Differentiation Summary

Here is a quick summary of what you will need to know (memorize) about basic differentiation.
If $f(x) = ag(x)$ (with $a =$ constant) then

$$f'(x) = ag'(x)$$

If $f(x) = g(x) + h(x)$ then

$$f'(x) = g'(x) + h'(x)$$

power rule

If $f(x) = x^n$ then

$$f'(x) = nx^{n-1}$$

Together, these rules allow us to differentiate any polynomial function.

For the next part, it is convenient to use the symbols y and u when we mean $y(x)$ and $u(x)$ and u' when we mean $u'(x)$

product and quotient rules

If $y = uv$ then ("this times the derivative of that, etc...")

$$y' = u'v + uv'$$

If $y = u/v$ then

$$y' = \frac{u'v - uv'}{v^2}$$

With this last one, it can be hard to remember which term gets the minus sign. Just check with $y = 1/x$

$$y' = \frac{0 \times x - 1 \times 1}{x^2} = -\frac{1}{x^2}$$

which agrees with the result from the power rule.

chain rule

We'll do this one with an example, and use the dy notation. Suppose $y = \sqrt{(1-x^2)}$. Substitute $u = 1 - x^2$ Then $y = \sqrt{u}$ and

$$\frac{dy}{du} = \frac{1}{2}\frac{1}{\sqrt{u}}$$

Furthermore, since $u = 1 - x^2$

$$\frac{du}{dx} = -2x$$

We want dy/dx, but that is just

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx} = \frac{1}{2}\frac{1}{\sqrt{1-x^2}}(-2x) = -\frac{x}{\sqrt{1-x^2}}$$

implicit differentiation

We'll do this one also with an example. Consider a circle with radius r and equation $x^2 + y^2 = r^2$. Imagine that x is a function of some other variable t (perhaps, time). Then y is also a function of time. Now take derivatives d/dt, by the chain rule

$$\frac{d}{dt}y^2 = 2y\frac{dy}{dt}$$

$$\frac{d}{dt}x^2 = 2x\frac{dx}{dt}$$

but since r is a constant

$$\frac{d}{dt}r^2 = 0$$

Putting it together

$$2y\,\frac{dy}{dt} + 2x\,\frac{dx}{dt} = 0$$

Now, *multiply by dt*

$$2ydy + 2xdx = 0$$

Rearrange to obtain

$$\frac{dy}{dx} = -\frac{x}{y}$$

trigonometric functions

If $f(x) = \sin x$, then

$$f'(x) = \cos x$$

while if $f(x) = \cos x$, then

$$f'(x) = -\sin x$$

As an exercise, you should try finding $f'(x)$ when $f(x) = \tan x$ using these definitions and the quotient rule from above.

exponential

Finally, if $f(x) = e^x$ then

$$f'(x) = e^x$$

while if $f(x) = log(x)$—mathematicians usually write the *natural* logarithm this way—then

$$f'(x) = \frac{1}{x}$$