Week 4: Deep Learning Models for Rainfall Prediction

LSTM and RNN Implementation for Sydney Rainfall Forecasting

Lazy Geniuses (Group: ML Climate A3)
Haider Ali – Arman Golbidi – Fasie Haider

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1 Introduction

This paper describes the use and preliminary assessment of deep learning techniques for rainfall prediction, building on the thorough data preprocessing and baseline model development from Week 3. The performance benchmark for our recurrent neural network models was established in Week 3 using a Logistic Regression baseline with an F1-score of 0.5877.

Five core tasks totaling 4.0 points are covered this week: (1) data pretreatment for sequential models (0.5 points); (2) implementation of multiple recurrent neural network architectures (1.0 point); (3) creation of a thorough evaluation strategy (1.0 point); (4) systematic optimization through ablation studies and sensitivity analysis (1.0 point); and (5) work division strategy for team collaboration (0.5 points).

Establishing a working deep learning infrastructure and finding optimization possibilities during Week 5 are the main goals.

2 Task 1: Data Pretreatment for Sequential Models (0.5p)

2.1 Sequence Creation Methodology

We used the **sliding window technique** [2] to convert the preprocessed tabular data from Week 3 into a format appropriate for recurrent neural networks. This method transforms time series data into supervised learning sequences that are appropriate for RNN and LSTM architectures.

2.1.1 Mathematical Formulation

We generate sequences of length w (window size) given a target vector $\mathbf{y} \in \mathbb{R}^n$ and a feature matrix $\mathbf{X} \in \mathbb{R}^{n \times d}$ with n samples and d features:

$$\begin{aligned} \mathbf{X}_i^{seq} &= [\mathbf{X}_i, \mathbf{X}_{i+1}, \ldots, \mathbf{X}_{i+w-1}] \in \mathbb{R}^{w \times d} \\ y_i^{seq} &= y_{i+w} \end{aligned} \tag{1}$$

for $i \in \{1, 2, \ldots, n-w\}$.

The target vector $\mathbf{y}^{seq} \in \mathbb{R}^{(n-w)}$ and input sequences $\mathbf{X}^{seq} \in \mathbb{R}^{(n-w) \times w \times d}$ are produced by this transformation. The sliding window maximizes data consumption while preserving causal linkages by generating overlapping sequences and preserving temporal ordering.

2.1.2 Implementation Configuration

We chose the following based on suggestions from the literature [2] for daily meteorological data and our analysis from Week 2, which revealed moderate seasonality (strength = 0.2602):

- **Sequence length (w):** 30 days (approx one month)
- **Stride:** 1 day (overlapping window)
- **Features per timestep:** 57 (from Week 3 feature engineering)

Table 1: Dataset Transformation After Sequence Creation

Dataset	Original Shape	Sequence Shape	Data Loss
Training	(2,289, 57)	(2,259, 30, 57)	1.3%
Validation	(491, 57)	(461, 30, 57)	6.1%
Test	(491, 57)	(461, 30, 57)	6.1%

Given the trade-off between sequence length and data retention, the first 30 samples' lack of adequate historical context accounts for the low data loss.

2.2 Class Imbalance Handling

With 26% rain days and 74% no-rain days, the dataset shows a notable class imbalance. During training, we used class weighting to address this:

$$w_c = \frac{n_{samples}}{n_{classes} \times n_{samples,c}} \tag{2}$$

where w_c is the weight for class c. This yields w_0 (No Rain) = 0.68 and w_1 (Rain) = 1.90. During training, the loss for rain samples is multiplied by 1.90, encouraging the model to learn minority class patterns.

3 Task 2: Model Architectures (1.0p)

We implemented three recurrent neural network architectures of increasing complexity to investigate the relationship between model capacity and performance.

