Hand calculating

perimeter

P first, then p. For a square on a circle of diameter 1, the outside perimeter is

$$P = 4$$

while the inside is

$$p = 2\sqrt{2}$$

Next: P (re-using the symbol):

$$P = \frac{2pP}{p+P} = \frac{16\sqrt{2}}{4+2\sqrt{2}} = \frac{8}{1+\sqrt{2}} = 3.3137$$

Then p:

$$p = \sqrt{pP} = \sqrt{2\sqrt{2} \cdot \frac{8}{1+\sqrt{2}}} = 4\sqrt{\frac{1}{1+1/\sqrt{2}}} = 3.0615$$

area

a first, then A. For a square on a circle of radius 1, the inside area is

$$a = \sqrt{2} \cdot \sqrt{2} = 2$$

while the outside is

$$A = 4$$

Next (re-using the symbol):

$$a = \sqrt{aA} = \sqrt{8} = 2\sqrt{2}$$

Then A

$$A = \frac{2aA}{a+A} = 16\frac{\sqrt{2}}{4+2\sqrt{2}} = \frac{8}{1+\sqrt{2}} = 3.3137$$

Then a:

$$a = \sqrt{aA} = \sqrt{2\sqrt{2}\cdot\frac{8}{1+\sqrt{2}}} = 4\sqrt{\frac{1}{1+1/\sqrt{2}}} = 3.0615$$

The calculations are identical, and will remain so from here on out.

sine and tangent

$n = 4$

$$n\sin\theta = 4\cdot\frac{1}{\sqrt{2}} = 2\sqrt{2}$$
$$n\tan\theta = 4\cdot 1 = 4$$

At this stage, the inner area by trigonometry is equal to the second value of a above, and the outer area is equal to the first value of A above.

Calculate S', etc:

$$C' = \sqrt{\frac{1}{2}(1+C)} = \sqrt{\frac{1}{2}(1+1/\sqrt{2})}$$

$$S' = S/2C' = \frac{1}{2\sqrt{2}}\cdot\sqrt{\frac{2}{1+1/\sqrt{2}}} = \frac{1}{2}\cdot\sqrt{\frac{1}{1+1/\sqrt{2}}}$$

$$T' = \frac{S}{1+C} = \frac{1/\sqrt{2}}{1+1/\sqrt{2}} = \frac{1}{1+\sqrt{2}}$$

so for $n = 8$

$$nS' = 4\sqrt{\frac{1}{1 + 1/\sqrt{2}}}$$

$$nT' = \frac{8}{1 + \sqrt{2}}$$

It continues. At this stage, the inner area is equal to the third value of a above, while the outer area is equal to the second value of A above.

It's the same calculation, in disguise.