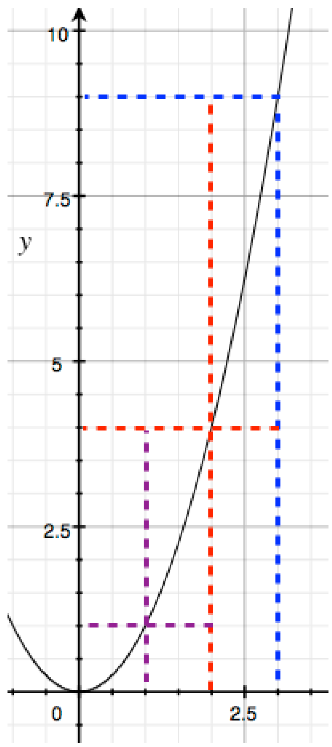
Area for $y = x^2$



The area under the curve is given by

$$\int_0^a x^2 dx = \frac{1}{3}x^3$$

For $x = 1, 2, 3, 4 \cdots$, the values are $\frac{1}{3}, \frac{8}{3}, \frac{27}{3}, \frac{64}{3} \cdots$. I was curious about the pattern for areas above and below the curve, bounded by the rectangle drawn around x, x^2 for $x = n, x' = n + 1$.

Between 0 and 1, we have $\frac{1}{3}$ below and $\frac{2}{3}$ above.

Between 1 and 2, we have $\frac{8}{3} - \frac{1}{3} - \frac{3}{3} = \frac{4}{3}$ below and $\frac{9}{3} - \frac{4}{3} = \frac{5}{3}$ above.

Between 2 and 3, we have $\frac{27}{3} - \frac{8}{3} - \frac{12}{3} = \frac{7}{3}$ below and $\frac{15}{3} - \frac{7}{3} = \frac{8}{3}$ above.
Between 3 and 4, we have $\frac{64}{3} - \frac{27}{3} - \frac{27}{3} = \frac{10}{3}$ below and $\frac{21}{3} - \frac{10}{3} = \frac{11}{3}$ above.

As x grows larger, the areas above and below the line approach each other, always differing by $\frac{1}{3}$ and totalling $x^2 - (x-1)^2 = 2x - 1 = (6x-3)/3$.