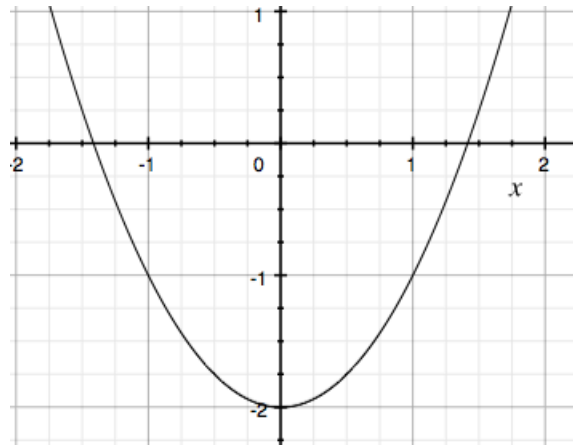


## Newton's method

Here is a method for finding roots of equations quickly, often called Newton's method, or the Newton-Raphson method. As an example, here is a plot of the function

$$f(x) = x^2 - 2$$



which is equal to zero when  $x = \pm\sqrt{2}$ . That is, we want the roots of the following equation

$$x^2 - 2 = 0$$

and more generally

$$x^2 - N = 0$$

to find the square root of some other number. Pick a point  $g$  (for guess). It doesn't have to be a particularly good guess. Then we need to construct the line tangent to the curve at that point, with slope

$m = f'(g)$  and ask, where does this line intercept the  $x$ -axis? The slope is  $\Delta y / \Delta x$ .

$$\frac{f(g) - 0}{g - g'} = f'(g)$$

$g'$  is the  $x$ -coordinate at the intercept. Rearrange

$$\frac{f(g)}{f'(g)} = g - g'$$

$$g' = g - \frac{f(g)}{f'(g)}$$

### square root problem

For this particular problem, we have

$$f(g) = g^2 - N$$

$$f'(g) = 2g$$

$$g' = g - \frac{g^2 - N}{2g} = \frac{1}{2}\left(g + \frac{N}{g}\right)$$

Which can be encapsulated into the following algorithm:

- Make a guess  $g$  and compute  $N/g$
- Average  $g$  and  $N/g$  to find a new guess
- Repeat until satisfied

The algorithm converges rapidly for most problems.

2

1.5

1.41666666667

1.41421568627

1.41421356237

1.41421356237