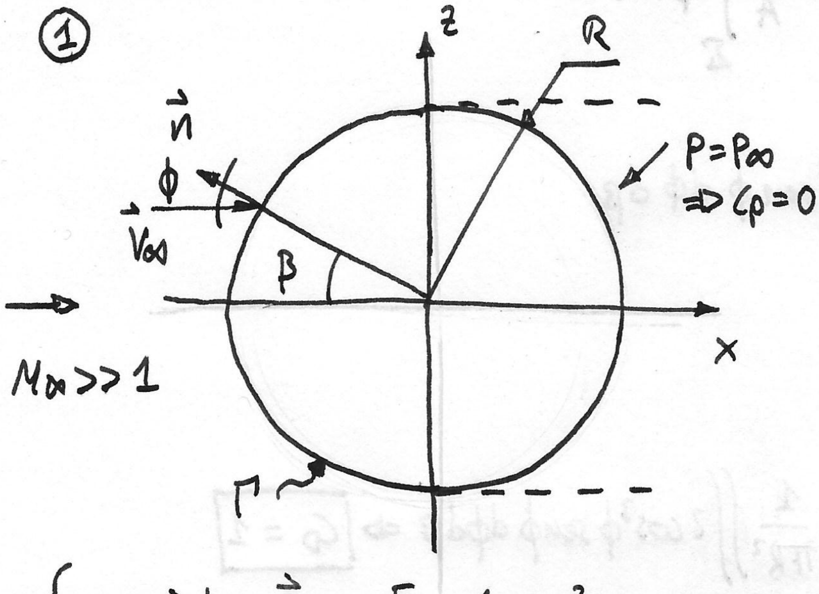
ANDRÉS PEDRAZA RODRÍGUEZ

1



$\phi = \beta$

$\vec{n} = \begin{pmatrix} -\cos\beta \\ \mathrm{sen}\,\beta \end{pmatrix}$

$\vec{V}_\infty = \begin{pmatrix} u_\infty \\ 0 \end{pmatrix}$

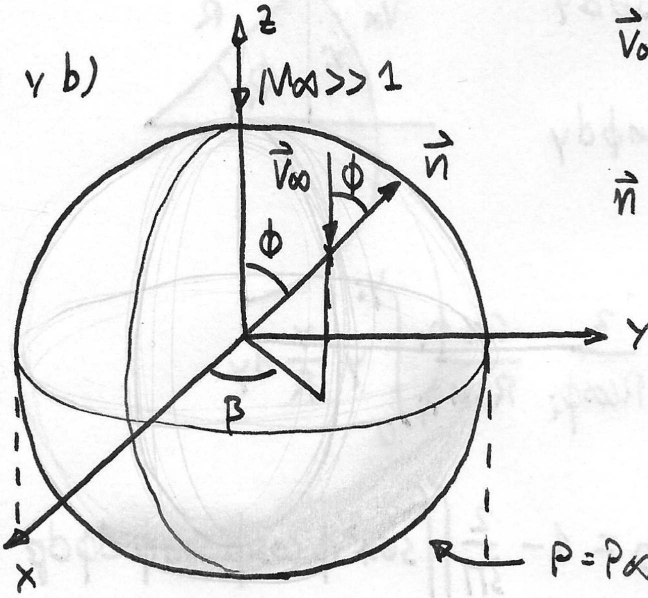
$$\int_\Sigma -pI\vec{n}\,ds = \vec{F} \;;\quad \frac{F_x}{A} = \tfrac{1}{2}\rho_\infty u_\infty^2 C_D \Rightarrow C_D = \frac{1}{A}\int_\Sigma C_p\, n_x\, d\sigma$$

$$C_D = \frac{1}{2R}\int_\Gamma -C_p(-\cos\beta)\,d\Gamma = \frac{1}{2R}\int_{-\pi/2}^{\pi/2} 2\cos^2(\beta)\cos\beta\; R\,d\beta$$

$$\boxed{C_D = \frac{4}{3}}$$

2

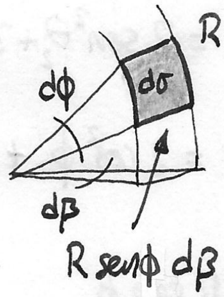
a) y b)



$P = P_\infty \Rightarrow C_p = 0$

$\vec{V}_\infty = \begin{pmatrix} 0 \\ 0 \\ -u_\infty \end{pmatrix}$

$\vec{n} = \begin{pmatrix} \mathrm{sen}\phi\cos\beta \\ \mathrm{sen}\phi\,\mathrm{sen}\beta \\ \cos\phi \end{pmatrix}$



$$\int_{\Sigma'} p\,\bar{\bar{I}}\cdot\vec{n}\,d\sigma = \vec{F}\ ;\quad \frac{F_z}{\frac{1}{2}\rho_\infty U_\infty^2 A} = C_D = \frac{1}{A}\int_{\Sigma} C_p\, n_z\, d\sigma$$

$$C_D = \frac{1}{\pi R^2}\int_{\phi=0}^{\phi=\pi/2}\int_{\beta=0}^{\beta=2\pi} C_p \cos\phi\, R^2 \operatorname{sen}\phi\, d\phi\, d\beta$$

