

Physical Transport Phenomena: index

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Lecture notes

Definition of $\Delta\mathcal{P}$

$$\Delta\mathcal{P} \equiv (p_0 - p_L) + \rho g[(\text{vertical position}_0 - (\text{vertical position}_L))] \quad (\text{Lecture notes 3.17})$$

Assumptions behind derivations in Chapter 2 (Re!)

see lecture notes (Lecture notes 3.22)

Chapter 1: Viscosity and the Mechanisms of Momentum Transport

Newton's law of viscosity

$$\tau_{yx} = -\mu \frac{dv_x}{dy} \quad (1.2-2)$$

Bingham plastic

$$\begin{cases} \tau_{yx} = -\mu_0 \frac{dv_x}{dy} + \tau_0 & \tau_{yx} \geq \tau_0 \\ \frac{dv_x}{dy} = 0 & -\tau_0 \leq \tau_{yx} \leq \tau_0 \\ \tau_{yx} = -\mu_0 \frac{dv_x}{dy} - \tau_0 & \tau_{yx} \leq -\tau_0 \end{cases} \quad (\text{Lecture notes 2.6})$$

Power-law fluid

$$\tau_{yx} = -m \left| \frac{dv_x}{dy} \right|^{n-1} \frac{dv_x}{dy} \quad (\text{Lecture notes 2.8})$$

Kinematic viscosity

$$\nu = \mu / \rho \quad (1.2-3)$$

Chapter 2: Shell Momentum Balances and Velocity Distributions in Laminar Flow

§2.2 Flow of a Falling Film (BSL1)

Momentum balance

$$LW\tau_{xz}|_x - LW\tau_{xz}|_{x+\Delta x} + LW\Delta x\rho g \cos\beta = 0 \quad (2.2-6)$$

Momentum flux distribution

$$\tau_{xz} = \rho g x \cos\beta + C_1 \quad (2.2-9)$$

Velocity distribution

$$v_z = \frac{\rho g \delta^2 \cos\beta}{2\mu} \left[1 - \left(\frac{x}{\delta} \right)^2 \right] \quad (2.2-16)$$

Maximum velocity

$$v_{z,\max} = \frac{\rho g \delta^2 \cos\beta}{2\mu} \quad (2.2-17)$$

Average velocity

$$\langle v_z \rangle = \frac{\rho g \delta^2 \cos\beta}{3\mu}$$

Volume flow rate

$$Q = \frac{\rho g W \delta^3 \cos\beta}{3\mu}$$

Film thickness

$$\delta = \sqrt[3]{\frac{3\mu\Gamma}{\rho^2 g \cos\beta}}$$

Force of the fluid on the surface

$$F_z = \rho g \delta L W \cos\beta$$

Momentum flux distribution with variable viscosity

$$\tau_{xz} = -\mu_0 e^{-\alpha(x/\delta)} \frac{dv_z}{dx} = \rho g x \cos\beta$$

Velocity distribution with variable viscosity

$$v_z = \frac{\rho g \delta^2 \cos\beta}{\mu_0} \left[e^\alpha \left(\frac{1}{\alpha} - \frac{1}{\alpha^2} \right) - e^{\alpha x/\delta} \left(\frac{x}{\alpha\delta} - \frac{1}{\alpha^2} \right) \right]$$

Average velocity with variable viscosity

$$\langle v_z \rangle = \frac{\rho g \delta^2 \cos\beta}{\mu_0} \left[e^\alpha \left(\frac{1}{\alpha} - \frac{2}{\alpha^2} + \frac{2}{\alpha^3} \right) - \frac{2}{\alpha^3} \right]$$

§2.3 Flow Through a Circular Tube (BSL1)

Momentum balance

see text

Momentum flux distribution

$$\tau_{rz} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L} \right) r + \frac{C_1}{r}$$

Velocity distribution

$$v_z = \frac{(\mathcal{P}_0 - \mathcal{P}_L)R^2}{4\mu L} \left[1 - \left(\frac{r}{R} \right)^2 \right]$$

