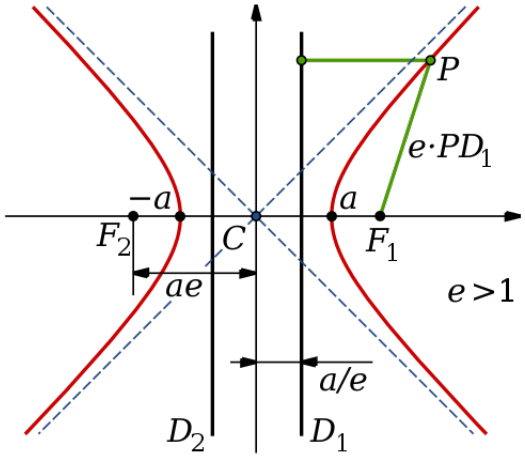
Hyperbola

Here is a hyperbola as shown in the wikipedia article on the subject.



Hyperbolas of this type (that open "east-west") have equations of the form

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

We can see that

$$\frac{x^2}{a^2} = 1 + \frac{y^2}{b^2}$$

so that the minimum value of x occurs when $y = 0$ and $x = a$. The conjugate hyperbola of this one is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

or equivalently

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

opens "north-south." And, although I want to wait to deal with this complication, we have to mention another very common hyperbola

$$xy = c$$

where it must be true that $x \neq 0$ and $y \neq 0$.

Another feature of hyperbolas is the asymptote, the straight line which is approached when $x, y >> a, b$. In the case of the first example

$$\frac{y^2}{b^2} = \frac{x^2}{a^2} - 1$$

$$y^2 = \frac{b^2}{a^2}x^2 - \frac{1}{a^2}$$

but for large x and y this approaches

$$y^2 = \frac{b^2}{a^2}x^2$$

$$y = \pm\frac{b}{a}x$$