Transformation of Cognitive Maps

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***Abstract*—Cognitive maps (CMs), fuzzy cognitive maps (FCMs), and dynamical cognitive networks (DCNs) are related tools for modeling the cognition of human beings and facilitating machine inferences accordingly. FCMs extend CMs, and DCNs extend FCMs. Domain experts often face the challenge that CMs/FCMs are not sufficiently capable in many applications and that DCNs are too complex. This paper presents a simplified DCN (sDCN) that extends the modeling capability of FCM/CM, yet maintains simplicity. Additionally, this paper proves that there exists a theoretical equivalence among models in the cognitive map family of CMs, FCMs, and sDCNs. It shows that every sDCN can be represented by an FCM or a CM, and *vice versa*; similarly, every FCM can be represented by a CM, and *vice versa*. The result shows that CMs, FCMs, and sDCNs are a family of cognitive models that differs from many extended models. This paper also provides a constructive approach to transforming one cognitive map model into other cognitive map models in the family. Therefore, domain experts are able to model applications with more descriptive sDCNs and leave theoretical analysis to the simpler CM forms. The existence of theoretical transformation links among the models provides strong support for their theoretical analysis and flexibility in their applications.**

***Index Terms*—Dynamical cognitive network (DCN), equivalence, fuzzy cognitive map (FCM), transformation.**

I. Introduction

COGNITIVE maps (CMs) [1], fuzzy cognitive maps (FCMs) [6], and dynamic cognitive networks (DCNs) [11] are related cognitive models. CMs are the first model of the family, which have laid a foundation for other models. CMs have been applied in various application domains, for example, analysis of electrical circuits [15], analysis and extension of graph-theoretic behavior [19], and modeling of control systems [5]. CMs are easy to use and straightforward models; however, they do not differentiate the strength of relationships. Rather, each node simply makes its decision based on the number of positive impacts and the number of negative impacts; hence, a CM is an oversimplified model for many applications.

FCMs extended CMs by introducing fuzzified weights to describe the strength of the relationships. Fuzzified concepts and fuzzified causal relationship modeling—including FCMs—have gained comprehensive recognition and been widely applied in entertainment [14], computer gaming [2], multiagent systems [7], social systems [3], ecosystems [4], financial systems [27], and earthquake risk/vulnerability analysis [28].

The construction of cognitive maps was mainly undertaken by domain experts in the early years. Recently, several data-oriented approaches—including Hebbian rules, the Swan algorithm, and differential techniques (see [21]–[26])—have been proposed to train maps or networks for knowledge representation. Nevertheless, models from domain experts remain a key source for cognitive maps because they are especially suitable to the highly nonlinear and discrete causal linked maps, which are widespread in decision-intensive causal systems.

It has been pointed out that FCMs are still not capable of handling complex causal systems (in [11]) since several core modeling capabilities are missing in the FCM model. FCMs do not differentiate the strength of factors, for example, an FCM may tell if a terrorism threat exists or not but does not differentiate serious threats from minor threats. Additionally, FCMs do not describe the dynamics of the relationships. There is no differentiation between long-term threats and immediate threats.

To address the limitations of CMs and FCMs, several extensions have been proposed [9]–[12]. These proposals introduced more values to concepts, including real-valued concepts, nonlinear weight, and time delays. DCNs are the most powerful model in the family; they systematically address the modeling defects of other cognitive maps and are able to handle complex dynamic causal systems. On the other hand, DCNs are also very complex models. Their complexity has restricted their application, as domain-knowledge experts are especially unfamiliar with dynamic DCN models. Much research [15], [17], [18], [20] and applications in recent years are still based on FCMs, because they are more capable than CMs, but their analysis and applications are less complex than those of DCNs.

DCNs are more capable than either FCMs or CMs in modeling cognitive knowledge; nonetheless, CMs have a binary form, which associates better with digital systems and logic foundations. Domain experts and researchers in the discipline would benefit greatly if the advantages of the models could be combined. To achieve this objective, this paper presents a simplified DCN, which is named as sDCN that balances the capability of DCNs and the simplicity of the widely familiar CM and FCM models. For simplicity, sDCNs neither support infinite-state sets or real intervals nor model the continuous dynamics of how causal impact is built up. In contrast, sDCNs are able to model the strength of causes and impacts, which is one of the main modeling capabilities missing from FCMs. (Dynamic relationships in sDCNs will be explored in a separate article balancing simplicity and generic DCNs.)

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A significant contribution of the paper is that it reveals certain theoretical mutual modeling characteristics among the CM, FCM, and sDCN cognitive models. Additionally, the paper provides a constructive approach to transform one cognitive model into other cognitive models using this approach; domain experts could use one model to capture the causal knowledge and transform it into another model for theoretical analysis.

The rest of the paper is organized as follows. Section II compares three major different cognitive models, followed by the proposal of sDCNs in Section III. Section IV proves the theoretical equivalence between CMs, FCMs, and sDCNs. Section V illustrates (by example) the analysis of sDCNs by examining the CM that represents the sDCN. Section VI concludes the paper.

II. Cognitive Maps

A. CM—An Over Simplified Model

CMs were used by Axelrod [1] to visualize causal relationships among factors to facilitate human cognitive thinking. CMs use binary concepts to model important factors and binary links to model their causal relationships. A CM can be viewed as a tuple

$$\mathbf{M} = \langle \mathbf{V}, \mathbf{A} \rangle \tag{1}$$

where $\mathbf{V}$ is the set of vertices representing the concepts, and $\mathbf{A}$ is the set of arcs representing the causal relationships among concepts.

$$\mathbf{V} = \{\langle v_1, f_{v_1}, \mathrm{S}(v_1)\rangle, \langle v_2, f_{v_2}, \mathrm{S}(v_2)\rangle, \ldots, \langle v_n, f_{v_n}, \mathrm{S}(v_n)\rangle\} \tag{2}$$

$$\mathbf{A} = \{\langle a(v_i, v_j), w(a(v_i, v_j))\rangle | v_i, v_j \in \mathbf{V}\} \tag{3}$$

where v_i ($i = 1, 2, \ldots, n$) are the vertices (concepts), n is the number of concepts, f_{v_i} is the decision function of v_i, $\mathrm{S}(v_i)$ is the state set of v_i, $a(v_i, v_j)$ is the arc from v_i to v_j, and $w\,(a(v_i, v_j))$ is the weight of arc $a(v_i, v_j)$, which can also be written as $w\,(v_i, v_j)$ or w_{ji}. For simplicity, we do not differentiate $v_i \in \mathrm{V}$ from $\langle v_i, f_{v_i}, \mathrm{S}(v_i)\rangle \in \mathrm{V}$ when there is no ambiguity.

