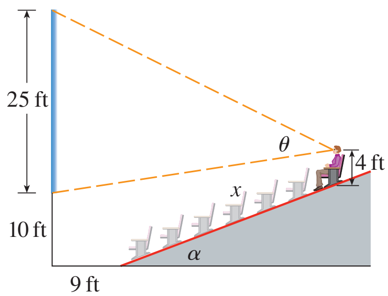
5.6 APPLIED PROJECT: CAS WHERE TO SIT AT THE MOVIES

This project can be completed anytime after you have studied Section 5.6 in the textbook.

A movie theater has a screen that is positioned 10 ft off the floor and is 25 ft high. The first row of seats is placed 9 ft from the screen and the rows are set 3 ft apart. The floor of the seating area is inclined at an angle of $\alpha = 20°$ above the horizontal and the distance up the incline that you sit is x. The theater has 21 rows of seats, so $0 \leqslant x \leqslant 60$. Suppose you decide that the best place to sit is in the row where the angle θ subtended by the screen at your eyes is a maximum. Let's also suppose that your eyes are 4 ft above the floor, as shown in the figure.



1. Show that

$$\theta = \arccos\left(\frac{a^2 + b^2 - 625}{2ab}\right)$$

where $a^2 = (9 + x\cos\alpha)^2 + (31 - x\sin\alpha)^2$

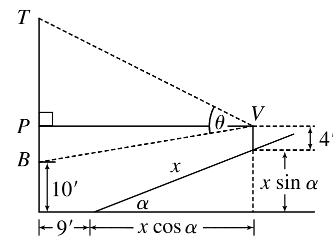
and $b^2 = (9 + x\cos\alpha)^2 + (x\sin\alpha - 6)^2$

2. Use a graph of θ as a function of x to estimate the value of x that maximizes θ. In which row should you sit? What is the viewing angle θ in this row?

3. Use your computer algebra system to differentiate θ and find a numerical value for the root of the equation $d\theta/dx = 0$. Does this value confirm your result in Problem 2?

4. Use the graph of θ to estimate the average value of θ on the interval $0 \leqslant x \leqslant 60$. Then use your CAS to compute the average value. Compare with the maximum and minimum values of θ.

SOLUTIONS



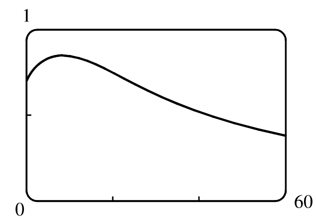
1. $|VP| = 9 + x\cos\alpha$, $|PT| = 35 - (4 + x\sin\alpha) = 31 - x\sin\alpha$, and $|PB| = (4 + x\sin\alpha) - 10 = x\sin\alpha - 6$. So using the Pythagorean Theorem, we have

$|VT| = \sqrt{|VP|^2 + |PT|^2} = \sqrt{(9 + x\cos\alpha)^2 + (31 - x\sin\alpha)^2} = a$,

and $|VB| = \sqrt{|VP|^2 + |PB|^2} = \sqrt{(9 + x\cos\alpha)^2 + (x\sin\alpha - 6)^2} = b$.

Using the Law of Cosines on $\triangle VBT$, we get $25^2 = a^2 + b^2 - 2ab\cos\theta \quad\Leftrightarrow$

$\cos\theta = \dfrac{a^2 + b^2 - 625}{2ab} \quad\Leftrightarrow\quad \theta = \arccos\left(\dfrac{a^2 + b^2 - 625}{2ab}\right)$, as required.



2. From the graph of θ, it appears that the value of x which maximizes θ is $x \approx 8.25$ ft. Assuming that the first row is at $x = 0$, the row closest to this value of x is the fourth row, at $x = 9$ ft, and from the graph, the viewing angle in this row seems to be about 0.85 radians, or about 49°.

3. With a CAS, we type in the definition of θ, substitute in the proper values of a and b in terms of x and $\alpha = 20^\circ = \frac{\pi}{9}$ radians, and then use the differentiation command to find the derivative. We use a numerical rootfinder and find that the root of the equation $d\theta/dx = 0$ is $x \approx 8.253062$, as approximated in Problem 2.

4. From the graph in Problem 2, it seems that the average value of the function on the interval $[0, 60]$ is about 0.6. We can use a CAS to approximate $\frac{1}{60}\int_0^{60} \theta(x)\,dx \approx 0.625 \approx 36^\circ$. (The calculation is much faster if we reduce the number of digits of accuracy required.) The minimum value is $\theta(60) \approx 0.38$ and, from Problem 2, the maximum value is about 0.85.