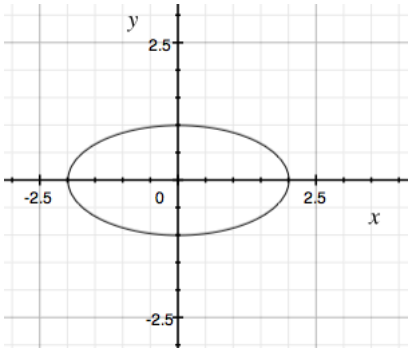
Volume of rotated ellipse

We want to calculate the volume generated by rotation of an ellipse (centered at the origin) about the x-axis.



The basic idea is that the cross-section of each little slice in the direction we are integrating is a circle with radius equal to f(x).

$$y = f(x)$$

The area of each slice is a function of x, given by

$$A = \pi\ y^2$$

We add up all those little slices by doing this integral

$$V = \pi \int y^2\ dx$$

For the general ellipse we have the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y^2 = b^2(1 - \frac{x^2}{a^2})$$

So the integral is

$$V = \pi \int y^2 dx = \pi \int b^2 (1 - \frac{x^2}{a^2})\ dx$$

which is just

$$V = \pi b^2\ (x - \frac{x^3}{3a^2})$$

evaluated between x = -a and x = a

$$V = \pi b^2 [(a - \frac{a}{3}) - (-a - \frac{-a}{3})] = \frac{4}{3}\pi b^2 a$$

We get a squared contribution for the b component, which describes the "stretching" of the ellipse in the direction of the axis of rotation. Rotation around the y-axis would give a formula containing a squared, and a bigger solid by a factor of a/b.