In CM, $w_{ji} \in \{-1, 0, +1\}$; -1 indicates a negative causal relationship, $+1$ represents a positive causal relationship, and 0 means no causal relationship exists. This paper does not differentiate $\mathrm{S}(v_i)$ and S_{v_i} when no ambiguities is involved. For simplicity, we also do not differentiate $v_i \in \mathbf{V}$ from $\langle v_i, f_{v_i}, \mathrm{S}(v_i)\rangle \in \mathbf{V}$ when there is no ambiguity.

For each concept v_i, $i = 1, 2, \ldots, n$, the decision-making function f_{v_i} is defined as follows:

$$f_{v_i}(u) = f_{v_i}\left(\sum_{j=1}^{n} w_{ij} \times x_j\right) \tag{4}$$

where x_j is the current state of vertex v_j, and u is the total impact v_i received. Based on the causal inputs, the decision function decides the following state of the concept. CM uses binary concepts

$$\mathrm{S}(v_i) = \{-1, 1\}.$$

In some literature, $\{0, 1\}$ is used instead.

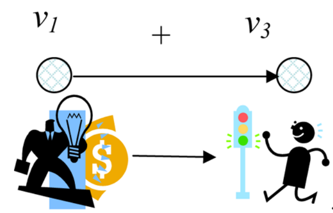

Fig. 1. CM of global/regional business demands (v_1) and organizing business trips (v_3).

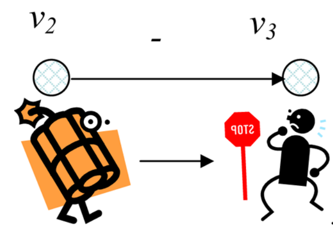

Fig. 2. CM of terrorism threats (v_2) and organizing business trips (v_3).

As CMs do not recognize the different strengths of the causal relationships, they are not suitable to model complex systems. For instance, consider a decision-making process related to *organizing business trips*. A CM models the causal relationship of *global/regional business demands* (v_1) and *organizing business trips* (v_3), as in Fig. 1.

That is to say, *global/regional business demands* have a positive impact on *organizing business trips*. When there are *global/regional business demands*, the decision to *organize business trips* would be made. The decision function of *organizing business trips* (v_3) is as follows:

$$f_{v_3}(u) = \begin{cases} 1, & u > 0 \\ 0/-1, & u <= 0 \end{cases} \tag{5}$$

where u is the impact v_3 receives. It tells that positive impact leads to the decision of organizing business trips, while negative impact leads to the opposite decision.

Similarly, CM can model the causal relationship between *terrorism threats* (v_2) and *organizing business trips* (v_3), as in Fig. 2.

CM is a visualized, easy to use model for capturing human causal knowledge. Unfortunately, it creates difficulties in real applications due to its oversimplicity. When *global/regional business demands* (v_1) and *terrorism threats* (v_2) coexist, the cognitive model fails to facilitate the decision making (see Fig. 3).

The concept *organizing business trips* receives a total impact of zero.

B. FCM—A Model for Simple Causal Links

FCMs [6], [13] are an extension of CMs created by introducing fuzzy weights to differentiate the strengths of causal relationships. An FCM can also be viewed as a tuple

$$\mathbf{M} = \langle \mathbf{V}, \mathbf{A} \rangle \tag{6}$$

where $\mathbf{V}$ is the set of vertices representing the concepts, and $\mathbf{A}$ is the set of arcs representing the causal relationships among

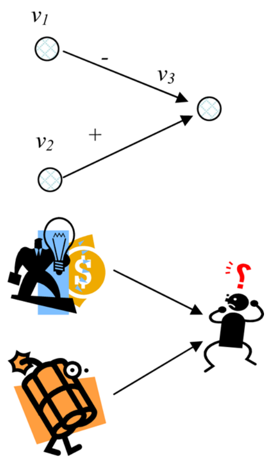

Fig. 3. CM of organizing business trips.

concepts.

$$\mathbf{V} = \{\langle v_1, f_{v_1}, \mathrm{S}(v_1)\rangle, \langle v_2, f_{v_2}, \mathrm{S}(v_2)\rangle, \ldots, \langle v_n, f_{v_n}, \mathrm{S}(v_n)\rangle\} \tag{7}$$

$$\mathbf{A} = \{\langle a(v_i, v_j), w(a(v_i, v_j))\rangle | v_i, v_j \in \mathbf{V}\} \tag{8}$$

where v_i $(i = 1, 2, \ldots, n)$ are the vertices (concepts), n is the number of concepts, f_{v_i} is the decision function of v_i, $\mathrm{S}(v_i)$ is the state set of v_i, $a(v_i, v_j)$ is the arc from v_i to v_j, and w $(a(v_i, v_j))$ is the weight of arc $a(v_i, v_j)$, which can also be written as w (v_i, v_j) or w_{ji}. In an FCM, w_{ji} becomes a fuzzy description of the causal relationship. $w_{ji} < 0$ indicates a negative causal relationship, $w_{ji} > 0$ represents a positive causal relationship, and $\|w_{ji}\|$ is the strength of the causal relationship.

For each concept v_i, $i = 1, 2, \ldots, n$, the decision-making function f_{v_i} is defined as follows:

$$f_{v_i}(u) = f_{v_i}\left(\sum_{j=1}^{n} w_{ij} \times x_j\right) \tag{9}$$

where x_j is the current state of vertex v_j. Based on the causal inputs, the decision function decides the next state of the concept. FCMs also use binary concepts[1]

$$\mathrm{S}(v_i) = \{-1, 1\}.$$

When FCM is applied in the decision-making process of organizing business trips, it uses weights to differentiate causal relationships. For example, the causal relationship between *global/regional business demands* (v_1) and *organizing business trips* (v_3) can be modeled as *global/regional business demands* have *high positive* impact on the decision of *organizing business trips* (see Fig. 4).

Here, *high* is modeled by a weight of 0.65 (depending on the definition of fuzzy descriptions).

Similarly, causal relationship between *terrorism threats* (v_2) and *organizing business trips* (v_3) can be modeled as *terrorism threats* (v_2) have *very high negative* impact on the decision of *organizing business trips* (v_3), as in Fig. 5.

[1] {0, 1}, or ternary value set {−1, 0,1} is adopted in some literatures.

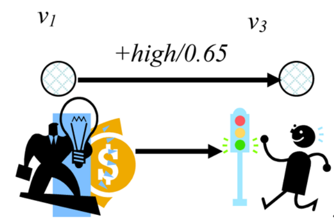

Fig. 4. FCM of global/regional business demands (v_1) and organizing business trips (v_3).

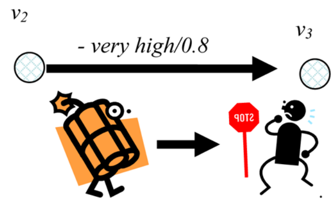

Fig. 5. FCM of terrorism threats (v_2) and organizing business trips (v_3).

