This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TIA.2021.3052858, IEEE Transactions on Industry Applications

Simplified Lambert W-function math equations when applied to photovoltaic systems modeling

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***Abstract*—In this paper, simplified and easy-to-work-with equations for the Lambert W-function are derived. This function is widely used to solve equations related to photovoltaic systems. More specifically, this mathematical function represents a useful tool when modeling solar cells/panels performance (that is, the current-voltage curve) by analytical approaches. However, the Lambert W-function has a complex solving process which might represent an unaffordable mathematical challenge for a great number of professionals/technicians in the photovoltaic industrial sector. Simple approximations for the Lambert W-function on both of its branches (positive and negative) are proposed in this work. The results of the present work show a simple but accurate way for photovoltaic systems modeling, even when these systems comprise a Maximum Power Point Tracking (MPPT) subsystem.**

Keywords*—Lambert W-function, solar cell, solar panel, photovoltaic systems performance, I-V curve, 1-Diode/2-Resistor model, MPPT

I. INTRODUCTION

Renewable energy plays a very important role in reducing fossil resources consumption [1], which is a present and urgent need for mankind due to problems such as global warming, climate change and air pollution [2]. Among the different renewable energy sources, solar energy is probably the most relevant, as it is clean, safe, and unlimited [3], [4].

The figures can be overwhelming. In one year the amount of energy received from the Sun is 10,000 times larger than the world's energy consumption [4]. The installed photovoltaic power increased from 100.9 GW in 2012 to 230 GW in 2015, reaching 400 GW in 2017 [1], [5]. This rate of increase has been possible due to a new generation of solar cells that allow production growth while reducing costs and environmental impact [6].

Modeling has become the key for photovoltaic system design and development, as it allows proper and accurate energy production forecasts [7]. The modeling of these photovoltaic systems (solar cells or solar panels) is normally carried out by using equivalent circuit models, whose equations are quite challenging as they are mathematically implicit expressions. Nevertheless, in the last decade the Lambert W-function has been revealed as a powerful tool to solve these equations.

The aim of the present paper is to derive simple expressions for the Lambert W-function, which is commonly used in photovoltaic systems, and describe how this function is required to solve equations related to these systems. The proposed expressions were derived by fitting well-known mathematical equations (polynomials, exponential functions, hyperbolic functions) to points on the Lambert W-function calculated numerically with the highest available accuracy. The work carried out in the IDR/UPM Institute has prompted the authors to carry out the present work, as simple solutions to analyze solar panel behavior are required when developing spacecraft missions in a Concurrent Design Facility (CDF) [8]–[11], or analyzing thermo-electric coupled behavior [12].

The innovation offered by the present work is to derive very simple expressions for the Lambert W-function that can be solved quickly with a pocket calculator, to model a photovoltaic systems performance (or that can be easily programmed for complex simulations, such as the thermo-electric analysis of space systems).

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The present paper is organized as follows. In Section II, the Lambert W-Function is described in relation to its use in photovoltaic systems modeling. In Section III, the different mathematical expressions derived to estimate the Lambert W-function branches W_{-1}, W_0^+, and W_0^-, are defined, together with their accuracy. In Section IV, the results of the use of these equations are shown. Finally, the conclusions are summarized in Section V.

II. The Lambert W-function

The Lambert W-function has proven to be a very useful tool for solving equations related to: time-delay differential problems, control stability, flight dynamics of projectiles, enzyme kinetics, and solid-state electronics modeling problems [13]. Solar cell/panel modeling sits within the latter discipline [14]–[21].

The Lambert W-function, $W(z)$, is defined as:

$$z = W(z)\exp\left(W(z)\right), \tag{1}$$

where z is a complex number. For a real variable x, the Lambert function is defined within the bracket [−1/e, ∞], having a double value within the bracket [−1/e, 0]. Two different branches are defined for this function: $W_0(x)$, for $W(x) \geq -1$, and $W_{-1}(x)$, for $W(x) \leq -1$. Additionally, the branch $W_0(x)$ is divided into two sections that can be better approached separately: $W_0^-(x)$, for $W_0(x) \leq 0$, and $W_0^+(x)$, for $W_0(x) \geq 0$, see Fig. 1.

This is quite a complex function that needs some computational aid to be solved, as it is defined by an explicit mathematical expression. Although easier approximations to this function can be found in the literature, such as the one proposed by Barry et al. [22], they are still too complex to be considered "direct" equations. Nevertheless, it is worth mentioning that the approximation from Barry et al. related to the sub-branch $W_0^+(x)$ and is far less complicated than the ones related to $W_{-1}(x)$ and $W_0^-(x)$ (see Appendix).

A. *The Lambert W-function when using 1-diode/2-resistor equivalent circuit model*

The 1-Diode/2-Resistor equivalent circuit model is the most popular way to model solar cell/panel behavior. The equation that relates the output current, I, to the output voltage, V, in this model (see Fig. 2) is the following [23]:

$$I = I_{pv} - I_0\left[\exp\left(\frac{V+IR_s}{naV_T}\right)-1\right] - \frac{V+IR_s}{R_{sh}}, \tag{2}$$

in which the first term is the photocurrent, the second term is the current through the diode, and the third term represents the current through the shunt resistor. V_T is the thermal voltage ($V_T = \kappa T/q$; κ being the Boltzmann constant, T the temperature and q the electron charge), a is the ideality factor of the diode and n is the number of series-connected cells within the panel.

Working with the above equation is not a simple task, as the parameters involved in the equation (I_{pv}, I_0, a, R_s, and R_{sh}) should be calculated to fit the equation to a specific *I-V* curve (see Fig. 3), which depends on both the solar irradiance, G, on the cell/panel and its temperature, T.

Equation (1) is an implicit mathematical expression. Therefore, for a given value of the output voltage, V, the calculation of the corresponding output current, I, is not immediate, which makes an iteration process necessary. Nevertheless, different methodologies have been developed to work with this equation depending on the available information [24]–[30].

