EEE 587 Optimal Control

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Discrete Time System

$$x(k+1) = x(k) + u(k)$$

Determine the optimal control policy which minimizes the performance:

$$J = \sum_{k=1}^{2} [2|x(k) - 0.1k^2| + |u(k-1)|]$$

Feasible states and controls are defined as follows:

$$0.0 \leq x(k) \leq 0.4, k = 0,1,2$$

$$-0.2 \leq u(k) \leq 0.2, k = 0,1$$

Dynamic Programming Equation

$$C^*_{\alpha x_i h} = J_{\alpha x_i} + J^*_{x_i h}$$

Cost to move from α to x_i — Minimum cost to reach final state h from x_i

Finding the optimal policy

Back propagation – Second Stage First (last stage)

K = 2

Cost = $C_{1,2}^*$ (x(1), u(1)) = $J_{1,2}$ (x(1), u(1)) + J_{22}^* = 2 |x(1) – 0.1 x 1^2| + 2 |x(2) – 0.1 x 2^2| + |u(1)|

x(k) = 0, 0.1, 0.2, 0.3, 0.4

u(k) = -0.2, -0.1, 0, 0.1, 0.2

Current State	Control	Next State	Cost	Minimum Cost	Optimal Control Applied
x(1)	u(1)	x(2) = x(1) + u(1)	$C_{1,2}^*$ [x(1), u(1)] = 2 |x(1) – 0.1 x 1^2| + 2 |x(2) – 0.1 x 2^2 | + |u(1)|	J*	u * x(1) at k = 1
0	0	0	1	0.8	0.2
	0.1	0.1	0.9		
	0.2	0.2	0.8		

0.1	-0.1	0	0.9	0.4	0.2
	0	0.1	0.6		
	0.1	0.2	0.5		
	0.2	0.3	0.4		
0.2	-0.2	0	1.2	0.4	0.2
	-0.1	0.1	0.9		
	0	0.2	0.6		
	0.1	0.3	0.5		
	0.2	0.4	0.4		
0.3	-0.2	0.1	1.2	0.5	0.1
	-0.1	0.2	0.9		
	0	0.3	0.6		
	0.1	0.4	0.5		
0.4	-0.2	0.2	1.2	0.6	0
	-0.1	0.3	0.9		
	0	0.4	0.6		

First Stage Second

$J^*_{12}(x(1))$ stored from previous stage

K = 1

Cost = $C_{0,2}^*(x(0), u(0)) = J_{0,1}(x(0), u(0)) + J_{12}^* = |u(0)| + J_{12}^*(x(1))$

Current State	Control	Next State	Cost	Minimum Cost	Optimal Control Applied
x(0)	u(0)	x(1) = x(0) + u(0)	$C_{0,2}^*[x(0), u(0)] = J_{12}^*(x(1)) + \|u(0)\|$	J*	u * x(1) at k = 0
0	0	0	0.8	0.5	0.1
	0.1	0.1	0.5		
	0.2	0.2	0.6		
0.1	-0.1	0	0.9	0.4	0
	0	0.1	0.4		
	0.1	0.2	0.5		
	0.2	0.3	0.7		
0.2	-0.2	0	1	0.4	0
	-0.1	0.1	0.5		
	0	0.2	0.4		
	0.1	0.3	0.6		
	0.2	0.4	0.8		
0.3	-0.2	0.1	0.6	{0.5, 0.5}	{-0.1, 0}
	-0.1	0.2	0.5		
	0	0.3	0.5		
	0.1	0.4	0.7		
0.4	-0.2	0.2	0.6	{0.6, 0.6, 0.6}	{-0.2, -0.1, 0}
	-0.1	0.3	0.6		
	0	0.4	0.6		

Question 1

Determine the optimal control law u*$(x(k), k)$

Ans.

Using the two tables above for the first and second stage, to find the optimal control law u* we find the action with the least cost for each state x(k)

Question 2

Determine the optimal control sequence {u*(0), u*(1)}, if the initial state value is x(0) = 0.2

Ans.

Initial State Value is x(0) is 0.2

We refer to first stage table (k=0) where optimal control action is 0 for x(0) = 0.2

Therefore u*(0) = 0

Next, referring to second stage table (k=1), optimal control action is 0.2 for x(1) = 0.2

Therefore u*(1) = 0.2

Hence, Optimal Control Policy is {u*(0), u*(1)} = {0, 0.2}