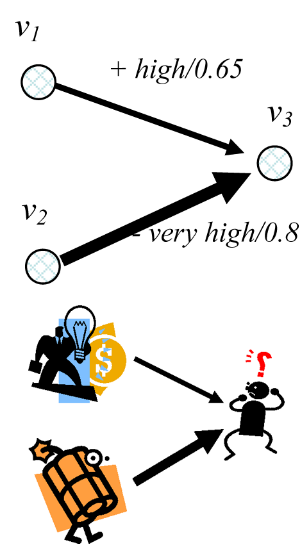

Fig. 6. FCM of organizing business trips.

When *global/regional business demands* and *terrorism threats* coexist (see Fig. 6), the decision maker receives a total impact (u) of

$$u = +0.65 \times 1 + (-0.8) \times 1 = -0.15.$$

Therefore, the decision toward *organizing business trips* is negative, because the decision function is as follows:

$$f_{v_3}(u) = \begin{cases} 1, & u > 0 \\ 0/-1, & u <= 0. \end{cases} \tag{10}$$

This example shows that FCMs are more capable in cognitive modeling than CMs. Nevertheless, FCM moves just one step forward, and if different factors connect to form a complex causal network, FCM may again fail to facilitate decision making. Fig. 6 shows that FCMs differentiate the strength of different links (different weights of the links) but fail to model the strength of the cause (no presentation of the different significances of the causes). Terrorist threats have been occurring frequently since 9/11, but people still travel; it does not mean that *terrorism threats* have a weaker causal link to the decision. Instead, people travel when *terrorism threats* are *minor*, therefore, the impact is not as significant as the impact of *global/regional business demands*. FCMs' inability to model the strength of the cause means they are unable to properly model the impact received by the decision function of *organizing business trips*. Additionally, FCM does not model how strong the decision of *organizing business trips* is, which affects the subsequent decisions. The inaccuracy could easily propagate through the causal network, which in the end leads to an unusable inference.

C. General DCN—A Comprehensive but Complex Model

DCNs [11] are general cognitive maps, which model the strength of the cause, the impact and the causal relationship, and the dynamics of how the causal impact is built up. A DCN can also be viewed as a tuple

$$\mathbf{M} = \langle \mathbf{V}, \mathbf{A} \rangle \tag{11}$$

where $\mathbf{V}$ is the set of vertices representing the concepts, and $\mathbf{A}$ is the set of arcs representing the causal relationships among concepts.

$$\mathbf{V} = \{\langle v_1, f_{v_1}, \mathrm{S}(v_1)\rangle, \langle v_2, f_{v_2}, \mathrm{S}(v_2)\rangle, \ldots, \langle v_n, f_{v_n}, \mathrm{S}(v_n)\rangle\} \tag{12}$$

$$\mathbf{A} = \{\langle a(v_i, v_j), w(a(v_i, v_j))\rangle | v_i, v_j \in \mathbf{V}\} \tag{13}$$

where v_i $(i = 1, 2, \ldots, n)$ are the vertices (concepts), n is the number of concepts, f_{v_i} is the decision function of v_i, $\mathrm{S}(v_i)$ is the state set of v_i, $\mathrm{S}(v_i)$ can be either a discrete set (with no restriction whether it has limited number of values), or a continuous interval, $a(v_i,\, v_j)$ is the arc from v_i to v_j, and w $(a(v_i,\, v_j))$ is the generic weight of arc $a(v_i,\, v_j)$, which can also be written as $w\ (v_i,\, v_j)$ or w_{ji}. A generic weight can be a scalar weight or a dynamic link. Several models can be applied in describing the dynamic link, one of which is the transfer function

$$y_{ij}(s) = w_{ij}(s) \times x_j(s) \tag{14}$$

$$x_i(t) = f_{v_i}(y_{i1}(t), y_{i2}(t), \ldots, y_{in}(t)) \tag{15}$$

where $y_{ij}(s)$ is the Laplace transform of $y_{ij}(t)$, $y_{ij}(t)$ is the impact from vertex j to vertex i, t is the time, $x_j(s)$ is the Laplace transform of $x_j(t)$, $w_{ij}(s)$ is the transfer function describing the dynamics of the impact, and f_{v_i} is the decision function of v_i. Decision functions in the DCN are no longer restricted as threshold functions. A detailed definition of the decision functions in DCNs can be found in [11].

DCNs avoid the structural nonrobustness of CMs and FCMs, thus are of modeling large complex causal systems [7]. Fig. 7 shows a DCN model of the decision of *organizing business trips*, where

$$y_{31}(s) = \frac{0.65}{s+1} \times x_1(s) \tag{16}$$

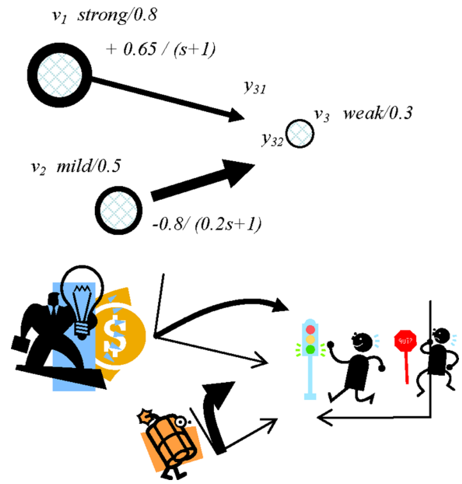

Fig. 7. DCN of organizing business trips.

$$y_{32}(s) = \frac{-0.8}{0.2s+1} \times x_2(s) \tag{17}$$

$$x_3(t) = f_{v_3}(y_{31}(t), y_{32}(t)) \tag{18}$$

$y_{31}(s)$, $y_{32}(s)$, $x_1(s)$, and $x_2(s)$ are Laplace transformations of $y_{31}(t)$, $y_{32}(t)$, $x_1(t)$, and $x_2(t)$, respectively; $y_{31}(t)$ is the impact from v_1 to v_3, and $y_{32}(t)$ is the impact from v_2 to v_3. As a general DCN allows continuous dynamics, the time is represented by t instead of step k. A transfer function based on a Laplace transform is one of the most widely used approaches for dynamics. How to use transfer functions in Laplace transformations to model dynamic relationships can be found in [11].

The DCN model indicates that when *global/regional business demands* are strong (0.8) (see the bigger icon in the Fig. 7), but the impact builds up gradually (smooth transition curve), the *terrorism threats* are *mild* (0.5) (smaller icon in the Fig. 7), but the impact builds up quickly (steeper transition curve); the link of the *terrorism threats* is also stronger, and the total final impact on the decision of *organizing business trips* is as follows:

$$+0.65 \times 0.8 + (-0.8) \times 0.5 = +0.24 \rightarrow 0.3. \tag{19}$$

For simplicity, (19) omitted the transition but just gives the total final impact. The decision function starts with a negative decision and later turns into a positive decision, and the strength of the final decision is *weak* (0.3). When the *terrorism threats* become more significant, the decision may turn against traveling until the *terrorism threats* level drops.

