Journal of Power Sources 227 (2013) 131–136

Contents lists available at SciVerse ScienceDirect

Journal of Power Sources

journal homepage: www.elsevier.com/locate/jpowsour

A new method for determining the characteristics of solar cells

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HIGHLIGHTS

► A new method is proposed to describe the current–voltage and power–voltage characteristics of solar cell and module in this article.
► By using the exact explicit analytical solutions, the I–V curves of the corresponding modules are calculated and simulated.
► To extract the parameters, an optimized technique is presented using Lambert W function and polynomial curve fitting.

ARTICLE INFO

Article history:
Received 6 May 2012
Received in revised form
13 July 2012
Accepted 19 July 2012
Available online 27 July 2012

Keywords:
Current–voltage characteristics
Parameters extraction
Lambert W function
Polynomial curve fitting

ABSTRACT

A new method is proposed to describe the characteristics of solar cell and module in this article. By using the exact explicit analytical solutions, the current–voltage and power–voltage curves of the corresponding modules are calculated and simulated. In order to extract the parameters of the solar cells, an optimized technique is presented by using Lambert W function and polynomial curve fitting. The accuracy of the model is compared with previous methods in other work. Results have the good agreement between the fitted current–voltage curves and the experimental data. Moreover, error and statistical analyses are carried out to illustrate the accuracy of the estimated parameters and procedure suitability.

1. Introduction

Due to the environmental and economic benefits obtained from photovoltaic generation, many research works have addressed the development of solar power system in recent years. One of the fundamental issues is to have access to model and parameters extraction for the solar cells. Over the years, several models were developed to describe the non-liner characteristics of the current–voltage (I–V) curve of the solar cells. The most popular approach is to utilize the electrical equivalent circuit, which is primarily based on the Shockley diode equation. In practical applications, the single diode model is the main equivalent circuit [1–6]. Moreover, the single diode model contains five parameters, which are called photocurrent, saturation current, series resistance, shunt resistance and ideality factor.

However, due to the transcendental nature of the current equation for solar model, significant computation effort is required to obtain all the model parameters [7]. To address this issue, several parameters extraction methods have been developed. One of the methods is numerical method, which typically uses polynomials to demonstrate the current–voltage relationship of PV panels and implements iterative method such as the recursive least-squares method [7] and Newton–Raphson method (NRM) [8] to obtain all the model parameters. However, the disadvantage of this approach is its dependency on the initial values used in the proposed iterative technique. Moreover, NRM does not always converge. Another solution technique is called analytic method [9] which is presented to express the transcendental current–voltage characteristic containing parasitic power consuming parameters like series and shunt resistances. Lambert W function is always applied to solve the transcendental equations [7,9–13], and has been widely used in many fields. For example, R.M. Corless et al. have given many available applications on the Lambert W function [14]. H. Abebe et al. used Lambert W function to develop an analytical compact model for the asymmetric lightly doped Double Gate (DG) MOSFET [9]. However, fewer researchers have given all the analytical solution for the five parameters of the single diode model. Although the analytical resolution is given, the data is still less accurate and common. Recently, various high accuracy algorithms techniques have been reported, such as particle swarm optimization (PSO) [15], differential evolution (DE) [16], genetic algorithm (GA) [17] and pattern search (PS) [18,19]. While the calculation process is more complicated, this is not conducive to master.

Considering the many advantages of numerical and analytic method, this paper proposes a new method for determining the I–V

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0378-7753/$ – see front matter © 2012 Published by Elsevier B.V.
http://dx.doi.org/10.1016/j.jpowsour.2012.07.061

