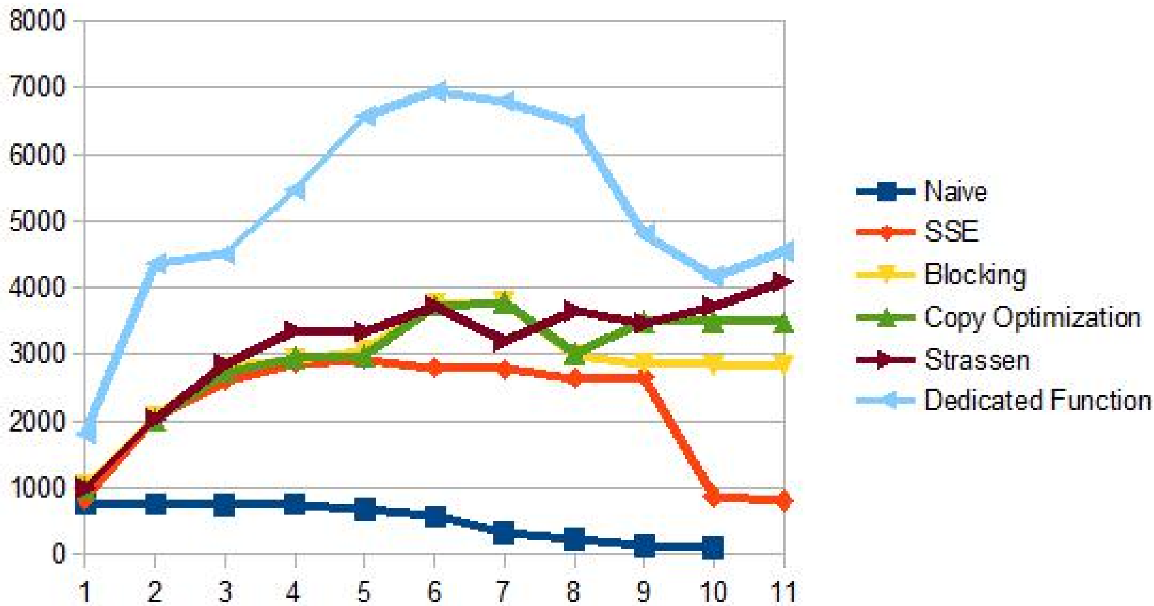
1 Summary

Below is the performance summary of our current implementation obtained with GCC compiler:

Table 1: Summary

Dim.	MFLOPS	Runtime	Instr completed	Total cycles	L1D misses	L2 misses	L1D accesses	L2 accesses	L1D miss rate	L2 miss rate	CPI
2	1805.83	0	44	21	0	0	22	0	0	0	0.48
4	4366.92	0	174	68	0	0	99	0	0	0	0.39
8	4503.28	0	1,150	528	0	0	623	0	0	0	0.46
16	5459.13	0	8,999	3,499	0	0	4,507	0	0	0	0.39
32	6572.47	0.01	55,409	23,259	12	0	26,694	18	0.04	0	0.42
64	6940.07	0.08	422,298	176,214	3,249	0	172,443	4,267	1.88	0	0.42
128	6794.34	0.62	3,377,742	1,434,701	43,028	0	1,376,330	70,856	3.13	0	0.42
256	6449.8	5.2	27,020,872	12,040,422	515,048	225	11,011,004	1,206,328	4.68	0.02	0.45
512	4812.29	55.78	247,304,246	129,315,203	11,542,820	115,943	88,459,296	20,492,626	13.05	0.57	0.52
1024	4161.53	516.03	1,670,373,328	1,192,039,842	44,462,792	5,039,958	833,376,675	102,891,767	5.34	4.9	0.71
2048	4554.29	3772.24	11,772,294,887	8,773,104,515	319,598,020	46,714,355	5,884,662,574	731,447,323	5.43	6.39	0.75

For 16×16 we get 5459.13 MFLOPS, 0.39 CPI, 0 cache miss rate. For 256×256 we get 6449.8 MFLOPS, 0.45 CPI, 4.68% L1D cache miss rate and 0.02% L2 cache miss rate. For 1024×1024 we get 4161.53 MFLOPS, 0.71 CPI, 5.34% L1D cache miss rate and 4.9% L2 cache miss rate. Below is a plot of improvements achieved via various optimizations:



