Risk Management

Prof. Luis Seco

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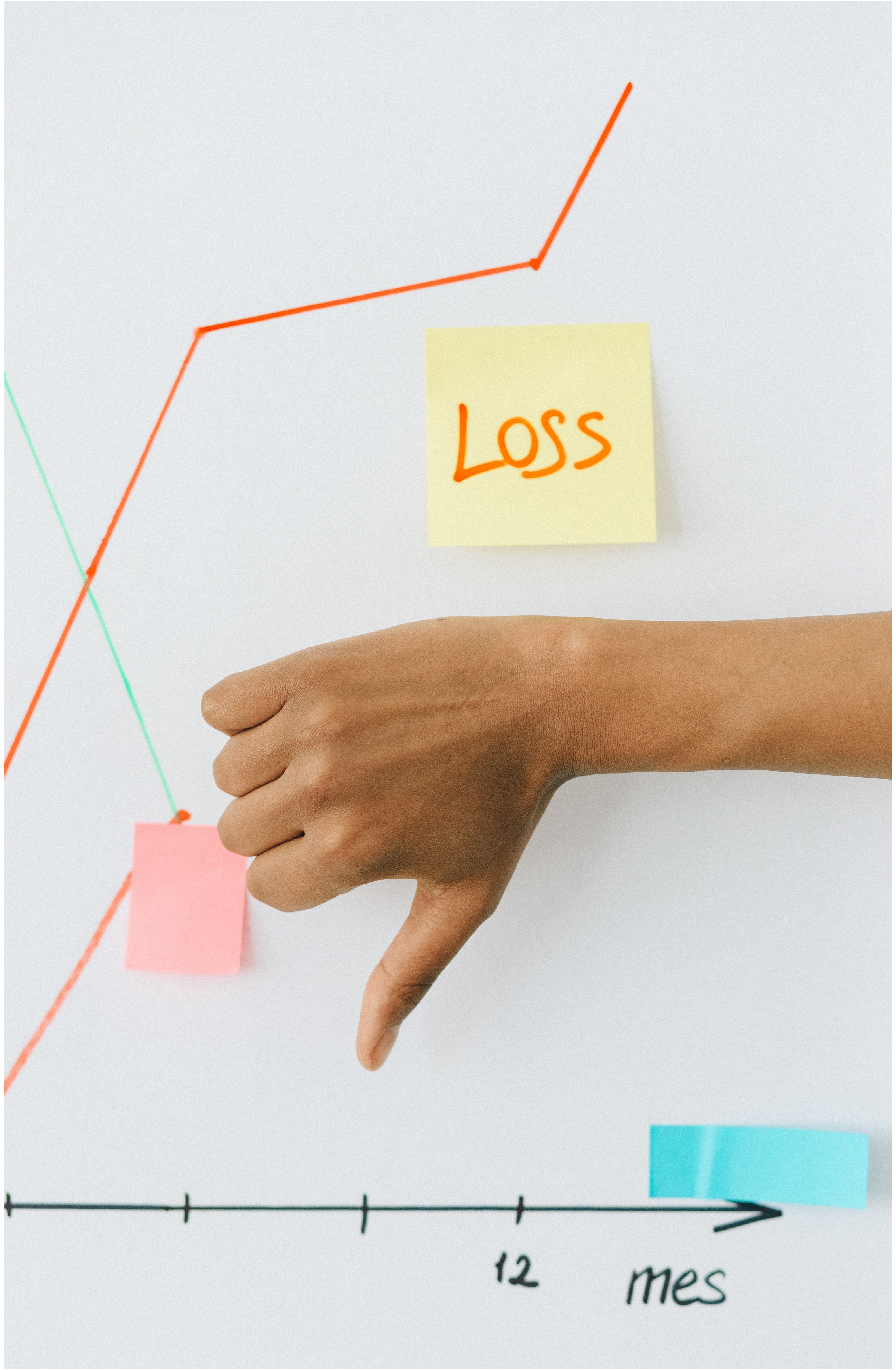
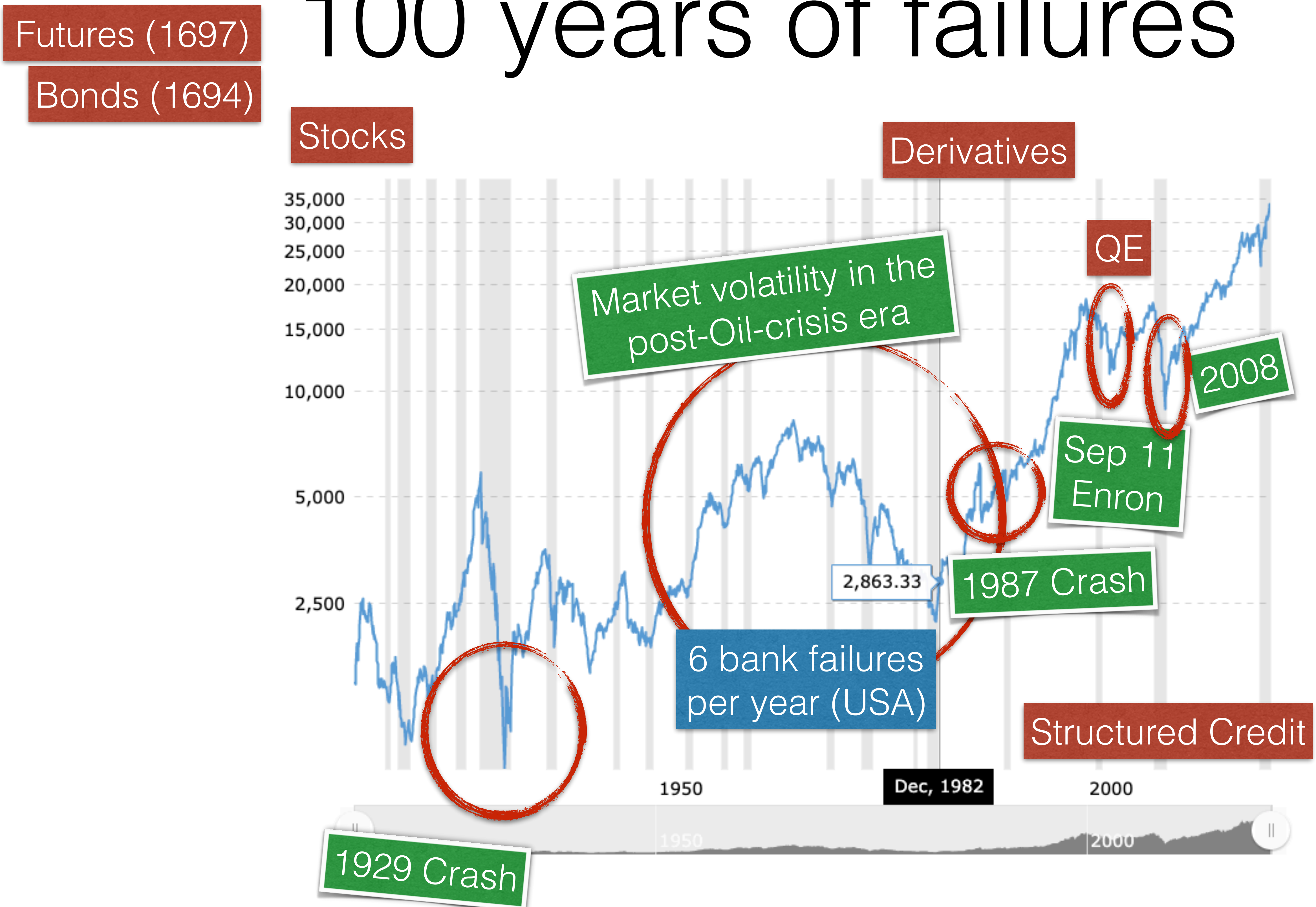


