adding fractions

Denominators are different and prime.

$$\frac{1}{2}+\frac{1}{3}, \qquad \frac{1}{3}+\frac{1}{5}, \qquad \frac{1}{2}+\frac{1}{13}, \qquad \frac{1}{a}+\frac{1}{b}=\frac{b+a}{ab}$$

One or both denominators not prime but have no common factors.

$$\frac{1}{9}+\frac{1}{10}, \qquad \frac{1}{11}+\frac{1}{15}, \qquad \frac{1}{15}+\frac{1}{16}, \qquad \text{same as above}$$

One is a multiple of the other.

$$\frac{1}{2}+\frac{1}{4}, \qquad \frac{1}{20}+\frac{1}{5}, \qquad \frac{1}{7}+\frac{1}{77}, \qquad \frac{1}{a}+\frac{1}{ab}=\frac{b+1}{ab}$$

They share a common factor.

$$\frac{1}{6}+\frac{1}{9}, \qquad \frac{1}{20}+\frac{1}{15}, \qquad \frac{1}{33}+\frac{1}{77}$$

$$\frac{1}{ab}+\frac{1}{bc}=\frac{bc+ab}{abc}$$

Similar to the last, but a common factor is found in the numerator afterward.

$$\frac{1}{10}+\frac{1}{15}=\frac{6+4}{60}=\frac{1}{6}$$

finding a common factor

◦ Method 1: write the *multiples* of each denominator.

```
 6 12 18 ..
 9 18 ..

20 40 60 ..
15 30 45 60 ..

33  66  99 132 165 198 231 ..
77 154 231 ..
```

◦ Method 2: find the *prime factors* of each denominator.

```
6 =   2.3
9 =     3.3
      2.3.3 = 18

20 =  2.2. .5
15 =      3.5
      2.2.3.5 = 60

33 =  3. .11
77 =    7.11
      3.7.11 = 231
```

◦ 3: Euclid's algorithm for the *greatest common divisor* (gcd):

```
20 = 1.15 + 5
15 = 3.5 +  0      5 is the gcd
```

Stop when the remainder is zero. 5 is the gcd of 20 and 15. Divide $20/5 = 4$ and then multiply $4 \cdot 15 = 60$.

```
77 = 2.33 + 11
```

```
33 = 3.11 +  0      11 is the gcd
```

Stop when the remainder is zero. 11 is the gcd of 33 and 77. Divide $77/11 = 7$ and then multiply $7 \cdot 33 = 231$.