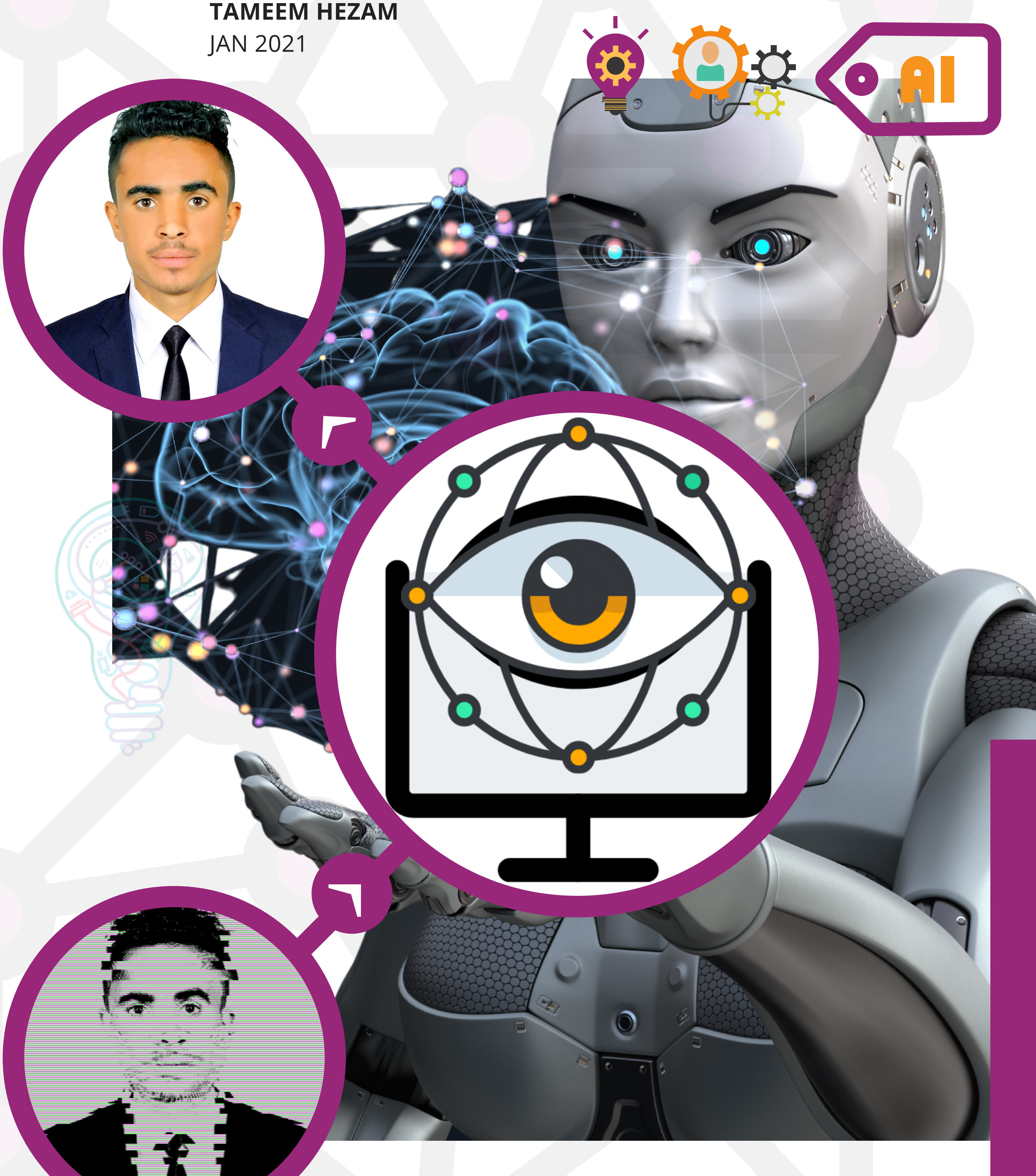
MAGE RESTORATION

TAMEEM HEZAM

JAN 2021



南京理工大学

NANJING UNIVERSITY OF SCIENCE & TECHNOLOGY

ACADEMIC REPORT

SUBMITTED TO PROFESSOR JIN CUN
ARTIFICIAL INTELLIGENCE TEACHER
IN NJUST

TAMEEM HEZAM
GRADUATE STUDENT IN NJUST
JAN 2021

REPORT CONTENTS

YOU CAN GET ALL CODING AND PROJECTDATA
https://github.com/tememway/image-restoration-report-.git

SCAN THIS CODE TO DOWENLOUD OUR PROJECT

Abstract

The purpose of image restoration is to "compensate for" or "undo" defects which degrade an image. Degradation comes in many forms such as motion blur, noise, and camera misfocus. In cases like motion blur, it is possible to come up with a very good estimate of the actual blurring function and "undo" the blur to restore the original image. In cases where the image is corrupted by noise, the best we may hope to do is to compensate for the degradation it caused. In this project, we will introduce and implement several of the methods used in the image processing world to restore images.

