scratch pad

In Chapter 2 of Hamming we find this: suppose we have the series

$$\frac{1}{2} + \cos x + \cos 2x + \cos 3x \ldots$$

Let s_n be the partial sum of the first n terms

$$s_n = \frac{1}{2} + \cos x + \cos 2x \cdots + \cos nx$$

Recall the addition formulas:

$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$

$$\sin(a-b) = \sin a \cos b - \sin b \cos a$$

Subtract

$$\sin(a+b) - \sin(a-b) = 2 \sin b \cos a$$

$$\sin b \cos a = \frac{1}{2} \; [\; \sin(a+b) - \sin(a-b) \;]$$

Multiply s_n by $\sin \frac{x}{2}$:

$$\sin \frac{x}{2} \cdot s_n = \frac{1}{2} \; [\; \sin \frac{x}{2} + \sin \frac{x}{2} \cdot \cos x + \sin \frac{x}{2} \cdot \cos 2x + \sin \frac{x}{2} \cdot \cos nx$$

Rewrite some typical terms using the result from above:

$$\sin \frac{x}{2} \cdot \cos x = \frac{1}{2} \; [\; \sin \frac{3}{2} x - \sin \frac{1}{2} x \;]$$

$$\sin \frac{x}{2} \cdot \cos 2x = \frac{1}{2} \; [\; \sin \frac{5}{2} x - \sin \frac{3}{2} x \;]$$

Notice that adding the cancellation of $\sin\frac{3}{2}x$. We have produced a *telescoping* series:

$$\sin\frac{x}{2}\cdot s_n = \frac{1}{2}\ [\ \sin(n+\frac{1}{2})x\ ]$$

$$s_n = \frac{\sin(n+\frac{1}{2})x}{2\sin\frac{x}{2}}$$

So

$$\frac{1}{2} + \cos x + \cos 2x \cdots + \cos nx = \frac{\sin(n+\frac{1}{2})x}{2\sin\frac{x}{2}}$$