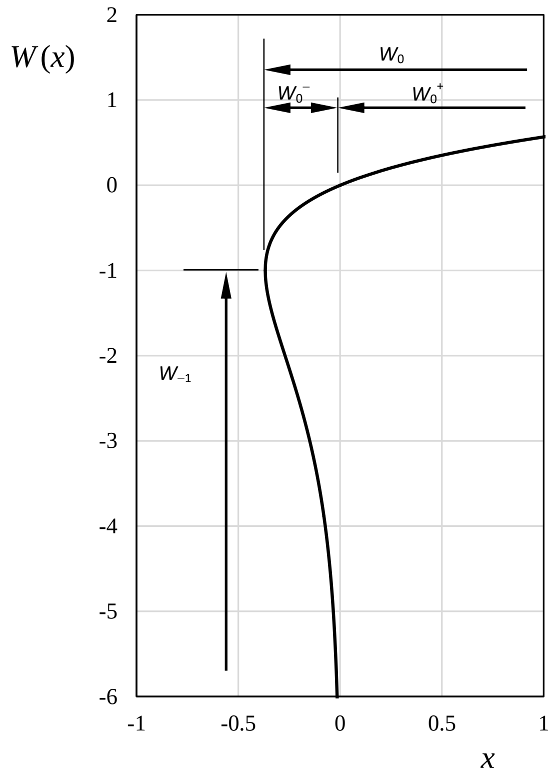

Fig. 1 Lambert W-function. The different branches are indicated in the graph.

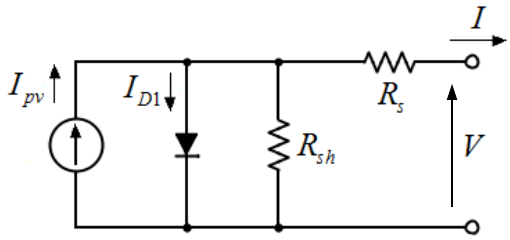

Fig. 2 Solar cell/panel 1-Diode/2-Resistor equivalent circuit.

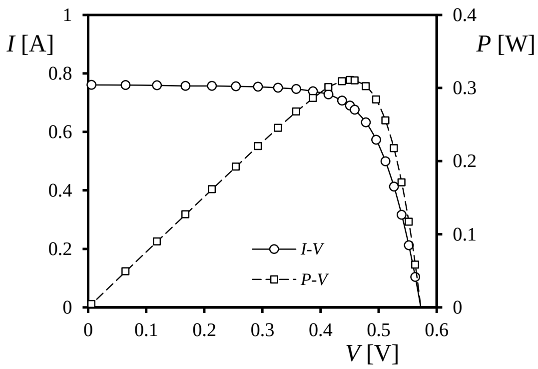

Fig. 3 *I-V* (current-voltage) and *P-V* (power-voltage) curves of a Si solar cell.

Among the aforementioned methodologies developed to fit equation (1) to a specific *I-V* curve, it is possible to

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mention numerical and analytical ones. Numerical techniques fit the equation by using different algorithms. These procedures obtain very accurate results. However, they require both high computational resources and skill. Additionally, they might be a drawback if coupled calculations are required (thermo-electric calculations of the behavior of solar panels in space are a good example of this sort of calculation [12]).

On the other hand, analytical procedures are simpler and based on limited information from the *I-V* curve, its characteristic points [31], [32]:

- short circuit current, I_{sc},
- open circuit voltage, V_{oc},
- and current and voltage at maximum power point, I_{mp} and V_{mp}.

Analytical methodologies are not as accurate as the numerical ones, but can offer quick and satisfactory results.

If the characteristic points of the *I-V* curve are known for certain values of sun irradiance, *G*, and the temperature, *T*, it is possible to calculate four of the parameters of the model in relation to the fifth one, which is the ideality factor, *a* [23], [33]:

$$I_{pv} = \frac{R_{sh} + R_s}{R_{sh}} I_{sc}, \tag{3}$$

$$I_0 = \frac{(R_{sh} + R_s) I_{sc} - V_{oc}}{R_{sh} \exp\left(\dfrac{V_{oc}}{naV_T}\right)}, \tag{4}$$

$$\frac{naV_T V_{mp}(2I_{mp} - I_{sc})}{\left(V_{mp}I_{sc} + V_{oc}(I_{mp} - I_{sc})\right)(V_{mp} - I_{mp}R_s) - naV_T(V_{mp}I_{sc} - V_{oc}I_{mp})} = \exp\left(\frac{V_{mp} + I_{mp}R_s - V_{oc}}{naV_T}\right), \tag{5}$$

$$R_{sh} = \frac{(V_{mp} - I_{mp}R_s)\left(V_{mp} - R_s(I_{sc} - I_{mp}) - naV_T\right)}{(V_{mp} - I_{mp}R_s)(I_{sc} - I_{mp}) - naV_T I_{mp}}. \tag{6}$$

Therefore, once the ideality factor *a*, has been estimated (according to its value within the bracket [1, 1.5] [34], [35]), it is possible to sequentially obtain R_s (from equation (5)), R_{sh} (from equation (6)), I_0 (from equation (4)), and I_{pv} (from equation (3)). Finally, the output current, *I*, can be defined for each value of the output voltage, *V*, by means of an iterative process, or the following equation [36]:

$$I = \frac{R_{sh}(I_{pv} + I_0) - V}{R_{sh} + R_s} - \frac{naV_T}{R_s} W_0\left(\frac{R_{sh}R_sI_0}{naV_T(R_{sh} + R_s)} \exp\left(\frac{R_{sh}R_s(I_{pv} + I_0) + R_{sh}V}{naV_T(R_{sh} + R_s)}\right)\right), \tag{7}$$

where W_0 is the positive branch of the Lambert W-function. In fact, the right-side sub-branch, W_0^+, is the one required.