A general DCN describes not only the strength of causes, impacts, and effects, but the dynamics of how impacts are built up as well. For example, the impact of *global/regional business demands* may take time to build up gradually, while a *terrorism threat* could have much more immediate effect. Such dynamics may affect the transition characteristics of the causal system, or even the final hidden pattern of the system, due to high nonlinearity. In the *global/regional business demands* example, the dynamics are modeled with transfer functions.

A general DCN is more complex than a CM or FCM. Although simplification of complex CMs and DCNs has been reported [8], applying DCNs is still more difficult than applying CMs and FCMs. Additionally, experts and engineers who apply cognitive map are normally from disciplines in which state space equations, differential equations, or transfer functions are not common tools. Regardless of the fact that DCNs allow scalar weight and binary concepts, the theory appears to be complex. All these issues have restricted the application of general DCNs; thus, development of a series of *special* DCNs for different needs is highly desirable.

This paper will present one sDCN, which includes fuzzified concepts and fuzzified causal relationships. It is a model designed to be easily adopted by domain experts who are familiar with FCM and CM and is substantially extended to avoid the problems described in Sections II-A and B. More importantly, this sDCN holds the important property that it can be transformed among CM and FCM models; therefore, it provides a tool that is easy for domain experts to use and guarantees the essential characteristics of CM and FCM models. This property is not present in many other CM and FCM extensions, including the generic DCNs. (The simplification of dynamic impacts without adopting advanced system tools like Laplace transformations will be reported separately.)

III. Simplified Dynamic Cognitive Network

A significant contribution of FCMs/CMs is that they allow circles, which represent the *feedback* in a real system. *Feedback/closed loops* are widespread in real systems, including causal systems; however, FCMs use binary vertices to represent concepts. A binary concept has two states—either *is* or *is not*; in contrast, a *closed loop* represents the causal processes that need a few rounds to reach the final state. The mismatch between binary concepts and *closed loops* can lead to contradictory inference in FCMs. The following paragraph uses an example to illustrate binary concepts with iterated inference.

Suppose *being rich* is a binary concept; in other words, one is either rich or not rich. If a man has \$1, he is *not rich*; if he is *not rich*, giving him \$1 does not make him *rich*. By iteration, after he receives dollars one by one until he has millions of dollars, he is still *not rich*. The contradiction is caused by the inability to differentiate different levels of being rich. If a fuzzy set is adopted, the membership of *being rich* can include fuzzy descriptions of *not rich*, *a little rich*, *somewhat rich*, and *rich*; thus, owing to the corresponding fuzzy logic, the contradiction can be avoided. This argument leads to the adoption of a cognitive model, which can be viewed as an sDCN.

An sDCN is defined as a tuple

$$\mathbf{M} = \langle \mathbf{V}, \mathbf{A} \rangle \tag{20}$$

where $\mathbf{V}$ is the set of vertices representing the concepts, and $\mathbf{A}$ is the set of arcs representing the causal relationships among concepts.

$$\mathbf{V} = \{\langle v_1, f_{v_1}, \mathrm{S}(v_1)\rangle, \langle v_2, f_{v_2}, \mathrm{S}(v_2)\rangle, \ldots, \langle v_n, f_{v_n}, \mathrm{S}(v_n)\rangle\} \tag{21}$$

$$\mathbf{A} = \{\langle a(v_i, v_j), w(a(v_i, v_j))\rangle | v_i, v_j \in \mathbf{V}\}. \tag{22}$$

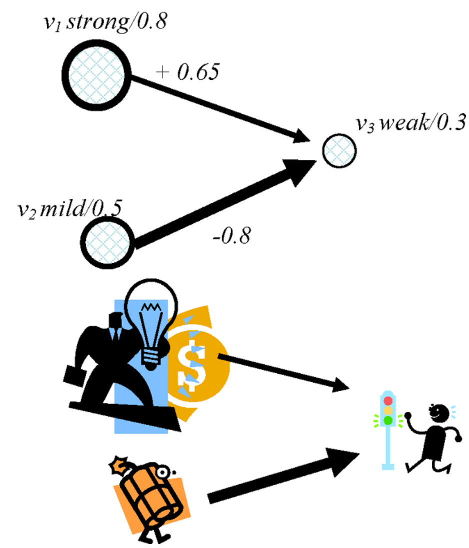

Fig. 8. sDCN of organizing business trips.

Here v_i ($i = 1, 2, \ldots, n$) are the vertices (concepts), n is the number of concepts, f_{v_i} is the decision function of v_i, $\mathrm{S}(v_i)$ is the finite-state set of v_i, $a(v_i, v_j)$ is the arc from v_i to v_j, and $w\,(a(v_i, v_j))$ is the weight of arc $a(v_i, v_j)$, which can also be written as $w\,(v_i, v_j)$ or w_{ji}. In sDCN, w_{ji} is the fuzzy description of the causal relationship. $w_{ji} < 0$ indicates a negative causal relationship, $w_{ji} > 0$ represents a positive causal relationship, and $\|w_{ji}\|$ is the strength of the causal relationship.

The decision-making function of sDCN is also defined as follows f_{v_i}:

$$f_{v_i}(u) = f_{v_i}\left(\sum_{j=1}^{n} w_{ij} \times x_j\right)$$
$$u = \sum_{j=1}^{n} w_{ij} \times x_j. \tag{23}$$

Based on the causal inputs, the decision function decides the following state of the concept. The state spaces of sDCN concepts are finite-value sets

$$\mathrm{S}(v_i) = \{x_i^1, x_i^2, \ldots, x_i^{R_i}\}$$
$$i = 1, 2, \ldots, n \tag{24}$$

where n is the number of the concepts, and R_i is the number of values the concept v_i has. In sDCN, each concept has its own value set with definitions depending on the needs of the system to be modeled. The state space of the sDCN is defined as the product of the vertices' state space

$$\mathrm{S(M)} = \prod_{i=1}^{n} \mathrm{S}(v_i). \tag{25}$$

Now, the sDCN is able to model cause and effect, and it is better in logical inference. For example, if the *terrorism threats* are minor and the *business need* is important and urgent, the travel should be arranged; otherwise, the trip should be deferred to a later time. The sDCN is shown in Fig. 8.

IV. Equivalence Between Cognitive Maps, Fuzzy Cognitive Maps, and Simplified Dynamical Cognitive Networks

A. Inclusive and Equivalence Relationships Between Models

In this paper, we examine a theoretical property of inference patterns of cognitive models, namely CMs, FCMs, and sDCNs. The inclusive relationship of a model in another model, e.g., model_A in model_B, refers to the fact that for each instance *M* of model_A, there always exists an instance *M′* of model_B, such that for any given initial state, the inference pattern of *M* is included in the inference pattern of *M′*. Definitions 1 and 2 are the formal definitions. Mutual inclusion of two models leads to another property of CMs, FCMs, and sDCNs, which is given in Definition 3.

