AESB3341-18 Petrophysics & Image Analysis

Teemu Weckroth, June 30, 2024

2. Constitution of rocks, rock-fluid behaviour

2.4 Geo-temperatures, geo-pressures, time-effects

2.4.1 In-situ temperatures and related salinities

Geothermal gradient

$$G_t = \frac{T_f - T_s}{D} \tag{2.1}$$

Arrhenius' equation

$$k = A\exp\left(-\frac{E_\mathrm{a}}{RT}\right) \quad \text{or} \quad \ln(k) = \ln(A) - \frac{E_\mathrm{a}}{RT} \tag{2.2}$$

Salinity conversion ratio

$$S_1 = \frac{S_2}{\rho_\mathrm{sol}} \cdot 1000$$

where:

S_1 = salinity in ppm
S_2 = salinity in $\mathrm{g\,mol^{-1}}$
P_sol = solution density in $\mathrm{g\,L^{-1}}$

Arp's empirical relation

Fahrenheit:

$$R_{\mathrm{w}T_2} = R_{\mathrm{w}T_1}\left(\frac{T_1 + 6.77\,°\mathrm{F}}{T_2 + 6.77\,°\mathrm{F}}\right)$$

Celsius:

$$R_{\mathrm{w}T_2} = R_{\mathrm{w}T_1}\left(\frac{T_1 + 21.5\,°\mathrm{C}}{T_2 + 21.5\,°\mathrm{C}}\right) \tag{2.4}$$

Arp's empirical relation for fresh water, $T_1 = 24\,°\mathrm{C}$

Fahrenheit:

$$R_{\mathrm{w}T_2} = R_{\mathrm{w}75}\left(\frac{81.77\,°\mathrm{F}}{T_2 + 6.77\,°\mathrm{F}}\right)$$

Celsius:

$$R_{\mathrm{w}T_2} = R_{\mathrm{w}24}\left(\frac{45\,°\mathrm{C}}{T_2 + 21.5\,°\mathrm{C}}\right) \tag{2.5}$$

NaCl concentration equivalents

$$C_\mathrm{sum} = \sum_{a=1}^{n} M_a C_{ai} \tag{2.6}$$

Water resistivity from known ion concentration

$$R_{\mathrm{w}75} = \left(2.74 \cdot 10^{-4} C_\mathrm{sum}\right)^{-1} + 0.0123 \tag{2.7}$$

2.4.2 In-situ pressures

Total overburden pressure

$$P_0 = P_r + P_f \tag{2.8}$$

Effective stress

$$\sigma_p = P_0 - P_f \tag{2.9}$$

2.4.3 Effects of time

$$G = \frac{T \cdot t \cdot P}{1000} \tag{2.10}$$

Maturity increment

$$\Delta M = \Delta T r^n \tag{2.11}$$

2.5 Rock bulk densities and matrix densities

2.5.1 Density definitions

True density (or matrix density)

$$\rho_m = \frac{\text{mass}}{V_\mathrm{total} - V_\mathrm{pore}} \tag{2.12}$$

Apparent specific gravity

$$G_\mathrm{sa} = \frac{\text{dry weight in air}}{\text{dry weight in air} - \text{submerged weight}}$$

Bulk density

$$\rho_b = \frac{\text{mass}}{\text{volume including all voids}}$$

or

$$\rho_b = \frac{\text{mass}}{\text{unit volume including the fluid}}$$

Bulk specific gravity

$$G_{sb} = \frac{\text{dry weight in air}}{\text{saturated weight} - \text{submerged weight}}$$

Grain density (or matrix density)

$$\rho_g = \sum_{i=1}^{n} \rho_i \cdot v_i \tag{2.13}$$

2.5.2 Laboratory measurement methods

Dry bulk density

$$\rho_b = \frac{W_g}{V_b} \tag{2.14}$$

Natural bulk density

$$\rho = \frac{W_g + W_w}{V_b} \tag{2.15}$$

Saturated bulk density

$$\rho_s = \frac{W_g + (V_p \cdot \rho_w)}{V_b} \tag{2.16}$$

Bulk density from the buoyancy method

$$\rho_b = \frac{W_1}{W_2 - W_3} \cdot \rho_w \tag{2.17}$$

Grain density from the buoyancy method

$$\rho_g = \frac{W_1}{W_1 - W_3} \cdot \rho_w \tag{2.18}$$

Grain volume from Boyle's law

$$(V_0 - V_\mathrm{ma}) \cdot P_1 = (V_0 - V_\mathrm{ma} + \mathrm{d}V) \cdot P_2 \tag{2.19}$$