100 years of failures



Events & Regulation

Event	Regulation
1929 Crash	Securities ACT (1933)
Enron	SOX (2002)
2008	Dodd-Frank (2010)
?	Data Regulation?
COVID-19	?
Climate Risk	Article 2.1.c of the Paris accord

How will data be regulated?

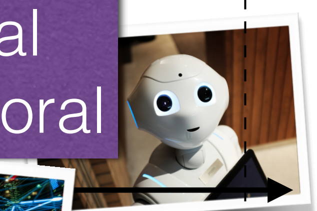
100 years of Risk Analysis

1930 *1950* *2000* *2020* *2030*

Harry Markowitz realized
no value was given to
RISK

"Portfolio Selection," *Journal of Finance*. 1952
languished on dusty library shelves
only four of the 14 pages contained text

ML
AI
ESG
NLP
Social
Behavioral



John Burr Williams,
The Theory of Investment Value
value of a stock given by
current value of future dividends.

A plethora of quant methods
started with the ubiquity of the PC

Find a good stock
buy it at the best price.

Climate Risk
article 2.1.c of the Paris Accord

Risks

- Market
    - Loss of NAV due to changes in asset prices
- Credit
    - Loss of NAV due to default events
- Liquidity
    - Delay turning asset value into cash: Example: LTCM, Hunt Brothers
- Gap risk
    - The risk that an investment's price will change from one level to another with no trading in between
- Legal Risk
    - Example: Bankers Trust
- Operational risk
    - Fat fingers, fraud, cybersecurity, etc.

Risk Classification

	Banking	Insurance	Asset Management
Market Risk	Basel I - VaR Basel III - FRTB	Solvency II - SCR	Concentration Exposures
Credit Risk	Basel II - CVaR Basel III - CVA (MAR50)	SCR	DDQ - Credit premia
Operational Risk	Fraud, fat fingers, etc.	internal systems, personnel, procedures, or controls (IAIS)	Fraud, systems, etc
Liquidity Risk	LCR (UK), ILG (Basel), HQLA, NSFR (US)	Governance, roles and responsibilities not clearly defined	Ad Hoc (UCITS, 30-ACT, etc.)
Gap risk	Asset valuation	ALM	Ø
Legal Risk	Marginal to the risk departments: i.e. Cat bonds?		IAA-1933: MNPI, Trading rules ESG
Cybersecurity	On-line banking	Disaster revenue generator	Data Protection Disaster recovery

Legend

Regulated

Unregulated

Revenue generator

Business cases

Case		Sector	Type	
Montreal - Sky resort	2003	Financial Services	Market. Credit, Style risk	A snow swap was created to shift risk from the city snow-cleaning operations to a ski resort
Orange County	1994	AM	Market risk	Bob Citron leveraged the County's fixed income portfolio
Metallgesellschaft	1993	AM	Market risk	Oil-futures contango $1.5Bn loss
Bankers Trust - P&G 1996	1996	Bank - AM	Market risk, legal risk	Swap, "rip-off" fee, and options as gambling
Amaranth	2006	AM	Market risk, credit risk	75% of the portfolio was on a single natural gas trade: $5Bn loss
LTCM	1998	AM - Banking	Liquidity	a $5Bn portfolio controlled more than $1Tn of bond nationals in the US
Malaysia Centra Bank	2018	Banking	Cybersecurity	Unauthorized fund transfers using falsified SWIFT messages, $81M loss
VW short squeeze	2008	AM - Banking	Market	Porche announced they owned shared in VW in excess of the short interest on the stock

Risk Management - international Banking

1987

1991

1997

2005

2017

Basel International Treaty

1997
JPM's CreditMetrics

Long Term Capital Management

1994
JPM's RiskMetrics

Basel I
(Market Risk)

Basel II
(Credit Risk)

Stock market crash
Driven by option trading

1998

Basel III
- Market
- Credit
- Operational

Options are non-linear instruments

Basel Committee on Banking Supervision

Timeline Risk & Regulation in Asset Management (USA)

1929

1933-34

2001

2002

2008

2009

SEC
Advisor act
Exchange regulation

Sarbanes-Oxley (SOX)

Dodd-Frank

Stock market crash

Insider trading was legal
Market manipulation easy

ENRON

The Great Recession

Asset management pre 1929

- The crash followed a speculative boom that had taken hold in the late 1920s. Net profits of companies showed an increase of 36.6% over 1928, itself a record half-year.