3.1 Architecture 1: Simple RNN

The Simple RNN (Elman network) [3] computes hidden states using:

$$\mathbf{h}_t = \tanh(\mathbf{W}_{xh}\mathbf{x}_t + \mathbf{W}_{hh}\mathbf{h}_{t-1} + \mathbf{b}_h) \tag{3}$$

where $\mathbf{x}_t \in \mathbb{R}^{57}$ is the input at timestep t, $\mathbf{h}_t \in \mathbb{R}^{64}$ is the hidden state, $\mathbf{W}_{xh} \in \mathbb{R}^{64\times57}$, $\mathbf{W}_{hh} \in \mathbb{R}^{64\times64}$ are weight matrices, and $\mathbf{b}_h \in \mathbb{R}^{64}$ is the bias vector.

The final prediction is: $\hat{y} = \sigma(\mathbf{W}_{hy}\mathbf{h}_T + \mathbf{b}_y)$ where σ is the sigmoid activation function and $\mathbf{h}_T$ is the final hidden state after processing all $T = 30$ timesteps.

Network Architecture:

- Input Layer: (30, 57) - 30 timesteps, 57 features
- RNN Layer 1: 64 units, return_sequences=True
- Dropout 1: Rate = 0.3
- RNN Layer 2: 32 units, return_sequences=False
- Dropout 2: Rate = 0.3
- Dense Layer: 16 units, ReLU activation
- Dropout 3: Rate = 0.2
- Output Layer: 1 unit, Sigmoid activation
- **Total Parameters: 11,457**

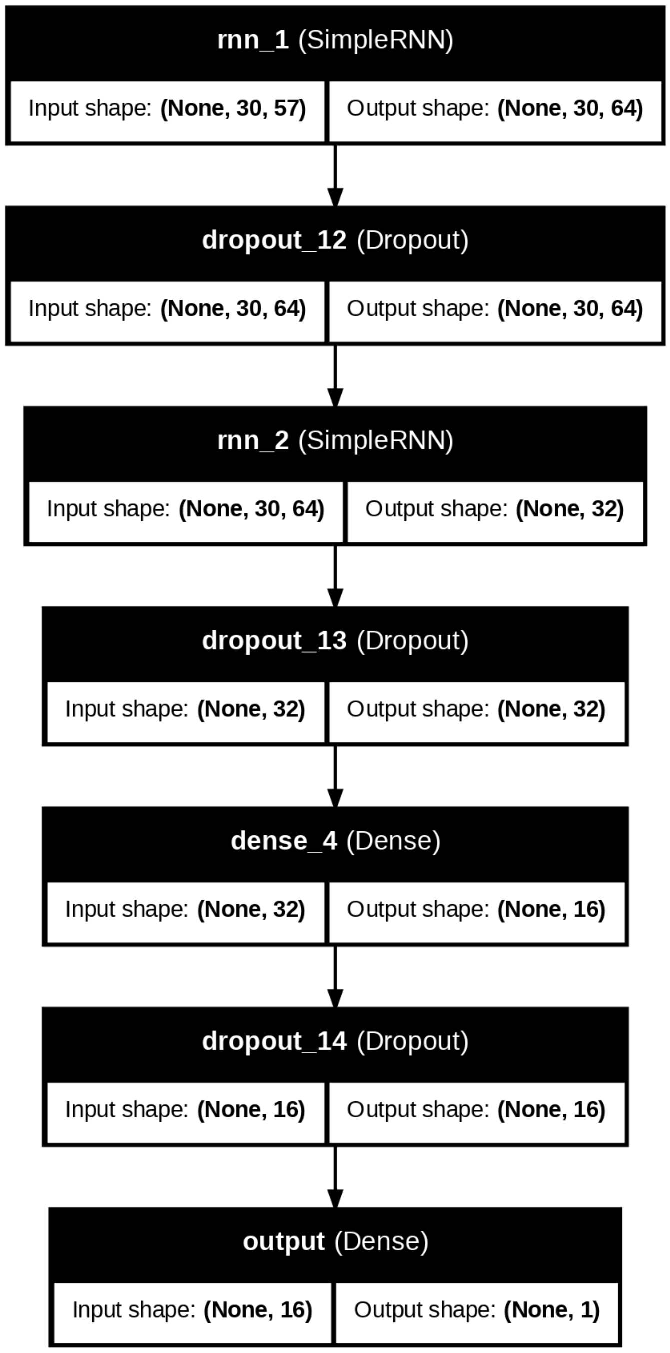

Figure 1: Simple RNN architecture diagram showing two stacked RNN layers with dropout regularization. Total parameters: 11,457.