Newton:

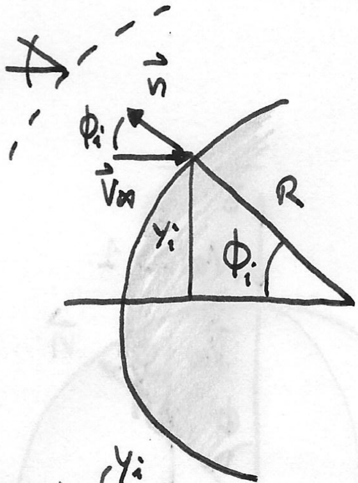
$$\boxed{C_p = 2\cos^2\phi} \Rightarrow C_D = \frac{1}{\pi}\iint 2\cos^3\phi \operatorname{sen}\phi\, d\phi\, d\beta \Rightarrow \boxed{C_D = 1}$$

Newton modificado:

$$\boxed{C_p = \left[\frac{(\gamma+1)^2}{4\gamma}\right]^{\gamma/\gamma-1}\left[\frac{4}{\gamma+1}\right]\cos^2\phi} \Rightarrow C_D = \frac{1}{\pi}\iint [\cdots][\cdots]\cos^3\phi \operatorname{sen}\phi\, d\phi\, d\beta$$

$$\Rightarrow \boxed{C_D = \frac{1}{2}\left[\frac{(\gamma+1)^2}{4\gamma}\right]^{\gamma/\gamma-1}\left[\frac{4}{\gamma+1}\right] \underset{\gamma=1'4}{\simeq} 0'9197}$$

Newton Busemann:



$$C_{p,i} = 2\operatorname{sen}^2\theta_2 + 2\left.\frac{d\theta}{dy}\right|_i \frac{\operatorname{sen}\theta_i}{y_i}\int_0^{y_i} y\cos\theta\, dy$$

$$C_{p,i} = 2\cos^2\phi_2 - 2\left.\frac{d\phi}{dy}\right|_i \frac{\cos\phi_i}{y_i}\int_0^{y_i} y\operatorname{sen}\phi\, dy$$

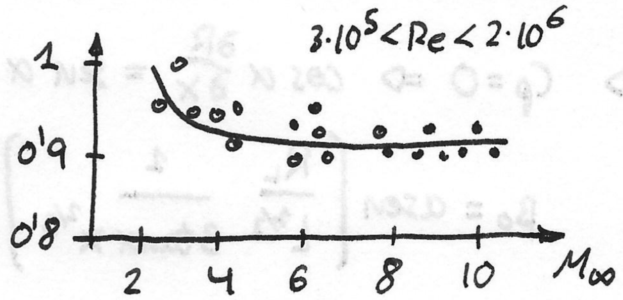
$$\left\{\begin{array}{l} y = R\operatorname{sen}\phi \\ \dfrac{d\phi}{dy} = \dfrac{1}{R\cos\phi}\end{array}\right. \Rightarrow C_{p,i} = 2\cos^2\phi_i - \frac{2}{R\cos\phi_i}\frac{\cos\phi_i}{R\operatorname{sen}\phi_i}\int_0^{y_i} y\,\frac{y}{R}\,dy$$

$$\boxed{C_{p,i} = 2\cos^2\phi - \frac{2}{3}\operatorname{sen}^3\phi} \Rightarrow C_D = 1 - \frac{2}{3\pi}\iint \operatorname{sen}^3\phi\cos\phi\operatorname{sen}\phi\, d\phi\, d\beta$$

$$\boxed{C_D = 0'7\overline{3}}$$

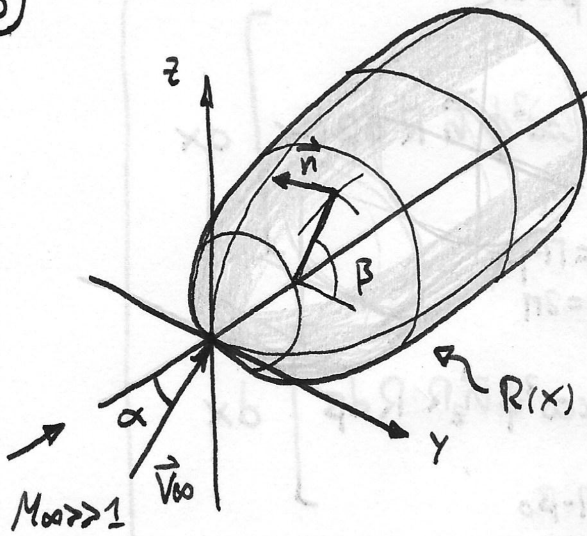
c)