4. Rock porosity, permeability, and capillary pressure

4.1 Porosity

4.1.2 Porosity definition

Total porosity

$$\phi_t = \frac{V_b - V_\mathrm{ma}}{V_b} \quad \text{or} \quad \phi_t = \frac{V_p}{V_b} \quad \text{or} \quad \phi_t = \frac{V_p}{V_\mathrm{ma} + V_p} \tag{4.1, 4.2, 4.3}$$

4.1.3 Porosity values and spatial characteristics

Trask sorting

$$S_0 = \sqrt{\frac{S_{25}}{S_{75}}} \tag{4.4}$$

4.1.5 Laboratory analysis of porosity (direct measurement)

Pore volume

$$V_\mathrm{pore} = \frac{W_2 - W_1}{\rho_\mathrm{fluid}} \tag{4.5}$$

Effective porosity

$$\phi_\mathrm{eff} = \frac{V_b - V_\mathrm{grain}}{V_b} \tag{4.6}$$

4.2 Permeability

4.2.2 Flow in tubes (Poiseuille)

Poiseuille's law for flow through a cylindrical tube

$$Q = \frac{\pi \cdot r^4 \cdot \Delta p}{8 \cdot \mu \cdot L} \tag{4.7}$$

4.2.3 Permeability measured on core samples

Darcy's law

$$Q = \frac{-k(h_2 - h_1)}{l} \tag{4.8}$$

Darcy's law generalization

$$Q = \frac{k \cdot A \cdot \Delta p}{\eta \cdot L} \tag{4.9}$$

4.2.4 Correction for a flowing medium in laboratory measurements

Darcy's law for gases

$$Q = \frac{k \cdot A \cdot \Delta p \cdot \overline{p}}{\eta \cdot L \cdot p_\mathrm{atm}} \quad \text{at } T = 20\,°\mathrm{C},\; p_\mathrm{atm} = 1\,\mathrm{atm} \tag{4.10}$$

4.2.5 Correction for gas slippage

Apparent permeability for a gas

$$k_a = k\left(1 + \frac{b}{\overline{p}}\right) \tag{4.11}$$

4.2.6 Correction for turbulence

Forchheimer's adaption of Darcy's law

$$\frac{\mathrm{d}p}{\mathrm{d}L} = \eta \cdot \frac{v}{k} + \beta \cdot \rho \cdot v^2 \tag{4.12}$$

4.2.8 Relation between pore space and permeability

Kozeny equation for parallel capillary tubes

$$k = \frac{n \cdot \pi \cdot r^4}{8A_c} \tag{4.13}$$

Porosity as the ratio of pore volume and bulk volume

$$\phi = \frac{V_p}{V_b} = \frac{n \cdot \pi r^2}{A_c} \tag{4.14}$$

Kozeny equation with porosity

$$k = \frac{\phi \cdot r^2}{8} \tag{4.15}$$

Internal surface area per unit pore volume

$$S_{V_p} = \frac{2}{r}$$

Total area in the pore space per unit of grain volume

$$S_{V_{\mathrm{gr}}} = S_{V_p} \cdot \frac{\phi}{1-\phi} \tag{4.16}$$

Combining equations 4.13 and 4.16 and substituting S_{V_p}

$$k = \left(\frac{1}{2S_{V_{\mathrm{gr}}}^2}\right) \cdot \frac{\phi^3}{(1-\phi)^2} \tag{4.17}$$

Tortuosity

$$\tau = \left(\frac{L_a}{L}\right)^2 \tag{4.18}$$

Insert tortuosity into equation 4.17

$$k = \left(\frac{1}{2\tau \cdot S_{V_{\mathrm{gr}}}^2}\right) \cdot \frac{\phi^3}{(1-\phi)^2} \tag{4.19}$$

General Kozeny relation for $2\tau = 5$

$$k = \left(\frac{1}{5S_{V_{gr}}^2}\right) \cdot \frac{\phi^3}{(1-\phi)^2} \tag{4.20}$$

4.2.9 Empirical relationships

Van Baaren's empirical relationship

$$k = 10D_{\mathrm{dom}}^2 \cdot C^{-3.64} \cdot \phi^{m+3.64} \tag{4.21}$$

4.2.10 Permeability from logs

Correlation of core permeability with core porosities (examples)

$$k = 10^{(C_1 + C_2 \cdot \log \phi)} \tag{4.22}$$

and

$$k = 10^{(C_1 + C_2 \cdot \phi)} \tag{4.23}$$

Wyllie and Rose equation

$$k = \left(100 \cdot \phi^2 \cdot \frac{1 - S_{\mathrm{wirr}}}{S_{\mathrm{wirr}}}\right)^2 \tag{4.24}$$