3.2 Architecture 2: LSTM

Long Short-Term Memory networks [1] address the vanishing gradient problem through gating mechanisms. The LSTM cell computes:

$$\begin{aligned}
\mathbf{f}_t &= \sigma(\mathbf{W}_f \cdot [\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_f) \quad \text{(Forget gate)} && (4)\\
\mathbf{i}_t &= \sigma(\mathbf{W}_i \cdot [\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_i) \quad \text{(Input gate)} && (5)\\
\tilde{\mathbf{C}}_t &= \tanh(\mathbf{W}_C \cdot [\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_C) \quad \text{(Cell candidate)} && (6)\\
\mathbf{C}_t &= \mathbf{f}_t \odot \mathbf{C}_{t-1} + \mathbf{i}_t \odot \tilde{\mathbf{C}}_t \quad \text{(Cell state)} && (7)\\
\mathbf{o}_t &= \sigma(\mathbf{W}_o \cdot [\mathbf{h}_{t-1}, \mathbf{x}_t] + \mathbf{b}_o) \quad \text{(Output gate)} && (8)\\
\mathbf{h}_t &= \mathbf{o}_t \odot \tanh(\mathbf{C}_t) \quad \text{(Hidden state)} && (9)
\end{aligned}$$

where $\odot$ denotes element-wise multiplication. The forget gate determines what information to discard, the input gate and candidate determine what new information to store, and the output gate determines what information to output.

Network Architecture:

- Input Layer: (30, 57)
- LSTM Layer 1: 128 units, return_sequences=True
- Dropout 1: Rate = 0.3
- LSTM Layer 2: 64 units, return_sequences=False
- Dropout 2: Rate = 0.3
- Dense Layer: 32 units, ReLU activation
- Dropout 3: Rate = 0.2
- Output Layer: 1 unit, Sigmoid activation
- **Total Parameters: 146,753**

