Integration problems

1

$$\int \frac{\cos x \; \ln(\sin x)}{\sin x} \; dx$$

whenever you see the log of "something" on top and "something" on the bottom, you should think about a substitution for "something", but first, here we see that if $u = \sin x$ then we have

$$du = \cos x \; dx$$

so we have

$$= \int \frac{\ln u}{u} \; du$$

now, do a second substitution to deal with the logarithm

$$v = \ln u$$

$$dv = \frac{1}{u} \; du$$

so the integral is

$$\int v \; dv = \frac{v^2}{2}$$

$$= \frac{1}{2} \; (\ln(\sin x))^2 + C$$

2

$$\int (\sin x + \cos x)^2 \; dx$$

Here I notice that when multiplied out we get

$$= \int \sin^2 x + 2 \sin x \cos x + \cos^2 x \; dx$$

$$= \int 2 \sin x \cos x \; dx$$

let $u = \sin x$ and then

$$du = \cos x \; dx$$

so we have

$$\int u \; du = \frac{1}{2} u^2 = \frac{1}{2} \sin^2 x + C$$

3

$$\int \frac{\cos^2 x}{1 + \sin x} \; dx$$

I see a trick. If we convert the bottom to a difference of squares, we'll have what's on top. That is

$$\frac{1}{1 + \sin x} \, \frac{1 - \sin x}{1 - \sin x} = \frac{1 - \sin x}{1 - \sin^2 x}$$

$$= \frac{1 - \sin x}{\cos^2 x}$$

So the integral becomes

$$\int 1 - \sin x \; dx = x + \cos x + C$$

4

$$\int \frac{\sin x}{1 + \sin x} \; dx$$

The same trick gives

$$\int \frac{\sin x \; (1 - \sin x)}{(1 + \sin x)(1 - \sin x)} \; dx$$

$$= \int \frac{\sin x \; (1 - \sin x)}{\cos^2 x} \; dx$$

$$= \int \frac{\sin x}{\cos^2 x}\ dx - \int \tan^2 x\ dx$$

So the first term is

$$-\int \frac{1}{u^2}\ du = \frac{1}{u}$$

$$= \frac{1}{\cos x} + C$$

and the second term is

$$\int \tan^2 x\ dx = \int \sec^2 x - 1\ dx$$

$$= \tan x - x + C$$

Altogether we have

$$= \frac{1}{\cos x} + \tan x - x + C$$

5

$$\int (2 + \tan x)^2\ dx$$

Just multiply it out

$$\int 2\ dx + \int 4 \tan x\ dx + \int \tan^2 x\ dx$$

For the last term, proceed as we did above. And for the middle term we have

$$4 \int \tan x\ dx = 4 \int \frac{\sin x}{\cos x}\ dx$$

let $u = \cos x$, so

$$du = -\sin x\ dx$$

$$= -4 \int \frac{1}{u}\ du = -4 \ln u = -4 \ln \cos x + C$$

Altogether we have

$$2x - 4 \ln \cos x + \tan x - x + C$$

$$= x - 4 \ln \cos x + \tan x + C$$