Alternatively, solving equation (5) to obtain the value of R_s requires an iterative process, this parameter can be obtained from [37]:

$$R_s = A\left(W_{-1}\left(B\exp(C)\right) - (D + C)\right), \tag{8}$$

where W_{-1} is the negative branch of the Lambert W-function and:

$$A = \frac{naV_T}{I_{mp}}, \tag{9}$$

$$B = -\frac{V_{mp}(2I_{mp} - I_{sc})}{V_{mp}I_{sc} + V_{oc}(I_{mp} - I_{sc})}, \tag{10}$$

$$C = -\frac{2V_{mp} - V_{oc}}{naV_T} + \frac{V_{mp}I_{sc} - V_{oc}I_{mp}}{V_{mp}I_{sc} + V_{oc}(I_{mp} - I_{sc})}, \tag{11}$$

$$D = \frac{V_{mp} - V_{oc}}{naV_T}. \tag{12}$$

B. *Explicit models solved with the Lambert W-function*

There are several explicit models to analyze the current-voltage performance of a solar cell/panel [38], [39]. Although these approximations do not preserve any physical aspect of the photovoltaic conversion process, they are interesting and accurate enough to generate new works from time to time [40]. Among the explicit models, the ones that reach a solution based on the Lambert W-function are:

- El-Tayyan's model:

$$I = I_{sc} - C_1 \exp\left(-\frac{V_{oc}}{C_2}\right)\left(\exp\left(\frac{V}{C_2}\right) - 1\right), \tag{13}$$

where:

$$C_1 = \frac{I_{sc}}{1 - \exp\left(-\dfrac{V_{oc}}{C_2}\right)}, \tag{14}$$

and, if $V_{oc}/C_2 >> 1$:

$$C_2 = \frac{V_{mp} - V_{oc}}{W_{-1}\left(\left(1 - \dfrac{V_{oc}}{V_{mp}}\right)\left(\dfrac{I_{mp}}{I_{sc}}\right)\right)}. \tag{15}$$

- Karmalkar & Haneefa's model:

$$\frac{I}{I_{sc}} = 1 - (1 - \gamma)\left(\frac{V}{V_{oc}}\right) - \gamma\left(\frac{V}{V_{oc}}\right)^m. \tag{16}$$

where:

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$$\gamma=\frac{2\left(\frac{I_{mp}}{I_{sc}}\right)-1}{(m-1)\left(\frac{V_{mp}}{V_{oc}}\right)^{m}},\tag{17}$$

$$m=\frac{W_{-1}\left(-\left(\frac{V_{oc}}{V_{mp}}\right)^{\frac{1}{K}}\left(\frac{1}{K}\right)\ln\left(\frac{V_{mp}}{V_{oc}}\right)\right)}{\ln\left(\frac{V_{mp}}{V_{oc}}\right)}+\frac{1}{K}+1,\tag{18}$$

with:

$$K=\frac{1-\left(\frac{I_{mp}}{I_{sc}}\right)-\left(\frac{V_{mp}}{V_{oc}}\right)}{2\left(\frac{I_{mp}}{I_{sc}}\right)-1},\tag{19}$$

- Das' model:

$$\frac{I}{I_{sc}}=\frac{1-\left(\frac{V}{V_{oc}}\right)^{k}}{1+h\left(\frac{V}{V_{oc}}\right)},\tag{20}$$

where:

$$k=\frac{W_{-1}\left(\left(\frac{I_{mp}}{I_{sc}}\right)\ln\left(\frac{V_{mp}}{V_{oc}}\right)\right)}{\ln\left(\frac{V_{mp}}{V_{oc}}\right)},\tag{21}$$

$$h=\left(\frac{V_{oc}}{V_{mp}}\right)\left(\frac{I_{sc}}{I_{mp}}-\frac{1}{k}-1\right).\tag{22}$$

C. *Modeling MPPT performance with the Lambert W-function*

The ambient conditions in which a solar panel works (temperature and solar irradiance), have a relevant effect on its performance [12], [37], [41]. This problem has led to the development of Maximum Power Point Tracking (MPPT) methods, in order to fix the working point of the solar panel at the Maximum Power Point (MPP) of the *I-V* curve (see Fig. 3), no matter the level of the irradiance on the solar panel or the temperature of its cells.

Among the different electronic architectures on which MPPT systems are based, DC-DC converters should be mentioned [42]–[45]. In this MPPT, the voltage at the primary winding (where the solar panel is connected, see Fig. 4) is changed in relation to the voltage at the secondary winding, to reach the aforementioned MPP [46]–[49]. Additionally, it should be noted that the process of setting the voltage at the primary winding of the DC-DC converter at the MPP of a solar panel, in relation to the irradiance and temperature conditions, is not an immediate task, since there are many procedures available in the literature [50]–[57].

The behavior of a solar panel can be defined by means of its efficiency, η, defined as the ratio between the input and the output power:

$$\eta=\frac{V_2 I_2}{V_1 I_1},\tag{23}$$

where V_1 is the input voltage, V_2 is the output voltage, I_1 is the input current, and I_2 is the output current (all variables in relation to the MPPT, see Fig. 4).

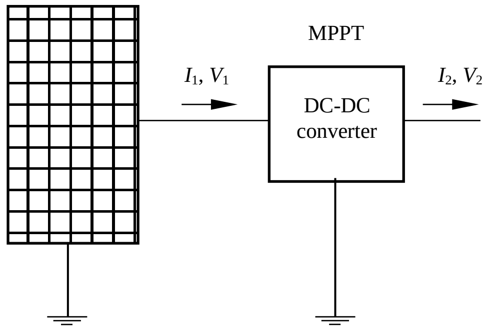

Fig. 4 Sketch of a solar panel connected to a DC-DC converter that works as MPPT.

