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Python for Bioinformatics

adventures in bioinformatics

Thursday, July 14, 2011

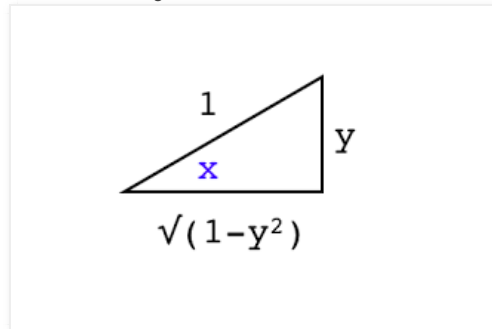
Euler's Gem 2

Here is a sketch of a second derivation of Euler's famous formula:

$$e^{i\theta} = \cos\theta + i \sin\theta$$

as presented by William Dunham in his [book](#) *Euler, The Master of Us All*. First post [here](#).

The first step is to recall a standard trigonometric substitution in calculus:



$$y = \sin x$$

$$x = \sin^{-1} y$$

$$\sqrt{1 - y^2} = \cos x$$

We're interested in the integral:

$$\int dy / \sqrt{1 - y^2}$$

Substituting with x we see that:

$$dy = \cos x \, dx$$

And the integral is

$$\int (1/\cos x) \cos x \, dx = \int dx = x$$

$$x = \int dy / \sqrt{1 - y^2}$$

Now Euler makes a complex change of variable:



Jackson's Mill WV

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$$\begin{aligned}
 y &= iz \\
 x &= \int dy / \sqrt{1 - y^2} \\
 &= \int i \, dz / \sqrt{1 - (iz)^2} \\
 &= i \int dz / \sqrt{1 + z^2} \\
 &= i \ln [\sqrt{1 + z^2} + z]
 \end{aligned}$$

The last step is another standard result from calculus which I will assume for the time being (more [here](#)).

Undo the substitution:

$$\begin{aligned}
 z &= y/i = \sin x / i \\
 z^2 &= -\sin^2 x \\
 \sqrt{1 + z^2} &= \sqrt{1 - \sin^2 x} \\
 &= \cos x
 \end{aligned}$$

$$x = i \ln (\cos x + \sin x / i)$$

We will use two identities involving i:

$$\begin{aligned}
 u / i &= -i u \\
 1 / (\cos u - i \sin u) &= (\cos u + i \sin u)
 \end{aligned}$$

(For the second one, see the previous post). Now:

$$\begin{aligned}
 x &= i \ln (\cos x + \sin x / i) \\
 x &= i \ln (\cos x - i \sin x) \\
 ix &= -\ln (\cos x - i \sin x) \\
 &= \ln [1 / (\cos x - i \sin x)] \\
 &= \ln (\cos x + i \sin x)
 \end{aligned}$$

Just eponentiate:

$$e^{ix} = \cos x + i \sin x$$

Wow, again!

Posted by telliott99 at 7/14/2011 09:30:00 AM
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About Me**telliott99**

I teach and do research in Microbiology. This blog started as a record of my adventures learning bioinformatics and using Python. It has expanded to include Cocoa, R, simple math and assorted topics. As bbum says, it's so "google can organize my head." The programs here are developed on OS X using R and Python plus other software as noted. YMMV

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