Your title here

vector fields

A vector field assigns to every point (x, y) in a region R in the plane a vector $\mathbf{F}(x, y)$ with two components

$$\mathbf{F}(x, y) = M(x, y)\mathbf{i} + N(x, y)\mathbf{i}$$

Each of the component functions M and N takes an ordered pair of real numbers and outputs a real number. As a shorthand, we could write:

$$\mathbf{F} = \langle M, N \rangle$$

In three dimensions we might have $\mathbf{F} = \langle M, N, P \rangle$ or $\langle P, Q, R \rangle$. We could also write the components of $\mathbf{F}$ explicitly as some functions of x and y, e.g.

$$\mathbf{R} = \langle x, y \rangle$$

This is the field of radial vectors, all of which point in the direction opposite to the origin, and have length $r = \sqrt{x^2 + r^2}$. We could also normalize to get $\mathbf{R}/r$ or even $\mathbf{R}/r^2$.

The spin field is $\mathbf{S} = \langle -y, x \rangle$. These can be normalized or divided by r^2 as for $\mathbf{R}$.

A *gradient field* is the gradient of some function f which is called the potential. Such fields are conservative, as we'll see later on. Recall that the gradient is

$$\nabla f = \langle f_x, f_y \rangle$$

where by f_x I mean the x-derivative of f. The radial fields are all gradient fields. Consider

$$f(x,y) = \frac{1}{2}(x^2 + y^2)$$

Clearly

$$\nabla f = \langle x, y \rangle = \mathbf{R}$$

The gradient is everywhere perpendicular to the level curves $f(x,y) = c$.

Some fields are gradient fields, and some are not.

What is the potential function for

$$\frac{\mathbf{R}}{r} = \frac{1}{\sqrt{x^2 + y^2}} \ \langle x, y \rangle$$

What function f has as its x-derivative $x/\sqrt{x^2 + y^2}$?

$$\frac{\partial}{\partial x} \ \sqrt{x^2 + y^2} = \ ?$$

That looks right.