Nomenclature

CMAE	maximum absolute current error (A)
$\mathrm{CMAE_{mp}}$	maximum absolute current error at the maximum power point (A)
CRMSE	root mean squared error at the maximum power point (A)
G_p	the conductance of the shunt resistance(Ω^{-1})
I	output current of the cell (A)
I_L	photocurrent (A)
I_{mp}	current at the maximum power point (A)
I_{sc}	short circuit current (A)
I_o	saturation current of the equivalent diode (A)
k	Boltzmann constant (1.38×10^{-23} J K^{-1})
n	ideality factor of the cell
N	number of measurements
R_s	series resistance of the cell (Ω)
R_p	shunt resistance of the cell (Ω)
T	cell temperature (K)
PMAE	maximum absolute power error (W)
$\mathrm{PMAE_{mp}}$	maximum absolute power error at the maximum power point (W)
PRMSE	root mean squared error at the maximum power point (W)
q	electron charge (1.6×10^{-19}C)
V	output voltage of the cell (V)
V_{ci}	computed values of the I–V curves for a given voltage point
V_{ei}	experimental values of the I–V curves for a given voltage point
V_{mp}	voltage at the maximum power point (V)
V_{oc}	open circuit voltage (V)
θ	angle between V_{oc} and I_{sc} (o)
θ_1	angle between V_{oc} and V_{mp} (o)
θ_2	angle between I_{mp} and I_{sc} (o)

characteristics of solar cells. To validate the accuracy of the new method, sets of comparative results were tested in matlab/simulink environment. Comparing with other competing methods, the obtained results have high accuracy. Meanwhile, the comparative results show that the proposed method gives a good representation at the maximum point.

2. Theory and application

2.1. Explicit analytic solutions

For single diode model of solar cells, the I–V characteristics can be expressed as:

$$I = I_L - I_o\left[\exp\left(\frac{V+IR_s}{nV_{th}}\right) - 1\right] - \frac{V+IR_s}{R_p} \tag{1}$$

Here V_{th} is the “thermal voltage” of a cell, which is given by:

$$V_{th}(T) = kT/q \tag{2}$$

The solution of Eq. (1) can be expressed by using Lambert W function [10]:

$$I = \frac{R_p(I_L+I_0)-V}{R_s+R_p} - \frac{nV_{th}}{R_s}W\left\{\frac{R_sR_pI_o}{nV_{th}(R_s+R_p)}\exp\left[\frac{R_p(R_sI_L+R_sI_o+V)}{nV_{th}(R_s+R_p)}\right]\right\} \tag{3}$$

$$V = R_p(I_L+I_o-I) - R_sI - nV_{th}\cdot W\left\{\frac{I_oR_p}{nV_{th}}\exp\left(\frac{R_p(I_L+I_o-I)}{nV_{th}}\right)\right\} \tag{4}$$

Similarly, the output power can be given explicitly in terms of V as follows:

$$P(V) = \frac{R_p(I_L+I_0)V-V^2}{R_s+R_p} - \frac{nVV_{th}}{R_s}W\left\{\frac{R_sR_pI_o}{nV_{th}(R_s+R_p)}\exp\left[\frac{R_p(R_sI_L+R_sI_o+V)}{nV_{th}(R_s+R_p)}\right]\right\} \tag{5}$$

The integration of Eq. (4) can be expressed as below [20]:

$$F(I,V) = \int_0^I V\,dI = \frac{1}{2A}\left[(-V-BI+C)^2-(-V+C)^2\right] - \frac{1}{2}BI^2 + ADI \tag{6}$$

Here the parameters of A, B, C, D are given by the following expressions:

$$A = R_p \tag{7}$$

$$B = R_s + R_p \tag{8}$$

$$C = nV_{th} + R_p(I_L+I_o) \tag{9}$$

$$D = I_L + I_o \tag{10}$$

From Eq. (1) and Eqs. (7)–(10), it is obtained:

$$R_p = A \tag{11}$$

$$R_s = B - A \tag{12}$$

$$n = (C - AD)/V_{th} \tag{13}$$

$$I_O = \frac{D - I - \dfrac{V+I(B-A)}{A}}{\exp\left[\dfrac{V+I(B-A)}{C-AD}\right] - 2} \tag{14}$$

$$I_L = D - I_o \tag{15}$$

Once A, B, C, D are determined, the five parameters that describe solar cells model behavior are calculated in sequence with Eqs. (11)–(15), respectively. Once, the five parameters are given, through Eqs. (3) and (5), the I–V and P–V curves of the corresponding modules are calculated and simulated.

