Dirac Delta function

"The Dirac delta function $\delta(x)$ is not really a 'function'. It is a mathematical entity called a distribution which is well defined only when it appears under an integral sign. It has the following defining properties:"

$$\delta(n) = \begin{cases} 0, & \text{if } x \neq 0 \\ \infty, & \text{if } x = 0 \end{cases}$$

$$\int_a^b \delta(x)\ dx = 1 \quad b < 0 < c$$

`http://www.math.oregonstate.edu/BridgeBook/book/math/deltaintro`

Going on, they say that by definition, a third property is

$$x\delta(x) \equiv 0$$

Kind of like rock-paper-scissors. :) $x = 0$ trumps $\delta(0) = \infty$.

Consequences include the following:

$$\int_{-\infty}^{\infty} f(x)\delta(x)dx$$

since $\delta(x)$ is zero everywhere other than $x = 0$, this is

$$= \int_{-\infty}^{\infty} f(0)\delta(x)dx =$$

but $f(0)$ is a constant so

$$f(0)\int_{-\infty}^{\infty}\delta(x)dx$$

$$= f(0)$$

The Dirac Delta function "picks out" the value of the function at 0. Second, it can be shifted easily by using $\delta(x-a)$

$$\int_{-\infty}^{\infty} f(x)\delta(x-a)dx = f(a)$$