Integrate z squared

Consider

$$\int_\gamma \frac{z^2}{4 - z^2}\ dz$$

where $\gamma = |z + 1| = 2$. So the denominator of the function is

$$\frac{1}{4 - z^2} = \frac{1}{(2 + z)(2 - z)}$$

It has zeroes as $z = \pm\ 2$. Only the point $z = 2$ is inside our contour. So if we split this by partial fractions

$$\frac{1}{(2 + z)(2 - z)} = \frac{1}{4}\ [\ \frac{1}{2 + z} + \frac{1}{2 - z}\ ]$$

so we can rewrite the integral as

$$I = \int_\gamma \frac{z^2}{4}\ [\ \frac{1}{2 + z} + \frac{1}{2 - z}\ ]\ dz$$

By Cauchy's Theorem, the first term is zero. The second one is:

$$I = \int_\gamma \frac{z^2}{4}\ (\frac{1}{2 - z})\ dz$$

and the value of I is

$$I = 2\pi i f(z_0)$$

where

$$f(z_0) = \frac{z^2}{4}\ \Bigg|_{z_0=2} = 1$$

so the integral is just $2\pi i$.