4.3 Capillarity

4.3.2 Surface tension

Surface energy force balance relations

$$F = 2\gamma l \tag{4.25}$$

and

$$\gamma = \frac{F}{2l} \tag{4.26}$$

Soap bubble surface tension

$$2\pi r \gamma = \pi r^2 \Delta p \tag{4.27}$$

4.3.4 Capillary pressures in a tube

Capillary pressure in terms of the radius of the tube

$$\Delta p = p_1 - p_2 = \frac{2 \cdot \gamma \cdot \cos(\theta)}{r} \tag{4.28}$$

Capillary pressure for air and water system

$$\Delta p = p_c = (\rho_{\mathrm{water}} - \rho_{\mathrm{air}}) \cdot g \cdot h \tag{4.29}$$

Capillary rise

$$h = \frac{p_c}{(\rho_{\mathrm{water}} - \rho_{\mathrm{air}}) \cdot g} \tag{4.30}$$

Capillary rise for air and water system

$$h = \frac{2 \cdot \gamma \cdot \cos(\theta)}{r \cdot g \cdot (\rho_{\mathrm{water}} - \rho_{\mathrm{air}})} \tag{4.31}$$

Capillary rise for air and oil system

$$h = \frac{2 \cdot \gamma \cdot \cos(\theta_{\mathrm{o/w}})}{r \cdot g \cdot (\rho_{\mathrm{water}} - \rho_{\mathrm{oil}})} \tag{4.32}$$

4.3.7 Conversion from laboratory to reservoir conditions

Corrections for differences in contact angles and interfacial tensions

$$p_C(\mathrm{Hg/air}) = \frac{2 \cdot \gamma \cdot \cos(\theta)}{r} = \frac{2 \cdot 480 \cdot 0.776}{r} \tag{4.33}$$

$$p_C(\mathrm{oil/air}) = \frac{2 \cdot \gamma \cdot \cos(\theta)}{r} = \frac{2 \cdot 35 \cdot 1}{r} \tag{4.34}$$

$$\frac{p_C(\mathrm{Hg/air\ at\ surface})}{p_C(\mathrm{oil/air\ in\ reservoir})} = \frac{480 \cdot 0.776}{35} = 10.5 \approx 10 \tag{4.35}$$

$$\frac{p_C(\mathrm{Hg/air\ at\ surface})}{p_C(\mathrm{gas/air\ in\ reservoir})} = \frac{480 \cdot 0.776}{72} = 5.1 \approx 5 \tag{4.36}$$

From lab capillary pressure to equivalent height above FWL

$$h = \frac{\gamma_{\mathrm{res}} \cdot \cos(\theta)_{\mathrm{res}} \cdot p_{C(\mathrm{max\ lab})} \cdot C}{\gamma_{\mathrm{lab}} \cdot \cos(\theta)_{\mathrm{lab}} \cdot g \cdot \Delta\rho} \tag{4.37}$$

4.5 Capillarity pressures, saturation height functions

Leverett's j-function

$$J = \frac{p_c \sqrt{\left(\frac{k}{\phi}\right)}}{\sigma \cdot \cos\theta} \tag{4.38}$$

Capillary pressure transposed from mercury data

$$\frac{p_{\mathrm{cw-o}}}{\sigma_{\mathrm{w-o}} \cdot \cos\theta^{\circ}} = \frac{p_{\mathrm{cw-a}}}{\sigma_{\mathrm{w-a}} \cdot \cos\theta^{\circ}} = \left(\frac{p_{\mathrm{cHg}}}{\sigma_{\mathrm{Hg}} \cdot \cos 140^{\circ}}\right) \cdot \sqrt{\frac{k}{\phi}} \tag{4.39}$$

Relationship between contact angle and saturation of water–oil systems

$$\cos\theta_{\mathrm{a-w}} = 1.0 = \left(\frac{p_{\mathrm{c-aw}}}{\sigma_{\mathrm{aw}}}\right) \cdot \left(\frac{r}{2}\right) = f(S_w) \tag{4.40}$$

