PHY 374 **Model Rocket Simulation** **Spring 2018**
Ragan **Due Wed. Feb.7**

In this assignment we use the Euler method to simulate the one-dimensional trajectory of a model rocket. The force on the rocket is given by

$$F_y(t) = -m(t)g - c_2 v(t)^2 + f(t) \tag{1}$$

where $m(t)$ is the mass of the rocket, $f(t)$ is the thrust, and c_2 is the quadratic air resistance coefficient,

$$c_2 = \tfrac{1}{2} C_D \rho A \tag{2}$$

where C_D is the drag coefficient. We shall take C_D =0.73+0.20= 0.93 which is a good approximation for a model with a straight slender body with a launch lug at subsonic speeds. (see the wind tunnel tests). A is the maximal frontal area and ρ is the density of air (=1.26 kg/m^3 at 25 ^{o}C) .

<u>Simulation Steps</u>

1. Determine the mass of the empty rocket m_r and of the propellant m_p. Determine its cross sectional area of the rocket (with fins) and calculate c_2.

2. Determine the piece-wise definition of your thrust curve for the A8-3 engine and write an if-else-end block that calculates $f(t_i)$. Plot your curve. Use $t_{\max} = 2.9\,\text{sec}$ for the engine burn time + ejection delay, and N=2900, i.e. $\Delta t = 1\,\text{msec}$.

3. Calculate the total impulse of your curve

$$I = \int_0^{t_{\max}} f(t)dt \approx \Delta t \sum_{i=1}^{N+1} f_i\ . \tag{3}$$

Does it match the stated impulse on the data sheet?

3. Inside your time loop calculate the mass of the propellant. We shall assume that the rate of change of the remaining propellant mass is proportional to the instantaneous thrust:

$$\tfrac{d}{dt} m_p(t) = -m_p(0) f / I \tag{4}$$

The total rocket weight is given by $m(t) = m_p(t) + m_0$, where m_0 is the mass of the empty rocket.

4. You're ready to use the Euler method. The initial conditions are

$$t(1) = 0$$
$$f(1) = 0$$
$$m(1) = mp(1) + mr$$
$$v(1) = 0$$
$$y(1) = 0$$
$$a(1) = -g + f(1)/m(1) \geq 0$$

The time-step equations are, starting with i=2:

$$t(i) = t(i-1) + \Delta t$$
$$v(i) = v(i-1) + a(i-1) * \Delta t$$
$$y(i) = y(i-1) + v(i-1) * \Delta t$$
$$m(i) = mr + mp(i)$$
$$a(i) = -g - \frac{c_2}{m(i)} v(i)^2 + \frac{f(i)}{m(i)}$$

There is a slight complication in that the normal force of the launch pad also acts on the rocket and insures that $a > 0$ until lift-off. See if you can add a condition and a 'logical variable' call liftoff to accomplish this.

5. Plot the *f*, *m*, *a*, *v*, and *y* curves. Then add a drag force plot.

6. Down load the file A3-8.txt from D2L and read the measured trajectory. Plot the measured trajectory as points on your $y(t)$ graph. Do they agree? Adjust your drag coefficient so they come in close agreement.

7. Now repeat your calculation for the C6-7 engine, and compare your max height to the measured value of 295 ± 5 meters.