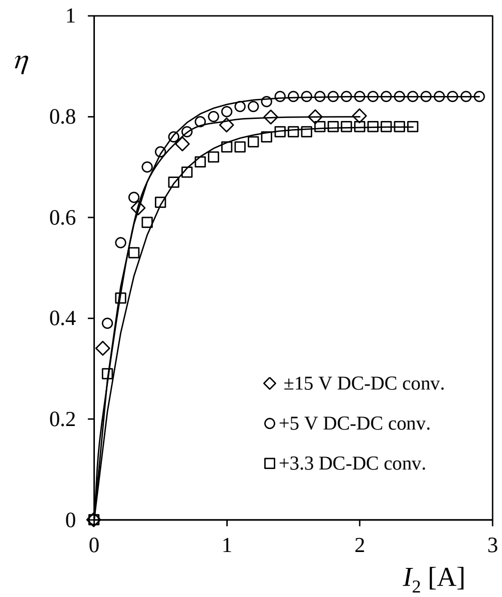

Fig. 5 Efficiency, η, of the UPMSat-2 power distribution subsystem DC-DC converters in relation to the output current, I_2. The fitting of equation (24) to the three data sets have also been plotted as solid lines.

The efficiency, η, of the three DC-DC converters used in the UPMSat-2 satellite power distribution subsystem is plotted in relation to the output current, I_2, in Fig. 5. These converters are:

- ±15 V converter: it converts the bus voltage into +15 V and –15 V (this converter has two output voltage levels in relation to ground).
- +5 V converter: it converts +15 V voltage into +5 V.
- +3.3 V converter: it converts +15 V voltage into +3.3 V.

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It is possible to fit each one of these curves to the following equation:

$$\eta = \eta_{\max}\left(1-\exp\left(g\left(I_2\right)\right)\right), \tag{24}$$

where $\eta_{\max}$ is the maximum value of the efficiency and $f(I_2)$ is a function (normally, a polynomial) which depends on the output current, I_2. If the simplest approach is selected:

$$g\left(I_2\right) = -\frac{I_2}{I_{ch}}, \tag{25}$$

equation (24) becomes the well-known solution of a first order system. In the above equation, I_{ch} is a characteristic current.

In Fig. 5, equation (24) has been fitted to the three sets of data, the values of the maximum efficiency and the characteristic current being: $\eta = 0.8$ and $I_{ch} = 0.22$ A (±15 V converter); $\eta = 0.84$ and $I_{ch} = 0.25$ A (+5 V converter); and $\eta = 0.78$ and $I_{ch} = 0.31$ A (+3.3 V converter).

Once the output voltage, V_2, the input variables of the MPPT (I_1 and V_1, i.e. the MPP), and the behavior of the MPPT (η and I_{ch}) are known, the problem that needs to be solved is finding the correct value of the output current, I_2. The following implicit equation defines this problem:

$$I_2 = \frac{V_1 I_1}{V_2}\eta_{\max}\left(1-\exp\left(-\frac{I_2}{I_{ch}}\right)\right). \tag{26}$$

As the equation from the solar panel 1-D/2-R equivalent circuit model, the above equation can be defined by using the left side of the Lambert W-function positive branch:

$$I_2 = I_{ch}\left(\lambda + W_0^-\left(-\lambda\exp\left(-\lambda\right)\right)\right), \tag{27}$$

where:

$$\lambda = \frac{V_1 I_1}{V_2 I_{ch}}\eta_{\max}. \tag{28}$$

III. Lambert W-function Simple Equations

Once the problems in which the Lambert W-function needs to be accurately estimated were identified (see previous subsections), they can be summarized as:

- 1-D/2-R equivalent circuit model:
  - Solving equation (8) to obtain the value of R_s, by using the negative branch of the Lambert W-function, W_{-1}.
  - Solving equation (7) to obtain the output current, I, in relation to the output voltage, V, by using the right-side sub-branch of the positive branch of the Lambert W-function, W_0^+.
- El-Tayyan, Karmalkar & Haneefa, and Das explicit models: solving equations (15), (18) and (21) by using the negative branch of the Lambert W-function, W_{-1}.
- MPPT (based on DC-DC converters) performance: solving equation (27) by using the left- sub-branch of the positive branch of the Lambert W-function, W_0^-.

A thorough review of the available literature was carried out, in order to obtain sufficiently large data to derive accurate mathematical expressions for the Lambert W-function within the appropriate x variable range (where this function needs to be calculated).

The relevant data (i.e. the five parameters of the 1-Diode/2-Resistors model I_{pv}, I_0, a, R_s, and R_{sh}, the number of cells series-connected, n, the temperature in which the *I-V* curve is measured or calculated, T, and the four characteristic points, I_{sc}, V_{oc}, I_{mp}, and V_{mp}), from 89 different photovoltaic devices (mostly solar panels), were found.

A. Lambert W-function for solving the 1-D/2-R equivalent circuit model implicit equation

If we go to equation (7), this allows us to calculate the current for a variable x that is written in relation to the aforementioned five parameters of the 1-Diode/2-Resistors model, the number of cells series-connected, the temperature and the output voltage, V:

$$x = f\left(I_{pv}, I_0, a, R_s, R_{sh}, n, T, V\right). \tag{29}$$

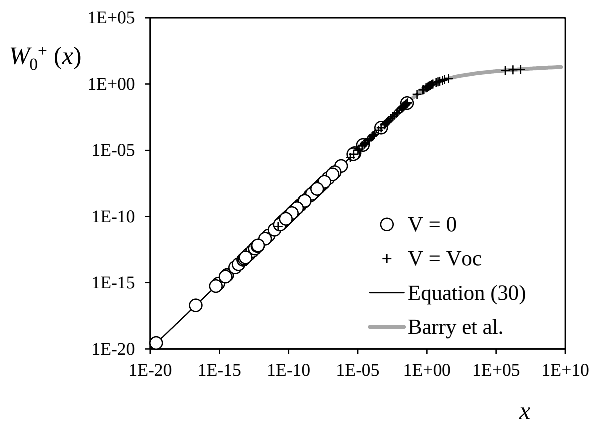

Fig. 6 Lambert W-function, W_0^+ from equation (7) calculated at $V = 0$ and $V = V_{oc}$ for the solar cells/solar panel data found in the available literature. The approximated equation (30) is also plotted in the graph.