Oil-displacing water capillary pressure curve

$$\cos\theta_{\mathrm{o-w}} = \left(\frac{p_{\mathrm{c-ow}}}{\sigma_{\mathrm{ow}}}\right) \cdot \left(\frac{r}{2}\right) = f(S_w) \tag{4.41}$$

Water–oil system contact angle as function of wetting phase saturation

$$\cos\theta_{\mathrm{o-w}} = \left(\frac{p_{\mathrm{c-ow}}}{\sigma_{\mathrm{ow}}}\right) \cdot \left(\frac{p_{\mathrm{c-aw}}}{\sigma_{\mathrm{aw}}}\right) = f(S_w) \tag{4.42}$$

5. Rock resistivity, conductivity, and natural electric potential

Rock resistivity in general

Resistance

$$r = \frac{E}{I} \tag{5.1}$$

Specific resistance or resistivity

$$R = R_0 = \frac{E \cdot A}{I \cdot L} \tag{5.2}$$

Conductivity

$$C = \frac{1}{R} \tag{5.3}$$

5.1 Resistivity of a multi-component system

5.1.2 The relation between F_R and rock porosity

Cross-sectional area of n capillary tubes

$$A_n = \phi A \tag{5.4}$$

Resistivity of brine in capillary pore space

$$R_{\mathrm{w\ cap}} = \frac{E \cdot A_n}{I_{\mathrm{w\ cap}} \cdot L} \tag{5.5}$$

Formation resistivity factor for capillary tubes system

$$F = F_R = \frac{R_0}{R_{\mathrm{w\ cap}}} = \frac{A}{A_n} \cdot \frac{I_{\mathrm{w\ cap}}}{I_0} \tag{5.6}$$

Dependency between formation resistivity factor and porosity

$$F = F_R = \frac{1}{\phi} \tag{5.7}$$

Resistivity for porous system consisting of grains

$$R_{\mathrm{w\ cap}} = \frac{E \cdot \phi \cdot A_n}{I_{\mathrm{w\ cap}} \cdot L} \tag{5.8}$$

Formation resistivity factor for porous system consisting of grains

$$F_R = \frac{1}{\phi} \cdot \frac{L_a}{L} = \frac{\sqrt{\tau}}{\phi} \tag{5.9}$$

5.1.3 Relation between F_R and matrix cementation/ compaction

First Archie equation

$$F = F_R = \frac{1}{\phi^m} = \frac{R_0}{R_w} = \frac{C_w}{C_0} \tag{5.10}$$

Straight-line relationship from first Archie equation

$$\log F = -m \log \phi \tag{5.11}$$

Humble Archie

$$F = F_R = \frac{a}{\phi^m} \tag{5.12}$$

In-situ m-values

$$m = \frac{-\log F}{\log \phi} \tag{5.13}$$

5.1.4 The relation between F_R and the water content S_W

Water saturation related to the porosity and resistivity index

$$S_w = \left(\frac{R_0}{R_t}\right)^{\frac{1}{n}} = \left(\frac{F_R \cdot R_w}{R_t}\right)^{\frac{1}{n}} \tag{5.14}$$

Resistivity index (amount of hydrocarbons in the pores)

$$I_R = \frac{R_t}{R_0} = \frac{C_0}{C_t} \tag{5.15}$$

Water saturation for less homogeneous textures

$$S_w = \left(\frac{a \cdot R_w}{\phi^m \cdot R_t}\right)^{\frac{1}{n}} \tag{5.16}$$

Second Archie equation

$$I_R = \frac{R_t}{R_0} = S_w^{-n} \tag{5.17}$$

5.1.5 Water saturation calculations using Archie; laboratory and wild life

General equation combining 5.10 and 5.16

$$C_t = \phi^m \cdot S_w^n \cdot C_w \quad \text{or} \quad R_t = \phi^{-m} \cdot S_w^{-n} \cdot R_w \tag{5.18}$$

5.2 Resistivity logging tools

5.2.2 Electrical surveys (ES)

Potential difference for a sphere

$$E_m - E_n = \sum_{AM}^{AN} \frac{I \cdot R}{4 \cdot \pi \cdot L^2} \, \mathrm{d}L = \frac{I \cdot R}{4 \cdot \pi \cdot AM} \tag{5.19}$$

