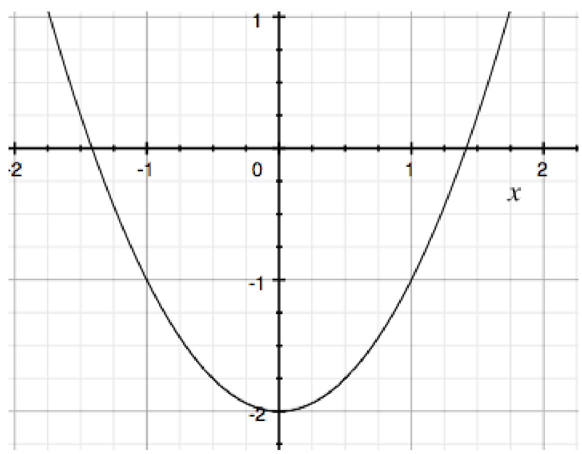
Newton square root

Here is a method for finding roots of equations quickly, often called Newton's method, or the Newton-Raphson method. As an example, here is a plot of the function

$$f(x) = x^2 - 2$$



which is equal to zero when $x = \pm\sqrt{2}$. That is, we want the roots of the following equation

$$x^2 - 2 = 0$$

and more generally

$$x^2 - N = 0$$

to find the square root of some other number.

Pick a point g (for guess). Then we need to construct the line tangent to the curve at that point, with slope $m = f'(g)$ and ask, where does this line intercept the x-axis?

The slope is $\Delta y / \Delta x$.

$$\frac{f(g) - 0}{g - r} = f'(g)$$

with r being the x-coordinate at the intercept. Rearrange

$$\frac{f(g)}{f'(g)} = g - r$$

$$r = g - \frac{f(g)}{f'(g)}$$

square root problem

For this particular problem, we have

$$f(g) = g^2 - N$$

$$f'(g) = 2g$$

$$r = g - \frac{g^2 - N}{2g} = \frac{1}{2}(g + \frac{N}{g})$$

In other words, r is the average of g and N/g.

Now set $g = r$ and repeat.

This can be encapsulated into the following algorithm:

- Make a guess g and compute N/g
- Average g and N/g to find a new guess
- Repeat until satisfied

The algorithm converges rapidly for most problems.

```
2
1.5
1.4166666666666665
1.4142156862745097
1.4142135623746899
1.414213562373095
```

notes

It is worthwhile to try to make a good first guess. If the method goes wrong (which it can, when equations have bumps or other issues), the problem can be fixed by making a better guess.

You can read more about the method here:

`http://www.math.ubc.ca/~anstee/math104/newtonmethod.pdf`

For the particular problem that we worked, finding the square root of N, the equation says

$$g' = \frac{1}{2}(g + \frac{N}{g})$$

find your next guess by averaging the current guess g and N/g.

This equation goes back at least as far as Heron of Alexandria (10-70 AD).