- Stock-exchange speculation which had led hundreds of thousands of Americans to invest heavily in the stock market, borrowing to buy more stocks. Brokers were routinely lending more than two-thirds of the value of the stocks. **Over $8.5 billion was out on loan, more than the entire amount of currency circulating in the U.S. at the time.**

- The average P/E ratio was 32.6 in September1929

- **Albert H. Wiggin, the head of Chase National Bank, shorted 40,000 shares of his own company, making a $4M profit.**

BCBS

- 1974: Basel Committee on Bank Supervision (BCBS)

… to enhance "financial stability by improving supervisory know-how and the quality of banking supervision worldwide."

- 1992: Capital to exceed 8% of risk-weighted assets (RWA)

Risk Weight	Instrument type
0%	cash, central bank and government debt, and any Organization for Economic Cooperation and Development (OECD) government debt.
20%	Development bank debt, OECD bank debt, OECD securities firm debt, non-OECD bank debt (under one year of maturity), non-OECD public sector debt and cash in collection
50%	residential mortgages, and the 100% category is represented by private sector debt, non-OECD bank debt (maturity over a year), real estate, plant and equipment, and capital instruments issued at other banks
100%	All other debt, such as corporate bonds, etc.

October 19, 1987

- Automatic trading of stock options created a volatility bubble which cause the market to crash 25% in a single day
- Options are non-linear instruments: capital ratios on RWA are linear functions, and hence not adequate in a world that uses derivatives increasingly
- Mathematical concepts made it into law

Value-at-Risk

VaR of a portfolio
95% confidence
equal to x

when the probability of losing more than x
is 5%

$$\int_{-\infty}^{-\mathrm{VaR}_\alpha} \rho(r)\, dr = \mathrm{Prob}\{\mathrm{Losses} \geq \mathrm{VaR}_\alpha\}$$

$$= 1 - \alpha$$

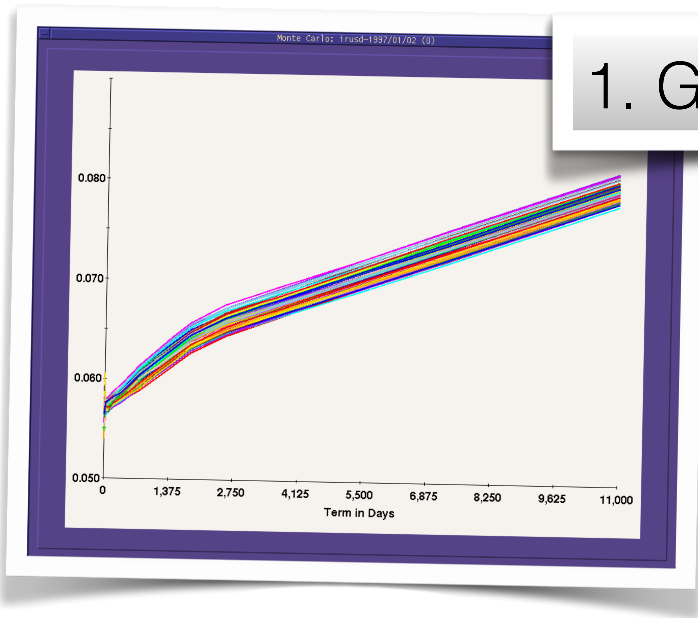
The lottery ticket

We issue 1000 lottery tickets
The winner gets $1M from us

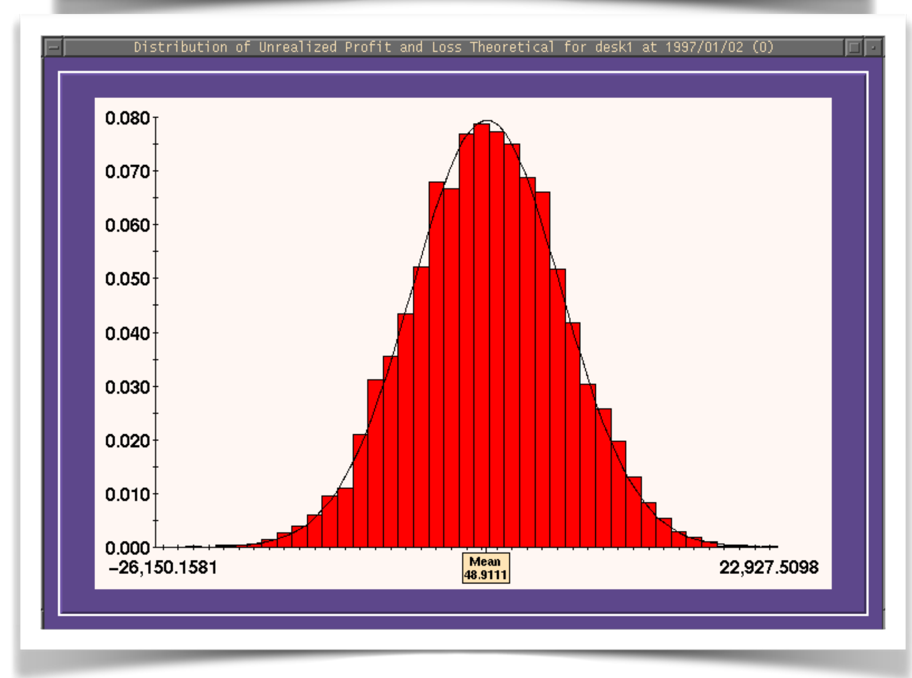
Number of tickets sold	Prob of losing $1M	VaR
1	0.1%	0
49	4.9%	0
50	5.0%	$1M
1000	100%	$1M

