PHY 374 **Spring2018**
Ragan

Simple Linear Regression: Radioactive Lifetime

Due Monday 12/29

Suppose we have some data and we want to fit it to a theoretical curve. For instance we might want to measure, say, a nuclear decay half-life from a Geiger counter, or a blackbody temperature from a spectrum. The method usually used is called the *method of least squares*, in which we try to minimize the quantity

$$S(\alpha) = \sum_{i=1}^{N} \left[y_i - f(x_i, \alpha) \right]^2 \qquad \text{Eq.1}$$

Where α is a parameter (or parameters) we are varying, y_i is the ith data point and $f(x_i, \alpha)$ is the theoretical value at x_i.

In this lab we will fit some Geiger counter data to the curve

$$R(t) = R_0 e^{-t/\tau}, \qquad \text{Eq.2}$$

We can make things a lot simpler by taking the natural log of Eq.2, which gives us

$$\ln R(t) = \ln R_0 - t/\tau \qquad \text{Eq.3}$$

which has the form of a line $y = ax + b$. Putting this form into Eq.1 we get

$$S(a,b) = \sum_i (y_i - ax_i - b)^2 \qquad \text{Eq.4}$$

We find the values of *a* and *b* that give the best fit, i.e. that minimizes *S*, by setting $\frac{\partial S}{\partial a} = 0$ and $\frac{\partial S}{\partial b} = 0$. This gives $0 = -2\sum_i x_i (y_i - ax_i - b)$ and $0 = -2\sum_i (y_i - ax_i - b)$, so we have two equations and two unknowns. Solving for *a* and *b* we get, for the slope

$$a = \frac{\sum_i x_i^2 \sum_i y_i - \sum_i x_i \sum_i x_i y_i}{N\sum_i x_i^2 - \left(\sum_i x_i\right)^2} \qquad \text{Eq.(5)}$$

and for the *y*-intercept

$$b = \frac{N\sum_i x_i y_i - \sum_i x_i \sum_i y_i}{N\sum_i x_i^2 - \left(\sum_i x_i\right)^2} \qquad \text{Eq.(6)}$$

In your personal directory create a directory named 'geiger' (You'll want to do this for each project). Download the program fakegeiger.m into your directory and run it. This will generate simulated Geiger counter data in file named 'geigerdata.txt' which has two columns, a time in minutes, and a Geiger count for each minute half-minute.

From the keyboard, read the data into an array with the statement `M=dlmread('`*`directorypath`*`\geigerdata.txt')`. You get the directory path from *Windows* by left clicking on the title bar of the directory window containing the data file. You can find the dimensions of M with the `size(M)` command. Look for the `M` array in your workspace.
Next create two column arrays with the commands `t=M(:,1)` and `R=M(:,2)`. The colon indicates that you want all the rows and the first or second columns for `t` and `R` repectively.

Next you can plot your data with `plot(t,R,'o')`. Now you are ready to write an m-file that calculates the lifetime of the radioactive source.

Follow these steps:

1) Start a new script
2) Start your m-file with the comment line that describes what it is going to do. For this first short program, comment each line.
3) Begin your m-file with following commands
   ```
   close all; clear all; clc;
   % closes figures, clears variables, clears command
   line
   ```
4) Read the data file into the variable `M`. (You won't need a directory path since the data file is in the same directory as your program.)
5) Create the `t` and `R` arrays.
6) Create an `LnR` array that is the natural log of the `R` array.
7) Plot the `LnR` array vs. `t` with circles.
8) Next calculate all of the sums (with `for-end` loops) that occur in Eqs.(5-6). Use variable names like `Sx, Sxx, Sy, Sxy`, etc.
9) Calculate a and b. What is the lifetime of the source? (how about the half-life?) What is the fitted initial count rate?

10) Plot your best fit line on the same figure as your data (use a solid line).
11) Format your graph with a title reporting the lifetime, axes labels, units and legend.
12) Finally, use the command `figure` to generate a new figure and plot the raw data (the R array) on the same graph as the fitted data (the exponential curve given by Eq.2).
13) Print out and turn in your program and the graphs.

Addendum

The following are the formulas for the uncertainty in the slope and in the intercept:

$$\Delta a = S\sqrt{\frac{N}{NS_{xx} - S_x^2}} \qquad \text{Eq.7}$$

and

$$\Delta b = S\sqrt{\frac{S_{xx}}{NS_{xx} - S_x^2}} \qquad \text{Eq.8}$$

where $S = \sqrt{\frac{\sum_i (y_i - ax_i - b)^2}{n-2}}$.

14) Calculate the uncertainties in your fitting parameters, are they reasonable? (You can plot additional lines on your plots showing the range of lines that fall within your uncertainties.)
15) Report your value of $\tau \pm \Delta\tau$ on your plot. Use the rule for propagation of uncertainties $\frac{\Delta f}{f} = \sqrt{\left(\frac{\partial f}{\partial a}\right)^2 + \left(\frac{\partial f}{\partial b}\right)^2 + \ldots}$