Maximum velocity

$$v_{z,\max} = \frac{(\mathcal{P}_0 - \mathcal{P}_L)R^2}{4\mu L}$$

Average velocity

$$\langle v_z \rangle = \frac{(\mathcal{P}_0 - \mathcal{P}_L)R^2}{8\mu L}$$

Volume flow rate

$$Q = W/\rho = \frac{\pi(\mathcal{P}_0 - \mathcal{P}_L)R^4}{8\mu L}$$

Force of the fluid on the surface

$$F_z = \pi R^2 (p_0 - p_L) + \pi R^2 L \rho g$$

Velocity distribution for Bingham flow

$$v_z^> = \frac{(\mathcal{P}_0 - \mathcal{P}_L)R^2}{4\mu_0 L} \left[1 - \left(\frac{r}{R} \right)^2 \right] - \frac{\tau_0 R}{\mu_0} \left[1 - \left(\frac{r}{R} \right) \right] \quad r \geq r_0$$

$$v_z^< = \frac{(\mathcal{P}_0 - \mathcal{P}_L)R^2}{4\mu_0 L} \left(1 - \frac{r_0}{R} \right)^2 \quad r \leq r_0$$

Volume flow rate for Bingham flow

see text

Equations for power-law fluids in Lecture notes 3.6.

Effective viscosity for tube flow

$$\mu_{\text{eff}} = \frac{\pi R^4 \Delta\mathcal{P}}{8LQ} \quad (\text{Lecture notes 3.8})$$

§2.4 Flow Through an Annulus (BSL1)

(2.2-18) *Same momentum balance as Circular Tube*
Momentum flux distribution

$$(2.2-19) \quad \tau_{rz} = \frac{(\mathcal{P}_0 - \mathcal{P}_L)R}{2L} \left[\left(\frac{r}{R} \right) - \left(\frac{1 - \kappa^2}{2 \ln(1/\kappa)} \right) \left(\frac{R}{r} \right) \right] \quad (2.4-12)$$

(2.2-20) Velocity distribution

$$(2.2-21) \quad v_z = \frac{(\mathcal{P}_0 - \mathcal{P}_L)R^2}{4\mu L} \left[1 - \left(\frac{r}{R} \right)^2 + \left(\frac{1 - \kappa^2}{\ln(1/\kappa)} \right) \ln \left(\frac{r}{R} \right) \right] \quad (2.4-13)$$

(2.2-23) *See text for more quantities*
Equations for Bingham plastics and Power-law liquids in Lecture notes 3.13.

§2.7 Flow Around a Sphere

Viscosity of fluid from terminal velocity of falling sphere

$$(2.2-26) \quad \mu = \frac{2}{9} \frac{R^2 (\rho_s - \rho) g}{v_t} \quad (2.7-17)$$

Suspension of particles in Bingham plastic

$$(2.3-8) \quad \tau_0 \geq \frac{4}{3\pi} R |\rho_s - \rho| g \quad (\text{Lecture notes 3.21})$$

2B.4 Laminar Flow in a Narrow Slit

(2.3-16) Momentum-flux distribution

$$(2.3-17) \quad \tau_{xz}(x) = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{L} \right) x \quad (2B.4-1)$$

Velocity distribution

$$(2.3-18) \quad v_z(x) = \frac{(\mathcal{P}_0 - \mathcal{P}_L)B^2}{2\mu L} \left[1 - \left(\frac{x}{B} \right)^2 \right] \quad (2B.4-2)$$

Volume flow rate

$$(2.3-19) \quad Q = \frac{2}{3} \frac{(\mathcal{P}_0 - \mathcal{P}_L)B^3 W}{\mu L}$$

Equations for Bingham plastics in Lecture notes 3.10.
Equations for Power-law liquids in Lecture notes 3.12.