Bearing in mind that, when evaluating the performance of a photovoltaic device, two extremes of the above variable, x, arise at short circuit ($V = 0$) and open circuit ($V = V_{oc}$) points, it is possible to define a characteristic interval for evaluating the Lambert W-function in relation to equation (7). The values of the Lambert W-function W_0^+ calculated at each point (equation (29)) with the data from each photovoltaic device found in the available literature, at $V = 0$ and $V = V_{oc}$, are shown in Fig. 6. Based on these points, included in the bracket $[10^{-20}, 10^{-5}]$, the following approximation is proposed:

$$W_0^+\left(x\right) = x - \exp\left(4.123\cdot 10^{-6}\ln\left(x\right)^2 + 2.0001\ln\left(x\right) + 1.64\cdot 10^{-4}\right). \tag{30}$$

The above equation has also been plotted in Fig. 7. It should be noted that this equation has 1.4% error for $x = 0.1$,

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the accuracy being improved for lower values of x. For larger values of x it is recommended to use the approximation proposed by Barry et al. [22] (see Appendix), as shown in Fig. 6.

B. Lambert W-function for solving the series resistor implicit equation from the analytcal solution of the 1-D/2-R equivalent circuit model

The negative branch of the Lambert W-function, W_{-1}, allows the user to calculate the series resistor of the 1-Diode/2-Resistor model, R_s, in relation to the characteristic points, the number of cells that are series-connected, the ideality factor and the temperature (equation (8)). Therefore, W_{-1} is required to be evaluated at:

$$x = f\left(I_{sc}, I_{mp}, V_{oc}, V_{mp}, a, n, T\right). \tag{31}$$

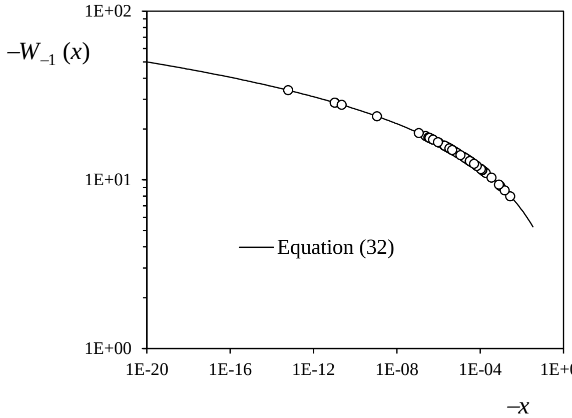

Fig. 7 Lambert W-function, W_{-1} from equation (8) calculated for the solar cells/solar panel data found in the available literature (open circles). The approximated equation (32) is also plotted in the graph.

In Fig. 7, the values of W_{-1}, calculated at the values of the above variable x resulting from the solar cells and solar panel data found in the literature, are plotted. The bracket in which the Lambert W-function W_{-1} needs to be evaluated is $[-10^{-3}, -10^{-20}]$. The following simple expression is proposed for these estimations:

$$\begin{gathered} W_{-1}(x) = 2.4978 \cdot 10^{-5} \ln(-x)^3 + \\ 2.8111 \cdot 10^{-3} \ln(-x)^2 + 1.1299 \ln(-x) - 1.4733 \end{gathered}. \tag{32}$$

This equation is also plotted in Fig. 7. The maximum error in equation (32) was proven to be below 0.4%, within the mentioned bracket.

C. Lambert W-function for solving the equations from El-Tayyan, Karmalkar & Haneefa, and Das explicit models

With regard to the explicit models that need the Lambert W-function (equations (15), (18) and (21)), the values of this function are plotted in Fig. 8, in relation to the corresponding figures of variable x:

$$x = f\left(I_{sc}, I_{mp}, V_{oc}, V_{mp}\right), \tag{33}$$

for each photovoltaic device found in the available literature. Based on these points, the following equation is proposed for the Lambert W-function, W_{-1}, in the bracket [–0.364, –0.1]:

$$\begin{gathered} W_{-1}(x) = \\ 248.42x^4 + 134.24x^3 + 4.4258x^2 - 14.629x - 4.9631 \end{gathered}. \tag{34}$$

The above equation has less than 1.6% error within the mentioned bracket.

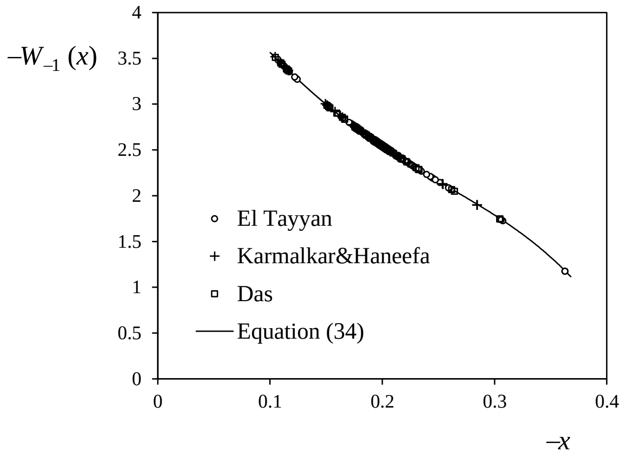

Fig. 8 Lambert W-function, W_{-1} from equations (15), (18), and (21) calculated for the solar cells/solar panel data found in the available literature (open circles, crosses, and open squares). The approximated equation (34) is also plotted in the graph.