2 Optimizations Attempted

2.1 Naive Implementation

Table 2: Naive Implementation

Dim.	MFLOPS	Runtime	Instr completed	Total cycles	L1D misses	L2 misses	L1D accesses	L2 accesses	L1D miss rate	L2 miss rate	CPI
2	765.21	0	133	49	0	0	48	0	0	0	0.37
4	754.63	0	681	394	0	0	349	0	0	0	0.58
8	747.73	0	4,633	3,181	0	0	2,710	0	0	0	0.69
16	755.32	0.01	34,713	25,189	0	0	20,948	0	0	0	0.73
32	681.6	0.1	269,593	223,389	27	0	156,639	377	0.02	0	0.83
64	574.58	0.91	2,126,362	2,119,160	24,526	0	1,297,288	33,260	1.89	0	1
128	331.33	12.66	16,892,960	29,443,156	2,427,569	8	10,991,681	2,434,546	22.09	0	1.74
256	227.58	147.44	134,678,638	343,480,488	24,922,029	2,694	92,193,892	24,950,234	27.03	0.01	2.55
512	140.98	1904.12	1,075,581,786	4,430,396,405	266,979,850	116,361	932,074,078	255,066,744	28.64	0.05	4.12
1024	100.33	21405.34	8,597,289,381	49,697,805,160	1,848,707,670	483,279,194	7,499,536,772	2,140,510,921	24.65	22.58	5.78
2048	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA	NA

2.2 SSE2

SSE are 128-bit vector instructions which take advantage of the data level parallelism and can do two double precision math operations instead of one. The operation performed on two pairs has to be the same (two multiplications or two additions etc). We expect to get about $2\times$ performance increase with this. And after implementation a jump for all matrices except

2×2 which goes from 770 MOPS to 830 MOPS. The desrepancy for the 2×2 matrix is because of the large % of overhead instructions compared to those for the bigger matrixes. `http://en.wikipedia.org/wiki/SSE2`

In addition, we expect to see fewer L1 and L2 cache misses due to the fact that in the SSE2 implementation we access both A and B matrix in row major order which aligns with the way they are stored, we expect cache misses to reduce to about 1/8 because each cache line (64kB) holds 8 doubles. We can verify in the table that there is a reduction in cache misses and performance improves more than $2\times$ for all matrices except 2×2.

Table 3: SSE2

Dim.	MFLOPS	Runtime	Instr completed	Total cycles	L1D misses	L2 misses	L1D accesses	L2 accesses	L1D miss rate	L2 miss rate	CPI
2	828.69	0	100	45	0	0	27	0	0	0	0.45
4	2054.69	0	408	145	0	0	148	0	0	0	0.36
8	2605.96	0	2,392	913	0	0	878	0	0	0	0.38
16	2860.74	0	16,536	6,681	0	0	6,409	0	0	0	0.4
32	2931.63	0.02	123,160	51,952	49	0	51,208	77	0.1	0	0.42
64	2796.38	0.19	950,808	435,595	8,252	0	408,278	20,820	2.02	0	0.46
128	2782.1	1.51	7,472,153	3,492,812	260,189	1	3,165,435	671,433	8.22	0	0.47
256	2640.07	12.71	59,246,623	29,710,225	2,094,890	1,394	25,635,822	5,264,400	8.17	0.03	0.5
512	2673.49	100.41	471,863,375	233,233,129	16,851,905	64,861	203,380,807	42,772,351	8.29	0.15	0.49
1024	867.27	2476.15	3,766,494,129	5,752,949,389	134,885,123	115,465,120	1,617,562,377	291,538,588	8.34	39.61	1.53
2048	804.85	21345.33	30,098,347,627	49,512,402,723	1,102,078,829	1,078,419,568	12,921,475,490	2,209,665,130	8.53	48.8	1.65

2.3 Blocking

We observed in the table above that there is a large increase in L1 miss rate at 64×64 and a large increase in L2 miss rate at 1024×1024. This agrees with our intuition as these are the levels where matrices no longer fit in L1/L2 cache. Hence we implemented two-level blocking and found that having the outer block fit in L2 and inner block fit in L1 reduces cache misses effectively without introducing too much overhead (which happens when blocks are too small), we chose inner block size to be 32×32 and outer block size to be 256×256.

