## VIENNA UNIVERSITY OF TECHNOLOGY

# 360.252 Computational Science on Many Core Architectures

Institute for Microelectronics

## Exercise 7

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### **Abstract**

Here documented the results of Exercise 7.

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### 1 Pipelined Conjugate Gradients (5/5 Points)

#### 1.1 Implement the Algorithm (3/3 Points)

In the listings below the Kernels for the blue and red colored font from lecture slides 7a. The task was to only execute 2 Kernel call's per iteration, with these two Kernels, this can be achieved.

#### 1.1.1 Blue Font Algorithm Part

#### Kernel for Blue Font Algorithm Part

```
__global__ void cuda_blue(int N, double *x, double *p, double *Ap, double *r, double *rip, double Alpha,
 1
         double Beta)
 2
3
       __shared__ double shared_memory[512];
 4
      double partial_dot_product = 0;
5
6
       for (int i = blockIdx.x * blockDim.x + threadIdx.x; i < N; i += blockDim.x * gridDim.x)
 7
        double p_thread = p[i];
        double Ap_thread = Ap[i];
8
9
        double r_{thread} = r[i] - Alpha * Ap_thread;
10
        x[i] += Alpha * p_thread;
11
        r[i] = r_{thread};
12
        p[i] = r_thread + Beta * p_thread;
13
         partial_dot_product += r_thread *r_thread;
14
15
      shared_memory[threadIdx.x] = partial_dot_product;
       for (int j = blockDim.x / 2; j > 0; j /= 2) {
16
         __syncthreads();
17
18
        if (threadIdx.x < j) {
          shared\_memory[threadIdx.x] += shared\_memory[threadIdx.x + j];
19
20
      }
21
       if (threadIdx.x == 0) r_ip[blockIdx.x] = shared_memory[0];
22
23
24
    }
```



#### 1.1.2 Red Font Algorithm Part

#### Kernels for Red Algorithm Part

```
1
         __global__ void cuda_blue(int N, double *x, double *p, double *Ap, double *r, double *rip, double Alpha,
              double Beta)
2
        {
           __shared__ double shared_memory[512];
3
           double partial_dot_product = 0;
4
5
           for (int i = blockIdx.x * blockDim.x + threadIdx.x; i < N; i += blockDim.x * gridDim.x){
6
            double p_{-}thread = p[i];
7
            double Ap_{thread} = Ap[i];
8
            double r_{thread} = r[i] - Alpha * Ap_{thread};
            x[i] += Alpha * p_thread;
9
            r[i] = r_{thread};
10
11
            p[i] = r_{thread} + Beta * p_{thread};
             partial_dot_product += r_thread *r_thread;
12
13
14
          shared\_memory[threadIdx.x] = partial\_dot\_product;
           for (int j = blockDim.x / 2; j > 0; j /= 2) {
15
             __syncthreads();
16
17
             if (threadIdx.x < j) {
              shared\_memory[threadIdx.x] += shared\_memory[threadIdx.x + j];
18
19
20
           if (threadIdx.x == 0) r_ip[blockIdx.x] = shared_memory[0];
21
22
          }
23
        }
```



#### 1.1.3 Bringing Everything together

In the fuction conjugate\_gradient\_pipelined() we use both the Kernels from before and iterate till convergence. In order for this algorithm to work, it needs initial  $\alpha_0$ ,  $\beta_0$  and also an  $Ap_0$ , this is done before the actual iterations in the while() loop. One can use the old functions from the last exercise like the dot product and the matrix vector product to obtain the two values and the vector. In the listing below shown the while() loop that iterates till convergence.

