problem 23

$$\int (\ln x)^2 \, dx$$

Let

$$u = (\ln x)^2$$

$$du = 2 \ln x \, \frac{1}{x} \, dx$$

$$dv = dx$$

$$v = x$$

We get

$$= x(\ln x)^2 - \int 2 \ln x \, dx$$

$$= x(\ln x)^2 - 2(x \ln x - x)$$

Evaluate from $x = 1 \Rightarrow x = 2$:

$$= 2(\ln 2)^2 - 2(2 \ln 2 - 2) + 2$$

$$= 2 \, [\, (\ln 2)^2 - 2 \ln 2 + 3 \,]$$

Check

$$\frac{d}{dx} \, [\, x(\ln x)^2 - 2(x \ln x - x) \,]$$

$$= (x) 2 \ln x \, \frac{1}{x} + (\ln x)^2 - 2(\ln x)$$

$$= (\ln x)^2$$