Potential difference for a sphere, rearranged

$$R = \frac{K_n \cdot \Delta E}{I} \tag{5.20}$$

5.2.5 Micro-resistivity devices

Flushed zone resistivity

$$R_{x0} = \frac{E_{M0} - E_{M1}}{I_0} \tag{5.21}$$

5.4 The electrochemical component

5.4.2 Membrane potential

$$E = k \log \frac{C_w}{C_{mf}} \tag{5.22}$$

$$E = E_j + E_m = (-71) \log \frac{R_{mf}}{R_{wf}} \tag{5.23}$$

5.6 The combination of SP components

$$E_{\text{total}} = I \cdot R_m + I \cdot R_{mc} + I \cdot R_{x0} + I \cdot R_t + I \cdot R_{sh} \tag{5.24}$$

$$E_{\text{total}} = E_m + E_j + E_{kmc} + E_{ksh} \tag{5.25}$$

5.7 Shale volume calculation

$$V_{sh} = \frac{PSP - SSP}{SSP}$$

5.10 A water saturation equation: practice

5.10.2 Water bearing reservoirs

Conductivity of the shaly water-bearing sand

$$C_0 = \frac{1}{F^*}(C_w + C_e) \tag{5.27}$$

Conductivity of the clay fraction

$$C_e = B \cdot Q_v \tag{5.28}$$

Empirical relation from 120 °F **to** 390 °F

$$B \cdot R_w = 13.5 \cdot Sal^{-0.70} \tag{5.29}$$

Empirical relation around 80 °F

$$B \cdot R_w = 6 \cdot Sal^{-0.64} \tag{5.30}$$

5.10.3 Hydrocarbon bearing reservoirs

$$Q'_v = \frac{Q_v}{S_w} \tag{5.31}$$

Corrected log reading of conductivity

$$C_t = \phi_t^{+m^*} \cdot S_{wt}^{+n^*} \cdot C_w \left(1 + \frac{R_w \cdot B \cdot Q_v}{S_{wt}}\right) \tag{5.32}$$

Juhasz normalized Q_v method

$$C_t = \phi_t^{+m^*} \cdot S_{wt}^{+n^*} \cdot C_w \left(1 + \frac{Q_{vn}}{S_{wt}}\left[\frac{C_{cw}}{C_w} - 1\right]\right) \tag{5.33}$$

6. Rock nuclear behaviour & applications

6.2 Natural radioactivity

6.2.5 The calculation of shale volumes

$$V_{\text{sh}} = \frac{GR - GR_{\text{min}}}{GR_{\text{sh}} - GR_{\text{min}}} \tag{6.1}$$

6.3 The gamma-gamma or density application

6.3.2 Interaction of gamma-rays and atoms

$$I = I_0 \cdot \exp(-\mu \cdot x) \tag{6.2}$$

or

$$\ln \frac{I}{I_0} = -\mu \cdot x \tag{6.3}$$

6.3.3 Density of the sedimentary rock

Electron density relation to bulk density

$$\rho_e = \frac{N \cdot Z \cdot \rho_b}{A} \tag{6.4}$$

Apparent bulk density relation to electron density

$$\rho_a = 1.07 \cdot \rho_e - 0.188 \tag{6.5}$$

6.3.4 Photo-electric effect of the reservoir

Photo-electric absorption cross-section

$$\tau = K \cdot Z^{4.6} \tag{6.6}$$

Photo-electric effect

$$P_e = (Z/10)^{3.6} \tag{6.7}$$

Effective photo-electric absorption cross-section per unit volume

$$U = \phi \cdot U_{fl} + (1 - \phi) \cdot U_{ma} \tag{6.8}$$

6.4 Neutron logs

6.4.2 Theoretical background

Hydrogen Index (HI)

$$HI = \frac{\text{number of H atoms}}{\text{volume} \cdot \text{number of H atoms in 1 cc H}_2\text{O}} \tag{6.9}$$

Porosity from Neutron log relation to true porosity

$$\phi_n = \phi \cdot (HI_{mf} \cdot S_{x0} + HI_{hc} \cdot (1 - S_{x0})) \tag{6.10}$$

Example: CH_4 with in-situ density $0.1\,\mathrm{g\,cm^{-3}}$ **and** $S_{xo} = 0.7$

$$\phi_n = \phi \cdot (1 \cdot 0.7 + 0.225 \cdot 0.3) = 0.77 \cdot \phi \tag{6.11}$$

6.4.3 Technical aspects and variety in neutron tools

Radium decays to Radon and Helium

$$_{88}\text{Ra}^{226} \longrightarrow {}_{86}\text{Rn}^{222} + {}_{2}\text{He}^{4} \tag{6.12}$$

α-particles from He bombard the Beryllium target

$$_{4}\text{Be}^{9} + {}_{2}\text{He}^{4} \longrightarrow {}_{6}\text{C}^{12} + {}_{0}\text{n}^{1} + \text{gamma rays} \tag{6.13}$$

