Logarithms: Change of Base

The formula for change of base is

$$log_a(x) = log_b(x)/log_b(a)$$

The way I use to remember this is to say to myself that $log_a(x)$ and $log_b(x)$ are related somehow—that's what we want—so they are on opposite sides of the equation. Then, there is a ratio with another log in the denominator, and *that log must be to the same base as in the numerator*. You can check it with easy bases like 2 and 4

$$log_2(16) = 4$$

$$log_4(16) = 2$$

$$log_4(2) = 1/2$$

so

$$log_2(16) = log_4(16)/log_4(2)$$

is correct. Here is a simple derivation:

$$x = b^y$$

$$y = log_b(x)$$

$$log_a(x) = log_a(b^y) = y\ log_a(b) = log_b(x)\ log_a(b)$$

$$log_a(x)/log_a(b) = log_a(b)$$