Fractional power rule

The standard rule for differentiation of powers of x also works for *fractional* powers

$$\frac{d}{dx}\ x^{p/q} = \frac{p}{q}\ x^{p/q-1}$$

This is often presented and used without proof. Here is a simple proof which relies on implicit differentiation. Write

$$y = x^{p/q}$$

Now raise both sides to the q power

$$y^q = x^p$$

Differentiate both sides. By the chain rule (and differentiating implicitly)

$$\frac{d}{dx}\ y^q = q\ y^{q-1}\ \frac{dy}{dx}$$

The derivative of x^p is as usual, so combining the results we obtain

$$q\ y^{q-1}\ \frac{dy}{dx} = p\ x^{p-1}$$

Solve for dy/dx

$$\frac{dy}{dx} = \frac{p\ x^{p-1}}{q\ y^{q-1}}$$

Multiply top and bottom by y

$$\frac{dy}{dx} = \frac{p\ x^{p-1}\ y}{q\ y^q}$$

Substitute for $y = x^{p/q}$ from above

$$\frac{dy}{dx} = \frac{p\ x^{p-1}\ x^{p/q}}{q\ y^q}$$

Substitute for $y^q = x^p$ from above

$$\frac{dy}{dx} = \frac{p\ x^{p-1}\ x^{p/q}}{q\ x^p}$$

Simplify

$$\frac{dy}{dx} = \frac{p}{q}\frac{x^{p/q}}{x} = \frac{p}{q}\ x^{p/q-1}$$