Chapter 6: Interphase Transport in Isothermal Systems

§6.1 Definition of Friction Factors

General definition of friction factor

$$F_k = AKf \quad (6.1-1)$$

§6.2 Friction Factors for Flow in Tubes

F_k for flow in conduits

$$F_k = (2\pi RL)\left(\frac{1}{2}\rho\langle\bar{v}_z\rangle^2\right)f \tag{6.1-2}$$

f for flow in conduits

$$f = \frac{1}{4}\left(\frac{D}{L}\right)\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{\frac{1}{2}\rho\langle\bar{v}_z\rangle^2}\right) \tag{6.1-4}$$

Correlations for f

$$f(\text{Re}) = \frac{16}{\text{Re}} \begin{cases} \text{Re} < 2100 \text{ stable} \\ \text{Re} > 2100 \text{ usually unstable} \end{cases} \tag{6.2-11}$$

Friction factor for tube flow

see figure

Friction factor for turbulent flow in non-circular tubes

$$f = \left(\frac{R_h}{L}\right)\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{\frac{1}{2}\rho\langle\bar{v}_z\rangle^2}\right) \tag{6.2-16}$$

Mean hydraulic radius for rectangular slit

$$R_h = B \tag{Lecture notes 9.6}$$

§6.3 Friction Factors for Flow Around Spheres

F_k for flow around submerged objects

$$F_k = (\pi R^2)\left(\frac{1}{2}\rho v_\infty^2\right)f \tag{6.1-5}$$

f for flow around submerged objects

$$f = \frac{4}{3}\frac{gD}{v_\infty^2}\left(\frac{\rho_{\text{sph}} - \rho}{\rho}\right) \tag{6.1-7}$$

Re for flow around spheres

$$\text{Re} \equiv \frac{Dv_\infty\rho}{\mu} = \frac{2Rv_\infty\rho}{\mu} \tag{6.3-8}$$

F_k for flow around spheres

$$F_k = (\pi R^2)\left(\frac{1}{2}\rho v_\infty^2\right)\left(\frac{24}{Dv_\infty\rho/\mu}\right) \tag{6.3-14}$$

Correlation for f

$$f \approx \frac{24}{\text{Re}} \qquad \text{for } \text{Re} \leq 0.1 \tag{6.3-15}$$

$$f \approx \left(\sqrt{\frac{24}{\text{Re}}} + 0.5407\right)^2 \qquad \text{for } \text{Re} \leq 6000 \tag{6.3-16}$$

$$f \approx 0.44 \qquad \text{for } 500 < \text{Re} < 100000 \tag{6.3-17}$$

Friction factor for spheres moving relative to fluid

see figure

Fig. 6.3-1

§6.4 Friction Factors for Packed Columns

Re for packed columns

$$\text{Re} = \frac{D_p v_0 \rho}{\mu} \left(\frac{1}{1 - \varepsilon}\right) \tag{Lecture notes 9.20}$$

Blake-Kozeny equation

$$\frac{\mathcal{P}_0 - \mathcal{P}_L}{L} = 150\left(\frac{\mu v_0}{D_p^2}\right)\frac{(1 - \varepsilon)^2}{\varepsilon^3} \tag{6.4-9}$$

Correlation for f

see figure

Fig. 6.4-2

More in lecture notes and §6.4

Chapter 7: Macroscopic Balances for Isothermal Flow Systems

§7.5 Estimation of the Viscous Loss

Macroscopic mechanical energy balance

$$\frac{1}{2}(v_2^2 - v_1^2) + g(h_2 - h_1) + \int_{p_1}^{p_2} \frac{1}{\rho} dp = \hat{W}_m - \hat{E}_v \tag{7.5-11}$$

For turbulent flow in tubes of circular cross section

see text

$$\tag{7.5-12}$$

Brief summary of friction-loss factors

see table

Table 7.5-1

Examples in Lecture Notes

Determination of Viscosity from Capillary Flow Data (BSL1 2.3-1)

Given $\Delta\mathcal{P}/L$, compute Q (Lecture notes 9.9)

Examples of trial-and-error method (Lecture notes 9.11)

Solving for unknown Re (Lecture notes 9.15)