Definition 1: Inclusive Relationship of cognitive model instances: Given two cognitive model instances *M* and *M′*, *M* is defined as being included in *M′* if there exist two constant α and β, for any value x^*, of any concept v of *M*, and all time step k, and there exists a value x^+ of a concept v' of *M′* such that for all $x(k) = x^*$, $k \in \{1, 2, \ldots,\} => x'(\alpha k + \beta) = x^+$.

The inclusive relationship of two cognitive model instances specifies that all the causal knowledge of the included instance exists in the inclusive instance. Any inference pattern of the included instance is actually a subsequence of the whole or part of the inference pattern of the inclusive instance.

If *M* is included in *M′*, it is denoted as $M \overset{e}{\subset} M'$.

Theorem 1: The inclusive relationship $\overset{e}{\subset}$ of cognitive model instances is a transitive relationship.

Proof: Given three cognitive model instances, M1 $\overset{e}{\subset}$ M2, M2 $\overset{e}{\subset}$ M3.

As M1 $\overset{e}{\subset}$ M2, there exist two constants α_1 and β_1, for any value x^*_{i1}, of any concept v_{i1} of M1, and there exists a value x^*_{i2}, of a concept v_{i2} of M2, that for all $x_{i1}(k) = x^*_{i1}$, $k \in \{1, 2, \ldots,\}$ $=> x_{i2}(\alpha_1 k + \beta_1) = x^*_{i2}$.

As M2 $\overset{e}{\subset}$ M3, there exist two constants α_2 and β_2, for value x^*_{i2}, of concept v_{i2} of M2, and there exists a value x^*_{i3}, of a concept v_{i3} of M3, that for all $x_{i2}\ (\alpha_1 k + \beta_1) = x^*_{i2}$, $k \in \{1, 2, \ldots,\}$ $=> x_{i3}(\alpha_2\ (\alpha_1 k + \beta_1) + \beta_2) = x^*_{i3}$.

That is, there exist two constants α_3 and β_3, for any value x^*_{i1}, of any concept v_{i1} of M1, and there exists a value x^*_{i3}, of a concept v_{i3} of M3, that for all $x_{i1}(k) = x^*_{i1}$, $k \in \{1, 2, \ldots,\}$ $=> x_{i3}(\alpha_3 k + \beta_3) = x^*_{i3}$, where $\alpha_3 = \alpha_1\alpha_2$ and $\beta_3 = \alpha_2\beta_1 + \beta_2$. By definition, this proves that M1 $\overset{e}{\subset}$ M3. ■

Definition 2: Given two cognitive models **M**[2] and **M′**, **M** is defined as being included in **M′** if for any instance of **M**, denoted as *M*, there exists an instance of **M′**, denoted as *M′*, such that $M \overset{e}{\subset} M'$.

The inclusive relationship of two cognitive models specifies that each instance of the included model can be included in an instance of the inclusive model. Obviously, **CM** is included in **FCM.**

Theorem 2: Inclusive relationship $\overset{e}{\subset}$ of cognitive models is a transitive relationship.

Proof: Given three cognitive models

M1 $\overset{e}{\subset}$ **M2**, **M2** $\overset{e}{\subset}$ **M3.**

As **M1** $\overset{e}{\subset}$ **M2**, for any M1 $\in$ **M1**, there exists

M2 $\in$ **M2**,

such that M1 $\overset{e}{\subset}$ M2.

As **M2** $\overset{e}{\subset}$ **M3**, there exists M3 $\in$ **M3**, such that

M2 $\overset{e}{\subset}$ M3.

By Theorem 1, it holds

M1 $\overset{e}{\subset}$ M3,

which proves that **M1** $\overset{e}{\subset}$ **M3.** ■

Definition 3: Two cognitive models, **M** and **M′**, are equivalent iff[3] **M** $\overset{e}{\subset}$ **M′** and **M′** $\overset{e}{\subset}$ **M**.

The equivalent relationship of two cognitive models specifies that each instance of one model can be included in an instance of the other model.

If **M** and **M′** are equivalent, it is denoted as **M** $\overset{e}{=}$ **M′**.

Theorem 3: An equivalent relationship $\overset{e}{=}$ of cognitive models is a symmetric relationship.

Proof: The proof can be derived directly from the Definition 3. ■

B. Equivalence Between CMs, FCMs, and sDCNs

Although CMs, FCMs. and sDCNs are different cognitive models, this section proves that equivalence exists among them. It is straightforward, by the Definition 2, that CM $\overset{e}{\subset}$ FCM and FCM $\overset{e}{\subset}$ sDCN. The interesting part is that if sDCN is also included in CM, the equivalence among all the three models holds, by the transitive characteristics of the $\overset{e}{\subset}$ relationship.

Theorem 4: For any CM, M, there exists an FCM M', $M \overset{e}{\subset} M'$ (CM $\overset{e}{\subset}$ FCM).

Proof: This proof will construct the M'according to the given M.

Let

$M' = \phi$, //[4] start with an empty FCM model.

For each $v_i \in M$, $i = 1, 2, \ldots, n$,

// M has n concepts.

add $v_i{}'$ to *M′*

$$M' = M' \cup \{\langle v'_i, f_{v'_i}, S(v'_i)\rangle\} \tag{26}$$

$$f_{v'_i}|_{v'_i \in M'} = f_{v_i}|_{v_i \in M} \tag{27}$$

$$S(v'_i)|_{v_i \in M'} = S(v_i)|_{v_i \in M} = \{0, 1\}.$$

Here, for the simplicity of presentation, we do not differentiate $v_i \in M$ and $\langle v_i, f_{v_i}, S(v_i)\rangle \in M$ if no ambiguity is caused. Similarly, for simplicity of presentation, $M' = M' \cup\{\langle v'_i, f_{v'_i}, S(v'_i)\rangle\}$ is used here to represent $V_{M'} = V_{M'} \cup\{\langle v'_i, f_{v'_i}, S(v'_i)\rangle\}$.

For each $\alpha(v_i, v_j) \in M$, *i*, *j* $= 1, 2, \ldots, n$,

add $\alpha(v'_i, v'_j)$ to *M′*

$$M' = M' \cup \{\langle \alpha(v'_i, v'_j), w(v'_i, v'_j) = w_{ji}|_{a(v'_i, v'_j) \in M}\rangle\}. \tag{28}$$

[2]Bold **M** is used to represent a cognitive model, and normal *M* is used to denote an instance of the model. For example, **CM** is cognitive map model, while CM is a cognitive map.

[3]*iff* is an abbreviation meaning *if and only if*.

[4]The symbol // leads a comment to the content.