Introdaction

The purpose of image restoration is to "compensate for" or "undo" defects which degrade an image. Degradation comes in many forms such as motion blur, noise, and camera misfocus. In cases like motion blur, it is possible to come up with a very good estimate of the actual blurring function and "undo" the blur to restore the original image. In cases where the image is corrupted by noise, the best we may hope to do is to compensate for the degradation it caused. In this report, we will introduce and implement several of the methods used in the image processing world to restore images. In the computational image, processing blurring is usually modelled by a convolution of image matrix and a blur kernel. A blur kernel, in this case, is a two-dimensional matrix which describes the response of an imaging system to a point light source or a point object. Another term for it is Point Spread Function (PSF). What does it mean? Let’s suppose we have three two-dimensional matrices: f(x,y) for the original image, h(m,n) for the blurring kernel and g(x,y) for the blurred image. Then we can write the convolution:

$$g(x,y) = \sum_{m}\sum_{n} h(m,n) \cdot f(x+m, y+n)$$

or, using an asterisk to denote convolution operation,

$$g = f * h.$$

The above equation is written in the spatial domain because we use spatial coordinates (x,y), but the convolution result can be described in a more convenient way in the frequency domain using frequency coordinates (u,w). From the convolution theorem the Discrete Fourier Transforming (DFT) of the blurred image is the point-wise product of the DFT of the original image and the DFT of the blurring kernel:G(u,w) = F(u,w) * H(u,w)where:F(u,w) = DFT{f (x,y)},G(u,w) = DFT{g(x,y)} and(u,w) = DFT{h (x,y)} for blurring kernel.The most common types of blur are motion blur, out-of-focus blur and Gaussian blur (it’s a good approximation of an image degrading by atmospheric turbulence).

There are several widely used techniques in image restoration, some of which are based on frequency domain concepts while others attempt to model the degradation and apply the inverse process.

The modelling approach requires determining the criterion of “goo

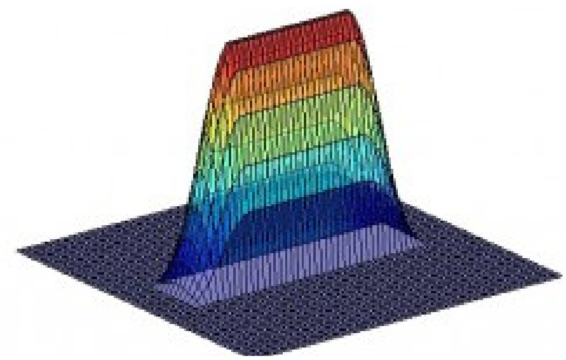
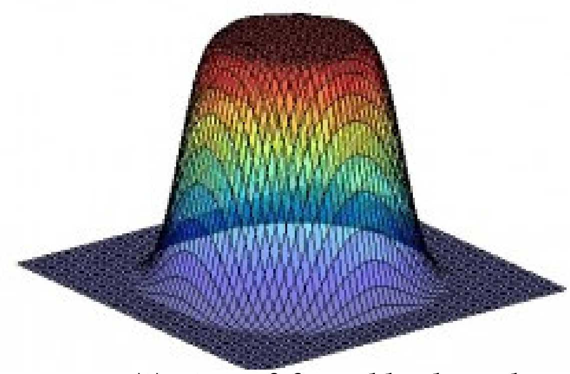


Image (1) : Motion blur kernel



Image(2) : Out-of-focus blur kernel

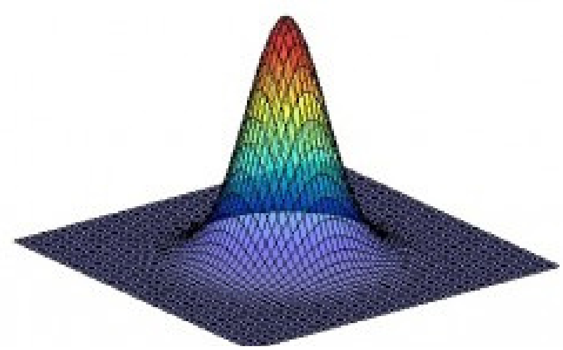


Image (3) : Gaussian blur kernel

In this report, we will demonstrate the simplest methods of image restoration when the actual spatial convolution filter (i.e., the type of the blur) used to degrade an image is known. It is important to understand that all the examples were artificially created (using motion blur kernel) to show the basic concept of image restoration techniques.