D. Lambert W-function for solving the implicit equation of the MPPT model

The left side of the Lambert W-function positive branch, required to solve the MPPT equation (see Section I, subsection D), needs to be evaluated at:

$$x = f\left(V_1, I_1, V_2, I_{ch}, \eta_{\max}\right), \tag{35}$$

see also equations (27) and (28). The following mathematical expressions are proposed to approach the aforementioned left side of the Lambert W-function positive branch:

$$W_0^-(x) = x\ ;\ x \in \left[-8 \cdot 10^{-3}, 0\right], \tag{36}$$

$$\begin{gathered} W_0^-(x) = 3.50621x^3 - 0.7188x^2 + 1.0104x\ ; \\ x \in \left[-2.15 \cdot 10^{-1}, -8 \cdot 10^{-3}\right] \end{gathered}, \tag{37}$$

$$\begin{gathered} W_0^-(x) = 1.56322\left(x + e^{-1}\right)^{\frac{1}{2.4}} - 1\ ; \\ x \in \left[-e^{-1}, -2.15 \cdot 10^{-1}\right] \end{gathered}. \tag{38}$$

The above equations have an error below 0.8%, 0.82% and 1.86% respectively.

IV. Results and Discussion

A. Modeling solar cells/panels

In order to check the expressions for the Lambert W-function obtained in the previous section, the well-known RTC France solar cell and Photowatt PWP 201 solar panel *I-V* curves from Easwarakhanthan et al. (1986) [58] were used (see Figs. 9 and 10). Since the publication of this work, these curves have become the standard for checking photovoltaic performance models and parameter extraction procedures [59], [60], [69], [61]–[68]. In Table I, the number of series-

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connected cells, n, and the characteristic points of both photovoltaic devices are included.

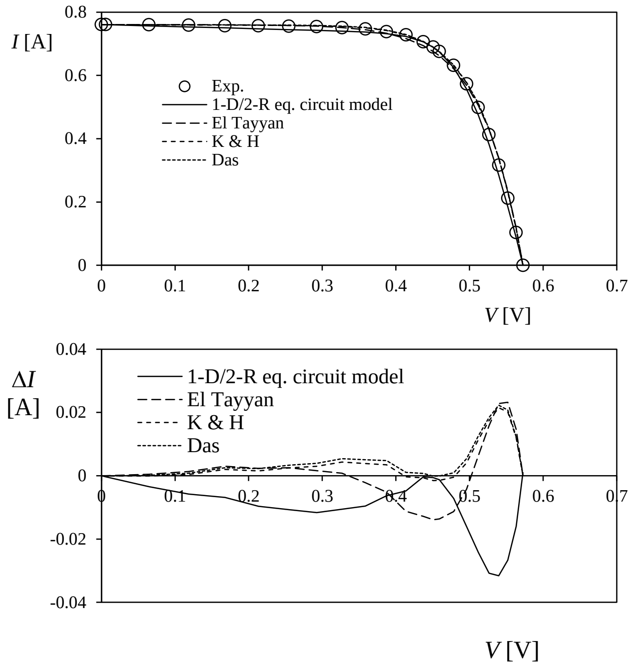

Fig. 9 RTC France solar cell [58]. 1-D/2-R equivalent circuit model and El Tayyan, Karmalkar & Haneefa, and Das explicit models fitted to the testing results (Exp.) (top). Error of the different models in relation to the testing results, $\Delta I = I - I_{exp}$ (bottom).

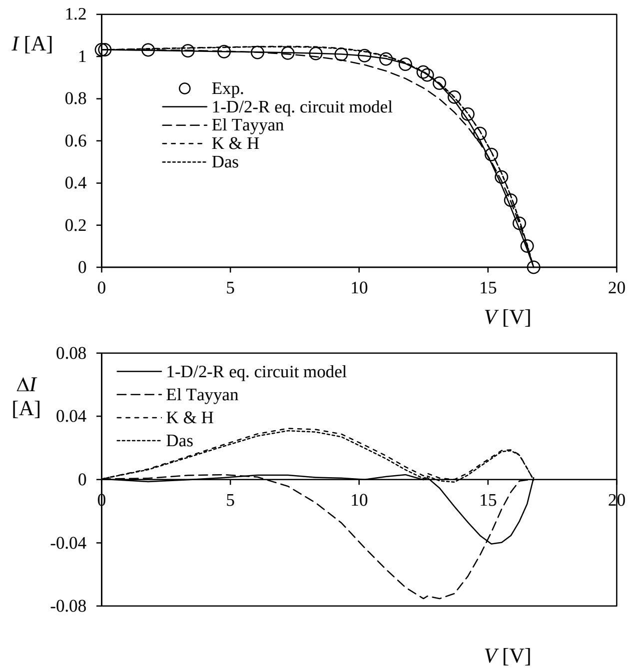

Fig. 10 Photowatt PWP 201 solar panel [58]. 1-D/2-R equivalent circuit model and El Tayyan, Karmalkar & Haneefa, and Das explicit models fitted to the testing results (Exp.). Error of the different models in relation to the testing results, $\Delta I = I - I_{exp}$ (bottom).

In Tables II and III, the parameters related to the 1-D/2-R equivalent circuit model fitted to the mentioned *I-V* curves are included. These parameters were first extracted by selecting a reasonable value for the ideality factor, $a = 1$, and then by using equations (8) to (12), (6), (4) and (3), combined with equation (32), to solve the Lambert W-function in equation (6). Taking the extracted values of Tables II and III, equation (7) was solved with the help of the Lambert W-function in equation (30), to obtain the output current, I, as a function of the output voltage, V. The results have been included as a solid line in Figs. 9 and 10. It can be observed that the results obtained match those from testing. The error with regard to the testing results, ΔI, is plotted in the bottom graphs of these figures, its maximum values being reduced in relation to I_{mp}: 1.7% (RTC) and 0.3% (PWP 201) for output voltages lower than V_{mp}, and 4.6% (RTC) and 4.4% (PWP 201) for output voltages larger than V_{mp}. Additionally, it should be noted that, logically, the proposed approach is less precise than the numerical one proposed by Easwarakhanthan et al. (1986), showing a larger RMSE. Nevertheless, it can also be underlined that this larger error in the proposed approach can be reduced by optimizing the initial value of the ideality factor, a, by an iterative process.

