

problem ?

$$\int \frac{\sin 2x}{1 + \cos^2 x} dx$$

The formulas to remember are:

$$\cos s + t = \cos s \cos t - \sin s \sin t$$

$$\sin s + t = \sin s \cos t + \cos s \sin t$$

If  $s = t$  the second one becomes:

$$\sin 2s = 2 \sin s \cos s$$

So our problem is now:

$$\int \frac{2 \sin x \cos x}{1 + \cos^2 x} dx$$

Notice that

$$\frac{d}{dx} \cos^2 x = -2 \cos x \sin x dx$$

That leads to the idea of letting

$$u = 1 + \cos^2 x$$

$$du = -2 \cos x \sin x dx$$

So the integral is

$$\begin{aligned} &= \int \frac{-du}{u} \\ &= -\ln u \\ &= -\ln(1 + \cos^2 x) + C \end{aligned}$$

We don't need absolute value signs because  $1 + \cos^2 \geq 0$ . Check by differentiating. Remember the minus sign, the rest is

$$\frac{1}{1 + \cos^2 x} (-2 \cos x \sin x)$$

so, with the minus sign we have our integrand back again. Check.