Vienna University of Technology

184.725 High Performance Computing

TU Wien Informatics

Exercise 1

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1 Exercise 1 - Closed Form Expressions

1.1 $\sum_{i=0}^{d} k^i$ for $k > 0$ (Ex1.1)

$$\begin{aligned}
\sum_{i=0}^{d} k^i &= \sum_{i=0}^{d} k^i \\
\sum_{i=0}^{d} k^i - k\sum_{i=0}^{d} k^i &= \sum_{i=0}^{d} k^i - k\sum_{i=0}^{d} k^i \\
\sum_{i=0}^{d} k^i - k\sum_{i=0}^{d} k^i &= \sum_{i=0}^{d} k^i - \sum_{i=1}^{d+1} k^i \\
\sum_{i=0}^{d} k^i (1-k) &= 1 - k^{d+1} \\
\sum_{i=0}^{d} k^i &= \frac{k^{d+1}-1}{k-1}
\end{aligned} \tag{1}$$

1.2 $\sum_{i=1}^{d} ik^i$ for $k > 0$ (Ex1.4)

$$\begin{aligned}
\sum_{i=1}^{d} ik^i &= \sum_{i=0}^{d} ik^i = \sum_{i=0}^{d} k\frac{\mathrm{d}}{\mathrm{d}k}k^i = k\frac{\mathrm{d}}{\mathrm{d}k}\overbrace{\sum_{i=0}^{d} k^i}^{\text{use (1)}} \\
\sum_{i=1}^{d} ik^i &= k\frac{\mathrm{d}}{\mathrm{d}k}\frac{1-k^{d+1}}{1-k} = \frac{dk^{d+2}-(d+1)k^{d+1}+k}{(1-k)^2}
\end{aligned} \tag{2}$$

1.3 $\sum_{i=1}^{d} i2^{d-i}$ (Ex1.3)

$$\begin{aligned}
&\sum_{i=1}^{d} i2^{d-i}, \quad \text{use k instead of 2} \\
&\sum_{i=1}^{d} ik^{d-i} = \sum_{i=0}^{d} dk^{d-i} - \sum_{i=0}^{d}(d-i)k^{d-i} \\
&\qquad = d\underbrace{\sum_{j=0}^{d} k^j}_{\text{use (1)}} - \underbrace{\sum_{j=0}^{d} jk^j}_{\text{use (2)}} \quad \text{with } j := d-1 \\
&\qquad = \frac{d(k^{d+1}-1)}{k-1} - \frac{dk^{d+2}-(d+1)k^{d+1}+k}{(1-k)^2} \quad \text{set } k \text{ back to 2} \\
&\qquad = d2^{d+1} - d - d2^{d+2} + d2^{d+1} + 2^{d+1} - 2 \\
&\sum_{i=1}^{d} i2^{d-i} = 2^{d+1} - 2 - d
\end{aligned} \tag{3}$$

1.4 $\sum_{i=1}^{d} i2^i$ (Ex1.2)

$$\sum_{i=1}^{d} i2^i = d2^{d+2} - (d+1)2^{d+1} + 2 \quad \text{with use of (2)} \tag{4}$$

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