TABLE I. NUMBER OF SERIES-CONNECTED CELLS AND CHARACTERISTIC POINTS FROM RTC FRANCE SOLAR CELL AND PHOTOWATT PWP 201 SOLAR PANEL.

	RTC France	Photowatt PWP 201
n	1	36
I_{sc} [A]	0.7605	1.0320
I_{mp} [A]	0.6894	0.9255
V_{mp} [V]	0.4507	12.493
V_{oc} [V]	0.5727	16.778

TABLE II. 1-D/2-R EQUIVALENT CIRCUIT MODEL FITTED TO THE RTC FRANCE SOLAR CELL *I-V* CURVE. PARAMETERS EXTRACTED IN THE PRESENT WORK AND IN EASWARAKHANTHAN ET AL. (1986) [58].

	RTC France solar cell	
	Easwrakhanthan et al. [58], [70]	Present work
a	1.4837	1
I_{pv} [A]	$7.608\cdot10^{-01}$	$7.638\cdot10^{-01}$
I_0 [A]	$3.223\cdot10^{-07}$	$2.692\cdot10^{-10}$
R_s [Ω]	$3.640\cdot10^{-02}$	$6.977\cdot10^{-02}$
R_{sh} [Ω]	$5.376\cdot10^{+01}$	$1.616\cdot10^{+01}$
RMSE [A]	$6.251\cdot10^{-03}$	$1.401\cdot10^{-02}$

TABLE III. 1-D/2-R EQUIVALENT CIRCUIT MODEL FITTED TO THE PHOTOWATT PWP 201 SOLAR PANEL *I-V* CURVE. PARAMETERS EXTRACTED IN THE PRESENT WORK AND IN EASWARAKHANTHAN ET AL. (1986) [58].

	Photowatt PWP 201 solar panel	
	Easwrakhanthan et al. [58], [70]	Present work
a	1.3458	1
I_{pv} [A]	1.0318	1.0358
I_0 [A]	$3.2876\cdot10^{-06}$	$4.1626\cdot10^{-08}$
R_s [Ω]	1.2057	1.9815
R_{sh} [Ω]	$5.4945\cdot10^{+02}$	$5.4292\cdot10^{+02}$
RMSE [A]	$7.805\cdot10^{-03}$	$1.799\cdot10^{-02}$

The results from the explicit models (El Tayyan, Karmalkar & Haneefa, and Das) applied to both the RTC France solar cell and the Photowatt PWP 201 solar panel, and solved with the proposed equations for the Lambert W-function, are included in Figs. 9 and 10. A relevant match with the testing results can be observed (with the exception of the fitting of El Tayyan's method to the PWP 201 experimental data), with reasonably reduced values of RMSE (Table IV). The current error, ΔI, in relation to the testing results, is plotted in the bottom graphs of Figs. 9 and 10. As shown by the 1-D/2-R equivalent circuit model, the maximum errors in relation to I_{mp} are not large: 0.62% (RTC) and 3.5% (PWP 201) for output voltages lower than V_{mp}, and 3.2% (RTC) and 2.1% (PWP 201) for output voltages larger than V_{mp}. These results exclude those from El-Tayyan's model, whose maximum errors are approximately twice those from the other two explicit models. This poorer performance of El Tayyan's method has already been suggested [39].

TABLE IV. PARAMETERS OF EL TAYYAN, KARMALKAR & HANEEFA, AND DAS EXPLICIT MODELS, FITTED TO RTC FRANCE SOLAR CELL AND PHOTOWATT PWP 201 SOLAR PANEL EXPERIMENTAL DATA [58].

Model parameters		RTC	PWP
El Tayyan	C_1	$7.6053\cdot10^{-01}$	1.03317
	C_2	$5.5678\cdot10^{-02}$	2.4751
	RMSE [A]	$1.029\cdot10^{-02}$	$4.092\cdot10^{-02}$
Karmalkar & Haneefa	γ	$9.9322\cdot10^{-01}$	1.0422
	m	$1.0014\cdot10^{+01}$	6.9996
	RMSE [A]	$8.159\cdot10^{-03}$	$1.649\cdot10^{-02}$
Das	k	$1.0021\cdot10^{+01}$	6.9669
	h	$4.2531\cdot10^{-03}$	$-3.822\cdot10^{-02}$
	RMSE [A]	$8.656\cdot10^{-03}$	$1.556\cdot10^{-02}$

Finally, it should be emphasized that the fairly accurate results obtained with the proposed methodology (i.e. the use of the Lambert W-Function when solving the 1-D/2-R equivalent circuit model and the explicit methods), are obtained by very simple calculations (in contrast with numerical methodologies) to solve the aforementioned models. This is the most important merit of the present work.

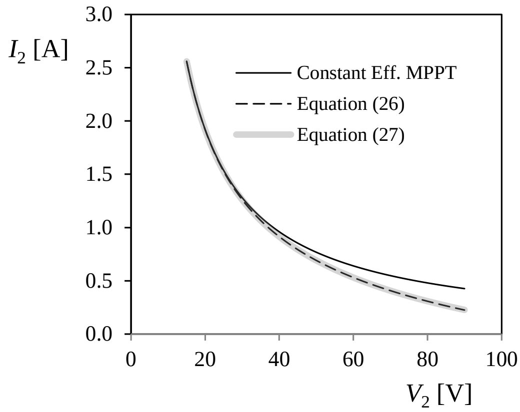

Fig. 11 Output current, I_2, from a MPPT system with the following characteristics: V_1 = V, I_1 = A, η_{max} = 0.8, and I_{ch} = 0.3 A (see equations (24) to (28)), in relation to the output voltage, V_2. The current, calculated by equations (26) and (27) is compared to the resulting one from a constant efficiency MPPT whose value is equal to η_{max}. (see equation (24)).

