

VIENNA UNIVERSITY OF TECHNOLOGY

184.725 HIGH PERFORMANCE COMPUTING

TU WIEN INFORMATICS

Exercise 1

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Abstract

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1 Exercise 1 - Closed Form Expressions

1.1 $\sum_{i=0}^d k^i$ for $k > 0$ (Ex1.1)

$$\begin{aligned}
 \sum_{i=0}^d k^i &= \sum_{i=0}^d k^i \\
 \sum_{i=0}^d k^i - k \sum_{i=0}^d k^i &= \sum_{i=0}^d k^i - k \sum_{i=0}^d k^i \\
 \sum_{i=0}^d k^i - k \sum_{i=0}^d k^i &= \sum_{i=0}^d k^i - \sum_{i=1}^{d+1} k^i \\
 \sum_{i=0}^d k^i (1 - k) &= 1 - k^{d+1} \\
 \sum_{i=0}^d k^i &= \frac{k^{d+1} - 1}{k - 1}
 \end{aligned} \tag{1}$$

1.2 $\sum_{i=1}^d i k^i$ for $k > 0$ (Ex1.4)

$$\begin{aligned}
 \sum_{i=1}^d i k^i &= \sum_{i=0}^d i k^i = \sum_{i=0}^d k \frac{d}{dk} k^i = k \frac{d}{dk} \sum_{i=0}^d k^i \quad \text{use (1)} \\
 \sum_{i=1}^d i k^i &= k \frac{d}{dk} \frac{1 - k^{d+1}}{1 - k} = \frac{d k^{d+2} - (d+1) k^{d+1} + k}{(1 - k)^2}
 \end{aligned} \tag{2}$$

1.3 $\sum_{i=1}^d i 2^{d-i}$ (Ex1.3)

$$\begin{aligned}
 \sum_{i=1}^d i 2^{d-i} &, \quad \text{use } k \text{ instead of } 2 \\
 \sum_{i=1}^d i k^{d-i} &= \sum_{i=0}^d d k^{d-i} - \sum_{i=0}^d (d-i) k^{d-i} \\
 &= d \sum_{j=0}^d k^j - \sum_{j=0}^d j k^j \quad \text{with } j := d - i \\
 &\quad \text{use (1)} \quad \text{use (2)} \\
 &= \frac{d(k^{d+1} - 1)}{k - 1} - \frac{d k^{d+2} - (d+1) k^{d+1} + k}{(1 - k)^2} \quad \text{set } k \text{ back to } 2 \\
 &= d 2^{d+1} - d - d 2^{d+2} + d 2^{d+1} + 2^{d+1} - 2 \\
 \sum_{i=1}^d i 2^{d-i} &= 2^{d+1} - 2 - d
 \end{aligned} \tag{3}$$

1.4 $\sum_{i=1}^d i 2^i$

$$\sum_{i=1}^d i 2^i = d 2^{d+2} - (d+1) 2^{d+1} + 2 \quad \text{with use of (2)} \tag{4}$$

2 Exercise 2 - Graph Tree's with Canonical Numbering

2.1 T_k^d with $k = 3$ and $d = 3$

2.2 B_k^d with $k = 3$ and $d = 4$

3 Exercise 3 - Planar Graph H_d

4 Exercise 4 - Gray Code Embedding

5 Exercise 5 - Inverse Gray Code

6 Exercise 6 -

7 Exercise 7 -