LRC Circuit 2

The idea of this short write-up is to look at the equations developed in Halliday and Resnick for LRC circuit and compare their solution to what we obtained in the case of a mass and spring system with friction.

There are three types of components: resistor with an impedance of

$$V = iR$$

capacitor with capacitance C

$$\frac{1}{C}q = V$$

$$\frac{1}{C}\dot{q} = \frac{1}{C}i = \dot{V}$$

inductor with L

$$V = L\dot{i}$$

I'm not yet sure why these make sense. Nevertheless, they write

$$U = \frac{1}{2}Li^2 + \frac{1}{2C}q^2$$

$$\dot{U} = Li\ \dot{i} + \frac{1}{C}q\ \dot{q} = -i^2R$$

Since $i = \dot{q}$, we can lose one factor of i

$$L\ \dot{i} + \frac{1}{C}q = -iR$$

And since $i = \dot{q}$, $\dot{i} = \ddot{q}$:

$$L\ \ddot{q} + \frac{1}{C} q = -\dot{q}R$$

Rearrange:

$$L\ \ddot{q} + \dot{q}R + \frac{1}{C} q = 0$$

$$\ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{LC} q = 0$$

Compare to what we had for the mass and spring with a small amount of friction:

$$\ddot{x} + \gamma\dot{x} + {\omega_0}^2 x = 0$$

$${\omega_0}^2 = k/m \sim \frac{1}{LC}$$

$$\omega' = \sqrt{{\omega_0}^2 - (\frac{\gamma}{2})^2} \sim \sqrt{\frac{1}{LC} - (\frac{R}{2L})^2}$$

$$x(t) = e^{(-\gamma/2)t}\ C\cos(\omega' t + \phi)$$

Write

$$q(t) = q_m e^{-(R/2L)t}\ \cos\omega' t$$

Looks like the same solution to me.