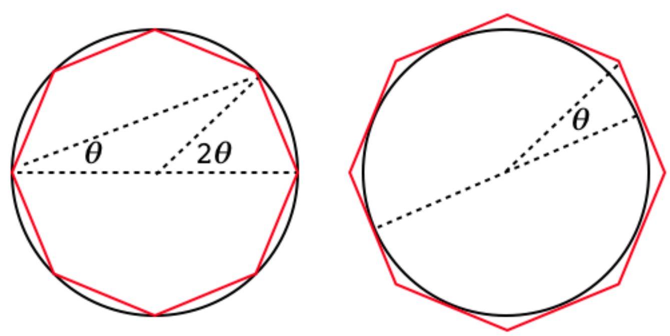
Area

I became aware later that there is yet another way to apply the method, and that is to calculate the *areas* of inscribed and circumscribed polygons. We'll go through this briefly.

For this approach we use a unit circle (radius 1) rather than a diameter of 1, as we did above. As before, we define θ to be the central angle of the half-sector (i.e. $\theta = 2\pi/2n$).



Rather than draw an entirely new figure, just imagine in the left panel that we draw the angle bisector of angle 2θ. The area of each new triangle is then $\sin\theta\cos\theta/2$ and the total area of the inner polygon is

$$a = n\sin\theta\cos\theta = nSC$$

in the notation we adopted previously in this chapter. And, as before, to progress to a' we have a factor of 2 as well as the new values S' and C':

$$a' = 2nS'C'$$

For the circumscribed or outer polygon, we just have what we had before, that the side length of the triangle in the right panel is $\tan\theta$ so the total area is

$$A = nT$$

Bring in the half-angle formulas as follows:

$$a' = 2nS'C' = 2n \cdot \frac{S}{2C'} \cdot C' = nS$$

That is slick, but we need an expression for nS:

$$aA = nSC \cdot n\frac{S}{C} = [nS]^2$$

$$aA = [a']^2$$

$$a' = \sqrt{aA}$$

This is like, and yet subtly different than what we had when calculating the perimeter.

Since

$$A = nT$$

and

$$A' = 2nT'$$

$$= 2n\frac{ST}{S+T} = 2\frac{nS \cdot nT}{nS + nT}$$

$$A' = 2\frac{a'A}{a' + A}$$

Compare

$$a' = \sqrt{aA} \qquad A' = 2\frac{a'A}{a' + A}$$

$$p' = \sqrt{pP'} \qquad P' = 2\frac{pP}{p + P}$$