Chapter 9: Thermal Conductivity and the Mechanisms of Energy Transport

§9.2 Conductive Heat-Flux Vector — Fourier’s Law

Fourier’s law of heat conduction (in one dimension)

$$\left(\frac{Q}{A} = \right) q_y = -k \frac{dT}{dy} \tag{9.2-2}$$

Conductive heat-flow vector

$$\vec{q} = -k\nabla T \tag{9.2-6}$$

Thermal diffusivity

$$\alpha = \frac{k}{\rho\hat{C}_p} \tag{9.2-7}$$

Prandtl number

$$\text{Pr} = \frac{\nu}{\alpha} = \frac{\hat{C}_p\mu}{k} \tag{9.2-8}$$

Development of steady-state temperature profile between plates

see figure

Fig. 9.2-1

Summary of units

see table

Table 9.2-1

§9.5 Thermal Conductivity Data from Experiments

References for values of k , \hat{C}_p , and Pr

see table

Tables 9.5-1 to 9.5-3

Chapter 10: Shell Energy Balances and Temperature Distributions in Solids and Laminar Flow

BSL1 §9.1 Shell Energy Balances: Boundary Conditions

Steady-state shell energy balance

see text

$$\tag{9.1-1}$$

Outline of shell energy balance approach

see lecture notes

Lecture notes 11.3

Newton’s law of cooling

$$q = h(T - T_{\text{fluid}}) \tag{9.1-2}$$

Preview of BSL1 chapter 9

see table

Lecture notes 11.4

BSL1 §9.2 Heat Conduction with an Electrical Heat Source

Rate of heat production per unit volume

$$S_e = \frac{I^2}{k_e} \tag{9.2-1}$$

Heat-flux distribution

$$q_r = \frac{S_e r}{2} \tag{9.2-9}$$

Temperature rise as a parabolic function of r

$$T(r) - T_0 = \frac{S_e R^2}{4k} \left[1 - \left(\frac{r}{R}\right)^2\right] \tag{9.2-13}$$

Maximum temperature rise (at $r = 0$)

$$T_{\text{max}} - T_0 = \frac{S_e R^2}{4k} \tag{9.2-14}$$

Average temperature rise

$$\langle T \rangle - T_0 = \frac{S_e R^2}{8k} \tag{9.2-15}$$

Heat flow at the surface

$$Q|_{r=R} = \pi R^2 L \cdot S_e \tag{9.2-16}$$

BSL1 §9.3 Heat Conduction with a Nuclear Heat Source

Volume source of thermal energy

$$S_n = S_{n0} \left[1 + b \left(\frac{r}{R^{(F)}}\right)^2\right] \tag{9.3-1}$$

Heat-flux distribution in the fissionable sphere

$$q_r^{(F)} = S_{n0} \left(\frac{r}{3} + \frac{b}{R^{(F)2}} \frac{r^3}{5}\right) \tag{9.3-12}$$

Heat-flux distribution in the spherical shell cladding

$$q_r^{(C)} = S_{n0} R^{(F)3} \left(\frac{1}{3} + \frac{b}{5} \right) \frac{1}{r^2} \quad (9.3-13)$$

Temperature profiles

$$T^{(F)} - T_0 = \text{see text} \quad (9.3-20)$$

$$T^{(C)} - T_0 = \frac{S_{n0} R^{(F)2}}{3k^{(C)}} \left(1 + \frac{3}{5} b \right) \left(\frac{R^{(F)}}{r} - \frac{R^{(F)}}{R^{(C)}} \right) \quad (9.3-21)$$

BSL1 §9.4 Heat Conduction with a Viscous Heat Source

Volume source of thermal energy

$$S_v = -\tau_{xz} \left(\frac{dv_z}{dx} \right) = \mu \left(\frac{dv_z}{dx} \right)^2 \quad (9.4-1)$$

BSL1 §9.5 Heat Conduction with Chemical Heat Source

Volume rate of thermal energy production by chemical reactions

$$S_c = S_{c1} \left(\frac{T - T^o}{T_1 - T^o} \right) \quad (9.5-1)$$

BSL1 §9.6 Heat Conduction Through Composite Walls: Addition of Resistances

Heat-flux distribution

$$q_0 = \frac{T_a - T_b}{\left(\frac{1}{h_0} + \sum_{i=1}^3 \frac{x_i - x_{i-1}}{k^{i-1,i}} + \frac{1}{h_3} \right)} \quad (9.6-15)$$