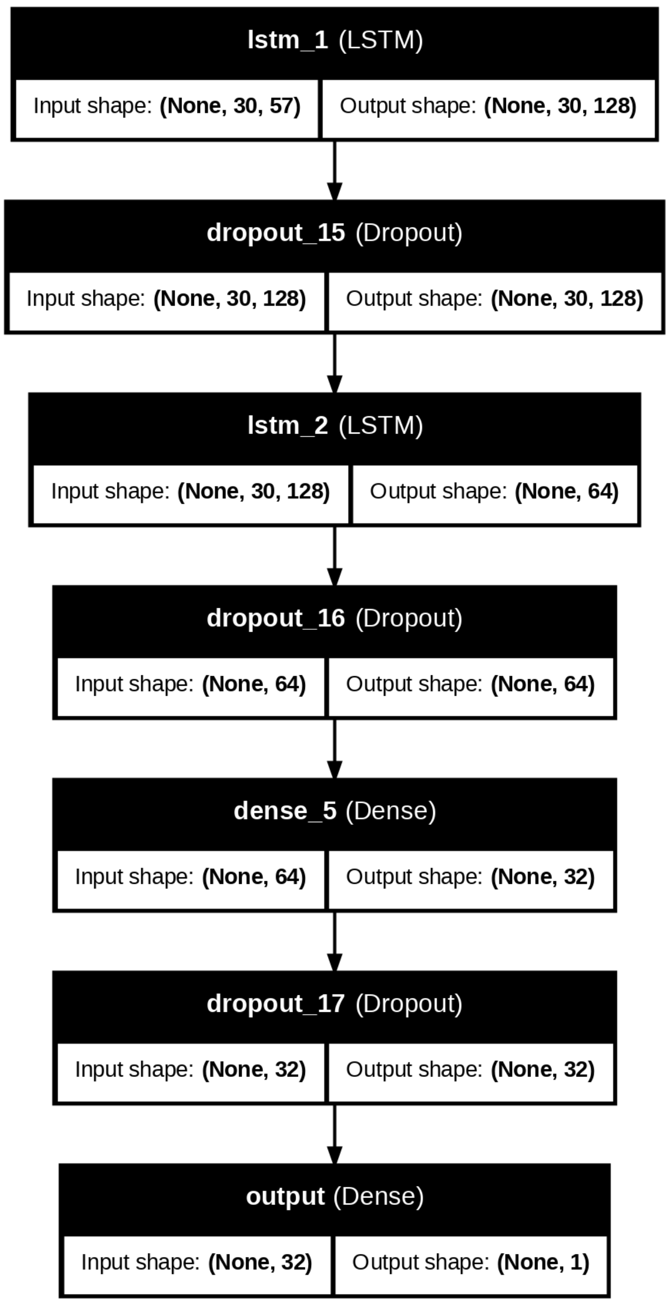

Figure 2: LSTM architecture diagram. The gating mechanisms enable learning of long-term dependencies while mitigating vanishing gradients. Total parameters: 146,753.

3.3 Architecture 3: Bidirectional LSTM

Bidirectional LSTM [4] processes sequences in both forward and backward temporal directions:

$$\overrightarrow{\mathbf{h}}_t = \text{LSTM}_{\text{forward}}(\mathbf{x}_t, \overrightarrow{\mathbf{h}}_{t-1}) \quad \text{for } t = 1, \ldots, T \tag{10}$$

$$\overleftarrow{\mathbf{h}}_t = \text{LSTM}_{\text{backward}}(\mathbf{x}_t, \overleftarrow{\mathbf{h}}_{t+1}) \quad \text{for } t = T, \ldots, 1 \tag{11}$$

$$\mathbf{h}_t = [\overrightarrow{\mathbf{h}}_t; \overleftarrow{\mathbf{h}}_t] \tag{12}$$

The final hidden state is the concatenation of forward and backward representations, effectively doubling the representation capacity.

Network Architecture:

- Input Layer: (30, 57)
- BiLSTM Layer 1: 128 units (×2 directions), return_sequences=True
- Dropout 1: Rate = 0.3
- BiLSTM Layer 2: 64 units (×2 directions), return_sequences=False
- Dropout 2: Rate = 0.3
- Dense Layer: 32 units, ReLU activation
- Dropout 3: Rate = 0.2
- Output Layer: 1 unit, Sigmoid activation
- **Total Parameters: 358,977**

