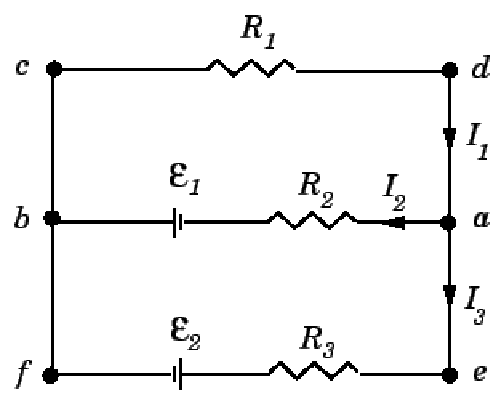
Kirchhoff

Here is a circuit problem from Fitzpatrick.



The resistances are given as $R_1 = 100\Omega$, $R_2 = 10\Omega$, and $R_1 = 5\Omega$. The voltages are given as $\epsilon_1 = 12V$ and $\epsilon_2 = 6V$. The directions for the currents are shown. The equations we have are then

$$I_1 = I_2 + I_3$$

from the junction rule (current in equals current out). Then, clockwise around the loop containing $R1$ and $R2$ (the high voltage side of ϵ_1 is on the left

$$-I_2R_2 + \epsilon_1 + I_1R_1 = 0$$

The other loop we will use contains R_2 and R_3.

$$-I_3R_3 - \epsilon_2 - \epsilon_1 + I_2R_2 = 0$$

Solution. Start by substituting into the second equation from the first

$$-I_2R_2 + \epsilon_1 + I_2R_1 + I_3R_1 = 0$$

and rearrange

$$I_2(R_1 - R_2) + \epsilon_1 + I_3R_1 = 0$$

Our plan is to remove I_3. Get equation 3 together with the above, and move the ϵ's to the other side in both

$$I_2(R_1 - R_2) + I_3R_1 = -\epsilon_1$$

$$I_2R_2 - I_3R_3 = \epsilon_1 + \epsilon_2$$

It's clear what to do, just a bit of a pain.

$$\frac{I_2(R_1 - R_2)}{R_1} + I_3 = -\frac{\epsilon_1}{R_1}$$

$$\frac{I_2R_2}{R_3} - I_3 = \frac{\epsilon_1 + \epsilon_2}{R_3}$$

So

$$\frac{I_2(R_1 - R_2)}{R_1} + \frac{I_2R_2}{R_3} = -\frac{\epsilon_1}{R_1} + \frac{\epsilon_1 + \epsilon_2}{R_3}$$

The coefficients for I_2 are

$$0.9 + 2 = 2.9$$

On the right-hand side we have

$$-0.12 + 3.6 = 3.48$$

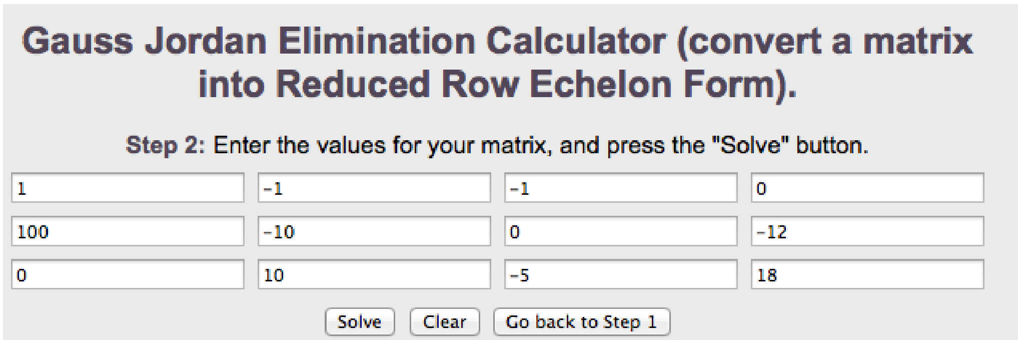
So we calculate $I_2 = 3.48/2.9 = 1.2$. From the second equation at the top we have

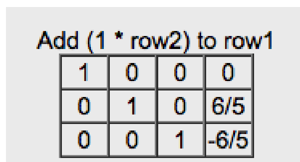
$$-1.2R_2 + \epsilon_1 + I_1R_1 = 0$$

$$-1.2(10) + 12 + I_1R_1 = 0$$

Notice: it doesn't matter what R_1 is. I_1 must equal 0. And then $I_3 = -I_2$. Alternatively you can just plug the numbers into an online solver like this one:

`http://www.gregthatcher.com/Mathematics/GaussJordan.aspx`





Add (1 * row2) to row1

1	0	0	0
0	1	0	6/5
0	0	1	-6/5

The solver shows all the steps, but I didn't keep that part.