Heat-flux distribution, rewritten

$$q_0 = U(T_a - T_0) \quad \text{or} \quad Q_0 = U(WH)(T_a - T_b) \quad (9.6-16)$$

Over-all heat transfer coefficient

$$U = \left(\frac{1}{h_0} + \sum_{i=1}^3 \frac{x_i - x_{i-1}}{k^{i-1,i}} + \frac{1}{h_3} \right)^{-1} \quad (9.6-17)$$

Composite cylindrical walls (Lecture notes 15.6)

$$Q_0 = 2\pi L r_0 q_0 = \dots \quad (9.6-29)$$

$$Q_0 = U_0(2\pi r_0 L)(T_a - T_b) \quad (9.6-30)$$

$$U_0 = r_0^{-1} \left(\frac{1}{r_0 h_0} + \sum_{i=1}^3 \frac{\ln r_i / r_{i-1}}{k^{i-1,i}} + \frac{1}{r_3 h_3} \right)^{-1} \quad (9.6-31)$$

BSL1 §9.7 Heat Conduction in a Cooling Fin

Dimensionless temperature, rewritten

$$\Theta = \frac{\cosh N(1 - \zeta)}{\cosh N} \quad (9.7-13)$$

BSL1 §9.8 Forced Convection

Velocity profile

$$v_z = v_{z,\max} \left[1 - \left(\frac{r}{R} \right)^2 \right] \quad (9.8-1)$$

Partial differential equation

$$\rho \hat{C}_p v_{\max} \left[1 - \left(\frac{r}{R} \right)^2 \right] \frac{\partial T}{\partial z} = k \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) \quad (9.8-12)$$

Chapter 11: The Equations of Change for Nonisothermal Systems

§11.5 The Equations of Change and Solving Problems with Two Independent Variables

Velocity distribution in dimensionless form for flow in the neighbourhood of a wall suddenly set in motion

see figure

Fig. 3.8-2 (b)

Temperature difference

$$\frac{T(y,t) - T_0}{T_1 - T_0} = 1 - \operatorname{erf} \frac{y}{\sqrt{4\alpha t}} \quad (11.5-10)$$

Wall heat flux

$$q_y|_{y=0} = -k \frac{\partial T}{\partial y} \Big|_{y=0} = \frac{k}{\sqrt{\pi\alpha t}} (T_1 - T_0) \quad (11.5-12)$$

Initial condition

$$\text{at } t \leq 0, \quad T = T_0 \text{ for all } y \quad (11.5-2)$$

Temperature profiles for unsteady-state heat conduction in a slab of finite thickness $2b$

see figure

Fig. 11.5-1

Temperature profiles for unsteady-state heat conduction in a cylinder of radius R

see figure

Fig. 11.5-2

Temperature profiles for unsteady-state heat conduction in a sphere of radius R .

see figure

Fig. 11.5-3

Chapter 14: Interphase Transport in Nonisothermal Systems

§14.1 Definitions of Heat-Transfer Coefficients

Heat-transfer coefficient

$$Q = hA\Delta T \quad (14.1-1)$$

Heat-transfer coefficients for the fluid in the heated section (Eq. I)

$$\begin{aligned} Q &= h_{\ln}(\pi DL) \left(\frac{(T_{01} - T_{b1}) - (T_{02} - T_{b2})}{\ln(T_{01} - T_{b1}) - \ln(T_{02} - T_{b2})} \right) \\ &\equiv h_{\ln}(\pi DL)\Delta T_{\ln} \end{aligned} \quad (14.1-4)$$

Energy balance in tube (Eq. II)

$$w\hat{C}_p T_{b1} - w\hat{C}_p T_{b2} + Q = 0 \quad \text{or} \quad Q = w\hat{C}_p(T_{b2} - T_{b1}) \quad (14.1-10)$$

Definition of h_{\ln}

$$h_{\ln} = \frac{w\hat{C}_p}{\pi D^2} \frac{(T_{b2} - T_{b1})}{(T_0 - T_b)_{\ln}} \left(\frac{D}{L} \right) \quad (14.1-14)$$