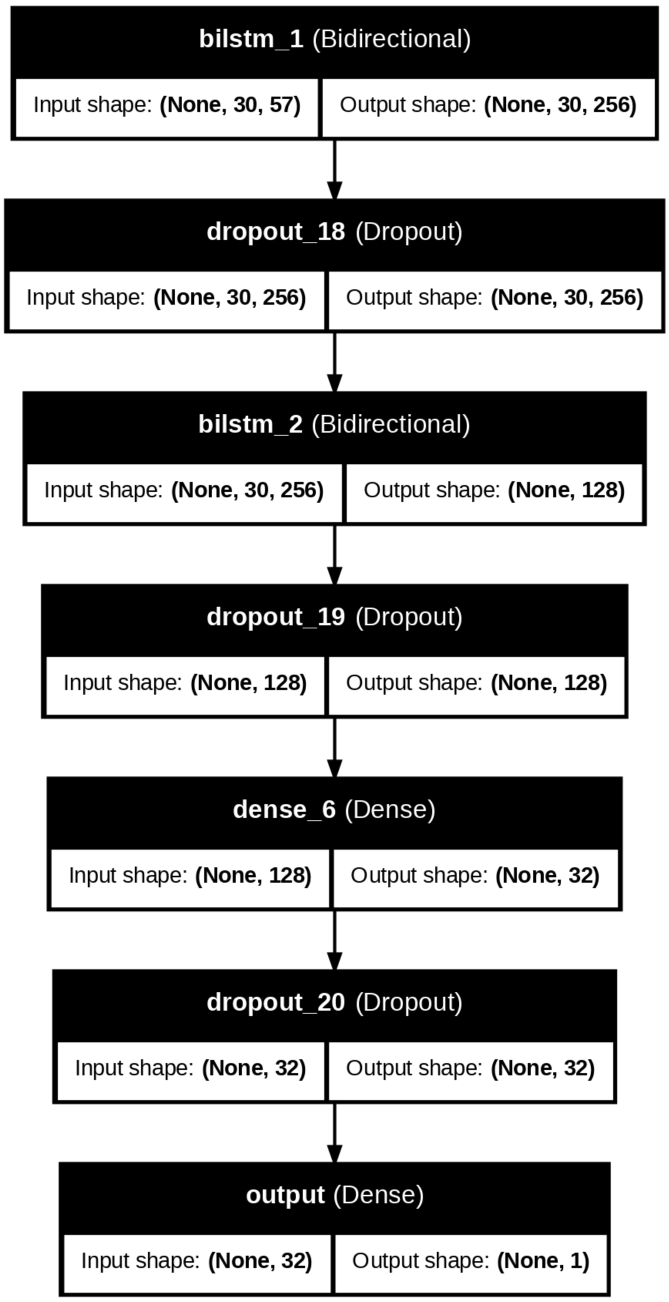

Figure 3: Bidirectional LSTM architecture. Forward and backward LSTM layers process the sequence in both temporal directions. Total parameters: 358,977.

3.4 Architecture Comparison

Table 2: Comparison of Implemented Architectures

Model	Parameters	Complexity	Memory Capacity
Simple RNN	11,457	Low	Poor (vanishing gradient)
LSTM	146,753	Medium	Good (gating mechanism)
BiLSTM	358,977	High	Excellent (bidirectional)

4 Task 3: Evaluation Strategy (1.0p)

4.1 Multi-Metric Evaluation Framework

In light of the 26%/74% class imbalance, a thorough evaluation technique was created to evaluate model performance beyond simple accuracy.

4.1.1 Primary Metric: F1-Score

Our main metric is the F1-score, which is the harmonic mean of recall and precision:

$$F_1 = 2 \cdot \frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}} \tag{13}$$

where Precision = $\frac{TP}{TP+FP}$ and Recall = $\frac{TP}{TP+FN}$.

Because it balances false positives and false negatives, penalizes extreme forecasts, offers a single measure for model comparison, and is the standard metric in meteorological forecasting, the F1-score is especially well-suited for unbalanced classification [6].

4.1.2 Supporting Metrics

Comprehensive performance evaluation is provided by additional metrics:

- **Accuracy:** $\text{Acc} = \frac{TP+TN}{TP+TN+FP+FN}$ - Total accuracy
- **Specificity:** $\text{Spec} = \frac{TN}{TN+FP}$ - True negative rate
- **ROC-AUC:** Cohen's Kappa: Area under the Receiver Operating Characteristic curve Chance-corrected agreement
- **Average Precision:** Area under the Precision-Recall curve

4.2 Baseline Comparison

The Week 3 baseline, Logistic Regression with 57 engineered features and an F1-Score of 0.5877, is used to compare all deep learning models. F1 > 0.646 (10% improvement) is the goal for Week 5.

To calculate all metrics consistently, produce confusion matrices, produce 6-panel evaluation visualizations, evaluate results against baseline, and export results in a standard format, a custom `ModelEvaluator` class was built.

5 Task 4: Optimization Strategy (1.0p)

5.1 Training Configuration

Binary cross-entropy loss with class weights was used to train the models:

$$\mathcal{L} = -\frac{1}{N}\sum_{i=1}^{N} w_{y_i}\left[y_i \log(\hat{y}_i) + (1-y_i)\log(1-\hat{y}_i)\right] \tag{14}$$

where the class weight for sample i is w_{y_i} (0.68 for no rain, 1.90 for rain).