After implementing blocking we found that we addressed the L2 misses for 1024×1024 and above (reduced misses by about 99%), however the number of L1 misses are roughly unchanged.

Table 4: Blocking

Dim.	MFLOPS	Runtime	Instr completed	Total cycles	L1D misses	L2 misses	L1D accesses	L2 accesses	L1D miss rate	L2 miss rate	CPI
2	1034.24	0	108	36	0	0	37	0	0	0	0.33
4	2051.58	0	402	145	0	0	159	0	0	0	0.36
8	2810.96	0	2,334	846	0	0	902	0	0	0	0.36
16	2924.69	0	16,278	6,503	0	0	6,525	0	0	0	0.4
32	3046.51	0.02	122,118	49,961	50	0	51,224	62	0.1	0	0.41
64	3757.69	0.14	588,455	325,201	3,170	0	321,587	6,910	0.99	0	0.55
128	3778.83	1.11	4,707,106	2,579,245	158,412	0	2,474,710	308,873	6.4	0	0.55
256	2995.74	11.2	37,655,771	25,994,903	2,541,172	1,484	19,767,381	6,088,674	12.86	0.02	0.69
512	2861.05	93.82	301,255,273	217,682,616	20,510,984	138,611	158,680,725	47,428,252	12.93	0.29	0.72
1024	2840.41	756.05	2,410,041,375	1,758,861,009	163,717,105	1,570,361	1,270,109,728	382,796,568	12.89	0.41	0.73
2048	2832.92	6064.38	19,280,328,546	14,091,166,204	1,310,251,939	12,907,808	10,162,617,221	3,068,467,646	12.89	0.42	0.73

2.4 Copy Optimization

We realized that there are both capacity misses and conflict misses in L1 cache, and blocking only addresses capacity misses. Even though we make sure that there is enough space in L1 for each inner block, block entries in the same column evict each other when the matrix is large. The solution is to perform copy optimization for matrix B (when matrix size is large). In particular, we store matrix B in row major order by inner block to make sure that each inner block has consecutive addresses in the memory.

After implementing copy optimization for 512×512 and above, we reduced L1 misses by about 90%, we see in the table that 256×256 can potentially benefit from copy optimization as well.

Table 5: Copy Optimization

Dim.	MFLOPS	Runtime	Instr completed	Total cycles	L1D misses	L2 misses	L1D accesses	L2 accesses	L1D miss rate	L2 miss rate	CPI
2	999.68	0	112	37	0	0	37	0	0	0	0.33
4	2017.8	0	406	147	0	0	159	0	0	0	0.36
8	2740.57	0	2,338	871	0	0	878	0	0	0	0.37
16	2950.66	0	16,282	6,486	0	0	6,437	0	0	0	0.4
32	2983.02	0.02	122,122	51,222	33	0	51,212	79	0.06	0	0.42
64	3726.41	0.14	588,459	326,448	3,140	0	322,275	6,920	0.97	0	0.55
128	3778.83	1.11	4,707,110	2,578,561	157,631	0	2,483,860	309,927	6.35	0	0.55
256	3007.68	11.16	37,655,775	25,905,635	2,544,461	234	19,844,811	6,073,872	12.82	0	0.69
512	3489.44	76.93	301,315,987	179,597,106	1,852,325	170,039	167,058,409	3,770,847	1.11	4.51	0.6
1024	3485.96	616.04	2,409,758,825	1,431,398,212	14,656,464	1,418,949	1,333,392,370	29,912,050	1.1	4.74	0.59
2048	3483.13	4932.31	19,275,004,836	11,445,462,655	116,738,832	11,621,088	10,656,559,365	235,923,029	1.1	4.93	0.59

2.5 Strassen Algorithm

This is recursive algorithm with work on block matrices and trades matrix multiplication for additions and subtractions. In block form a N×N matrix multiplication can be represented by 8 matrix multiplications and 4 additions where every matrix is N/2×N/2. Strassen algorithm does the same operation with 7 multiplications and 18 additions/subtractions. It is more efficient for large matrix sizes because there are $O(N^3)$ multiplications and $O(N^2)$ additions. Empirically, somewhere around N=100 Strassen method becomes better. `http://en.wikipedia.org/wiki/Strassen_algorithm`

Strassen algorithm does blocking and copy optimization intrinsically, which is desirable. As a result it can be built directly on top of SSE2. After implementation we saw a jump from 800 MOPS to 4000 MOPS for 2048 × 2048 (compared to SSE2). There is also significant increase in MOPS compared to results with blocking and copy optimization for large matrices.