2.2. Polynomial curve fitting

Eq. (6) shows that the integral of the solar voltage $F(I,V)$ is the polynomial function on A, B, C and D. And the $F(I,V)$ can be obtained from the experimental data by polynomial curve fitting. Therefore, substituting four different groups of value $F(I,V)$ into Eq. (6), the A, B, C and D can be determined by solving the algebraic equations.

In order to accurately represent the characteristics of solar cells, a sixth-order polynomial [8] is defined as:

$$V(I_i) = a_0 + a_1 I_i + a_2 I_i^2 + a_3 I_i^3 + a_4 I_i^4 + a_5 I_i^5 + a_6 I_i^6 \tag{16}$$

Substituting Eq. (16) in Eq. (6), $F(I,V)$ can be obtained:

$$\begin{aligned} F(I,V) &= a_0 I + \frac{1}{2}a_1 I^2 + \frac{1}{3}a_2 I^3 + \frac{1}{4}a_3 I^4 + \frac{1}{5}a_4 I^5 + \frac{1}{6}a_5 I^6 + \frac{1}{7}a_6 I^7 \\ &= \frac{1}{2A}\left[(-V-BI+C)^2 - (-V+C)^2\right] - \frac{1}{2}BI^2 + ADI \end{aligned} \tag{17}$$

For evaluating the error of this method, the measure of error is given by:

$$S = \|e\|_2^2 = \left(\sqrt{\sum_{i=0}^{6}[V(I_i)-V_i]^2}\right)^2 = \sum_{i=0}^{6}[V(I_i)-V_i]^2 \tag{18}$$

To minimize the error, the derivative with respect to the sample points is zero.

$$f(i,v)\frac{\partial S}{\partial a_j} = 2\sum_{i=0}^{6}\left(\sum_{k=0}^{6} a_k I_i^k - V_i\right) I_i^j = 0, \quad j = 0,1,\ldots,6 \tag{19}$$

And Eq. (19) can be written as:

$$\sum_{i=0}^{6}\left(\sum_{k=0}^{6} I_i^{j+k}\right) a_k = \sum_{i=0}^{m} I_i^j V_i, \quad j = 0,1,\ldots,6 \tag{20}$$

For convenience of representation, the following notations of vectors and matrix are introduced:

$$Y = \begin{bmatrix} \sum_{i=0}^{6} V_i \\ \sum_{i=0}^{6} I_i V_i \\ \vdots \\ \sum_{i=0}^{6} I_i^6 V_i \end{bmatrix}$$

$$X = \begin{bmatrix} 6+1 & \sum_{i=0}^{6} I_i & \cdots & \sum_{i=0}^{6} I_i^6 \\ \sum_{i=0}^{6} I_i & \sum_{i=0}^{6} I_i^2 & \cdots & \sum_{i=0}^{6} I_i^7 \\ \vdots & \vdots & & \vdots \\ \sum_{i=0}^{6} I_i^6 & \sum_{i=0}^{6} I_i^7 & \cdots & \sum_{i=0}^{6} I_i^{12} \end{bmatrix}$$

$$\theta = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_6 \end{bmatrix}$$

Then, the measured output vector Y can be represented by a simple regression model:

$$Y = X\theta \tag{21}$$

According to the theorem of the matrix transformation, the vector of the estimated parameters θ is given by:

$$\theta = X^{-1}Y \tag{22}$$

Due to the existing shape similarity between the characteristics of solar cells and the step response of a first order system [23], the seven experimental data points which were used for polynomial curve fitting, can be chosen by the following formula, as shown in Fig. 1.

$$I_0 = \tan\theta_1 I_{mp}$$

$$I_1 = \frac{1}{2}I_{mp}\left[1 - \frac{1}{4}(1-\cos\theta)\right]$$

$$I_2 = I_{mp} - \frac{1}{4}(1-\cos\theta)I_{mp}$$

$$I_3 = I_{mp}$$

$$I_4 = I_{mp} + \frac{1}{4}(1-\cos\theta)(I_{sc} - I_{mp})$$

$$I_5 = \frac{1}{2}I_{sc} - \frac{1}{2}\left[I_{mp} + \frac{1}{4}(1-\cos\theta)(I_{sc}-I_{mp})\right]$$