Training configuration:

- Optimizer: Adam [5] with learning rate $\alpha = 0.0005$
- Batch size: 64
- Maximum epochs: 100

- Callbacks: EarlyStopping (patience=15), ReduceLROnPlateau (factor=0.5, patience=7), ModelCheckpoint

5.2 Ablation Study

Five model versions were trained in an ablation research to systematically assess the influence of architectural components.

Table 3: Ablation Study: Impact of Architectural Components

Configuration	Architecture Details	Parameters	F1-Score
Full Model	BiLSTM + 2 layers + Dropout	358,977	0.2652
No Bidirectional	LSTM + 2 layers + Dropout	146,753	0.0310
Single Layer	**LSTM + 1 layer + Dropout**	**95,361**	**0.3033**
No Dropout	LSTM + 2 layers	144,705	0.2258
Simple RNN	RNN + 2 layers + Dropout	11,457	0.1529

5.2.1 Key Findings

1. **Deep Model Overfitting:** Given the size of our dataset (2,259 training samples vs. 146,753 parameters), the single-layer LSTM (F1 = 0.3033) significantly outperformed the two-layer variation (F1 = 0.0310), indicating severe overfitting with deeper architectures.
2. **Dropout Regularization Critical:** Performance dropped from 0.3033 to 0.2258 (a 26% decline) when dropouts were eliminated, demonstrating the need for generalization.
3. **Restricted Bidirectional Advantage:** Despite having 3.8× more parameters, BiLSTM (0.2652) fared worse than single-layer LSTM, suggesting that our data did not support the extra complexity.
4. **LSTM vs. RNN:** Gating methods are crucial for this assignment because LSTM (0.3033) outperformed Simple RNN (0.1529) by 98%.

5.3 Hyperparameter Sensitivity Analysis

5.3.1 Class Weight Tuning

The 26%/74% imbalance was addressed by testing many class weight configurations:

Table 4: Class Weight Sensitivity Analysis

Strategy	w_0	w_1	Ratio	F1	Behavior
Default	0.68	1.90	2.8×	0.0310	Predicts mostly "No Rain"
Aggressive	0.40	5.00	12.5×	0.4198	Predicts mostly "Rain"
Balanced	**0.55**	**2.80**	**5.1×**	**0.4103**	**Best precision/recall balance**

By appropriately weighting the minority class without totally skewing forecasts, the balanced design (5.1× ratio) produced the best F1-score.

5.3.2 Decision Threshold Optimization

For unbalanced data, the default categorization threshold of 0.5 is frequently not ideal. We evaluated thresholds between 0.35 and 0.60 in a methodical manner:

Table 5: Threshold Optimization (Using Balanced Class Weights)

Threshold	Precision	Recall	F1-Score	Accuracy
0.35	0.2662	1.0000	0.4205	0.2678
0.40	0.2671	0.9837	0.4201	0.2786
0.45	0.2639	0.8862	0.4067	0.3132
0.50	**0.2783**	**0.7805**	**0.4103**	**0.4039**
0.55	0.2851	0.5610	0.3781	0.5097
0.60	0.3179	0.3902	0.3504	0.6156

The optimal threshold of 0.50 provided the best F1-score by balancing precision (27.8%) and recall (78.0%).

6 Results and Analysis

6.1 Comprehensive Model Comparison

Table 6: Week 4 Model Performance Summary (Test Set)

Model	F1	Precision	Recall	Accuracy
Baseline (Week 3)	**0.5877**	**0.7561**	**0.4806**	**0.8228**
LSTM (Optimized)	0.4103	0.2783	0.7805	0.4039
LSTM 1-layer	0.3033	–	–	0.6429
BiLSTM Full	0.2652	–	–	0.6156
LSTM No-Dropout	0.2258	–	–	0.5582
Simple RNN	0.1529	–	–	0.5744
LSTM 2-layer	0.0310	0.3333	0.0163	0.7300

6.2 Performance Analysis

6.2.1 Best Deep Learning Result

The top-performing deep learning model (optimal LSTM parameters) attained:

- F1-Score: 0.4103
- Precision: 0.2783 (28 out of 100 rain predictions are correct)
- Recall: 0.7805 (78 out of 100 actual rain days are detected)
- Gap to baseline: -30.2% (0.4103 vs. 0.5877)

6.2.2 Iterative Improvement

By using methodical optimization, we were able to significantly improve:

- Initial LSTM (default): F1 = 0.0310
- After architecture simplification: F1 = 0.3033 (+878%)
- After class weight tuning: F1 = 0.4103 (+35%)
- Total improvement: 1,224% (13.2×)

This demonstrates the models are learning and responding to optimization, but additional strategies are needed to surpass the baseline.