Set

$$x_{v_i'}(0)|_{v_i' \in M'} = x_{v_i}(0)|_{v_i \in M}. \tag{29}$$

It can be verified that

$$x_{v_i'}(k)|_{v_i' \in M'} = x_{v_i}(k)|_{v_i \in M}. \tag{30}$$

By definition, it is proven that $M \overset{e}{\subset} M'$ and that CM $\overset{e}{\subset}$ FCM. ■

Similarly, it holds that FCM $\overset{e}{\subset}$sDCN.

Theorem 5: For any FCM *M*, there exists an sDCN M', $M \overset{e}{\subset} M'$ (FCM $\overset{e}{\subset}$ sDCN).

Proof: This proof will construct the M' according to the given M.

Let

$M' = \phi$, // start with an empty sDCN model.

For each $v_i \in M$, $i = 1, 2, \ldots, n$,

// M has n concepts.

$$M' = M' \cup \{\langle v_i', f_{v_i'}, S(v_i') \rangle\} \tag{31}$$

$$f_{v_i'}|_{v_i' \in M'} = f_{v_i}|_{v_i \in M} \tag{32}$$

$$S(v_i')|_{v_i \in M'} = S(v_i)|_{v_i \in M} = \{0, 1\}. \tag{33}$$

For each $\alpha(v_i, v_j) \in M$, $i, j = 1, 2, \ldots, n$

$$M' = M' \cup \{\langle \alpha(v_i', v_j'), w(v_i', v_j') = w_{ji}|_{a(v_i', v_j') \in M} \rangle\}. \tag{34}$$

Set

$$x_{v_i'}(0)|_{v_i' \in M'} = x_{v_i}(0)|_{v_i \in M}. \tag{35}$$

It can be verified that

$$x_{v_i'}(k)|_{v_i' \in M'} = x_{v_i}(k)|_{v_i \in M}. \tag{36}$$

By definition, it is proven that $M \overset{e}{\subset} M'$ and that FCM $\overset{e}{\subset}$ sDCN. ■

Note that, although the sDCN has been extended from a CM and achieved the same goals as extended models of FCMs (like DCNs), the equivalence property still holds. Theorem 6 will prove that sDCN $\overset{e}{\subset}$ CM. The loop is then closed for the models to be mutually transformed because an inclusive relationship is transitive.

Theorem 6: For any sDCN *M*, there exists a CM M', $M \overset{e}{\subset} M'$.

Proof: Let

$M' = \phi$, // start with an empty CM model.

First, $\forall\, x_i^{r_i} \in S(v_i) = \{x_i^1, x_i^2, \ldots, x_i^{R_i}\}$, $v_i \in M$, $i = 1, 2, \ldots, n$

// for each state value of a concept in M, $\forall$ notes "*for any*"

// M has n concepts. $S(v_i)$ has R_i values, and $x_i^{r_i}$ is a state value from the set.

$$M' = M' \cup \{\langle \overline{v}_{1i}^{r_i}, f_{\overline{v}_{1i}^{r_i}}, S(\overline{v}_{1i}^{r_i}) \rangle\}. \tag{37}$$

// add concept $\overline{v}_{1i}^{r_i}$ to M'.

Define

$$f_{\overline{v}_{1i}^{r_i}}(u) = \begin{cases} 1, & u \geq 1 \\ 0, & u < 1 \end{cases} \tag{38}$$

where u is the sum of inputs of concept $\overline{v}_{1i}^{r_i}$

$$S(\overline{v}_{1i}^{r_i}) = \{0, 1\}. \tag{39}$$

Second, $\forall\, X_p \in \boldsymbol{S}(M)$

$$X_p = \begin{bmatrix} x_1^{r_1} \\ x_2^{r_2} \\ \cdots \\ x_n^{r_n} \end{bmatrix} \tag{40}$$

// for each state of M, X_p

$$\begin{aligned} M' = M' &\cup \{\langle \overline{v}_{2p}, f_{\overline{v}_{2p}}, S(\overline{v}_{2p}) \rangle\} \\ &\cup \{\langle a(v_{11}^{r1}, \overline{v}_{2p}), w(v_{11}^{r1}, \overline{v}_{2p}) \rangle \\ &\langle a(v_{12}^{r2}, \overline{v}_{2p}), w(v_{12}^{r2}, \overline{v}_{2p}) \rangle \\ &\ldots, \langle a(v_{1n}^{r_n}, \overline{v}_{2p}), w(v_{1n}^{r_n}, \overline{v}_{2p}) \rangle\} \end{aligned} \tag{41}$$

// add concept $\overline{v}_{2p}$ to M',

// add arcs from $v_{11}^{r1}, v_{12}^{r2}, \ldots, v_{1n}^{r_n}$ to $\overline{v}_{2p}$.

Set

$$w(a(v_{1i}^{r_i}, \overline{v}_{2p})) = +1 \tag{42}$$

(Set the arc as a positive impact link.)

where $i = 1, 2, \ldots, n$.

$$f_{\overline{v}_{2p}}(u) = \begin{cases} 1, & u \geq n \\ 0, & u < n \end{cases} \tag{43}$$

where u is the sum of inputs of concept $\overline{v}_{2p}$.

$$S(\overline{v}_{2p}) = \{0, 1\} \tag{44}$$

Third, $\forall\, X_q \in \boldsymbol{S}(\mathrm{D}(M))$, $X_q = \begin{bmatrix} x_1^{r_1} \\ x_2^{r_2} \\ \cdots \\ x_n^{r_n} \end{bmatrix}$.

// for each derived state of M and X_q (a derived state is a state that can be the resultant/derived state inferred from an initial state)

$$\begin{aligned} M' = M' &\cup \{\langle \overline{v}_{3q}, f_{\overline{v}_{3q}}, S(\overline{v}_{3q}) \rangle\} \\ &\cup \{\langle a(\overline{v}_{3q}, \overline{v}_{11}^{r1}), w(\overline{v}_{3q}, \overline{v}_{11}^{r1}) \rangle \\ &\langle a(\overline{v}_{3q}, \overline{v}_{12}^{r2}), w(\overline{v}_{3q}, \overline{v}_{12}^{r2}) \rangle \\ &\langle a(\overline{v}_{3q}, \overline{v}_{1n}^{rn}), w(\overline{v}_{3q}, \overline{v}_{1n}^{rn}) \rangle\}. \end{aligned} \tag{45}$$

// add concept $\overline{v}_{3q}$ to M',

// add arcs from $\overline{v}_{3q}$ to $\overline{v}_{11}^{r1}, \overline{v}_{12}^{r2}, \ldots, \overline{v}_{1n}^{rn}$.

Set

$$w(a(\overline{v}_{3q}, \overline{v}_{1i}^{r_i})) = +1. \tag{46}$$

// set the arc as a positive impact link.

where $i = 1, 2, \ldots, n$.

