

The Rocket Equation

Suppose we have a large rocket of mass m which accelerates by firing hot gas out to the rear with exhaust velocity v_e . The mass that is ejected is dm . The initial velocity of the rocket is v_i and the final velocity is $v_i + dv = v_f$.

Conservation of momentum gives us this equation

$$mv = (m - dm)(v + dv) + (dm)(v - v_e)$$

$$mv = mv + m dv - dm v - dm dv + dm v - dm v_e$$

$$0 = m dv - dm v - dm dv + dm v - dm v_e$$

$$0 = m dv - dm dv - dm v_e$$

Ignore the term with two differentials

$$m dv = dm v_e$$

$$dv = v_e \frac{dm}{m}$$

$$\Delta v = v_e \ln \frac{m}{m_0}$$

There is a sign problem here. $m < m_0$??