Calculation of VaR

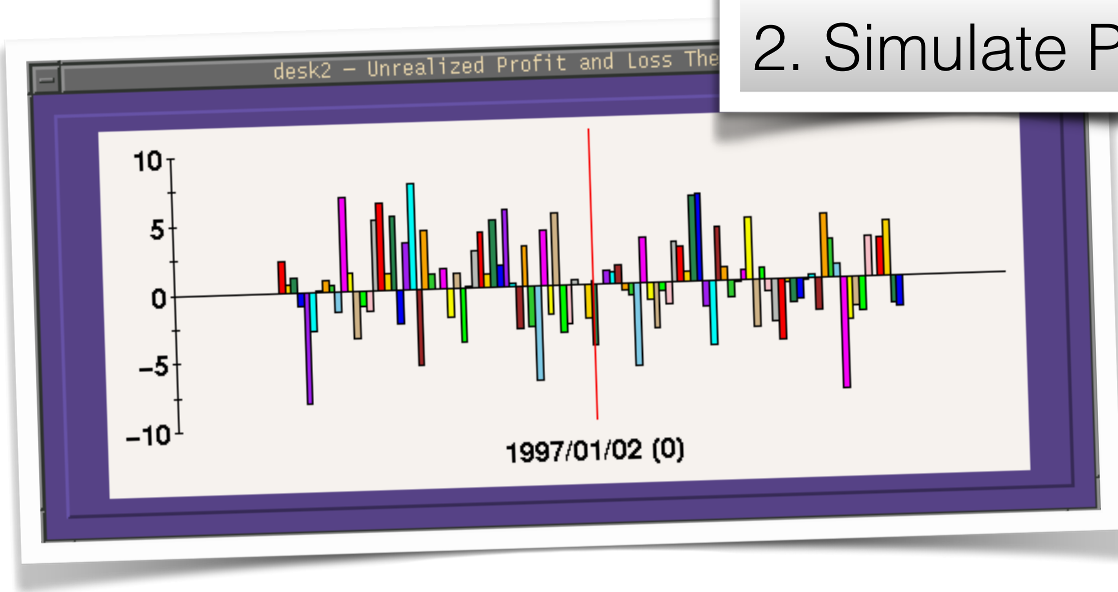
1. Generate scenarios



3. Calculate P&L statistics



2. Simulate P&L



Gaussian VaR - 1 D

$$\text{Prob}\{\text{P \& L} \leq a\} = \frac{1}{\sqrt{2\pi\sigma^2 \cdot T}} \int_{-\infty}^{a} e^{-(x-\mu)^2/(2T\sigma^2)}\, dx$$

If we denote by $\phi(x)$ the commutative distribution of the Gaussian(0,1), then

$$\text{Prob}\{\text{P \& L} \leq a\} = \phi(\frac{a-\mu}{\sigma})$$

$$\text{VaR}_\alpha = \sigma \cdot z_\alpha - \text{Expected return}$$

$z_{0.95} = 1.65$

$z_{0.99} = 2.33$

Example

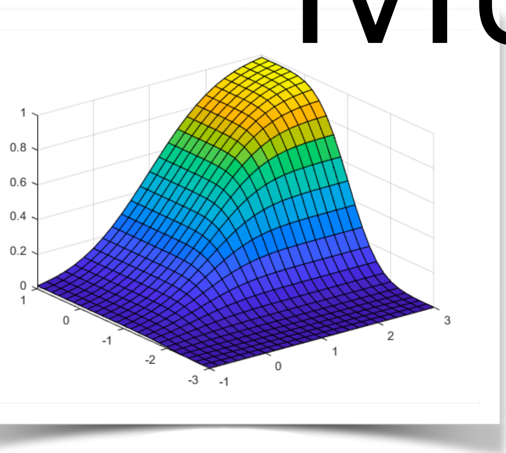
- Assume a portfolio with one assets of $5 million allocated to stock ABC
- Assume the price volatility of stock ABC 2.98% for a one-day period.
- The 1-day 95%-VaR would be

$$\$1.65 * 5 * 0.0298M = \$246k$$

The one-year 95%-VaR would then be

$$\sqrt{365} * 0.246 = \$4.69\,M$$

Multivariate Gaussians



Correlation

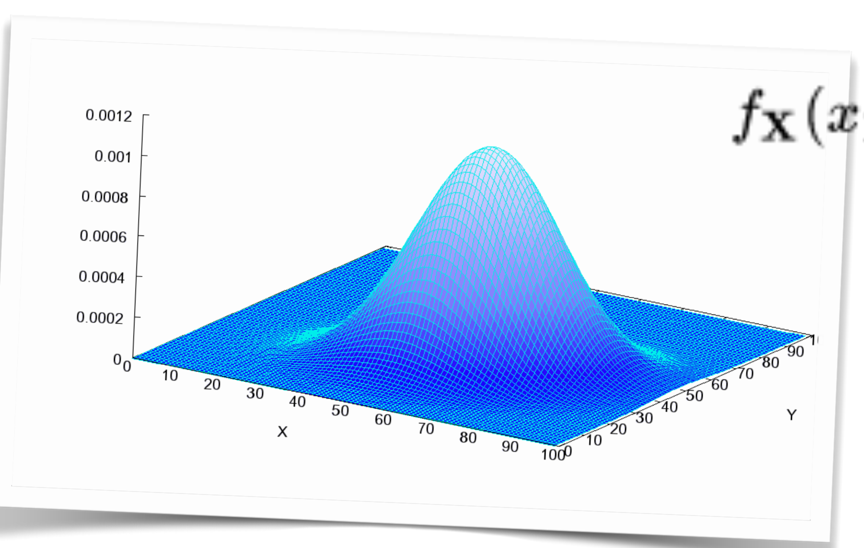
Marginal means

$$f(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \exp\left(-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_X)^2}{\sigma_X^2} + \frac{(y-\mu_Y)^2}{\sigma_Y^2} - \frac{2\rho(x-\mu_X)(y-\mu_Y)}{\sigma_X\sigma_Y}\right]\right)$$

