted θ.

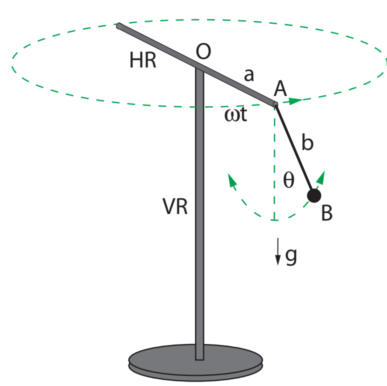

Figure 4: Rotating pendulum

a) Find the Cartesian coordinates (x, y, z) of a fixed reference frame expressed in terms of the coordinates (x', y', z) of a rotating frame, which rotates with the horizontal rod AB. Assume O to be the origin of both coordinate systems, and choose the x'-axis to point along the rod.

b) Use the results from a) to find the Cartesian coordinates (x, y, z) of the pendulum bob expressed in terms of θ and t, and use that to determine the Lagrangian $L = T - V$ as a function of θ and $\dot{\theta}$.

Problem 1.5

A particle with mass m moves in three-dimensional space under the influence of a constraint. The constraint is expressed by the equation

$$e^{-(x^2+y^2)} + z = 0 \tag{1}$$

for the Cartesian coordinates (x, y, z) of the particle.

a) Explain why the number of degrees of freedom of the particle is 2. Use x and y as generalized coordinates and find the expression for the position vector $\mathbf{r}$ of the particle in terms of x and y.

b) A virtual displacement is a change in the position of the particle $\mathbf{r} \to \mathbf{r} + \delta\mathbf{r}$ which is caused by an inifinitesimal change in the generalized coordinates, $x \to x + \delta x$ and $y \to y + \delta y$. Find $\delta\mathbf{r}$ expressed in terms of δx and δy.

c) The constraint can be interpreted as a restriction for the particle to move on a two-dimensional surface in three-dimensional space. Any virtual displacement $\delta\mathbf{r}$ is a tangent vector to the surface while the constraint force $\mathbf{f}$ which acts on the particle is perpendicular to the surface. Use this to determine $\mathbf{f}$ as a function of x and y, up to an undetermined normalization factor (the length of the vector).

d) Make a drawing of a section through the surface for $y = 0$. Indicate in the drawing the direction of the two vectors $\mathbf{f}$ and $\delta\mathbf{r}$ for a chosen point on the surface.