A mole of popcorn

In re-reading Walter Isaacson's biography of Einstein, I came across this about Avogadro's number:

> A current estimate is approximately 6.02214×10^{23}. (This is a big number: that many unpopped popcorn kernels when spread across the United States would cover the country nine miles deep.)[29]

Such an estimate is not particularly surprising, since 10^{23} is a pretty big number. "All the grains of sand on all the beaches in the world" is estimated to be 5×10^{21}.

`http://www.thenakedscientists.com/forum/index.php?topic=19016.0`

and a single grain of sand contains more atoms than that. Anyway, let's check the estimate.

Calculation

According to wikipedia

`https://en.wikipedia.org/wiki/United_States`

the surface area of the the United States is $9,826,675$ km^2, which is approximately 9.83×10^{18} mm^2. One mile is 1609 meters or 1.61×10^6 mm, so the volume of the United States to a depth of one mile is about 1.58×10^{25} mm^3.

If we take take the diameter of an (unpopped) popcorn kernel to be 4 mm, then its volume is

$$\frac{4}{3}\pi r^3 = \frac{4}{3} \times 3.14 \times 8 = 33.5 \text{ mm}^3$$

`https://en.wikipedia.org/wiki/Sphere_packing`

Analysis of sphere packing suggests that the densest packing of small spheres in a volume where the boundaries are insignificant is about 74%, hence we require about 45.2 mm^3 for each kernel.

Therefore, the number of kernels we can pack into that one-mile of depth is:

$$\frac{1.58 \times 10^{25}}{45.2} \approx 3.5 \times 10^{23}$$

We seem to be a little short—more like a little bit under two miles.

If we consider just the *land area*, then the volume in the numerator is reduced by a factor of

$$\frac{9.16}{9.83} \approx 0.93$$

which helps some, but it still doesn't get us to 9 miles. More significant is our estimate of the volume of a kernel. At our house we are big on popcorn (well, everyone but me), but it is this foo-foo stuff with really small kernels, which is not what I'm looking for here.

According to this

`https://answers.yahoo.com/question/index?qid=20090430104639AAc4XIv`

the diameter is more like 1/4 inch or 6.5 mm, giving a volume of 144 mm^3, so with these corrections I obtain:

$$\frac{1.58 \times 10^{25} \times 0.93}{144/0.74} \approx \frac{1.47 \times 10^{25}}{194} \approx 0.75 \times 10^{23}$$

which works out to a depth of

$$\frac{6.022}{0.75} \approx 8 \text{ miles}$$

Close enough.