Integrating products of sine and cosine

This one is easy:

$$\int sin^4\ x\ cos\ x\ dx = \int u^4\ du$$

Still easy:

$$\int sin^4\ x\ cos^3\ x\ dx = \int sin^2\ x\ (1 - sin^2\ x) cos\ x\ dx$$

The general case is only a problem when m and n are both even.

$$\int sin^m\ x\ cos^n\ x\ dx$$

The classic:

$$\int cos^2\ x\ dx =$$

Two ways to do cosine squared. Use the double angle formula:

$$cos^2\ x = \frac{1}{2}[1 + oos(2x)]$$

Or (my favorite) let

$$u = cos\ x; du = -sin\ x\ dx; dv = cos\ x\ dx; v = sin\ x$$

The integral becomes

$$\int u\ dv = uv - \int v\ du = sin\ x\ cos\ x + \int sin^2\ x\ dx = sin\ xcos\ x + \int (1 - cos^2\ x)\ dx$$

That looks like no progress, but we just move the cosine squared term to the lhs:

$$2\int cos^2\ x\ dx = sin\ x\ cos\ x + \int 1\ dx = sin\ x\ cos\ x + x$$

$$\int cos^2\ x\ dx = \frac{1}{2}(sin\ x\ cos\ x + x)$$