Integration—summary

The derivatives and integrals of trig functions are fundamental. Start with sine and cosine:

$$\frac{d}{dx}\ \sin x = \cos x$$

$$\int \cos x\ dx = \sin x + C$$

The cosine is just a question of changing sign.

As you know, application of the quotient rule gives:

$$\frac{d}{dx}\ \tan x = \sec^2 x$$

So

$$\int\ \sec^2 x\ dx = \tan x$$

because the quotient rule gives minus uv' and so $\sin^2 x + \cos^2 x$ on top, leaving only $1/v^2$ in the end.

The secant is

$$\frac{d}{dx}\ \sec x = \sec x \tan x$$

Recalling

$$(\frac{u}{v})' = \frac{u'v - uv'}{v^2}$$

We get $\sin x$ on top from the $-uv'$ part, and multiply that by $1/v^2$.

So

$$\int \sec x \tan x = \sec x + C$$

which seems really odd, but there it is.

Finally, the cotangent and cosecant are related to their non-"co" friends, but with a minus sign.

$$\frac{d}{dx} \cot x = -\csc^2 x$$

$$\int \csc^2 x \; dx = -\cot x + C$$

$$\frac{d}{dx} \csc x = -\csc x \cot x$$

$$\int \csc x \cot x \; dx = -\csc x + C$$

You need to know these! The same folks who take a simple calculus concept and turn it into a complicated arithmetic problem also find it amusing to ask about cosecant rather than cosine, even though you would almost never see it in real life.

integrating

There isn't anything new in thinking about $\int \sin x \; dx$. What about

$$\int \tan x \; dx$$

If we substitute $u = \cos x$ we see that $du = -\sin x \; dx$ so we have

$$\int \tan x \; dx = -\int \frac{1}{u} \; du$$

$$= -\ln \; \cos x + C$$

This should be written as an absolute value

$$\int \tan x \; dx = -\ln \; |\cos x| + C$$

The next interesting one is the secant

$$\int \sec x \; dx$$

There is a trick to this one, multiply top and bottom by $\sec x + \tan x$

$$\int \sec x \; \frac{\sec x + \tan x}{\sec x + \tan x} dx$$

You see that $\sec^2 x$ is the derivative of $\tan x$ and $\sec x \tan x$ is the derivative of $\sec x$ so this is just

$$\int \frac{1}{u} \; du$$

again, namely

$$\int \sec x \; dx = \ln |\sec x + \tan x \; | + C$$

As before, the "co" versions are similar but for a minus sign.

$$\int \cot x \; dx = \ln \; |\sin x| + C$$

$$\int \csc x \; dx = -\ln |\csc x + \cot x \; | + C$$

The other most important ones are the inverse of the sine, tangent, and secant, which I've covered elsewhere.