Heat balance with definition of h_{\ln} (Eq. III)

$$\frac{h_{\ln} D}{k} \equiv \text{Nu}_{\ln} = \frac{(T_{b2} - T_{b1})}{(T_0 - T_b)_{\ln}} \left(\text{RePr} \frac{D}{4L} \right) \quad \text{Lecture notes 15.2}$$

For uniform T_0 along tube ($T_{01} = T_{02} \equiv T_0$) (ALWAYS assumed!)

$$\frac{(T_{b2} - T_{b1})}{(T_0 - T_b)_{\ln}} = \ln \left(\frac{T_0 - T_{b1}}{T_0 - T_{b2}} \right) = \ln \frac{\Delta T \text{ at inlet}}{\Delta T \text{ at outlet}} \quad \text{Lecture notes 15.2}$$

Heat transfer in a circular tube

see figure

Fig. 14.1-1

§14.2 Heat-Transfer Coefficients For Forced Convection Through Tubes and Slits Obtained From Solutions of the Equations of Change

Nusselt number for fully developed, laminar flow of Newtonian fluids with constant physical properties

see figure

Fig. 14.2-1

§14.3 Empirical Correlations for Heat-Transfer Coefficients for Forced Convection in Tubes

Nusselt number

$$\text{Nu}_{\ln} \equiv \frac{h_{\ln} D}{k} \quad \text{Lecture notes 15.4}$$

Reynolds number

$$\text{Re} \equiv \frac{Dv\rho}{\mu} = \frac{DG}{\mu} \quad \text{Lecture notes 15.4}$$

Prandtl number

$$\text{Pr} \equiv \frac{\hat{C}_p \mu}{k} \quad \text{Lecture notes 15.4}$$

Nusselt number for highly turbulent flow

$$\text{Nu}_{\ln} = 0.026 \text{Re}^{0.8} \text{Pr}^{1/3} \left(\frac{\mu_b}{\mu_0} \right)^{0.14} \quad (14.3-16)$$

Nusselt number for laminar flow

$$\text{Nu}_{\ln} = 1.86 \left(\text{RePr} \frac{D}{L} \right)^{1/3} \left(\frac{\mu_b}{\mu_0} \right)^{0.14} \quad (14.3-17)$$

Mean hydraulic radius for non-circular tubes

$$4R_h = 4(S/Z) = D \quad \text{Lecture notes 15.5}$$

Conservation equation (modified form of Eq. III)

$$\left[\frac{U_0 D_0}{k} \right] = \ln \left(\frac{T_f - T_{b1}}{T_f - T_{b2}} \right) \text{RePr} \frac{D_0}{4L} \quad \text{Lecture notes 15.6}$$

Heat-transfer coefficients for fully developed flow in smooth tubes

see figure

Fig. 14.3-2

Mass Transfer

Diffusivity

$$\mathcal{D}_{AB} \quad \text{Lecture notes 19.1}$$

Molar flux

$$N_A = \frac{\mathcal{D}}{\sqrt{\pi \mathcal{D} t}} (c_{s1} - c_{s0}) \quad \text{Lecture notes 19.3}$$

Mass transfer in tubes

Definition of mass-transfer coefficient (Eq. Im)

$$\mathcal{W}_A^{(m)} = k_{x,\ln}(\pi DL)(x_{A0} - x_{Ab})_{\ln} \quad \text{Lecture notes 20.1}$$

Mass-conservation equation (Eq. IIm)

$$\mathcal{W}_A^{(m)} = (c_{AB2} - c_{AB1})\pi R^2 \langle v \rangle \quad \text{Lecture notes 20.1}$$

(Eq IIIm)

$$\frac{k_{x,\ln} D}{c \mathcal{D}_{AB}} = \text{Sh} = \ln \left[\frac{c_{A0} - c_{AB1}}{c_{A0} - c_{AB2}} \right] \text{ReSc} \frac{D}{4L} \quad \text{Lecture notes 20.1}$$

Correlations

see notes

Lecture notes 20.3