Marginal Standard Deviations

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_X \\ \mu_Y \end{pmatrix}, \quad \boldsymbol{\Sigma} = \begin{pmatrix} \sigma_X^2 & \rho\sigma_X\sigma_Y \\ \rho\sigma_X\sigma_Y & \sigma_Y^2 \end{pmatrix}.$$

Covariance Matrix



$$f_{\mathbf{X}}(x_1,\ldots,x_k) = \frac{\exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)}{\sqrt{(2\pi)^k|\boldsymbol{\Sigma}|}}$$

Matrix Inverse

Matrix Determinant

Gaussian VaR - n D

- If the variables $X_1, X_2, \ldots, X_n$ are jointly gaussian, their distribution density is given by

$$f_{\mathbf{X}}(x_1, \ldots, x_k) = \frac{\exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^{\mathrm{T}}\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)}{\sqrt{(2\pi)^k|\boldsymbol{\Sigma}|}}$$

- A linear combination of the $\sum_{i=1}^{n} c_i X_i$ is a 1-d Gaussian with

$$\mu = \sum_{i=1}^{n} c_i \cdot \mu_i, \qquad \sigma = \vec{c} \cdot \Sigma \cdot \vec{c}^{\,t}$$

- Therefore, for a portfolio with gaussian factors as above

$$\mathrm{VaR}_\alpha = \sigma \cdot z_\alpha - \text{Expected return}$$

$z_{0.95} = 1.65$

$z_{0.99} = 2.33$

Application: RiskMetrics

- RiskMetrics is a method for calculating an approximation to the Value-at-Risk (VaR) of a single investment or an investment portfolio.
- It assumes that an investment's returns follow a normal distribution over time.

Delta Normal VaR

Portfolio value

Risk Factor today
-known value-
(interest rate, stock price, etc.)

$$P(R) \approx P(R_0) + \underbrace{\left.\frac{\partial P}{\partial R}\right|_{R=R_0}}_{\delta}(R - R_0) + \frac{1}{2}\underbrace{\left.\frac{\partial^2 P}{\partial R^2}\right|_{R=R_0}}_{\Gamma}(R - R_0)^2$$

Future Risk Factor
-Unknown value-
(interest rate, stock price, etc.)

Portfolio Sensitivities

Bond: duration

$$\delta = \left.\frac{\partial(N_0 e^{-R})}{\partial R}\right|_{R=R_0} = -N_0 e^{-R_0}.$$

Delta Normal VaR

If Risk Factors are gaussian…

$$\mu = \sum_{i=1}^{n} c_i \cdot \mu_i, \qquad \sigma = \vec{c} \cdot \Sigma \cdot \vec{c}^{\,T}$$

$$P(R) \approx P(R_0) + \underbrace{\left.\frac{\partial P}{\partial R}\right|_{R=R_0}}_{\delta} (R - R_0)$$

… the P&L is gaussian

$$\mathrm{VaR}_\alpha = \sigma \cdot z_\alpha - \text{Expected return} \qquad z_{0.95} = 1.65 \qquad z_{0.99} = 2.33$$

Example

- Assume a portfolio with two assets, one of $5 million allocated to stock ABC and another of $8 million allocated to stock DEF.
- Respectively, the price volatility of stock ABC and stock DEF is 2.98% and 1.67% for a one-day period.
- The correlation between the two stocks is 0.67.
- The daily volatility of the portfolio is

$$\sqrt{5^2 \cdot 0.0298^2 + 8^2 \cdot 0.0167^2 + 2 * 5 * 8 * 0.0298 * 0.0167 * 0.067} = \$0.21M$$

therefore the 1-day 95%-VaR would be

$$\$1.65 * 0.21M = \$340k$$

The one-year 95%-VaR would then be

$$\sqrt{365} * 340 = \$6.5M$$

Greeks - delta

P&L reflects changes in prices. Portfolio sensitivities are efficient ways of tracking *“small”* price changes

- Delta (δ, Δ) measures the rate of change of the portfolio value with respect to changes in the risk factors. Delta is the first derivative of the value of the portfolio with respect to the risk factors

$$\delta = \frac{\partial V}{\partial r}$$

- When there are several risk factors $(r_1, \ldots, r_k)$ delta becomes a vector

$$\vec{\delta} = \left(\frac{\partial V}{\partial r_1}, \ldots, \frac{\partial V}{\partial r_k} \right)$$

Greeks - Gamma

- Gamma (Γ) measures the rate of change of the portfolio delta respect to changes in the risk factors.

- Gamma is the second derivative of the value of the portfolio with respect to the risk factors

$$\Gamma = \frac{\partial^2 V}{\partial r^2}$$

- When there are several risk factors $(r_1, \ldots, r_k)$ Γ becomes a matrix

$$\Gamma = \left(\frac{\partial^2 V}{\partial r_i \, \partial r_j} \right)_{i,j=1\ldots,k}$$

Greeks - Vega

- Vega (ν), measures sensitivity to volatility. Vega is the derivative of the portfolio value with respect to the volatility of the underlying risk factors.

- Vega is not the name of any Greek letter.

$$\nu = \frac{\partial V}{\partial \sigma}$$

- Often times, the volatility of the risk factor is not observable, but implied.

