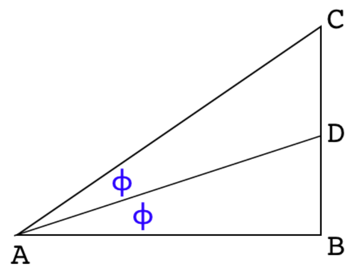
Euclid's proof of angle bisector corollary

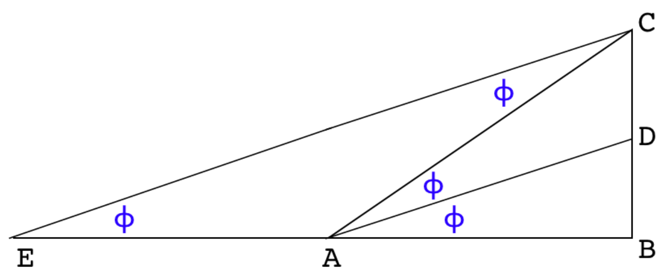
Consider the following $\triangle ABC$, with $\angle BAC$ bisected by AE.



The angle bisector theorem says that $AB : BD = AC : DC$, and the corollary says that $AB + AC : BC$ is the same ratio.

Euclid's proof

Euclid's proof is Book 6, proposition 3.



Draw CE parallel to DA. Then, $\angle DAC = \angle ACE$ (by alternate interior angles).

We are given that $\angle DAB = \angle CAD$, and now see that $\angle AEC$ and $\angle DAB$ are equal as corresponding angles of a line cutting two parallel lines. Therefore all four angles (marked ϕ) are equal.

Therefore, $\triangle ACE$ is isosceles, with $AC = AE$.

Furthermore, $\triangle ABD$ is similar to $\triangle EBC$. Therefore, $AB : BD = EB : BC$.

And since $EB = EA + AB$ and $AC = AE$, we have that $AB : BD = AB + AC : BC$.

reverse

This proof just reverses each statement above. We are given $AB : AC = BD : DC$.

Both are equal to $AB : AE$, by similar triangles, so $AC = AE$.

Therefore $\triangle ACE$ is isosceles with $\angle ACE = \angle AEC$.

But $\angle AEC = \angle BAD$ (isosceles)

and $\angle ACE = \angle CAD$ (alternate interior angles)

so $\angle CAD = \angle BAD$.

$\square$