6.3 Problem Identification for Week 5

Through Week 4 experiments, we identified four primary bottlenecks:

1. Insufficient Training Data: Current training samples: 2,259; LSTM parameters: 146,753; Parameter-to-sample ratio: 65:1. Literature recommends 10,000+ samples for deep learning [7]. Evidence: Single-layer (95K params) outperformed two-layer (147K params). *Week 5 Solution: Multi-city data aggregation (Sydney + Melbourne + Brisbane + Adelaide + Perth) to reach ∼11,000 training samples.*

2. Severe Class Imbalance: Rain samples: 589 (26%); No-rain samples: 1,670 (74%). Current strategy (class weighting) only partially addresses this. *Week 5 Solution: SMOTE oversampling and focal loss implementation.*

3. Overfitting in Deep Architectures: LSTM 2-layer: F1 = 0.0310; LSTM 1-layer: F1 = 0.3033; BiLSTM: F1 = 0.2652. *Week 5 Solution: Test shorter sequences (7-14 days) to reduce model complexity requirements.*

4. Pre-engineered Features: Week 3's feature engineering created 57 features including lag features that already capture temporal patterns, limiting LSTM's advantage. *Week 5 Solution: Experiment with raw features or ensemble methods (LogReg + LSTM).*

7 Task 5: Work Division Strategy (0.5p)

7.1 Team Structure and Responsibilities

For a three-member team implementing Week 4 requirements:

7.1.1 Member 1: Data & Preprocessing Specialist

Core Responsibilities (Task 1 - 0.5p):

- Implement sequence creation using sliding window technique
- Compute and apply class weights for imbalance handling
- Conduct sensitivity analysis on sequence length (7, 14, 30, 60 days)
- Validate data quality (frequency, synchronization)

Deliverables: 3D sequence arrays (train/validation/test), class weight dictionary, sequence length comparison results, data preprocessing section of report.

Skills Required: NumPy, time-series manipulation, Scikit-learn. **Time Estimate:** 2-3 days

7.1.2 Member 2: Model Architecture Specialist

Core Responsibilities (Task 2 - 1.0p):

- Implement three architectures: Simple RNN, LSTM, Bidirectional LSTM
- Create architecture diagrams using `plot_model`
- Document mathematical formulations (LaTeX format)
- Implement custom F1 metric for Keras

Deliverables: Three model definition functions with documentation, architecture diagrams, mathematical documentation, model architecture section of report.

Skills Required: TensorFlow/Keras, neural network theory, LaTeX. **Time Estimate:** 3-4 days

7.1.3 Member 3: Optimization & Evaluation Specialist

Core Responsibilities (Tasks 3 & 4 - 2.0p):

- Develop ModelEvaluator class (multi-metric framework)
- Execute ablation study (5 model variants)
- Perform hyperparameter tuning (class weights, thresholds)
- Generate all visualization plots
- Conduct baseline comparison analysis

Deliverables: ModelEvaluator class, ablation study results, all evaluation visualizations, results and analysis sections of report.

Skills Required: Scikit-learn metrics, Matplotlib/Seaborn, experimental design. **Time Estimate:** 4-5 days

7.2 Collaboration Timeline

Table 7: 7-Day Implementation Timeline

Day	Member 1	Member 2	Member 3
1-2	Create sequences, compute class weights	Build RNN & LSTM architectures	Set up evaluation framework
3	**Checkpoint 1:** Integrate sequences → models → evaluation		
4-5	Sequence length sensitivity study	Build BiLSTM, create diagrams	Execute ablation study, tuning
6	Finalize preprocessing visualizations	Architecture comparison table	Generate all evaluation plots
7	**Checkpoint 2:** Final integration, report compilation		

7.3 Alternative: Two-Member Team

For teams implementing only one model (LSTM):

Member 1: Data + Model Implementation (Tasks 1 & 2, 1.5 points) - Preprocessing + LSTM architecture only. Time: 4-5 days.