- The term is used often in banking, in asset management we find often the terms “long-vol” or “short-vol” to denote the sign of a portfolio vega. Short vol strategies are most common, long-vol strategies are often desired.

Greeks - Vega

- Theta (θ, Θ) measures the sensitivity of the value of a portfolio to the passage of time, or the "time decay."

$$\theta = \frac{\partial V}{\partial t}$$

Greeks - Rho

- Rho (ρ) measures sensitivity to the interest rate:

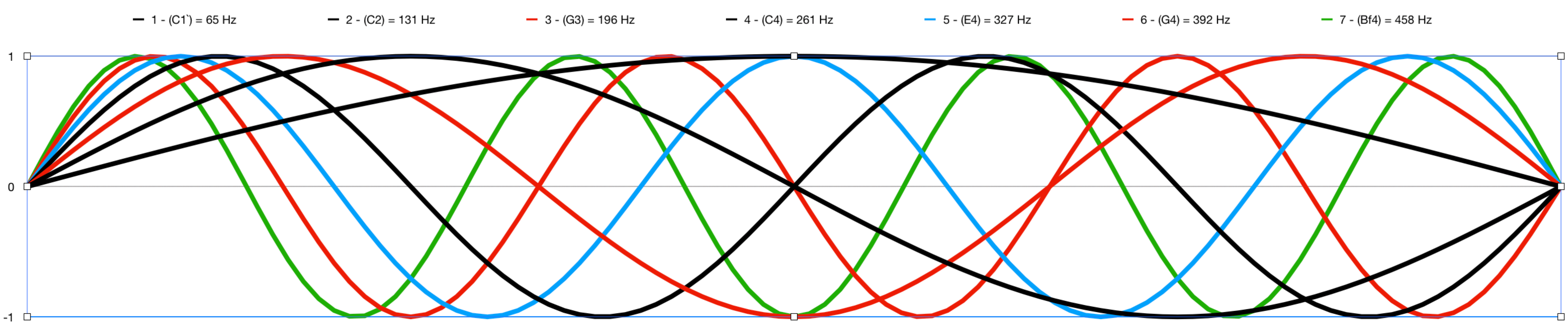
$$\rho = \frac{\partial V}{\partial r}$$

Eigenvalues in infinite data sets

Wave operator

Fundamental Frequency

$$-\partial_x^2 \, \sin(n \cdot x) = n^2 \, \sin(n \cdot x)$$



Eigenvalues and Gaussians

$$\Sigma = O \cdot \underbrace{\begin{pmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & 0 & \cdots \\ & & \cdots & \\ 0 & \cdots & 0 & \lambda_n \end{pmatrix}}_{D} \cdot O^{-1}$$

$$\Sigma = H \cdot H^* \qquad H = O\sqrt{D}$$

Cholesky Decomposition

Eigenvalues and Gaussians

$$\Sigma = H \cdot H^*$$

$$f_{\mathbf{X}}(x_1, \ldots, x_k) = \frac{\exp\left(-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu})\right)}{\sqrt{(2\pi)^k |\boldsymbol{\Sigma}|}}$$

Diagonalizing a Gaussian

Assume a multivariate gaussian distribution with a $n \times n$ Variance/Covariance matrix A, with a data set given by

$$x_i \in \mathbf{R}^n, \quad i = 1, \ldots$$

Then, the dataset given by $H^{-1}x_i$ is gaussian with covariance matrix given by

$$\begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots \\ & & \cdots & \\ 0 & \cdots & 0 & 1 \end{pmatrix}$$

Principal Components

- Even when a multivariate data set is not given by a multivariate Gaussian distribution, we can still transform the dataset as before $H^{-1}x_i$, with the result being uncorrelated marginals.

- We can the move on to analyze each marginal with one-dimensional methods, and combine the results transforming the data by the matrix H.

Cholesky Decomposition

$$\Sigma = H \cdot H^T \qquad\qquad \Sigma^{-1} = (H^T)^{-1} \cdot H^{-1}$$

$$\begin{bmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{bmatrix} = \overbrace{\begin{bmatrix} L_{00} & 0 & 0 \\ L_{10} & L_{11} & 0 \\ L_{20} & L_{21} & L_{22} \end{bmatrix}}^{\text{Lower Triangular L}} \underbrace{\begin{bmatrix} L_{00} & L_{10} & L_{20} \\ 0 & L_{11} & L_{21} \\ 0 & 0 & L_{22} \end{bmatrix}}_{\text{Transpose of L}}$$

Multivariate Gaussian

Linear dataset transformation

$$H^{-1} \cdot (x - \mu) = y \implies (x - \mu)^T \Sigma^{-1} \cdot (x - \mu) = y^T \cdot y$$

$$dx = |H|\, dy = |\Sigma|^{1/2} dy$$

$$f_{\mathbf{X}}(x_1, \ldots, x_k) = \frac{\exp\left(-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu})\right)}{\sqrt{(2\pi)^k |\boldsymbol{\Sigma}|}} \quad = (2\pi)^{-k/2}\, e^{-\frac{1}{2}|y|^2} \frac{dy}{dx}$$

