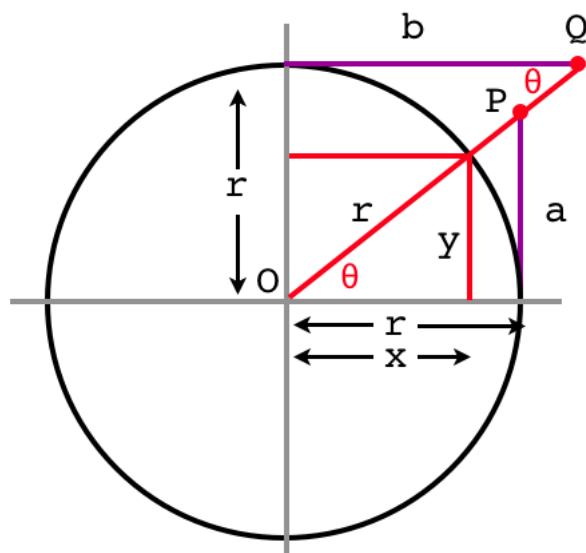


## All 6 trig functions



Consider a unit circle with radius  $r = 1$  and  $y/r = \sin \theta$  and  $x/r = \cos \theta$ . Extend the radius with the angle  $\theta$  and then draw the vertical connector  $a$  and horizontal connector  $b$ . The original triangle with sides  $x, y, r$  is similar to the triangle with sides  $r, a, OP$ , and both are similar to the triangle with sides  $b, r, OQ$ .

$$x, y, r \sim r, a, OP, \sim b, r, OQ$$

By similar  $\triangle$

$$a/r = y/x = \tan \theta$$

But  $r = 1$  so

$$a = \tan \theta$$

If you imagine a point moving around the circle  $a$  will get very large as  $\theta \rightarrow \pi/2$ , and in fact, becomes  $\infty$  there.

The segment  $OP$  is (by similar  $\triangle$ ) to  $a$  as

$$OP/a = r/y = 1/y = 1/\cos \theta = \sec \theta$$

The horizontal from the y-axis to Q is  $b$ . Consider  $\theta$  near the top of the figure. By similar  $\triangle$ , the relations we had were

$$x, y, r \sim r, a, OP, \sim b, r, OQ$$

$$r/b = y/x = \tan \theta$$

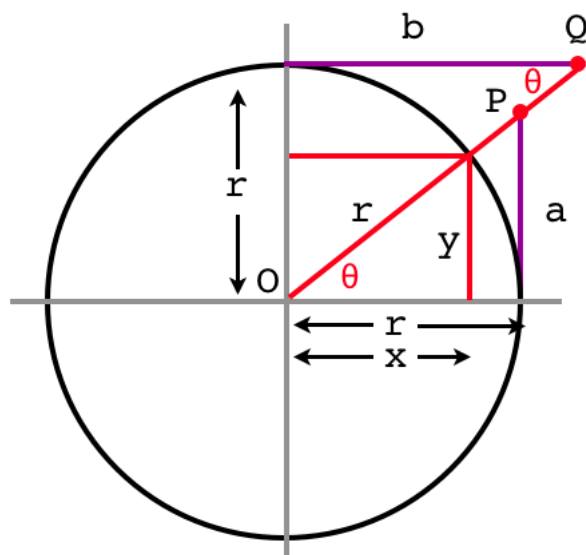
since  $r = 1$

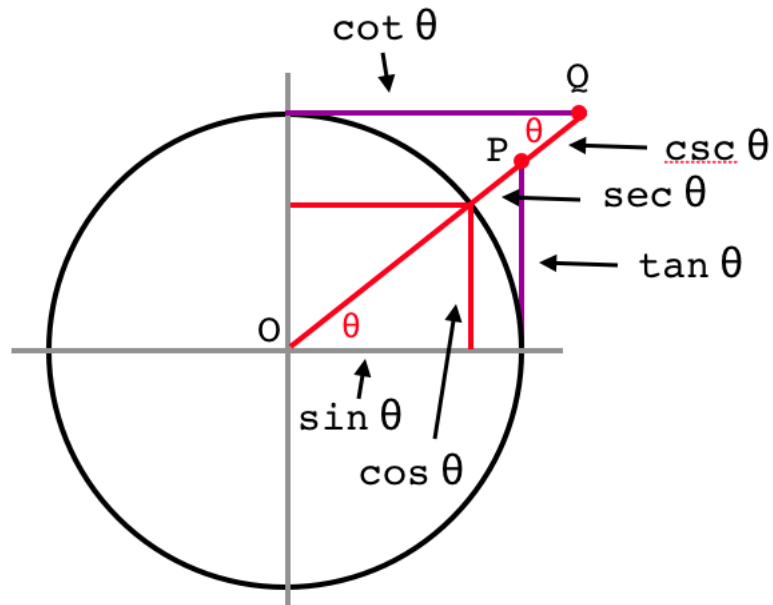
$$b = r/\tan \theta = 1/\tan \theta = \cot \theta$$

Finally

$$r/OQ = 1/OQ = \sin \theta$$

$$OQ = 1/\sin \theta = \csc \theta$$





Note: the above diagram has an error, sine and cosine are switched. I have to find the original figure (or redraw) to change this.