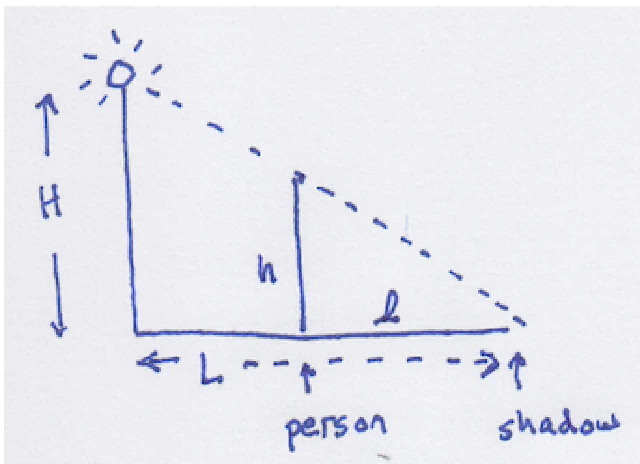
# Lamppost problem

Here's a classic problem: there is a light on top of a post and a person is walking away from it at some speed. At what speed is the tip of the person's shadow moving?




Obviously the two speeds are related, but it seems clear that they depend on the angle to the light or current distance to the person.

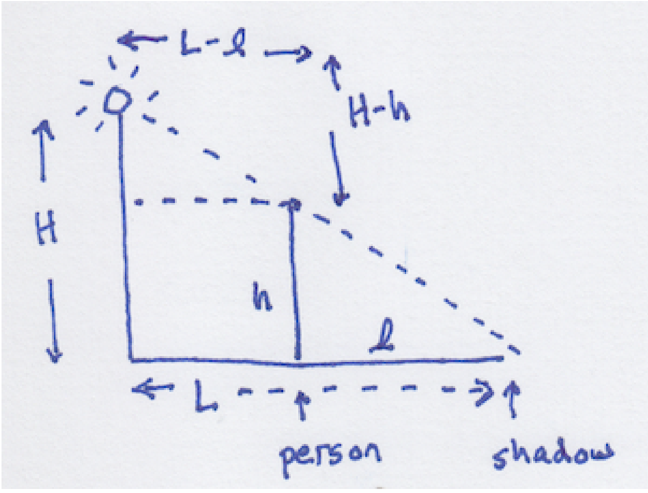
From similar triangles we get that

$$\frac{H}{L} = \frac{h}{l}$$

$H$ and $h$ are fixed. The speed of the shadow is $dL/dt$. But the person's speed is not $dl/dt$.

Instead, the person's current distance from the lamppost is the difference $L - l$. How to express this difference in terms of the other values?

One answer is to find another similar triangle:




$$\frac{H-h}{L-l} = \frac{H}{L} = \frac{h}{l}$$

Rearranging the first and last terms:

$$\frac{H-h}{h} = \frac{L-l}{l}$$

We want

$$\frac{dl}{dt} = \frac{h}{H-h} \cdot \frac{d(L-l)}{dt}$$

The speed of the shadow is $H/(H-h)$ times the speed of the person.

So, for example, if $H = 5$ m and the person is 1.8 m then

$$\frac{h}{H-h} = \frac{1.8}{3.2} = 0.5625$$

So if the person's speed is 1.2 m/s the shadow's speed is 0.675 m/s.

The seems counter-intuitive, but suppose the person's height were one-half the lamppost. Then $h/(H-h) = 1$. The distance from the post is always equal to the length of the shadow, so the speed is the same.