Nahin Imaginary Tale Ch 2

We start with a problem in analytical geometry: to construct a line segment whose measure is the square root of a given line segment.

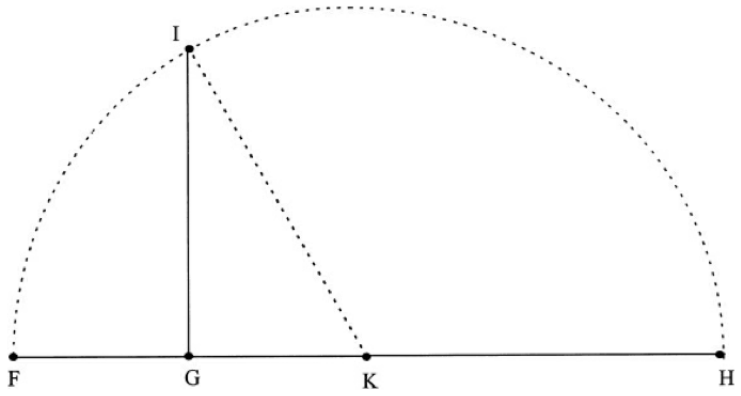


Figure 2.1. Constructing the square root of a line
$(IG = \sqrt{GH})$.

We are given GH. Construct the rest of the diagram by extending GH to F by an as yet undetermined distance, find the midpoint, and then draw the circle. Finally, extend the altitude GI.

We will show that, for a particular value of $FG = x$, GI is the square root of GH.

Let R be the radius of the circle and a be the altitude GI.

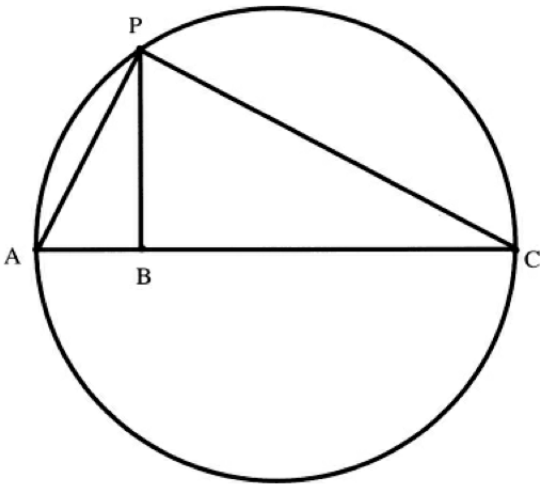
The base of the right triangle shown as $GK = R - x$. Then Pythagoras

says that:

$$a^2 + (R - x)^2 = R^2$$
$$a^2 = 2Rx - x^2$$

In the particular case $x = 1$ (if FG is unity), a^2 is equal to $2R - 1$, which is just GH. $\square$

The next problem brings out another feature (of the same diagram, with labels changed). Draw AP. Recall the famous theorem that $\triangle APC$ is a right triangle.



By similar triangles:

$$\frac{AB}{BP} = \frac{BP}{BC}$$
$$BP^2 = AB \cdot BC$$

This makes the solution to the previous problem quite obvious, substituting those labels we had

$$a^2 = AB \cdot x$$

and we set $x = 1$ to get the solution.