# RichardsNitscheSignorini — Proof Chain
## Main.lean → Theorem 4.10



**Legend:**
- Constitutive axiom
- Established result
- Novel result
- Proved theorem
- Main theorem

**Prop 4.7** splitting $O(\sqrt{\Delta t / Gr})$

- Split ↓ w/ Gr ↑Gr → ↓error
- Split vanishes error → 0
- Prop 4.8 Godunov sub $O(h^{1/2})$
- Diffusion error $O(h)$
- Prop 4.9 [Novel] Godunov sat $O(h^{m/(m+1)})$

- Split accuracy ↑Gr → ↑accuracy
- Thm 4.10(a) sub-saturated $O(\min(\Delta t, \sqrt{\Delta t/Gr}) + h^{1/2} + h)$
- Sublinear rate $0 < m < 1$
- Sat rate worse $m/(m+1) < 1/2$
- Thm 4.10(b) saturated $O(\min(\Delta t, \sqrt{\Delta t/Gr}) + h^{m/(m+1)} + h)$

- $K(0) = 0$ Mualem 1976
- $K(1) = Ks$ Mualem 1976
- C asymp dry eq. (4.13)
- K asymp dry eq. (4.11)

- Hölder at sat eq. (4.20)
- Prop 4.3 RH speed $s = Ks/\Delta\theta$
- $K^{-1}(1) = 1/Ks$ from K_Se_one
- Prop 4.2 D blow-up sat
- K strict mono vG 1980
- Prop 4.1 D ~ Se^β dry
- Prop 4.4b L1 contraction

- Cor 4.6a $K^{-1}$ blow-up
- Cor 4.6b cond. number
- K continuous vG 1980
- Prop 4.3 Lax entropy
- $K \geq 0$ Mualem 1976
- Cor 4.2 D → ∞ sat
- $K' \to \infty$ sat eq. (4.21)
- Cor 4.1 D → 0 dry
- $K' \to 0$ dry eq. (4.12)
- Prop 4.4c uniqueness

- Prop 4.4a existence
- Prop 4.5 front ratio
- K Lipschitz compact [0,1)