$$I_6 = \tan\theta_2 I_{sc}$$

Previous studies have shown that R_s impacts the I–V curve near the maximum power point, and R_p determines the slope of I–V curve between V_{oc} and V_{mp}. A typical approach [5] is to estimate R_s and R_p value by using the slopes at the V_{oc} and I_{sc}, respectively. I_O and I_L are always calculated at the points of (V_{oc}, 0)and (0,I_{sc}). And n is determined by the inherent characteristics of the solar cells [2]. Thus, to accurately obtain the five parameters, the four different groups of value $F(I,V)$ are chosen at I_0, I_2, I_4,I_6. Substituting the four groups of value $F(I,V)$ into Eq. (17), the A, B, C and D can be determined by solving the algebraic equations.

3. Validation and discussions

In an attempt to confirm the validity of our method, a practical data of a silicon solar cell and module was used. And the experimental data was reported by Easwarakhanthan et al. [21]. In addition, another different type of solar cell called plastic solar cell was used [22]. Moreover, the proposed algorithm was implanted in Matlab/Simulink environment. Sixth-order polynomial curve fitting and well-picked four groups of value $F(I,V)$ were employed for parameters extraction from commercial solar cells. To provide a comprehensive evaluation, the obtained results were also compared with other methods by using the same reference parameters, and they were given in Tables 1–3. Through Eqs. (3)

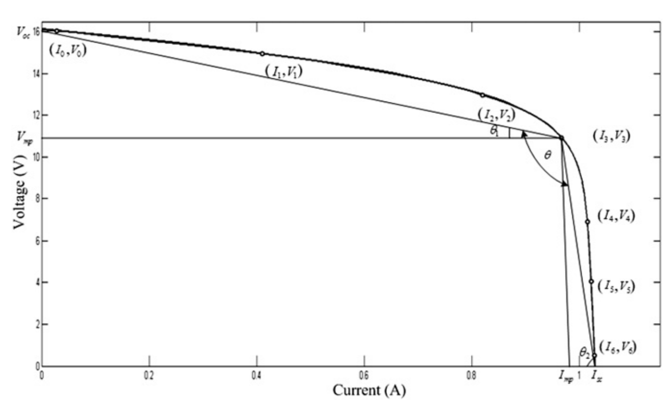


Fig. 1. The seven experimental data used for polynomial curve fitting.

Table 1
Parameters for the silicon solar cell and module using different methods.

Item	Silicon solar cell (33 °C)				Silicon solar module (45 °C)			
	Ref. [19]	Ref. [20]	Ref. [16]	Our method	Ref. [19]	Ref. [20]	Ref. [16]	Our method
I_L (A)	0.7608	0.7603	0.7617	0.7609	1.0318	1.0300	1.0313	1.0313
I_O (μA)	0.3223	0.3374	0.9980	0.3220	3.2876	6.3986	3.1756	3.2212
n	1.4837	1.4841	1.6000	1.4837	48.4500	50.9900	48.2889	48.3221
R_s (Ω)	0.0364	0.0376	0.0313	0.0364	1.2057	1.1619	1.2053	1.2132
G_p (Ω^{-1})	0.0186	0.0094	0.0156	0.0185	0.0018	0.0014	0.0014	0.0016

Table 2
Statistical results for the silicon solar cell and module using different methods.

Item	Silicon solar cell (33 °C)			Silicon solar module (45 °C)		
	Ref. [20]	Ref. [16]	Our method	Ref. [20]	Ref. [16]	Our method
CMAE	0.01550	0.02819	0.01258	0.01731	0.004423	0.005152
$CMAE_{mp}$	6.998×10^{-5}	7.934×10^{-3}	1.551×10^{-3}	0.0090	0.0043	0.0007
CRMSE	0.0051	0.0091	3.543×10^{-3}	0.0314	0.0028	0.0026
PMAE	0.008834	0.01607	0.007174	0.2770	0.05394	0.08244
$PMAE_{mp}$	3.149×10^{-5}	3.570×10^{-3}	6.980×10^{-3}	0.1098	0.0535	0.0089
PRMSE	0.0026	0.0048	9.552×10^{-4}	0.3865	0.0325	0.0382

Table 3
Parameters and statistical results for the plastic solar cell using different methods.