Uncorrelated marginals

Montecarlo

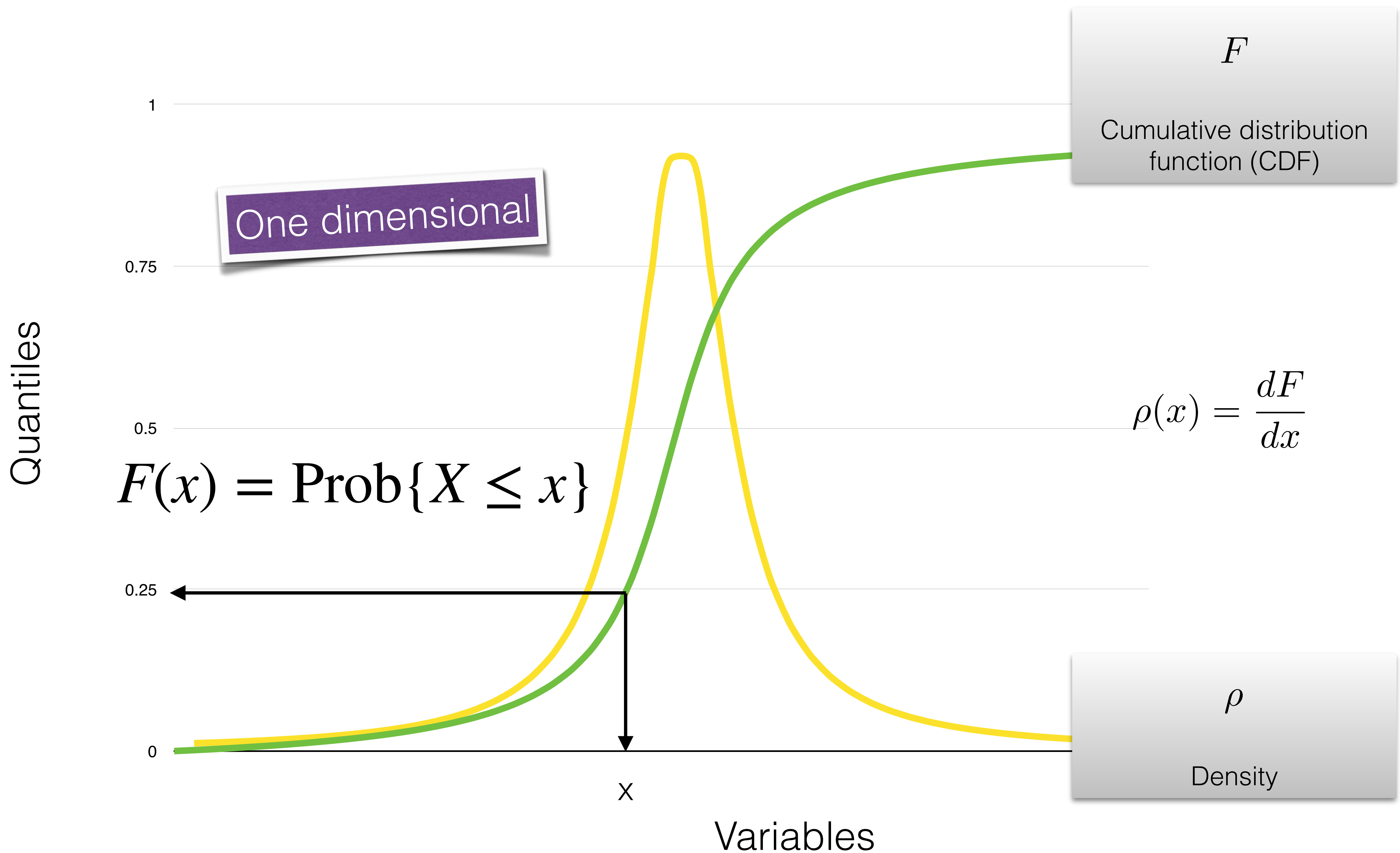
Question: Monte Carlo: given a data distribution with probability density ρ, how can we produce a sample $x_i,\ i = 1,\ldots,k$, so that the values x follow the distribution given?

Montecarlo

Answer: If F is the cumulative density corresponding to the probability density ρ, and x_i, $i = 1, \ldots, k$, is a uniformly distributed sample, $F^{-1}(x_i)$ is distributed like ρ.

(F^{-1} is the functional inverse of F)

