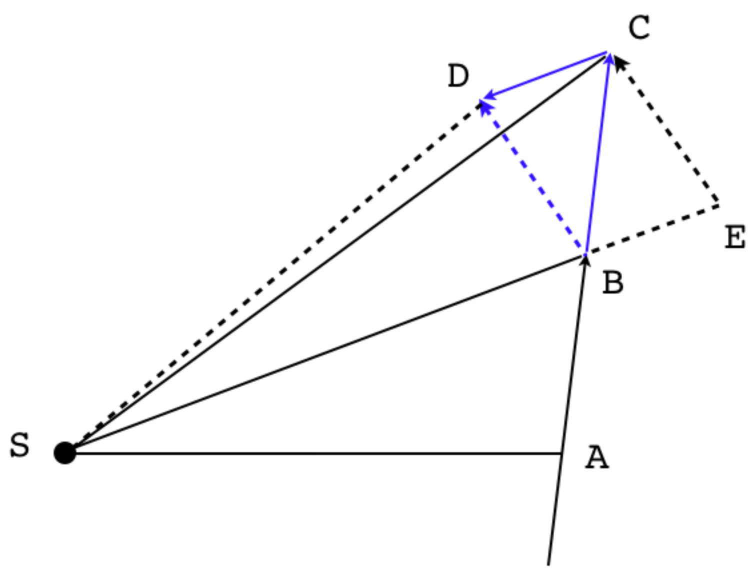
Kepler (part 1): Newton

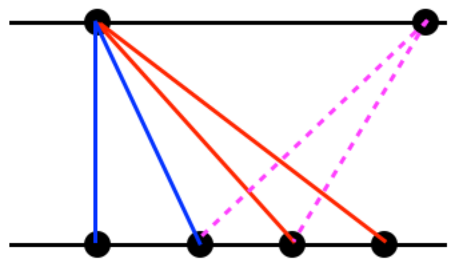
This is part one of ten, about deriving Kepler's Laws for planetary motion from Newton's Laws. In this part, we look at the geometric proof of K2 used by Newton.



We diagram the sun S and a planet at A. Imagine that the force toward the sun is applied discretely. That is, for a small interval Δt, the planet travels from A to B at constant velocity and if undisturbed, would travel to C in the next unit of time.

In the absence of a force, the velocity would be constant and so the length of AB is the same as that of BC, and since AB is on the same line as BC, the area of $\triangle ABS$ is equal to the area of $\triangle BCS$.

Proof: draw the vertical line from S to the line containing ABC. The area of either triangle is one-half the length of that altitude times the distance, either AB or BC. The principle is illustrated in the next figure.



Given two parallel lines separated by a distance h, pick two points on one line separated by a distance d and *any* point on the other line. The triangles drawn using those points will all have equal area, namely $(1/2)dh$.

Now, suppose the force is applied at B *toward the sun* along EBS. As a result, the trajectory BC is modified by the change in velocity resulting from application of the force toward the sun. The new path is the additional velocity times Δt. Call the length CD and add it to BC to give the actual trajectory, BD.

CD is parallel to SBE. Therefore, every point on CD has an altitude with respect to SBE of the same length. So any point on CD can be used to draw a triangle with the same base SB and the result will have the equal area no matter which point is chosen.

In particular, the area of $\triangle BDS$ is equal to the area of $\triangle BCS$, which was found earlier to be equal to the area of $\triangle ABS$. Since the two triangles from the actual motion have the same area, the area is constant.