Define

$$f_{\overline{v}_{3q}}(u) = \begin{cases} 1, & u \geq 1 \\ 0, & u < 1 \end{cases} \tag{47}$$

where u is the sum of inputs of concept $\overline{v}_{3q}$

$$\mathrm{S}(\overline{v}_{3q}) = \{0, 1\}. \tag{48}$$

Finally

$$M' = M' \cup \{\langle a(\overline{v}_{2p}, \overline{v}_{3q}), w(\overline{v}_{2p}, \overline{v}_{3q})\rangle \,|F_M(X_p) = X_q, \quad X_p \in S(M), X_q \in S(M)\} \tag{49}$$

where

$$F_M = \begin{bmatrix} f_1 \\ f_2 \\ \dots \\ f_n \end{bmatrix}$$

is the decision function vector of model M.

// for all x_q, x_p, such that $F_M(X_p) = X_q$,

// add arc $a(\overline{v}_{2p}, \overline{v}_{3q})$ to M'.

Set

$$w(a(\overline{v}_{2p}, \overline{v}_{3q}))_{|F_M(X_p) = X_q} = +1. \tag{50}$$

// set the arc as a positive impact link.

Suppose

$$X_M(0) = \begin{bmatrix} x_1^{\gamma_1} \\ x_2^{\gamma_2} \\ \dots \\ x_n^{\gamma_n} \end{bmatrix}$$

define $\mathrm{V}_I = \{\overline{v}_{11}^{\gamma_1}, \overline{v}_{12}^{\gamma_2}, \dots, \overline{v}_{1n}^{\gamma_n}\} \subset M'$.

For simplicity of presentation, if no ambiguity is caused, we use $\{\overline{v}_{11}^{\gamma_1}, \overline{v}_{12}^{\gamma_2}, \dots, \overline{v}_{1n}^{\gamma_n}\} \subset M'$ to represent

$$\{\langle \overline{v}_{11}^{\gamma_1}, f_{\overline{v}_{11}^{\gamma_1}}, S(\overline{v}_{11}^{\gamma_1})\rangle, \langle \overline{v}_{12}^{\gamma_2}, f_{\overline{v}_{12}^{\gamma_2}}, S(\overline{v}_{12}^{\gamma_2})\rangle, \dots \langle \overline{v}_{1n}^{\gamma_n}, f_{\overline{v}_{1n}^{\gamma_n}}, S(\overline{v}_{1n}^{\gamma_n})\rangle\} \subset M'.$$

For all $v \in V_I$, $x_v(0) = 1$.
For all $v \notin V_I$, $x_v(0) = 0$.

It can be verified that for any value $x_i^{\gamma_i}$ of any concept v_i, if

$$x_i|_{v_i \in M}(k) = x_i^{\gamma_i} \tag{51}$$

it holds that

$$x_{\overline{v}_{1i}^{\gamma_i}}|_{\overline{v}_{1i}^{\gamma_i} \in M'}(2k) = 1. \tag{52}$$

By Definition 1, $M \overset{e}{\subset} M'$. ■

Theorem 7: CM $\overset{e}{=}$ FCM $\overset{e}{=}$ sDCN.

Proof: The proof can be derived from the definitions; therefore, it has been omitted.

V. Applications of Simplified Dynamical Cognitive Network and the Transformation

A. Transforming FCM to a Representative CM

To prove the existence of an inclusive model, there are fundamentally two types of proofs: One type of proof is a proof, which shows that such a model exists. By analogy, given $f(x)$ continuous, if $f(x1) > 0$ and $f(x2) < 0$, then there exists a root between *x1* and *x2* because the continuous $f(x)$ has to pass zero. However, this proof does not tell what the root is, and there can be more than one root between *x1* and *x2*. The second type of proof is a constructive approach, which is an approach to construct an inclusive model. Therefore, the existence of an inclusive model is proven. The proof in the paper is constructive. It provides an approach from which, given an instance map of one model, the corresponding inclusive model can be constructed.

For the clarity of presentation, this section provides a case study of constructing the CM that represents the FCM of *global/regional business demands* in Fig. 6. The transformation among sDCN and FCM or CM can be performed similarly according to the result in Section IV.

Let the FCM M denote as follows:

$$\mathrm{V}(M) = \{v_1, v_2, v_3\}, \text{// simplified presentation} \tag{53}$$

$$\mathrm{A}(M) = \{\langle a(v_1, v_3), w(v_1, v_3) = w_{31} = 0.65\rangle \quad \langle a(v_2, v_3), w(v_2, v_3) = w_{32} = -0.8\rangle\} \tag{54}$$

$$f_{v_3}(u) = \begin{cases} 1, & u = w_{31} \times x_1 + w_{32} \times x_2 > 0 \\ 0/-1, & u = w_{31} \times x_1 + w_{32} \times x_2 <= 0. \end{cases} \tag{55}$$

Note that f_{v_1}, f_{v_2} are not used in this case study and have been omitted. The corresponding CM can be constructed as follows:

$$\mathrm{V}(M') = \{v_{11}^1, v_{11}^2, v_{12}^1, v_{12}^2, v_{21}, v_{22}, v_{23}, v_{24} \quad v_{13}^1 \text{ or } v_{31}, v_{13}^2 \text{ or } v_{32}\}.$$

A technical simplicity has been performed to simplify the presentation, where one node can represent both v_{13}^1 and v_{31} (as used in the Theorem 6). Similarly, one node can represent both v_{13}^2 and v_{32}

$$x_{11}^1|_{M'} = 1 \text{ if } x_1|_M = 0, x_{11}^1|_{M'} = 0 \text{ if } x_1|_M = 1 \tag{56}$$

$$x_{11}^2|_{M'} = 1 \text{ if } x_1|_M = 1, x_{11}^2|_{M'} = 0 \text{ if } x_1|_M = 0 \tag{57}$$

$$x_{12}^1|_{M'} = 1 \text{ if } x_2|_M = 0, x_{12}^1|_{M'} = 0 \text{ if } x_2|_M = 1 \tag{58}$$

$$x_{12}^2|_{M'} = 1 \text{ if } x_2|_M = 1, x_{12}^2|_{M'} = 0 \text{ if } x_2|_M = 0 \tag{59}$$

$$\begin{aligned} \mathrm{A}(M') = \{&\langle a(v_{11}^1, v_{21}), w(v_{11}^1, v_{21}) = 1\rangle \\ &\langle a(v_{11}^1, v_{22}), w(v_{11}^1, v_{22}) = 1\rangle \\ &\langle a(v_{11}^2, v_{23}), w(v_{11}^2, v_{23}) = 1\rangle \\ &\langle a(v_{11}^2, v_{24}), w(v_{11}^2, v_{24}) = 1\rangle \\ &\langle a(v_{12}^1, v_{21}), w(v_{12}^1, v_{21}) = 1\rangle \\ &\langle a(v_{12}^1, v_{23}), w(v_{12}^1, v_{23}) = 1\rangle \\ &\langle a(v_{12}^2, v_{22}), w(v_{12}^2, v_{22}) = 1\rangle \\ &\langle a(v_{12}^2, v_{24}), w(v_{12}^2, v_{24}) = 1\rangle \\ &\langle a(v_{21}, v_{13}^1), w(v_{21}, v_{13}^1) = 1\rangle \\ &\langle a(v_{22}, v_{13}^1), w(v_{22}, v_{13}^1) = 1\rangle \\ &\langle a(v_{23}, v_{13}^2), w(v_{23}, v_{13}^2) = 1\rangle \\ &\langle a(v_{24}, v_{13}^2), w(v_{24}, v_{13}^2) = 1\rangle\} \end{aligned} \tag{60}$$

$$f_{v_{21}}(u) = \begin{cases} 1, & u \ge 2 \\ 0, & u < 2 \end{cases} \tag{61}$$

$$f_{v_{22}}(u) = \begin{cases} 1, & u \ge 2 \\ 0, & u < 2 \end{cases} \tag{62}$$