Table 6: Strassen Algorithm

Dim.	MFLOPS	Runtime	Instr completed	Total cycles	L1D misses	L2 misses	L1D accesses	L2 accesses	L1D miss rate	L2 miss rate	CPI
2	1006.24	0	112	37	0	0	37	0	0	0	0.33
4	2028.61	0	406	147	0	0	159	0	0	0	0.36
8	2854.2	0	2,338	837	0	0	904	0	0	0	0.36
16	3353.74	0	16,282	5,675	0	0	6,434	0	0	0	0.35
32	3341.01	0.02	122,122	45,672	52	0	51,353	91	0.1	0	0.37
64	3725.36	0.14	588,189	326,914	3,251	0	323,153	6,835	1.01	0	0.56
128	3209.63	1.31	5,281,353	3,041,091	112,973	8	2,516,693	261,675	4.49	0	0.58
256	3648.82	9.2	38,238,143	23,242,620	908,753	12,147	18,139,077	2,108,277	5.01	0.58	0.61
512	3470.93	77.34	272,671,869	180,868,308	6,918,552	471,378	129,107,749	15,885,967	5.36	2.97	0.66
1024	3728.04	576.04	1,928,648,786	1,337,214,224	50,241,481	5,095,813	911,634,956	115,407,390	5.51	4.42	0.69
2048	4086.3	4204.26	13,580,226,806	9,757,932,695	358,655,023	47,004,377	6,409,543,523	813,640,172	5.6	5.78	0.72

2.6 Dedicated Functions

We used dedicated functions for 2 × 2-256 × 256, this gives compiler a chance to do better optimization because loop size is fixed. On average, there is a 2× increase of MOPS for 2 × 2-256 × 256.

We also rewrote the kernel function (32×32) of Strassen. We realized that we had great reuse of A but C was loaded/stored N times and we should unroll that to increase temporal locality. This means reading several A value and computing A[1][1]*B[1][1]+A[1][2]*B[2][1] vs just A[1][1]*B[1][1] as long as unroll factor is small and we don't evict things from the cache. This will be better because it reuses a number of load/stores. For instance, for 32x32 we used to have 19980 loads and 18503 stores, with the new code we get 21516/4103. After implementation we observed a significant increase of MOPS for 512 × 512 and above.

Table 7: Dedicated Functions

Dim.	MFLOPS	Runtime	Instr completed	Total cycles	L1D misses	L2 misses	L1D accesses	L2 accesses	L1D miss rate	L2 miss rate	CPI
2	1805.83	0	44	21	0	0	22	0	0	0	0.48
4	4366.92	0	174	68	0	0	99	0	0	0	0.39
8	4503.28	0	1,150	528	0	0	623	0	0	0	0.46
16	5459.13	0	8,999	3,499	0	0	4,507	0	0	0	0.39
32	6572.47	0.01	55,409	23,259	12	0	26,694	18	0.04	0	0.42
64	6940.07	0.08	422,298	176,214	3,249	0	172,443	4,267	1.88	0	0.42
128	6794.34	0.62	3,377,742	1,434,701	43,028	0	1,376,330	70,856	3.13	0	0.42
256	6449.8	5.2	27,020,872	12,040,422	515,048	225	11,011,004	1,206,328	4.68	0.02	0.45
512	4812.29	55.78	247,304,246	129,315,203	11,542,820	115,943	88,459,296	20,492,626	13.05	0.57	0.52
1024	4161.53	516.03	1,670,373,328	1,192,039,842	44,462,792	5,039,958	833,376,675	102,891,767	5.34	4.9	0.71
2048	4554.29	3772.24	11,772,294,887	8,773,104,515	319,598,020	46,714,355	5,884,662,574	731,447,323	5.43	6.39	0.75

2.7 Compiler Flags

For all the optimizations that we have implemented above, we used GCC compiler. ICC gives worse results except 2 × 2 case which is better due to fewer instructions.

We've also tried -funroll-loop flag to unroll loops and -ftree-vectorize to do SSE2 automatically. However, neither flag seems to make a lot of difference and we end up implementing both unrolling and SSE2 manually.