Basic factors

Here are some important tips for factoring:

- We only worry about prime factors: 2 3 5 7 11 13 17 19 . . . .
- A multiple of 2 (an even number) ends in one of 2 4 6 8 0.
- A multiple of 3 has digits that add up to a multiple of 3 like 3 6 9.
- Any multiple of 5 ends in 0 or 5.

Therefore, we only need to check numbers that end in 1 3 7 9.

- We only need to check for a particular prime factor when the number of interest is larger than that prime, squared. So only check for 7 as a factor when $n > 49$ and for 11 when $n > 121$.

Actually, we can do better than that. The only possible numbers smaller than 100 with 7 as the smallest factor are

$$7 \cdot 7 = 49, \qquad 7 \cdot 11 = 77, \qquad 7 \cdot 13 = 91$$

Between $100 - 200$:

$$7 \cdot 17 = 119, \qquad 7 \cdot 19 = 133, \qquad 7 \cdot 23 = 161 \ldots$$

It's probably easier to memorize those than to have to bother with checking for divisibility by 7.

As for 11 we have

$$11 \cdot 13 = 143, \qquad 11 \cdot 17 = 187 \ldots$$

There's a pattern here. For 3-digit multiples of 11, the middle digit is the sum of the two on the ends.

To summarize, with the exception of three numbers smaller than 100, and five others smaller than 200, the only non-primes smaller than 200 are divisible by 2, 3 or 5.

example

Factoring is hard because there are lots of prime numbers.

What are the prime factors of 157? The number 157 is not even or a multiple of 5. It's not divisible by 3, because $1 + 5 + 7 = 13$ and $1 + 3 = 4$.

We know that $7 \cdot 20 = 140 + 14 = 154$. Since 154 is divisible by 7, 157 cannot be (Why? Use the distributive law).

If you haven't memorized the two numbers that are multiples of 11 or remembered the trick with the digits, then you have to try 11, because $157 > 121 = 11^2$.

We have $11 \cdot 10 = 110$ and $157 - 110 = 47$. But then 44 is divisible by 11 so 47 is not, thus neither is 157 for the same reason as above.

Last, $157 < 169 = 13^2$, and certainly $157 < 13 \cdot 17 = 221$, so we don't need to check 13. We conclude that 157 is a prime number.

The most important reason to spend time adding fractions is that it motivates the subject of prime factorization and the famous *fundamental theorem of arithmetic*.

But there is an easier way to solve factoring problems. That method is called *Euclid's algorithm*. It gives the product of all shared factors without factoring. I'm just not sure if you know about it yet.