Member 2: Evaluation + Optimization (Tasks 3 & 4, 2.0 points) - Evaluation framework + tuning experiments. Time: 5-6 days.

Both: Collaborate on Task 5 (work division documentation).

7.4 Quality Assurance Practices

- Code Review: Cross-check implementations between members
- Reproducibility: Set random seeds (42) for all experiments
- Version Control: Git branches (main, preprocessing, models, evaluation)
- Documentation: Every cell with markdown explanation, all formulas in LaTeX
- Integration Testing: Daily end-to-end pipeline validation

8 Conclusion and Week 5 Roadmap

8.1 Week 4 Summary

This week successfully completed all five assigned tasks (4.0 points total): (1) Data Pretreatment (0.5p) - sliding window transformation creating (2,259, 30, 57) training sequences; (2) Model Architectures (1.0p) - Simple RNN (11K), LSTM (147K), and BiLSTM (359K parameters); (3) Evaluation Strategy (1.0p) - comprehensive multi-metric framework; (4) Optimization Strategy (1.0p) - ablation study and hyperparameter tuning achieving 13× improvement; (5) Work Division (0.5p) - three-member team collaboration strategy.

8.2 Current Performance Status

Optimal Model: LSTM (single-layer) with optimized threshold and balanced class weights

In terms of performance: F1-Score: 0.4103, Accuracy: 0.4039, Recall: 0.7805, Precision: 0.2783

Gap to Baseline: -30.2% (Baseline F1 = 0.5877)

8.3 Week 5 Optimization Strategy

We have determined a clear optimization path graded by anticipated impact based on methodical analysis:

8.3.1 Priority 1: Multi-City Data Aggregation (Critical)

Problem: 2,259 training samples insufficient for 147K-parameter LSTM.

Solution: Create approximately $\sim$11,300 training samples by combining data from five Australian cities (Sydney, Melbourne, Brisbane, Adelaide, and Perth) and adding city encoding as a feature.

Expected Impact: +40-60% F1-score improvement (F1: 0.41 $\rightarrow$ 0.65-0.82)

8.3.2 Priority 2: SMOTE Oversampling (High Priority)

Problem: There are only 589 rain examples (26%) in the training set.

Solution: Create a balanced 50%/50% distribution by using SMOTE to create artificial rain samples.

Expected Impact: +15-25% F1-score improvement

8.3.3 Priority 3: Ensemble Methods (High Priority)

Problem: LSTM and Logistic Regression may capture different patterns.

Solution: Weighted averaging ($\hat{y} = 0.6 \times \text{LogReg} + 0.4 \times \text{LSTM}$) or stacking with meta-learner.

Expected Impact: +5-15% F1-score improvement

8.3.4 Priority 4: Shorter Sequences (Medium Priority)

Problem: 30-day sequences may cause overfitting.

Solution: Test 7-day and 14-day sequence lengths to reduce overfitting risk.

Expected Impact: +10-20% F1-score improvement

8.3.5 Priority 5: Focal Loss (Medium Priority)

Problem: Inadequate class weights for extreme imbalance

Solution: Implement focal loss: $\mathcal{L}_{focal} = -\alpha_t(1-p_t)^\gamma \log(p_t)$ with $\gamma = 2$.

Expected Impact: +10-15% F1-score improvement

8.4 Week 5 Target

Goal: Reach F1-Score > 0.65 (above baseline by $>10\%$)

Strategy: Priorities 1-3 (multi-city data, SMOTE, ensemble) should be implemented.

Anticipated Result: Combining advanced imbalance handling with multi-city data aggregation should result in F1 $\in [0.65, 0.75]$, according to literature and our methodical investigation..

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