Image (4) : Original image Lenna

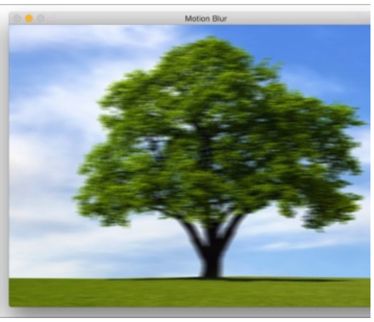


Image (5) : Degraded image (motion blur is applied here)

Form (1) Comparison

Inverse Filtering

The idea of the inverse filtering method is to recover the original image from the blurred image by inversing blurring filter. We assume that no additional noise is present in the system. We need to find such a filter kernel r(x,y) that its convolution with the blurred image could produce a result close to the original image:

(g * r) = f,

(f * h) * r = f,

or since the convolution operation is associative and has multiplicative identity,

h * r = Or',

where Or(x,y) is the Kronecker's delta-function.Then, applying the convolution theorem in frequency domain we obtain:

H(u,w) * R(u,w) = 1.

Thus, dividing the DFT of the blurred image by the DFT of the kernel, we can recover (in theory) the original image. Then the inverse filter in the frequency domain is simply R(u,w) = Rinv(u,w) in the frequency domain is 1 / H(u,w).

Using point-wise multiplication of inverse filter Rinv(u,w) with Fourier transform of the blurred image G (u,w), we get Fourier to transform of the restored image. Now using inverse DFT we obtain the final restored image.

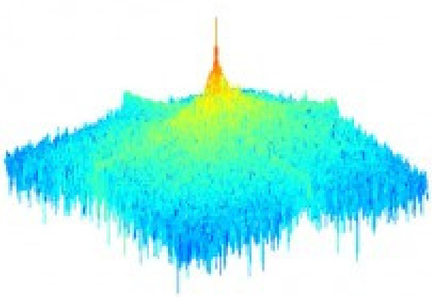


Image (6) : Spectrum of the original image

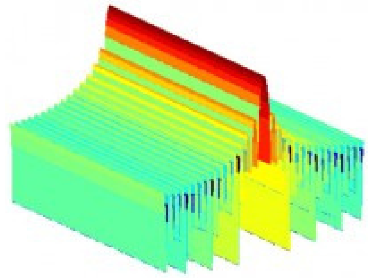


Image (7) : Spectrum of the motion blur kernel

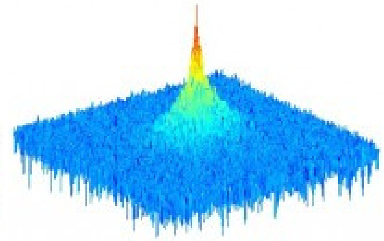


Image (8) : Spectrum of the degraded (blurred) image

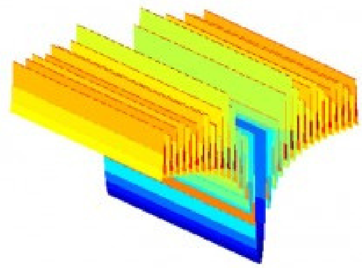


Image (9) : Spectrum of the inverse filter

Wiener Filtering

The second technique widely used in image restoration is known as a Wiener filtering. This restoration method assumes that noise which is present in the system is additive white Gaussian noise and it minimizes mean square error between original and restored images. Wiener filtering normally requires prior knowledge of the power spectra (spectral power densities) of the noise and the original image. Spectral power density is a function that describes power distribution over the different frequencies.A simplified equation of the Wiener filter Rw is given below:

$$R_W(u,w) = \frac{H(u,w)^*}{|H(u,w)|^2 + \dfrac{S_n(u,w)}{S_f(u,w)}}$$

where Sn(u,w) is the spectral power density of the noise and Sf(u,w) is the spectral power density of the image.Then using inverse DFT we obtain the restored image.

