Exponentials

We start to explore functions more systematically. You already know powers like x^2 and x^3 and in general, we can write

$$y = ax^n$$

where a is some constant, x is the independent variable, and n is a positive integer. So if $a = 2$ and $n = 3$

$$y = 2x^3$$

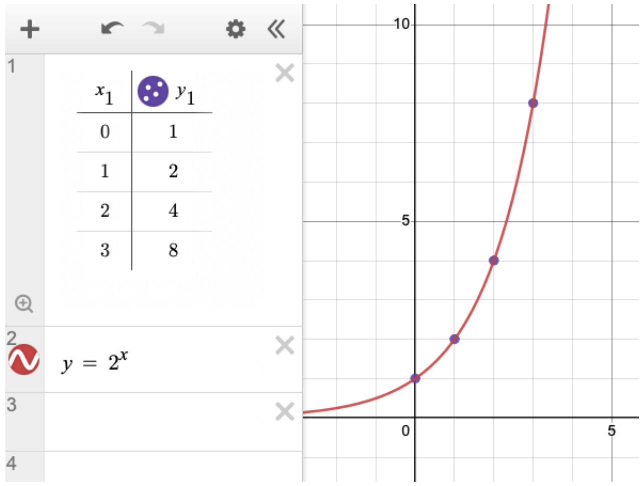
is a *cubic* function because of that power of 3. Various powers of x can be combined into what are called *polynomials* like

$$y = ax^2 + bx + c$$

The next major class of functions are the *exponential* functions like $y = b^x$, where b is some constant called the *base* and x is again, the independent variable. A simple example is

$$y = 2^x$$

We can get some idea about this function by trying different values of x.



We sketch the graph by filling in between the points $(x, 2^x)$ obtained for integer values of x. It's important to remember that x doesn't have to be an integer. For example $x = 1/2$ is perfectly legal, so then $y = 2^{1/2}$.

What does it mean to say that $y = 2^{1/2}$? The meaning becomes clear when we square both sides.

$$y^2 = 2^{1/2} \cdot 2^{1/2} = 2$$

The last step follows from the basic rule for exponents of a common base. Then

$$y = \sqrt{y^2} = 2^{1/2} = \sqrt{2}$$

So that's the first step in thinking about exponentials as functions: x can have values that are not integers.

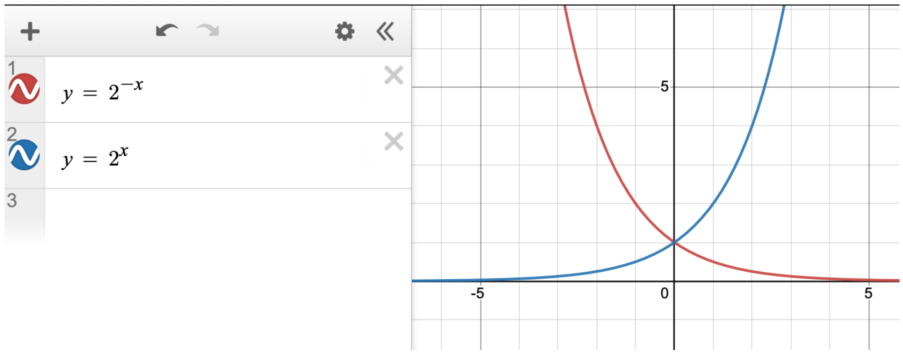
The exponent be negative, as well.

$$y = 2^{-x}$$

We can get some idea about this one by multiplying on top and bottom as follows:

$$y = 2^{-x} \cdot \frac{2^x}{2^x} = \frac{2^0}{2^x} = \frac{1}{2^x}$$

So 2^{-x} is the inverse of 2^x.



In this figure the red curve is the graph of $y = 2^{-x}$ and the blue curve is $y = 2^x$. They are mirror images.

This form of the exponential is called the negative exponential or exponential decay.