Método	Resultado (C_D)	Datos experimentales (C_D) *
Newton	1	
Newton modificado	0'9197	
Newton Busemann	$0'7\overline{3}$	



* Véase: "Measurements of Sphere Drag from Hypersonic". Rand Corporation (1960)

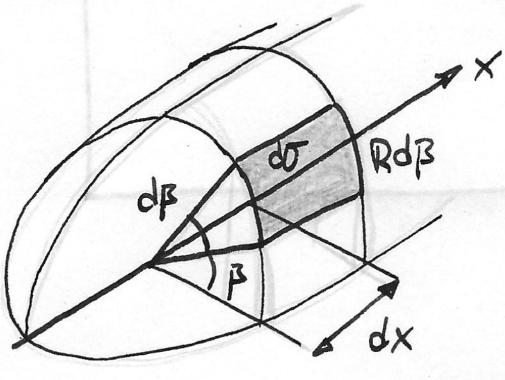
③



$$\vec{V}_\infty = \begin{pmatrix} U_\infty \cos\alpha \\ 0 \\ U_\infty \operatorname{sen}\alpha \end{pmatrix}$$

$$\vec{n} = \begin{pmatrix} -\partial R/\partial x \\ \cos\beta \\ \operatorname{sen}\beta \end{pmatrix} \frac{1}{\sqrt{1+\left(\frac{\partial R}{\partial x}\right)^2}}$$

$$\begin{cases} R = R_L\left(\dfrac{x}{L}\right)^{1/3} & x\in[0,L] \\ \dfrac{\partial R}{\partial x} = \dfrac{R_L}{L^{1/3}}\,\dfrac{1}{3x^{2/3}} \end{cases}$$



$$d\sigma = R\,d\beta\,dx$$

$$C_p = \hat{C}_p \cos^2\phi$$

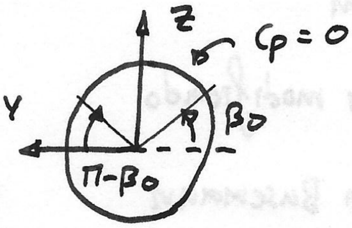
$$\cos\phi = \frac{\vec{V}_\infty\cdot\vec{n}}{|\vec{V}_\infty|} \Rightarrow \boxed{\cos^2\phi = \left[-\cos\alpha\frac{\partial R}{\partial x} + \operatorname{sen}\alpha\operatorname{sen}\beta\right]\frac{1}{\sqrt{1+\left(\frac{\partial R}{\partial x}\right)^2}}}$$

$$\begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} = \frac{\hat{C}_p}{\pi R_L^2}\int_{x=0}^{x=L}\int_{\beta ini}^{\beta fin} \cos^2(\phi(\alpha,\beta,x))\,\vec{n}(x,\beta)\,R(x)\,d\beta\,dx$$

$$\text{donde } \hat{C}_p = \begin{cases} 2 & \text{para Newton} \\ \left[\dfrac{(\gamma+1)^2}{4\gamma}\right]^{\gamma/\gamma-1}\left[\dfrac{4}{\gamma+1}\right] & \text{para Newton modificado} \end{cases}$$

Solo se puede integrar hasta $c_p \geq 0$ por el desprendimiento

$$\Rightarrow \quad C_p = 0 \Rightarrow \cos\alpha \frac{\partial R}{\partial x} = \operatorname{sen}\alpha \operatorname{sen}\beta_0$$



$$\beta_0 = \operatorname{asen}\left(\frac{R_L}{L^{2/3}} \frac{1}{3\tan\alpha\, x^{2/3}}\right)$$

Por tanto:

$$\boxed{\begin{aligned} \vec{C_f} &= \frac{\hat{C}_p}{\pi R_L^2}\int_{x=0}^{x=L}\left[\int_{\beta=0}^{\beta=\beta_0} \cos^2\phi\, \vec{n}\, R\, d\beta + \int_{\beta=\pi-\beta_0}^{\beta=2\pi} \cos^2\phi\, \vec{n}\, R\, d\beta\right] dx \\ C_m^0 &= \frac{\hat{C}_p}{\pi R_L^2}\int_{x=0}^{x=L}\left[\int_{\beta=0}^{\beta=\beta_0} \cos^2\phi\, n_z\, x\, R\, d\beta + \int_{\beta=\pi-\beta_0}^{\beta=2\pi} \cos^2\phi\, n_z\, x\, R\, d\beta\right] dx \\ x_{cp} &= \frac{C_m^0}{C_z} \end{aligned}}$$