Plastic solar cell(27.3 °C)											
Item	I_L (A)	I_O (μA)	n	R_s (Ω)	G_p (Ω^{-1})	CMAE (μA)	$CMAE_{mp}$ (μA)	CRMSE (μA)	PMAE (μW)	$PMAE_{mp}$ (μW)	PRMSE (μW)
Ref. [13]	0.0079	0.0136	2.31	8.59	0.0051	0	0	0	0	0	0
Ref. [22]	0.0079	0.0329	2.59	8.586	0.0050	920.93	106.74	288.89	841	59.775	205.89
Our method	0.0079	0.0136	2.3101	8.5884	0.0051	1.33	0.188	0.394	0.999	0.106	0.277

and (5), the I–V and P–V curves of the corresponding modules are calculated and simulated. The results for both the silicon solar cell and module are given in Figs. 2 and 3. The errors in different methods are shown in Figs. 4 and 5. Moreover, the results for the plastic solar cell are given in Fig. 6.

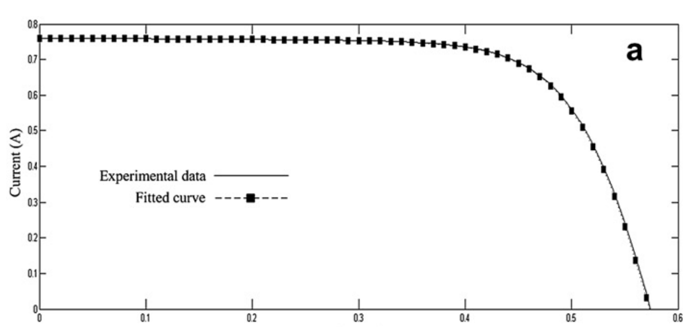
With Tables 1 and 3, it is easy to see that parameters extracted by using the proposed method, are very close to those reported in the other two references. Figs. 2, 3 and 6 show that the new method can achieve high accuracy, which are in good agreement with the experimental results. In order to make this study as general as possible, the same statistical analysis is performed [17]. The absolute error (AE_i) and the root mean squared error (RMSE) are given by:

$$AE_i = |V_{ci} - V_{ei}| \quad (17a)$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N} AE_i^2} \quad (18a)$$

Figs. 3 and 4 show that, the new method can represent the cell and module characteristics with the small absolute errors of current and power. In addition, it is interesting to find that, in different voltage ranges the absolute error varies with different methods. Almost before the maximum power point, the error of three algorithms is basically the same, after which there is a clear distinction. Moreover, comparisons with simulation results, it has shown that the proposed method has a smaller error value after the maximum power point. The results indicate that the parallel resistance has a greater impact on the maximum power point.

In addition, it can be concluded from Table 2 that, this new method can accurately access the I–V characteristics of the solar cell and module with much lower errors. For the maximum voltage and current, $CMAE_{mp}$ and $PMAE_{mp}$ values are less than 0.0043 A and 0.1098 W, respectively. The root means squared errors at current and power do not exceed 0.01258 A and 0.0382 W. Comparisons with simulation results under different methods in Table 2, it has shown that the proposed method reduces the overall estimation error at a very low standard deviation.



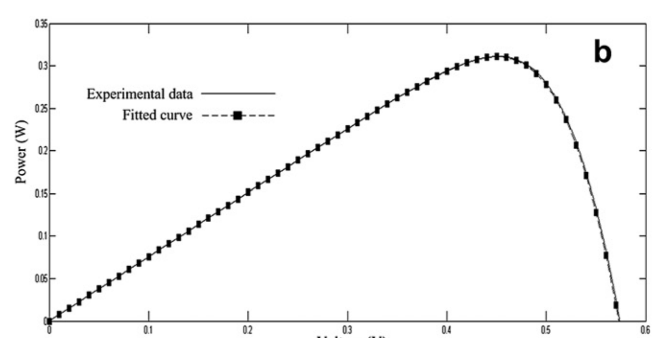


Fig. 2. Experimental data and the fitted curve for silicon solar cell: (a) I–V characters and (b) P–V characters.