The capturing nucleus becomes excited and emits γ-rays

$$_{1}\text{H}^{1} + {}_{0}\text{n}^{1} \longrightarrow {}_{1}\text{H}^{2} + \text{gamma rays} \tag{6.14}$$

7. Rock acoustical and related mechanical behaviour

7.1 General introduction

Acoustic transit time

$$\Delta t = \frac{1 \cdot 10^6}{v} \tag{7.1}$$

7.2 Basic concept

7.2.1 Stiffness of rock

Longitudinal and transverse trains

$$\varepsilon_l = \frac{\Delta L}{L} \quad \text{and} \quad \varepsilon_t = \frac{\Delta D}{D} \tag{7.2, 7.3}$$

Shear strain

$$\varepsilon_s = \frac{\Delta L}{L} = \tan\theta \tag{7.4}$$

Young's modulus

$$E = \frac{F \cdot L}{A \cdot \Delta L} \tag{7.5}$$

Poisson's ratio

$$\mu = \frac{\varepsilon_t}{\varepsilon_l} \tag{7.6}$$

Poisson's ratio in the case of a cylinder

$$\mu = \left(\frac{\Delta d}{d}\right) / \left(\frac{\Delta L}{L}\right) \tag{7.7}$$

Shear modulus

$$G = \frac{F}{A \cdot \theta} \tag{7.8}$$

Bulk modulus

$$K = \frac{p \cdot V}{\Delta V} \tag{7.9}$$

Relations between elastic constants E, μ, G, and K

$$G = \frac{E}{[2(1+\mu)]} \tag{7.10}$$

and

$$K = \frac{E}{[3(1-2\mu)]} \tag{7.11}$$

Wave propagation in elastic bodies

$$\sigma = A_0 \cos 2\pi\,[f_t - (x/\lambda)] \tag{7.12}$$

and

$$v = \lambda \cdot f \quad \text{and} \quad f = 1/t \tag{7.13, 7.14}$$

Absorption and wave amplitude

$$A = A_0 \cdot \exp(-\alpha x) \tag{7.15}$$

Wave propagation and attenuation effects

$$\sigma = A_0 \cdot \exp(-\alpha x) \cdot \cos 2\pi\,[f_t - (x/\lambda)] \tag{7.16}$$

Velocity of compression propagation

$$v_p = [(K + 4/3)G/p]^{1/2} = \left\{\frac{(E/\rho)(1-\mu)}{(1-2\mu)(1+\mu)}\right\}^{1/2} \tag{7.17, 7.18}$$

Velocity of shear propagation

$$v_s = (G/\rho)^{1/2} = \left[\frac{E/\rho}{2(1+\mu)}\right]^{1/2} \tag{7.19, 7.20}$$

Comparison between compression- and shear-wave velocities

$$\frac{v_p}{v_s} = [(4/3) + (K/G)]^{1/2} = \left\{\frac{2(1-\mu)}{1-2\mu}\right\}^{1/2} \tag{7.21, 7.22}$$

$$v_p > \sqrt{2v_s} \tag{7.23}$$

or

$$\Delta t_s > \sqrt{2\Delta t_p} \tag{7.24}$$

Snell's law

$$\frac{\sin\alpha_1}{v_1} = \frac{\sin\alpha_{21}}{v_2} = \frac{\sin\alpha_3}{v_3} \tag{7.25}$$

Angle α_2 is expressed as

$$\sin\alpha_2 = \left(\frac{v_1}{v_2}\right)\sin\alpha_1 \tag{7.26}$$

when

$$\sin\alpha_1 = \left(\frac{v_1}{v_2}\right) = \sin\alpha_C \tag{7.27}$$

Compressional critical angle

$$\sin\alpha_{pc} = \left(\frac{v_{p1}}{v_{2p}}\right) \tag{7.28}$$

Shear critical angle

$$\sin\alpha_{sc} = \left(\frac{v_{p1}}{v_{s2}}\right) \tag{7.29}$$

Relation between critical angles

$$\frac{\sin\alpha_{sc}}{\sin\alpha_{pc}} = \left(\frac{v_{p2}}{v_{s2}}\right) \tag{7.30}$$