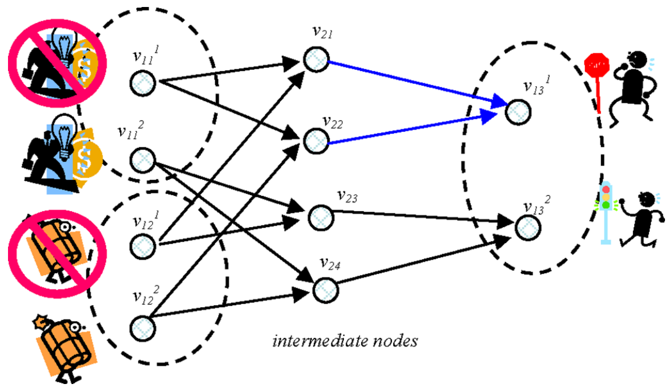

Fig. 9. Constructed CM representing the FCM in Fig. 6.

$$f_{v_{23}}(u) = \begin{cases} 1, & u \geq 2 \\ 0, & u < 2 \end{cases} \tag{63}$$

$$f_{v_{24}}(u) = \begin{cases} 1, & u \geq 2 \\ 0, & u < 2 \end{cases} \tag{64}$$

$$f_{v_{13^1}}(u) = \begin{cases} 1, & u \geq 1 \\ 0, & u < 1 \end{cases} \tag{65}$$

$$f_{v_{13^2}}(u) = \begin{cases} 1, & u \geq 1 \\ 0, & u < 1. \end{cases} \tag{66}$$

Note that CM is binary; therefore, the "$f = 1$ if $u \geq 2$" means that both of the causes should be present; "$f = 1$ if $u \geq 1$" means that either of the causes should be present. It can be verified that for any $k = 1, 2, 3, \ldots$

if $x_i\ (k) = 0$, then $x_{1i}^1(2k) = 1$, else $x_{1i}^1(2k) = 0$; $i = 1, 2, 3$

if $x_i\ (k) = 1$, then $x_{1i}^2(2k) = 1$, else $x_{1i}^2(2k) = 0$; $i = 1, 2, 3$.

Therefore, the constructed CM can represent the FCM. Fig. 9 illustrates the constructed CM.

B. Analyzing the sDCN via the Constructed CM

Different models have different merits. sDCNs are more descriptive and allow domain experts to model complex systems easily; CMs have simpler structure and are more appropriate for theoretical analysis. The equivalence among the models provides the freedom to select the most suitable form (model) for a given task.

The rest of the section illustrates the application of the transformation by proving that the final hidden patterns of sDCNs are either a static state or a limit cycle.

Theorem 8: For any sDCN M, with any initial state x_0, the hidden pattern (final inference pattern) is either a static state or a limit cycle.

Proof: As sDCN $\stackrel{e}{=}$ CM, there exists a CM, M', such that

$$M \stackrel{e}{\subset} M'.$$

As M' is a CM, the final hidden pattern of M' is either a static state or a limit cycle. Therefore, the final hidden pattern of M is either a static state or a limit cycle. ■

Similarly, further research into sDCNs or FCMs can be carried out by studying the inclusive models of other types. For example, [29] has several theoretical analysis results on CM, including the length of the result limit cycle, where the corresponding analysis on sDCN would be rather complicated. Now, the analysis can be performed through the inclusive CM.

C. Possible Hardware Implementations of Cognitive Maps

To date, there has been no significant implementation of cognitive map tools or platforms. This paper shows that the construction of a CM corresponding to an FCM and sDCN is possible. The result proves that the implementation of CM platforms can be based solely on CMs. CMs have binary concept values and binary relationship values; such a simple structure can significantly reduce the implementation cost and make hardware implementations possible. This is similar to computers, which have relatively simple instruction sets at the assembly language level or the hardware level but are able to support complex higher level languages like Java, C#, or prolog.

VI. Conclusion and Discussions

It is common for a real causal system to have loops. Feedback needs a mechanism to indicate the strength of impacts. The binary concepts of CMs and FCMs fail to meet this requirement—a CM or FCM with circles can produce contradictory inference. To address the problem, several extensions of CMs and FCMs have been made, including DCNs. However, DCNs are too complex for many domain experts.

This paper proposes a simplified DCN model named sDCN. sDCNs achieve several goals that motivated the earlier extensions to CMs and FCMs. sDCNs are able to model the strength of causes, the strength of causal relationships, and the degree of impacts. Node state sets of sDCNs are limited as finite-state value sets. Real intervals used in general DCNs are no longer used in sDCN to achieve the simplicity. sDCNs are normally capable enough for domain experts to model their cognitive knowledge and, at the same time, avoid unnecessary complexities. This paper proves that sDCNs, CMs, and FCMs can be grouped as a family of cognitive models by the inclusive relationship discovered in this paper, while other cognitive map extensions do not necessarily have this property.

This paper defines an inclusive relationship. Model instance M1 is said to be included in model instance M2 if all inference patterns of M1 are represented in the inference patterns of M2. If every model instance of a cognitive model (e.g., **Ma)** can find an inclusive model instance of another cognitive model (e.g., **Mb)**, then **Ma** is said to be included in **Mb**. The inclusive model instance is called a transformation of the original model instance in **Mb**. This paper proves that the CM, the FCM, and the sDCN are mutually inclusive. The proof of the corresponding results is constructive for any given model instance in the family, because it provides an approach to construct the corresponding inclusive model.

This result enables domain experts to model applications with more descriptive sDCNs and leave theoretical analysis to the simpler CM forms. In Section V, an example is given to prove the possible final patterns of sDCNs via the transformation. The length of the cycles in the final inference pattern can also be analyzed through an inclusive CM.

Given a model instance, its inclusive model instances are not unique.

The transformation results enable possible hardware implementation of cognitive maps. The fundamental structure of the hardware can be the simplest CM, with binary nodes and binary links. It can thus be manufactured with standard simple units. Various FCM and sDCN model instances can be constructed through their inclusive CM model.

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