B. Modeling MPPT based on DC-DC converters

In Fig. 11, the output current, I_2, is shown in relation to the output voltage, V_2, for a solar panel - MPPT system with the following characteristics: V_1 = V, I_1 = A, η_{max} = 0.8, and I_{ch} = 0.3 A. This current was calculated:

- by solving equation (26) numerically (plotted with a dashed line), and
- by equation (27), also taking into account equations (36) to (38) (solid grey line).

Finally, these curves are compared to the current from an ideal MPPT system with a constant efficiency equal to η_{max} (plotted with a solid line). The effect of the efficiency reduction can be clearly observed for the lower values of the output current. The percentage error of the output current, I_2, calculated by equation (27), is plotted in relation to that from equation (26), see Fig. 12:

$$err = \frac{I_{2,\text{ eq. (27)}} - I_{2,\text{ eq. (26)}}}{I_{2,\text{ eq. (26)}}}\,. \tag{39}$$

Fig. 12 shows that the error related to equation (27), solved with the suggested approach to the Lambert W-function defined by equations (36) to (38), is lower than 1%.

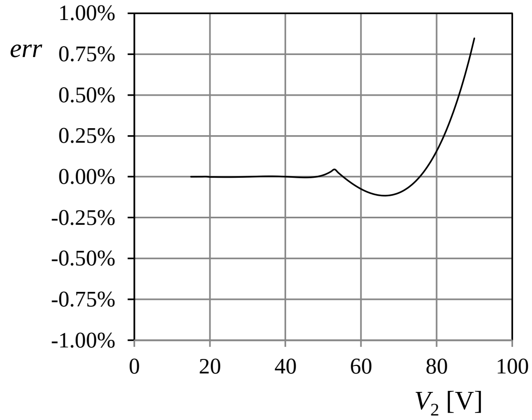

Fig. 12 Percentage error (err, see equation (39)) of the output current, I_2, calculated by equation (27) (and the proposed approach to the left side of the Lambert W-function positive branch, equations (36) to (38)), with regard to the current calculated by solving equation (26). The error, err, is plotted in relation to the output voltage, V_2. See also Fig. 11.

V. CONCLUSIONS

The Lambert W-function has revealed itself to be a relevant tool for solving the implicit equations that arise when analyzing photovoltaic systems performance. However, working with this mathematical function can be a challenge as it is not represented by any direct equation.

In the present work, simple equations are successfully derived for the Lambert W-function in the following cases:

- Direct calculation of the photovoltaic output current, I, as a function of the output voltage, V, by using the 1-Diode/2-Resistor equivalent circuit model (with all the five parameters being well defined).
- Calculation of the series resistor parameter, R_s, from the 1-Diode/2-Resistor equivalent circuit model in relation to the characteristic points of the I-V curve (I_{sc}, I_{mp}, V_{oc}, and V_{mp}), the ideality factor, a, the number of the series-connected cells of the photovoltaic device, and the thermal voltage, V_T.
- Calculation of the parameters from the explicit methods by El Tayyan, Karmalkar & Hannefa, and Das.
- Calculation of the output current of a photovoltaic system composed of a solar panel and an MPPT based on a DC-DC converter, when the efficiency of this Maximum Peak Power Tracking is modeled by a simple first-order system equation.

The proposed Lambert W-function equations were checked with well-known standard information from the RTC solar cell and the Photowatt PWP 201 solar panel (I-V

curves from Easwarakhanthan et al. (1986)). The results obtained from fitting the 1-D/2-R equivalent circuit model (with an initial condition a = 1) indicate a 4.6% maximum error (in relation to I_{mp}). This is quite a relevant result, bearing in mind that this accuracy can be improved by optimizing the ideality factor, a, and how easily it was obtained by using the proposed equations for the Lambert W-function. Additionally, the fitting of three explicit models to the aforementioned testing data was carried out with the derived equations. The results from the Karmalkar & Haneefa and Das models indicate a 3.5% maximum error (in relation to I_{mp}). The results related to El Tayyan's model are less accurate, the lack of accuracy being related to the model itself.

The results regarding the Lambert W-function equations when analyzing MPPT behavior, were also checked by a direct comparison with the numerical solution. The results obtained by the use of the procedure described in the present work have a maximum difference of 1% in relation to the numerical solution.

VI. Appendix

Analytical approximation to the Lambert W-function proposed by Barry et al. [22]:

$$W_0^-(x) = -1 + \frac{\sqrt{\varphi}}{1+\left(\left(N_1\sqrt{\varphi}\right)/N_2+\sqrt{\varphi}\right)}, \quad (40)$$

where:

$$\varphi = 2(1+ex), \quad (41)$$

$$N_1 = \left(1-\frac{1}{\sqrt{2}}\right)\left(N_2+\sqrt{2}\right), \quad (42)$$

$$N_2 = 3\sqrt{2}+6-\frac{\left(2237+1457\sqrt{2}\right)e-4108\sqrt{2}-5764}{\left(215+199\sqrt{2}\right)e-430\sqrt{2}-796}\varphi. \quad (43)$$

$$W_0^+(x) = 1.4586887\ln\left(\frac{1.2x}{\ln\left(2.4x/\ln(1+2.4x)\right)}\right) - 0.4586887\ln\left(\frac{2x}{\ln(1+2x)}\right). \quad (44)$$

$$W_{-1}(x) = -1-\sigma-5.95061\left(1-\frac{1}{1+f(\sigma)}\right), \quad (45)$$

where:

$$f(\sigma) = \frac{0.23766\sqrt{\sigma}}{1-0.0042\sigma\exp\left(-0.0201\sqrt{\sigma}\right)}, \quad (46)$$

$$\sigma = -1-\ln(-x). \quad (47)$$

Acknowledgment

We are indebted to the Director of the IDR/UPM Institute, Prof. Angel Sanz-Andres, for his constant support in relation to all research activities carried out in this institution. We are indebted to Anna Maria Ballester-Bohn for her kind help in improving the text style. We are also grateful to the Reviewers, whose comments helped to improve this paper.

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