Summed travel time

$$\frac{1}{v_b} = \frac{\phi}{v_f} = \frac{1-\phi}{v_{\text{ma}}} \tag{7.31}$$

or

$$\Delta t = \Delta t_f \phi + \Delta t_{\text{ma}}(1-\phi) \tag{7.32}$$

Porosity as a function of transit times

$$\phi = \frac{(\Delta t - \Delta t_{\text{ma}})}{(\Delta t_f - \Delta t_{\text{ma}})} \tag{7.33}$$

7.3 The practical method of approach

7.3.2 Limitations of acoustic logging

Critical shear velocity

$$\frac{\sin(\phi_{\text{formation}})}{\sin(\phi_{\text{mud}})} = \frac{V_{\text{mud}}}{V_{\text{formation}}} \tag{7.34}$$

9. Evaluation of minerals, fluids, and in-situ environments

9.2 Evaluations for oil and gas

9.2.6 Porosity determination

Bulk density as linear relation between matrix and fluid points

$$\rho_b = (1-\phi) \cdot \rho_{\text{ma}} + \phi \cdot \rho_{\text{fl}} \tag{9A.1}$$

can be rearranged to

$$\phi = \frac{\rho_{\text{ma}}\rho_b}{\rho_{\text{ma}} - \rho_{\text{fl}}} \tag{9A.2}$$

Fluid density when pores contain a mixture of mud filtrate and HCs

$$\rho_{\text{fl}} = S_{x0} \cdot \rho_{\text{mf}} + (1 - S_{x0}) \cdot \rho_{\text{hc}} \tag{9A.3}$$

Transit times according to Wyllie equation

$$\Delta T = \phi \cdot \Delta T_{\text{fl}} + (1-\phi) \cdot \Delta T_{\text{ma}} \tag{9A.4}$$

or

$$\phi = \frac{\Delta T - \Delta T_{\text{ma}}}{\Delta T_{\text{fl}} - \Delta T_{\text{ma}}} \tag{9A.5}$$

Porosity from Neutron log relation to true porosity

$$\phi_n = \phi \cdot (HI_{\text{mf}} \cdot S_{x0} + HI_{\text{hc}} \cdot (1 - S_{x0}))$$

9.2.7 Gas effects

Apparent bulk density relation to electron density

$$\rho_a = 1.07 \cdot \rho_e - 0.188$$

9.2.8 Lithology

Effective porosity for shaly sands

$$\phi_e = 1 - V_{\text{sa}} - V_{\text{sh}} \tag{9A.6}$$

Equations for neutron-density cross-plots

$$\rho_b = (1 - \phi_e - V_{\text{sh}}) \cdot \rho_{\text{ma}} + \phi_e \rho_{\text{mf}} + V_{\text{sh}} \cdot \rho_{\text{sh}} \tag{9A.7}$$

and

$$\phi_n = \phi_e + V_{\text{sh}} \cdot \phi_{\text{nsh}} \tag{9A.8}$$

Multiple mineral log evaluations, neutron and density tool

$$\rho_b = V_{\text{ma1}} \cdot \rho_{\text{ma1}} + V_{\text{ma2}} \cdot \rho_{\text{ma2}} + \phi \cdot \rho_{\text{fl}} \tag{9A.9}$$

$$\phi_n = V_{\text{ma1}} \cdot \phi_{\text{nma1}} + V_{\text{ma2}} \cdot \phi_{\text{nma2}} + \phi \cdot c \tag{9A.10}$$

$$1 = V_{\text{ma1}} + V_{\text{ma2}} + \phi \tag{9A.11}$$

Multiple mineral log evaluations, Sonic–Pe combination

$$\Delta T = V_{\text{ma1}} \cdot \Delta T_{\text{ma1}} + V_{\text{ma2}} \cdot \Delta T_{\text{ma2}} + \phi \cdot \Delta T_{\text{fl}} \tag{9A.12}$$

$$P_{\text{eb}} \cdot \rho_b = V_{\text{ma1}} \cdot U_{\text{ma1}} + V_{\text{ma2}} \cdot U_{\text{ma2}} + \phi \cdot U_{\text{fl}} \tag{9A.13}$$

9.2.9 Saturation determination from logs

Conductivity from Archie equations

$$C_t = \phi^m \cdot S_w^n \cdot C_w \tag{9A.14}$$

Estimate mud-filtrate conductivity

$$E = (-71) \cdot \log \frac{C_w}{C_{\text{mf}}} \tag{9A.15}$$

For $S_w = 1$

$$C_w = C_t/\phi^m \tag{9A.16}$$

9.2.11 Hydrocarbon reserves volume estimation

$$HCIIP = V_b \cdot \frac{N}{G} \cdot \phi \cdot S_{\text{hc}} \cdot \frac{1}{B_0} \tag{9A.17}$$

$$\text{recoverable reserves} = HCIIP \cdot RF \tag{9A.18}$$

9.3 General introduction on the evaluation of coal and water

9.3.4 Ash content

$$\rho_{\text{bulk}} = \rho_{\text{ash}} \cdot V_{\text{ash}} + \rho_{\text{carb}} \cdot V_{\text{carb}} \tag{9B.1}$$