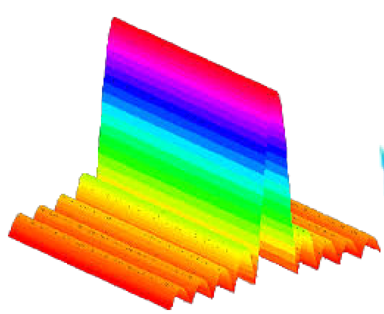


Image (10) : Spectrum of the Wiener filter

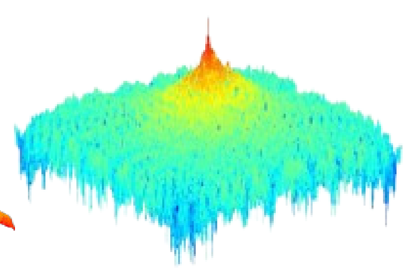


Image (11) : Spectrum of the Wiener-filtered image

Often, we can assume that

$$\frac{S_n(u,w)}{S_f(u,w)} = const = K$$

frequency aspect of the Wiener filter. The Wiener filter behaves as a bandpass filter, where the high pass filter is an inverse filter and the low pass filter is specified by the parameter K. Note how Wiener filter becomes an inverse filter when K = 0.

Below you can see the comparison of restoration results obtained by using an inverse filter and Wiener filter.

Image (12) : Image restored with inverse filter

Image (13) : Image restored with Wiener filter)

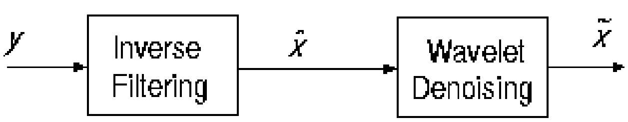
Form (2) Comparison

- **Wavelet-based Image Restoration**

frequency aspect of the Wiener filter. The Wiener filter behaves as a bandpass filter, where the high pass filter is an inverse filter and the low pass filter is specified by the parameter K. Note how Wiener filter becomes an inverse filter when K = 0.

Theory

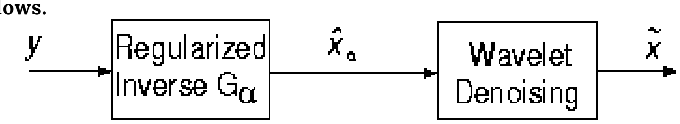
Although the Wiener filtering is the optimal tradeoff of inverse filtering and noise smoothing, in the case when the blurring filter is singular, the Wiener filtering actually amplify the noise. This suggests that a denoising step is needed to remove the amplified noise. Wavelet-based denoising scheme, a successful approach introduced recently by Donoho, provides a natural technique for this purpose. Therefore, the image restoration contains two separate steps: Fourier-domain inverse filtering and wavelet-domain image denoising. The digram is shown as follows.



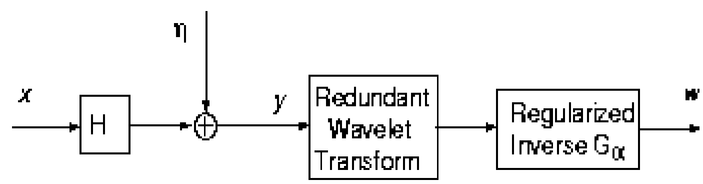
Donoho's approach for image restoration improves the performance, however, in the case when the blurring function is not invertible, the algorithm is not applicable. Furthermore, since the two steps are separate, there is no control over the overall performance of the restoration. Recently, R. Neelamani et al. proposed a wavelet-based deconvolution technique for ill-conditioned systems. The idea is simple: employ both Fourier-domain Wiener-like and wavelet-domain regularization. The regularized inverse filter is introduced by modifying the Wiener filter with a new-introduced parameter:

$$G_\alpha = \frac{H^* S_{xx}}{|H|^2 S_{xx} + \alpha S_{\eta\eta}}.$$

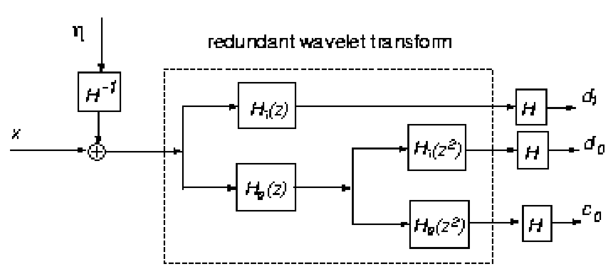
The parameter can be optimally selected to minimize the overall mean-square error. The diagram of the algorithm is displayed as follows.