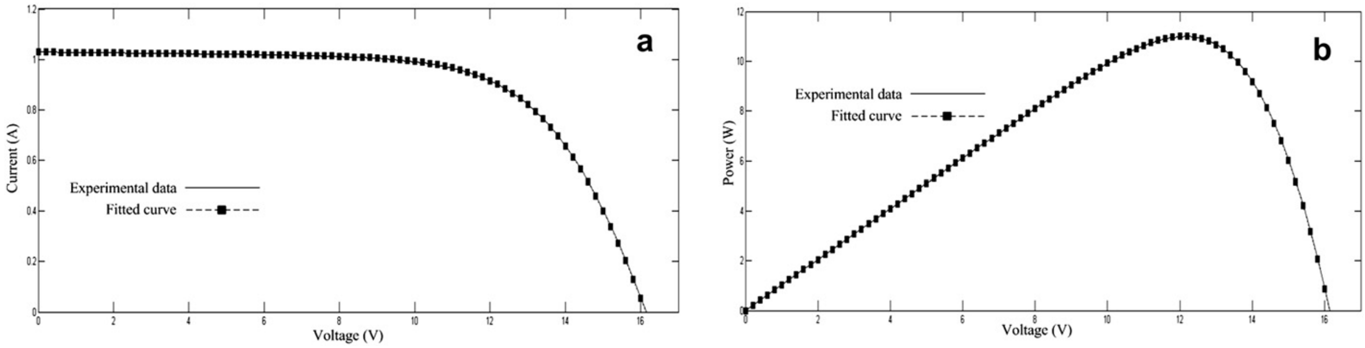


Fig. 3. Experimental data and the fitted curve for silicon solar module: (a) *I–V* characters and (b) *P–V* characters.

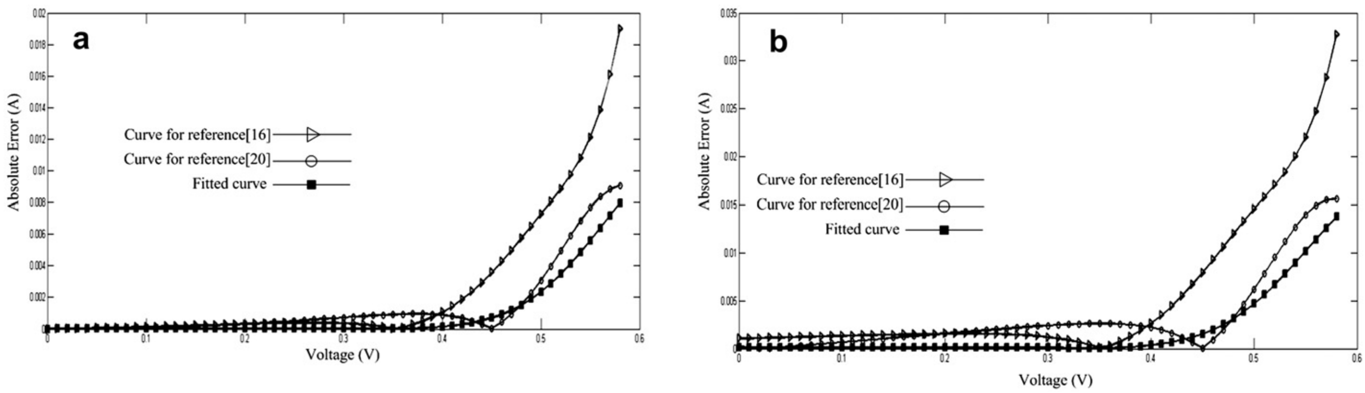


Fig. 4. Absolute errors for silicon solar cell: (a) absolute current errors and (b) absolute power errors.

