funny coord

Integrate the unit square in polar coordinates! Notice the bounds for r, the outer limit is $\sqrt{1+\sin^2\theta}$.

$$\iint_R dA = \int_0^{\pi/2} \int_0^{\sqrt{1+\sin^2\theta}} r \; dr \; d\theta$$

inner integral

$$= \frac{1}{2} r^2 \Big|_0^{\sqrt{1+\sin^2\theta}}$$

$$= \frac{1}{2}(1+\sin^2\theta)$$

outer integral

$$= \int_0^{\pi/2} \frac{1}{2}(1+\sin^2\theta) \; d\theta$$

Look it up

$$= \frac{1}{2} \left[\; \theta + \frac{1}{2}(\theta - \frac{1}{2}\sin 2\theta) \; \right] \Big|_0^{\pi/2}$$

The first θ gives us $\pi/4$ and the rest is

$$= \frac{1}{4}(\theta - \frac{1}{2}\sin 2\theta) \;] \Big|_0^{\pi/2}$$

The second θ gives us $\pi/8$ leaving

$$= -\frac{1}{8}\sin 2\theta \Big|_0^{\pi/2}$$

which is 0. So we have a total of $3/8\pi$, which is wrong!