The implementation of the regularized inverse filter involves the estimation of the power spectrum of the original image in the spatial domain. Since wavelet transforms have good decorrelation property, the wavelet coefficients of the image can be better modeled in a stochastic model, and the power spectrum can be better estimated. This inspires a new approach: changing the order of the regularized inverse filtering and the wavelet transform. (See the following digram)



This way the both inverse filtering and noise smoothing can be performed in wavelet domain. Specifically, the power spectrum of the image in a same subband can be estimated under the assumption that the wavelet coefficients are independent. Therefore, the power spectrum is just the variance of the wavelet coefficients. We note that the exchange of the order of inverse filtering and wavelet transform is valid only when undecimated wavelet transform is used and the blurring function is separable. Therefore, for interpretation we can exchange the order of the blurring operation and the wavelet transform, which means that the inverse filtering cancels the blurring in the wavelet domain. So, wavelet thresholding results in a reasonable estimate. The above explanation can be visualized using the following figure.



Simulation

As usual, we corrupted the standard 256x256 Lena test image by convolving wit the simple 4x4 square blurring filter

$$H = \frac{1}{16}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix},$$

and adding zero-mean white Gaussian noise of variance 100. The three introduced wavelet-based image restoration algorithms are applied to the corrupted image, and the results are reported in the following table. According to the visual performance and the mean square error, the algorithms improve the restoration performance. However, the denoising step uses wavelet thresholding to remove the noise, the images are blurred a little bit again, although the MSE is improved.

Image (14): Blurred Lena ImagePSNR = 23.2993, MSE = 304.1938

Image (15): Restored Lena Image (Donoho)PSNR = 16.8552, MSE = 1341.4

Image (16): Restored Lena Image (WaRD)PSNR = 19.5115, MSE = 727.7

Image (17): Restored Lena Image (Subband)PSNR 20.1223, MSE = 632.2

Form (3) Comparison

BLIND DECONVOLUTION

To this point, we have studied restoration techniques assuming that we knew the blurring function h . Actually, we have also assumed that we knew the image spectral density Suu and Spectral noise Snn as well. This section will focus on some techniques for estimating h based on our degraded image. For comparison, we will demonstrate how the MSE between our restored image and the original image changes depending on whether or not we know h, Suu, or Snn.Two restoration filters will be the basis for our procedures. The first is the Wiener Filter, which exhibits the optimal tradeoff (in the MSE sense) between inverse filtering and noise smoothing. The second filter tries to restore the power spectrum of the degraded image, and is known as Power Spectrum Equalization [Lim].We use as our degradation model the standard idea that our input image is blurred through convolution with a low pass LSI filter (h) and then Gaussian Noise is added to the result. Moreover, because Power Spectrum Equalization (PSE) works best assuming h is phaseless, so we generate our h to have zero phase. This is not too unrealistic because common degradations such as camera misfocus, uniform motion (linear phase), and atmospheric turbulence can all be modelled with zero phase filters. Note also that because we used a different filter for our degradation model than for Inverse Filtering and Wavelet Denoising, the MSEs of our restored images for this section should not be compared to those from the previous sections. Let's begin bv recalling the Wiener filter:

$$G = \frac{H^* S_{uu}}{|H|^2 S_{uu} + S_{\eta\eta}}$$

where H is the Fourier Transform of h, and Suu and Snn are defined as above. The following example shows lenna.256 degraded with our phaseless filter and AGN with variance 80. Suu is estimated as the magnitude squared of the Fourier Transform of the input image (lenna.256).

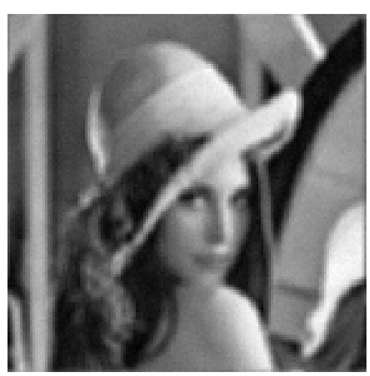


Image (18): Lenna after Blurring Plus Noise

Mean Squared Error = 1.0660e+05

Image (19): Lenna restored using Wiener Filter

Mean Squared Error = 123.2

Form (3) Comparison

Simulation

As usual, we corrupted the standard 256x256 Lena test image by convolving wit the simple 4x4 square blurring filter

$$H = \frac{1}{16}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix},$$

and adding zero-mean white Gaussian noise of variance 100. The three introduced wavelet-based image restoration algorithms are applied to the corrupted image, and the results are reported in the following table. According to the visual performance and the mean square error, the algorithms improve the restoration performance. However, the denoising step uses wavelet thresholding to remove the noise, the images are blurred a little bit again, although the MSE is improved.

