Wicked integrals

$$\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\ dx = \ ?$$

Going to the cotangent doesn't help me.

Here's the trick:

$$\frac{\sin x}{\sin x + \cos x}$$

$$= \frac{1}{2} \cdot \ \frac{\sin x + \sin x}{\sin x + \cos x}$$

Now, add and subtract $\cos x$ in the numerator!

$$= \frac{1}{2} \cdot \ \frac{\sin x + \cos x + \sin x - \cos x}{\sin x + \cos x}$$

If you don't see it yet, this should help

$$= \frac{1}{2}\ [\ 1 - (\frac{\cos x - \sin x}{\sin x + \cos x})\ ]$$

But now the term in parentheses, together with dx, is

$$\int \frac{du}{u}$$

So the answer is

$$I = \frac{1}{2}\ [\ x - \ln(\sin x + \cos x)\ ]$$

And since $\sin x + \cos x = 1$ at both bounds, the logarithm term is

$$\ln 1 = 0$$

at both bounds, so we get just

$$= \frac{1}{2}x \,\bigg|_0^{\pi/2} = \frac{\pi}{4}$$