#### CG while() loop

```
int iters = 0;
 1
 2
        cudaDeviceSynchronize();
 3
        timer.reset();
 4
        while (1) {
 5
          // Line 2-4 and partial of line 6, The blue colored part in the algorithm:
 6
          cuda_blue<<<br/>blocks_lnch,thrds_block>>>(N, cuda_solution, cuda_p, cuda_Ap, cuda_r, cuda_ip_rr, alpha,
               beta);
          // cudaMemcopy for blockwise inner product < r_i, r_i > for CPU calculations of alpha and beta
 7
 8
          cudaMemcpy(bwip_rr, cuda_ip_rr, sizeof(double) * blocks_lnch, cudaMemcpyDeviceToHost);
          // Line 5 and 6, the red colored part in the algorithm
 9
          cuda_red<<<blocks_Inch,thrds_block>>>(N, csr_rowoffsets, csr_colindices, csr_values, cuda_p, cuda_Ap,
10
               cuda_ip_ApAp, cuda_ip_pAp);
11
          // cudaMemcopy for blockwise inner product <Ap_i, Ap_i> and <p_i, Ap_i> for CPU calculations of alpha
               and beta
12
          cudaMemcpy(bwip_pAp, cuda_ip_pAp, sizeof(double) * blocks_Inch, cudaMemcpyDeviceToHost);
13
          cudaMemcpy(bwip_ApAp, cuda_ip_ApAp, sizeof(double) * blocks_Inch, cudaMemcpyDeviceToHost);
14
          // CPU summation of blockwise inner products.
15
           ip_r = bwip_r [0];
          ip\_ApAp = bwip\_ApAp[0];
16
17
          ip_pAp = bwip_pAp[0];
          for ( size_t i=1; i<blocks_lnch; ++i)
18
19
          {
20
             ip_rr += bwip_rr[i];
21
            ip\_ApAp += bwip\_ApAp[i];
22
            ip_pAp += bwip_pAp[i];
23
24
          //Check if convergence criterion is fulfilled .
25
           if (std::sqrt(ip_rr / initial_residual_squared ) < 1e-6) {
26
            break:
27
          }
          // Computation of alpha and beta for next while iteration .
28
          alpha = ip_rr / ip_pAp;
29
30
          beta = (alpha*alpha*ip_ApAp - ip_rr) / ip_rr;
31
32
           if (iters > 10000)
33
            break; // solver didn't converge
34
           ++iters;
35
        }
```



#### 1.2 Comparison of Pipelined CG and Classical CG on both GPU's (2/2 Points)

#### 1.2.1 CG Algorithm Performance on RTX3060

For this specific implementation, the runtime for the pipelined CG algorithm on the RTX3060 outperforms the classic one only for  $\sqrt{N}$  larger than approximately 300. After this point, the imporvements of the pipelined algorithm appear to be only minor.

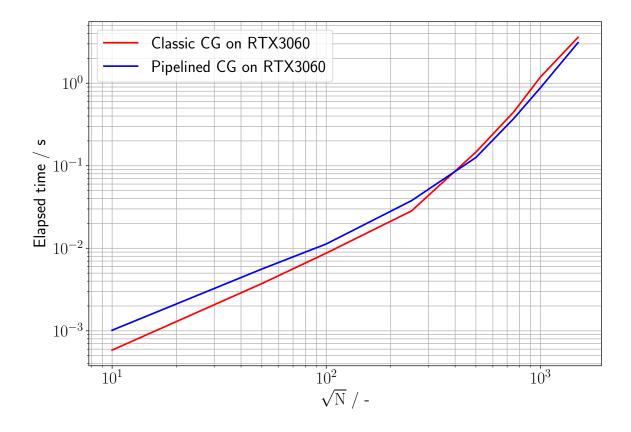


Figure 1: Elapsed time for Classical and Pipelined CG Algorithm on the RTX3060 GPU



#### 1.2.2 CG Algorithm Performance on Tesla K40

For this specific implementation, the runtime for the pipelined CG algorithm on the Tesla K40 outperforms the classic one only for  $\sqrt{N}$  larger than approximately 350. After this point, the imporvements of the pipelined algorithm appear to be only minor.

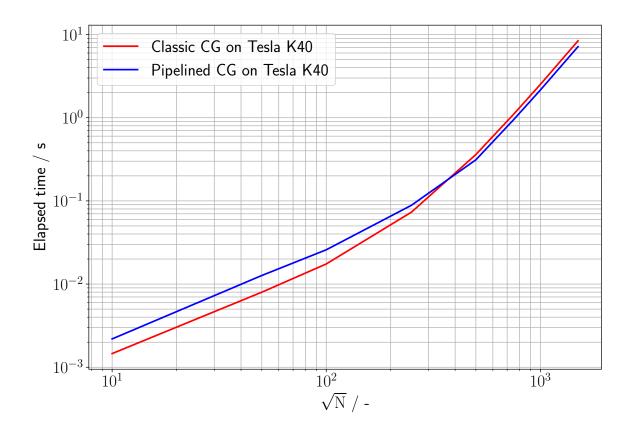


Figure 2: Elapsed time for Classical and Pipelined CG Algorithm on the Tesla K40 GPU