Image (14): Blurred Lena ImagePSNR = 23.2993, MSE = 304.1938

Image (15): Restored Lena Image (Donoho)PSNR = 16.8552, MSE = 1341.4

Image (16): Restored Lena Image (WaRD)PSNR = 19.5115, MSE = 727.7

Image (17): Restored Lena Image (Subband)PSNR 20.1223, MSE = 632.2

Form (3) Comparison

BLIND DECONVOLUTION

To this point, we have studied restoration techniques assuming that we knew the blurring function h . Actually, we have also assumed that we knew the image spectral density Suu and Spectral noise Snn as well. This section will focus on some techniques for estimating h based on our degraded image. For comparison, we will demonstrate how the MSE between our restored image and the original image changes depending on whether or not we know h, Suu, or Snn.Two restoration filters will be the basis for our procedures. The first is the Wiener Filter, which exhibits the optimal tradeoff (in the MSE sense) between inverse filtering and noise smoothing. The second filter tries to restore the power spectrum of the degraded image, and is known as Power Spectrum Equalization [Lim].We use as our degradation model the standard idea that our input image is blurred through convolution with a low pass LSI filter (h) and then Gaussian Noise is added to the result. Moreover, because Power Spectrum Equalization (PSE) works best assuming h is phaseless, so we generate our h to have zero phase. This is not too unrealistic because common degradations such as camera misfocus, uniform motion (linear phase), and atmospheric turbulence can all be modelled with zero phase filters. Note also that because we used a different filter for our degradation model than for Inverse Filtering and Wavelet Denoising, the MSEs of our restored images for this section should not be compared to those from the previous sections. Let's begin by recalling the Wiener filter:

$$G = \frac{H^* S_{uu}}{|H|^2 S_{uu} + S_{\eta\eta}}$$

where H is the Fourier Transform of h, and Suu and Snn are defined as above. The following example shows lenna.256 degraded with our phaseless filter and AGN with variance 80. Suu is estimated as the magnitude squared of the Fourier Transform of the input image (lenna.256).

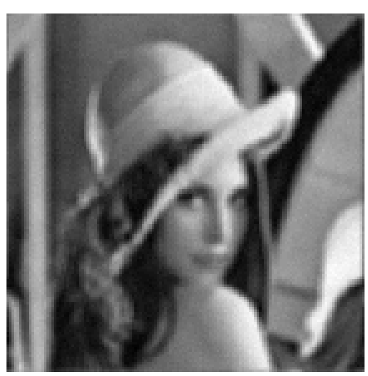


Image (18): Lenna after Blurring Plus Noise

Mean Squared Error = 1.0660e+05

Image (19): Lenna restored using Wiener Filter

Mean Squared Error = 123.2

Form (3) Comparison

The above images were generated using wien.m. Much of the blockiness is due to the compression we used on the gifs to store the images. This will be true for all the images in this section. The important things to note are the excellent reduction of MSE, and the improved definition in the boa of the restored image. In general, the blurriness of the degraded image has been removed.Next we will examine the effectiveness of Power Spectrum Equalization. The equation is as follows:

$$G = \left[\frac{S_{uu}}{|H|^2 S_{uu} + S_{\eta\eta}}\right]^{1/2}$$

Note the similarity to Wiener Filtering, but we only use the magnitude of H. The following example demonstrates restoration using the same specs as for the Wiener Filter above.

Image (20): Lenna after Blurring Plus Noise
Mean Squared Error = 1.0660e+05

Image (21): Lenna restored using PSE
Mean Squared Error = 419.5

Form (4) Comparison

The above images were generated using pse.m. We have again removed a great deal of the blur, but our restoration is not as good as with Wiener Filtering. In general, Wiener filtering is the optimal restoration technique, and this should be remembered later on. Next, we will perform the same restoration using estimated spectral noise. For this, we will assume that the noise is white and therefore has a flat (constant) spectral density. Experimentation showed us that overestimating the noise was better than underestimating the noise. This is due to the high pass characteristic of inverse filtering during restoration. When in doubt, suppress the inverse filter. The degraded images below were generated the same way as before, but Snn was estimated as 100.

Image (22): Lenna restored using Wiener Filtering
Mean Squared Error = 252.6

Image (23): Lenna restored using PSE
Mean Squared Error = 1.3932e+05

Form (5) Comparison

The above images were generated using pse.m. We have again removed a great deal of the blur, but our restoration is not as good as with Wiener Filtering. In general, Wiener filtering is the optimal restoration technique, and this should be remembered later on. Next, we will perform the same restoration using estimated spectral noise. For this, we will assume that the noise is white and therefore has a flat (constant) spectral density. Experimentation showed us that overestimating the noise was better than underestimating the noise. This is due to the high pass characteristic of inverse filtering during restoration. When in doubt, suppress the inverse filter. The degraded images below were generated the same way as before, but Snn was estimated as 100.

