Analysis: definitions

• The set X is *bounded above* $\iff \exists\ M \in \mathbb{R} \mid \forall\ x \in X, x \leq M$.

A closely related definition is the following:

• A function $f : X \to \mathbb{R}$ is *bounded above* on X $\iff \exists\ M \in \mathbb{R} \mid \forall\ x \in X, x \leq M$.

As an example $f(x) = x^2$ is *not bounded above*. The reason is that reading the definition, we must pick M first. Given M, we can always find a value of x so that $f(x) > M$.

If we were allowed to pick x first, then of course we could always find M such that $f(x) \leq M$. But that is not how the game is played.