9.3.5 Moisture content

If the coal is assumed to consist entirely of carbon, ash, and moisture:

$$\Delta T = \Delta T_{\mathrm{fl}} \cdot V_{\mathrm{mois}} + \Delta T_{\mathrm{ash}} \cdot V_{\mathrm{ash}} + \Delta T_{\mathrm{carb}} \cdot V_{\mathrm{carb}} \tag{9B.2}$$

$$\rho_b = \rho_{\mathrm{fl}} \cdot V_{\mathrm{mois}} + \rho_{\mathrm{ash}} \cdot V_{\mathrm{ash}} + \rho_{\mathrm{carb}} \cdot V_{\mathrm{carb}} \tag{9B.3}$$

$$1 = V_{\mathrm{mois}} + V_{\mathrm{ash}} + V_{\mathrm{carb}} \tag{9B.4}$$

9.3.6 Floats/sinks, calorific value, volatiles, sulphur

Relation between gas and ash content

$$V_{\mathrm{gas}} = a - b \cdot V_{\mathrm{ash}} \tag{9B.5}$$

9.4 Evaluation of groundwater

9.4.3 Parameters regarding the condition of water

Total dissolved solids

$$TDS = 7 \cdot C_w \tag{9B.6}$$

Conversion to standard conditions

$$C_w = C_0\,(1 + 0.0226 \cdot (T - T_0)) \tag{9B.7}$$

Variation in Hardness H between countries

$$H(^\circ\mathrm{D}) = 0.14 \cdot \mathrm{Ca}^{++}(\mathrm{mg\,L^{-1}}) + 0.231 \cdot \mathrm{Mg}^{++}(\mathrm{mg\,L^{-1}}) \tag{9B.8}$$

$$H(^\circ\mathrm{E}) = 100 \cdot \mathrm{CaCO_3}(\mathrm{grains/gallon}) \approx 0.80\,H(^\circ\mathrm{D}) \tag{9B.9}$$

$$H(^\circ\mathrm{F}) = 100 \cdot \mathrm{CaCO_3}(\mathrm{mg\,L^{-1}}) \approx 0.56\,H(^\circ\mathrm{D}) \tag{9B.10}$$

9.4.4 The interpretation of log information

Difference of SP potentials E between shale and sand

$$E = (-71) \cdot \log \frac{R_{\mathrm{mf}}}{R_{\mathrm{w}}} \tag{9B.11}$$

Empirical relations for the calculation of R_w

$$R_{\mathrm{w}} = R_{\mathrm{mf}} / 10^{(\mathrm{SP}/-70.7)} \tag{9B.12}$$

$$R_{\mathrm{w}}^{\mathrm{NaCl}} = 0.825 \cdot \left(R_{\mathrm{we}}^{1.227}\right) \tag{9B.13}$$

$$R_{\mathrm{w}}^{\mathrm{NaHCO_3}} = 1.18 \cdot R_{\mathrm{w}}^{\mathrm{NaCl}} \tag{9B.14}$$

Formation factor for 100% water (Humble/Archie)

$$F = \frac{R_0}{R_{\mathrm{w}}} = \frac{a}{\phi^m} \tag{9B.15}$$

Empirical relation for unconsolidated Rhine and dune sands

$$F = \frac{1.26}{\phi^{1.2}} \tag{9B.16}$$

Empirical oil-field relation for hard rocks (hilly areas)

$$F = \frac{0.62}{\phi^{2.15}} \tag{9B.17}$$

Empirical relation for known grain-sizes

$$F = 33 - 15\log(D_{\mathrm{dom}}) \tag{9B.18}$$

Porosity from Wyllie equation

$$\phi = \frac{\Delta T - \Delta T_{\mathrm{ma}}}{\Delta T_{\mathrm{fl}} - \Delta T_{\mathrm{ma}}} \tag{9B.19}$$

Other

Fahrenheit to Celsius

$$^\circ\mathrm{C} = \frac{5}{9}(^\circ\mathrm{F} - 32)$$