Image (24): Lenna restored using Wiener Filtering
Mean Squared Error = 132.4

Image (25): Lenna restored using PSE
Mean Squared Error = 371.9

Form (6) Comparison

The above images were generated using wien3.m and pse3.m, respectively. Again, we see that the Wiener filter has a lower MSE than the PSE filter. Also, for the Wiener Filter, our MSE isn't much worse than when we estimate Suu from the original image. For the PSE, our MSE has actually improved! We attribute this to the fact that our estimation of Suu isn't exact in either case. So generally, using the degraded image to estimate Suu won't hurt us with either restoration filter. The biggest unknown that hurts us so far is Snn for the PSE filter.

BLIND DECONVOLUTION

Many different methods were attempted to restore our image when we don't explicitly know h. Most of them had very little success. The reasons will be explained as we explain the general approach we used. The methods for estimating h are known as Blind Deconvolution because our inverse filtering (deconvolution) is being performed without knowledge of our blurring function. The methods we used were all homomorphic ideas. In general, our degradation is modelled as a convolution plus noise. In the frequency domain, convolution becomes multiplication. If we ignore the additive noise, we can take the log of the multiplication and get addition. Thus, the log of the FT of our degraded image DI is equal to the log of the FT of the original image OI plus the log of the Transfer Function H. Now that we have added, we can use statistical estimation to estimate H and thus solve for OI. The problem with this method is that in practice we can't ignore the noise. Therefore, we need ways to estimate the log of the multiplication of OI and H plus the Noise Spectrum. The first approach we had success with came from the Jain text. It uses the following estimate for H.

$$\log|H| = \frac{1}{M}\sum_{k=1}^{M}[\log|V_k| - \log|U_k|]$$

Uk and Vk are obtained by breaking the input image (u) and degraded image (v) into M smaller blocks and computing their Fourier Transforms. H is then used with Snn and Suu to compute the Wiener Filter. Notice that this method only computes Magnitude of H, so its best for phaseless LSI filters. This necessitated our filter design of a phaseless h. The following pictures show the Magnitude Plots of the actual and estimated transfer functions. Our image degradation model is the same as always, and we calculated H using the above equation with M = 16. This broke down the images into 64x64 pixel blocks.

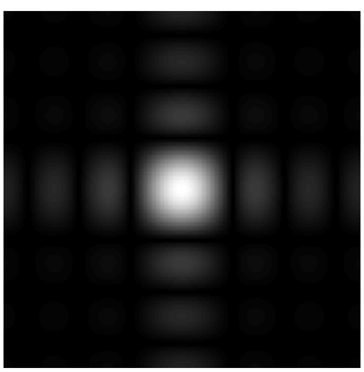


Image (24): Magnitude Plot of Blurring Filter

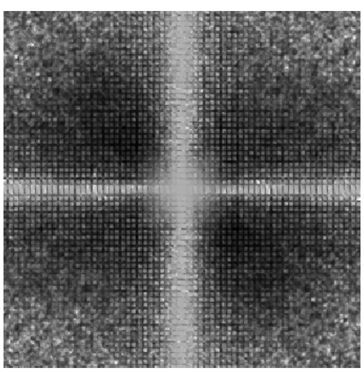


Image (25): Magnitude Plot of Estimated Blurring Filter

Form (6) Comparison

Note that we capture the form of the degradation filter, but we have a lot more noise. Surprisingly, our restoration MSE isn't too bad, depicted below. Both the above and below images were created using jain.m with noise variance 80. Also, since we are restoring using Wiener Filtering, we estimate Snn and Suu for restoration.

Image (26): Lenna after Blurring Plus Noise
Mean Squared Error = 1.0660e+05

Image (27): Lenna restored using Jain Approach
Mean Squared Error = 283.1

Form (7) Comparison

We see that our MSE has gone from 256 with unknown Snn to 283 with all three Wiener Filter componenents (h, Suu, * Snn). We can see that some of the blurring has been reduced in our restored image. Lines can be seen in the band around the hat and the boa is a bit clearer. However, even though our MSE isn't too bad, this is clearly the worst image restoration thus far with respect to visual aspects. Most of the restoration was magnitude restoration. But this was by far our greatest success at blind deconvolution.Our second approach came from the Lim text. It was proposed by Stockham. The homomorphic idea of taking the log in the frequency domain is again present, as is the notion of breaking up the picture into M sub-blocks. The equation estimates the denominator of the PSE filter as follows:

$$[|H|^2 S_{uu} + S_{\eta\eta}]^{1/2} = e^{-C/2}\prod_{i=1}^{M}|S_{vv}|^{1/M}$$

where C is Euler's constant (0.57221...). Note that H is never calculated directly. Instead, we use this estimation to choose our PSE filter and restore our image. Our attempt at restoration with this method is shown below. Again, variance is 80, but Suu and Snn are assumed known. Recall that if Snn is not explicitly known, then our PSE restoration fails.

