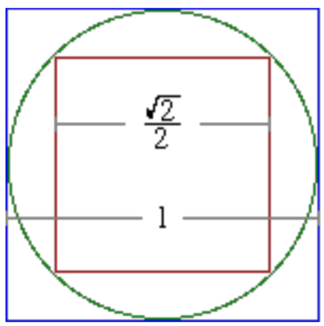
Let's try Archimedes' algorithm

$$P_{n+1} = \frac{2\,p_n P_n}{p_n + P_n} \quad \text{and} \quad p_{n+1} = \sqrt{p_n P_{n+1}}$$

starting with inscribed and circumscribed squares



$$p_2 = 2\sqrt{2} = 2.8284\ldots$$

$$P_2 = 4$$

Here are the results of 12 iterations of the algorithm.

n	p_n	P_n
2	2.8284271247461901	4
3	3.0614674589207182	3.3137084989847604
4	3.1214451522580523	3.1825978780745281
5	3.1365484905459393	3.1517249074292561
6	3.1403311569547529	3.1441183852459043
7	3.1412772509327729	3.1422236299424568
8	3.1415138011443011	3.1417503691689665
9	3.1415729403670914	3.1416320807031818
10	3.1415877252771597	3.1416025102568089
11	3.1415914215112	3.1415951177495891
12	3.1415923455701177	3.1415932696293073
13	3.1415925765848727	3.1415928075996446
14	3.141592634338563	3.141592692092254 4

These computations were done using a spreadsheet with only limited accuracy (ostensibly, 15 decimal places). Nevertheless, notice that either of the last two entries agree with the actual value of π to at least 6 places. Another half dozen iterations would yield π to 9 places. Not bad!

A better starting estimate would obviously have helped our situation here. Archimedes started with regular hexagons (an inscribed perimeter of 3 and a circumscribed perimeter of $2\sqrt{3} = 3.4641...$).

Archimedes' algorithm falls is one of a larger class of **related algorithms**.

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