Problems

example

$$\int e^x \cos x \; dx$$

Let

$$u = e^x, \quad du = e^x \; dx$$

$$dv = \cos x \; dx, \quad v = \sin x$$

Then the integral is $uv - \int v \; du$ or

$$= e^x \sin x - \int e^x \sin x \; dx$$

which looks like no help, but keep going. Lather, rinse, repeat:

$$u = e^x, \quad du = e^x \; dx$$

$$dv = -\sin x \; dx, \quad v = \cos x$$

Then the second integral is

$$-\int e^x \sin x \; dx = e^x \cos x - \int e^x \cos x \; dx$$

Putting the answers together:

$$\int e^x \cos x \; dx = e^x \sin x + e^x \cos x - \int e^x \cos x \; dx$$

so

$$2 \int e^x \cos x \; dx = e^x \sin x + e^x \cos x$$

$$\int e^x \cos x \; dx = \frac{1}{2} \; e^x \; [\; \sin x + \cos x \;]$$

Check by differentiating. Leave the factor of $1/2$ aside for the moment:

$$\frac{d}{dx} \; e^x \; [\; \sin x + \cos x \;] \; = e^x \; [\; \sin x + \cos x \;] + e^x \; [\; \cos x - \sin x \;]$$

The $\sin x$ terms cancel, giving 2 terms of $\cos x$ but we have the factor of $1/2$ so that finally gives:

$$\frac{d}{dx} \; e^x \; [\; \sin x + \cos x \;] \; = e^x \cos x$$