Image (28): Lenna after Blurring Plus Noise
Mean Squared Error = 1.0656e+05

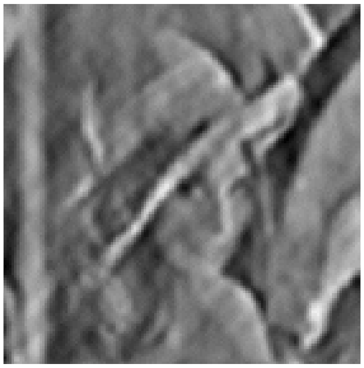


Image (29): Lenna restored using Stockham Approach
Mean Squared Error = 7.4031e+07

Form (7) Comparison

The above images were generated using Stockham.m. Clearly, our restoration has further degraded our image. This was seen earlier when we attempted PSE restoration without knowledge of Snn. In this case, we are trying to estimate the entire denominator, not just Snn. Our restoration suffers greatly.

We conclude that Power Spectrum Equalization is just a poor restoration choice in general. It fails for unknown Snn and for our attempt at unknown h. The lack of inverse filtering makes the denominator extremely important in restoration, so no estimation will be very robust. Supposedly, there are cases where PSE is preferable to Wiener Filtering [Castleman], but our models do not fit them.

Conclusions

We can describe image blurring process in most common cases as its convolution with some blur kernels. To restore degraded image we use deconvolution techniques such as inverse filtering and Wiener filtering. But we can observe that even in our examples (where the image was first artificially blurred by convolution) we didn’t get the expected results — restored images do not look exactly like the originals (especially for inverse filtering). This happens because small values of the Fourier transformed blur kernel turn to big ones of the restoration filter which significantly amplifies the noise.

References

- Wiener N (1964).Extrapolation, Interpolation, and Smoothing of Stationary Time Series. Cambridge, Mass: MIT Press. ISBN 0-262-73005-7.
- K. R. Castleman, Digital Image Processing, Prentice-Hall, 1996
- A. K. Jain, Fundamentals of Digital Image Processing, Prentice-Hall,
- J. S. Lim, Two-Dimensional Signal and Image Processing, Prentice-Hall, 1990
- M. R. Banham and A. K. Katsaggelos, "Spatially Adaptive Wavelet-Based Multiscale Image Restoration," IEEE Trans. Image Processing, vol. 5, no. 4, pp. 619-634, April 1996.
- J. Biemond and A. K. Katsaggelos, ``A New Iterative Restoration Scheme for Noisy Blurred Images,'' Proc. Conf. on Math. Methods in Signal Processing, pp. 74-76, Aachen, W. Germany, September 1984.
- J. Biemond and A. K. Katsaggelos, ``Iterative Restoration of Noisy Blurred Images,'' Proc. 5th Inf. Theory Symposium in the Benelux, pp. 11-20, Aalten, The Netherlands, May 1984.
- A. K. Katsaggelos, ``A General Formulation of Adaptive Iterative Image Restoration Algorithms,'' Proc. 1986 Conf. Inf. Sciences and Systems, pp. 42-47, Princeton, NJ, March 1986.
- A. K. Katsaggelos, J. Biemond, R. M. Mersereau and R. W. Schafer, ``An Iterative Method for Restoring Noisy Blurred Images,'' Proc. 1984 Int. Conf. Acoust., Speech, Signal Processing, pp. 37.2.1-37.2.4, San Diego, CA, March 1984.
- R. Neelamani, H. Choi, and R. G. Baraniuk, Wavelet-based deconvolution for ill-conditioned systems, IEEE Conference on Acoustics, Speech, and Signal Processing (ICASSP), vol. 6, pp. 3241-3244, Phoenix, AZ, March 15-19, 1999. Abstract.

MAGE RESTORATION REPORT

Written by : TAMEEM HEZAM

GRADUATE STUDENT IN NANJING UNIVERSITY OF SCIENCE AND TECHNOLOGY.

Written by TAMEEM HEZAM . **PROFESSOR** Jin Cun.

南京理工大学

NANJING UNIVERSITY OF SCIENCE & TECHNOLOGY

https://github.com/tememway/image-restoration-report-.git