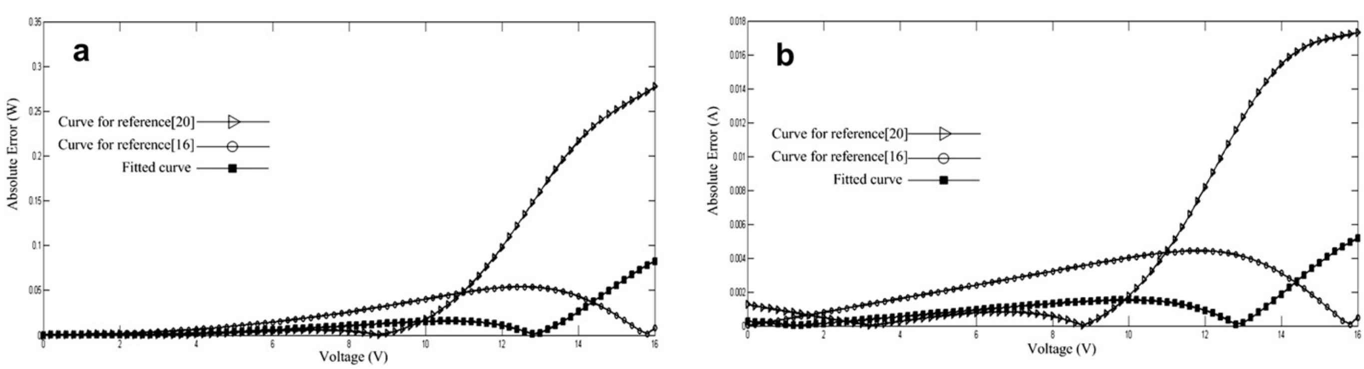


Fig. 5. Absolute errors for silicon solar module: (a) absolute current errors and (b) absolute power errors.

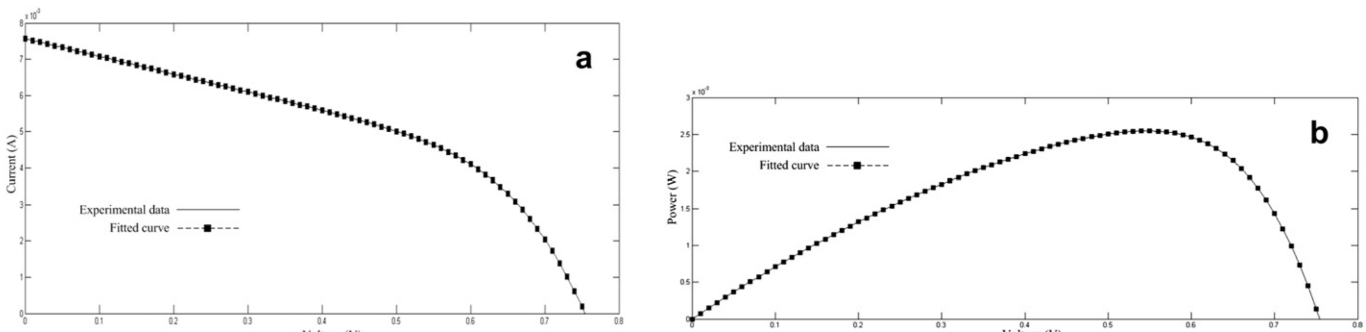


Fig. 6. Experimental data and the fitted curve for plastic solar cell: (a) *I–V* characters and (b) *P–V* characters.

Therefore, there are three aspects which have been addressed. The first question involves the I–V characteristics, the second problem relates to the parameters extraction, the third aspect deals with the error and statistical analyses. We tentatively put forward that, the quality standard of a good method has not only a small current error, but also small deviation of power, especially at the maximum power point.

4. Conclusions

We have presented a new method for determining the I–V and P–V characteristics of solar cells. By using the exact explicit analytical solutions, the I–V and P–V curves of the corresponding modules are calculated and simulated. A new technique is presented by using Lambert W function and subsection polynomial curve fitting to extract the parameters. The method has been successfully applied to commercial solar cell and module. The results obtained are in good agreement with those published previously. In addition, the quality standard of a good method has not only a small current error, but also small deviation of power, especially at the maximum power point.

Acknowledgement

This work was supported by the National golden sun demonstration project (Grant Nos. 20090718047), and the Fundamental Research Funds for the Central Universities (Grant Nos. 11D10301).

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