RMS voltage

I got to thinking about how voltage is typically reported. According to wikipedia

`https://en.wikipedia.org/wiki/Mains_electricity`

in the US and Canada the nominal voltage for a standard circuit should be 120V at source ($\pm$ 5 %). They go on to say that

> Historically 110 V, 115 V and 117 V have been used at different times and places in North America.

In a circuit run with alternating current (AC), the voltage varies as a sine wave. So it's obvious that we need something a bit more sophisticated to talk about *the* voltage.

One way would be to talk about the average voltage. In calculus, we define the average of a function over an interval as

$$\frac{\int_{x_1}^{x_2} f(x)\ dx}{x_2 - x_1}$$

This has a remarkably simple value for the sine and cosine functions. Namely, since $\int \sin x\ dx = -\cos x$, the integral over the interval $[0..\pi/2]$ is

$$I = -\cos x \Big|_0^{\pi/2} = (-0) - (-1) = 1$$

Because of symmetry we can pick any interval that is a multiple of $\pi/2$.

The average value is the maximum value (1) divided by $\pi/2$, that is to say, multiplied by $2/\pi$. When thinking about this it is helpful to remember that the average is less than the maximum.

correct calculation

The problem is that this is not what is done. For historical reasons, we calculate the RMS, the *root mean square*.

This means exactly what it says, namely, square the values (for a discrete problem), calculate the mean, and then take the square root.

For a continuous function like the sine, we use integration to find the average. The first part of the calculation is to integrate

$$I = \int \sin^2 x \; dx$$

then divide the result I by the interval to find the mean, and then take the square root.

This integral is probably the first non-trivial integral encountered in learning calculus, and can be solved in several ways. We will do it by guessing. With a prime to indicate the derivative, use the formula

$$[uv]' = u'v + uv'$$

Here what we want is to find the derivative of this product:

$$[\sin x \cos x]' = -\cos^2 x + \sin^2 x$$

$$= -1 + 2\sin^2 x$$

Integrate both sides and all of a sudden, we have the integral we want

$$\sin x \cos x = -x + 2 \int \sin^2 x \; dx$$

so

$$I = \int \sin^2 x \; dx = \frac{1}{2}(x + \sin x \cos x)$$

Over the interval $[0..\pi/2]$ we have simply

$$I = \frac{1}{2} \; [\; (\pi/2 + 0) - (0 + 0) \;] = \frac{\pi}{4}$$

Dividing by the length of the interval gives $1/2$ and then taking the square root we get $1/\sqrt{2}$

In other words the maximum and nominal voltage are simply related by this factor, $\sqrt{2}$. The maximum voltage for a 120V circuit is about 170V.

Of course the actual voltage depends on a number of practical factors, which we ignore.