Statistics

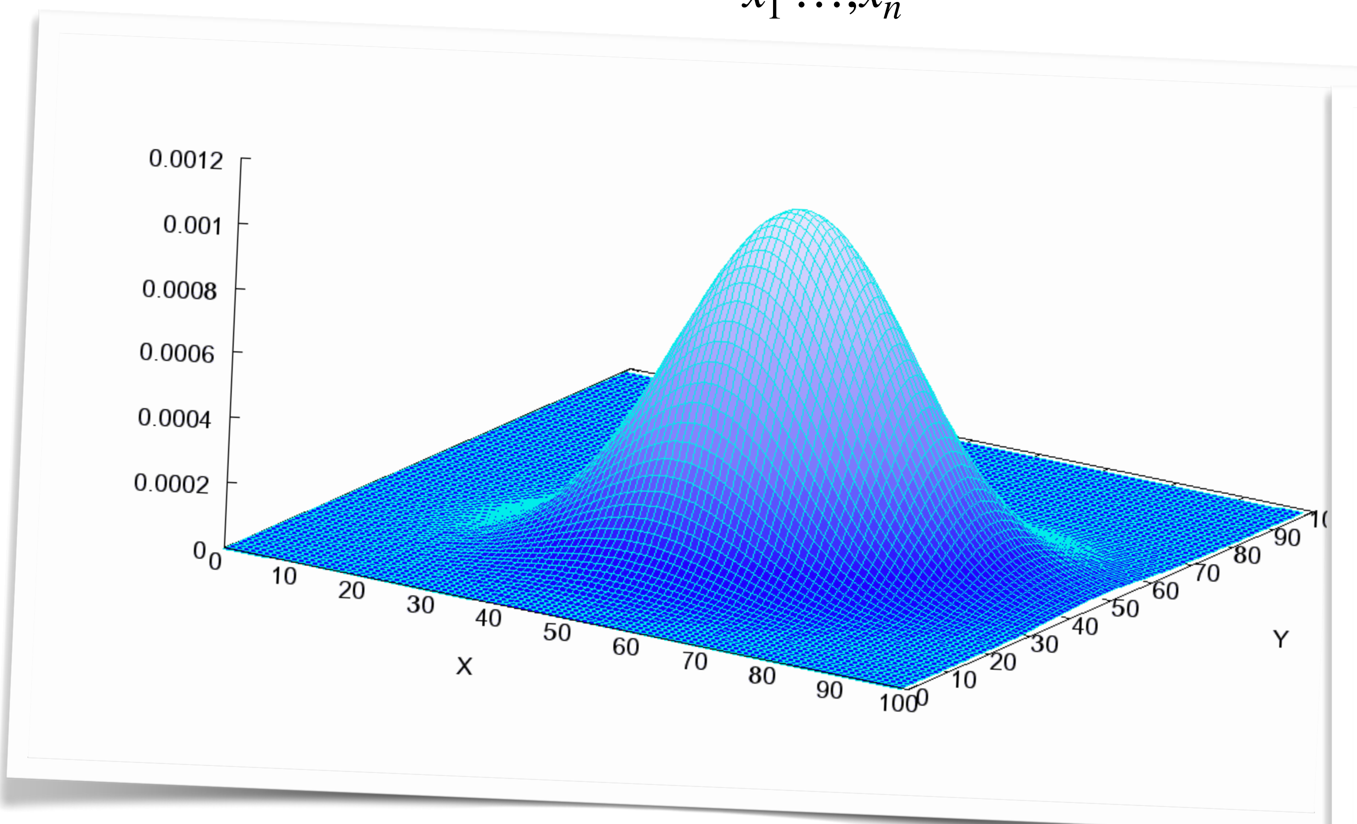


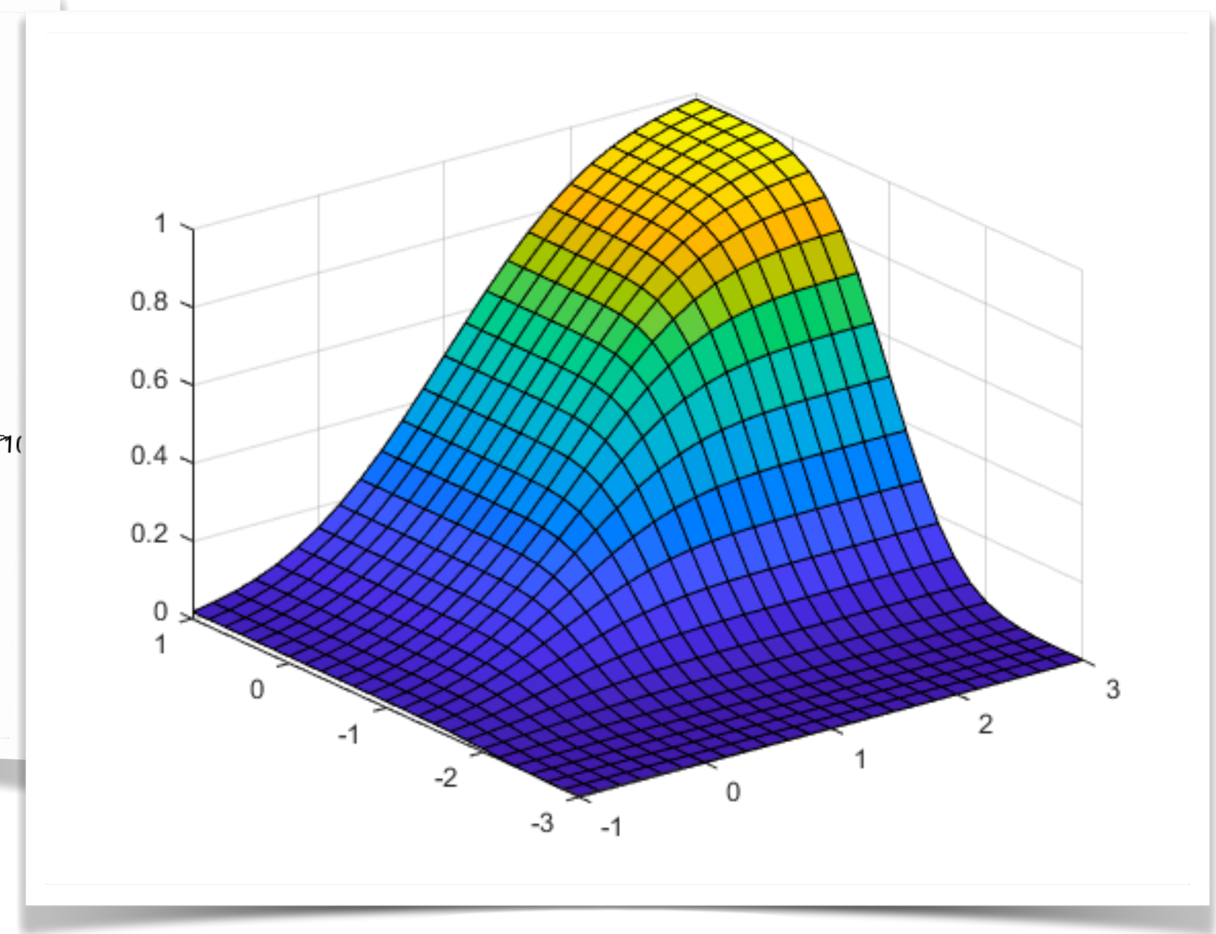
F
Cumulative distribution function (CDF)
1
0.75
0.5
0.25
0
One dimensional
Quantiles
$F(x) = \text{Prob}\{X \leq x\}$
$\rho(x) = \frac{dF}{dx}$
ρ
Density
x
Variables

Multivariate

$$\rho(x_1 \ldots, x_n) = \frac{\partial^n F(x_1 \ldots, x_n)}{\partial_{x_1 \ldots, x_n}}$$

$$F(x_1, \ldots, x_n) = \text{Prob}\{X_1 \leq x_1, \ldots, X_n \leq x_n\}$$

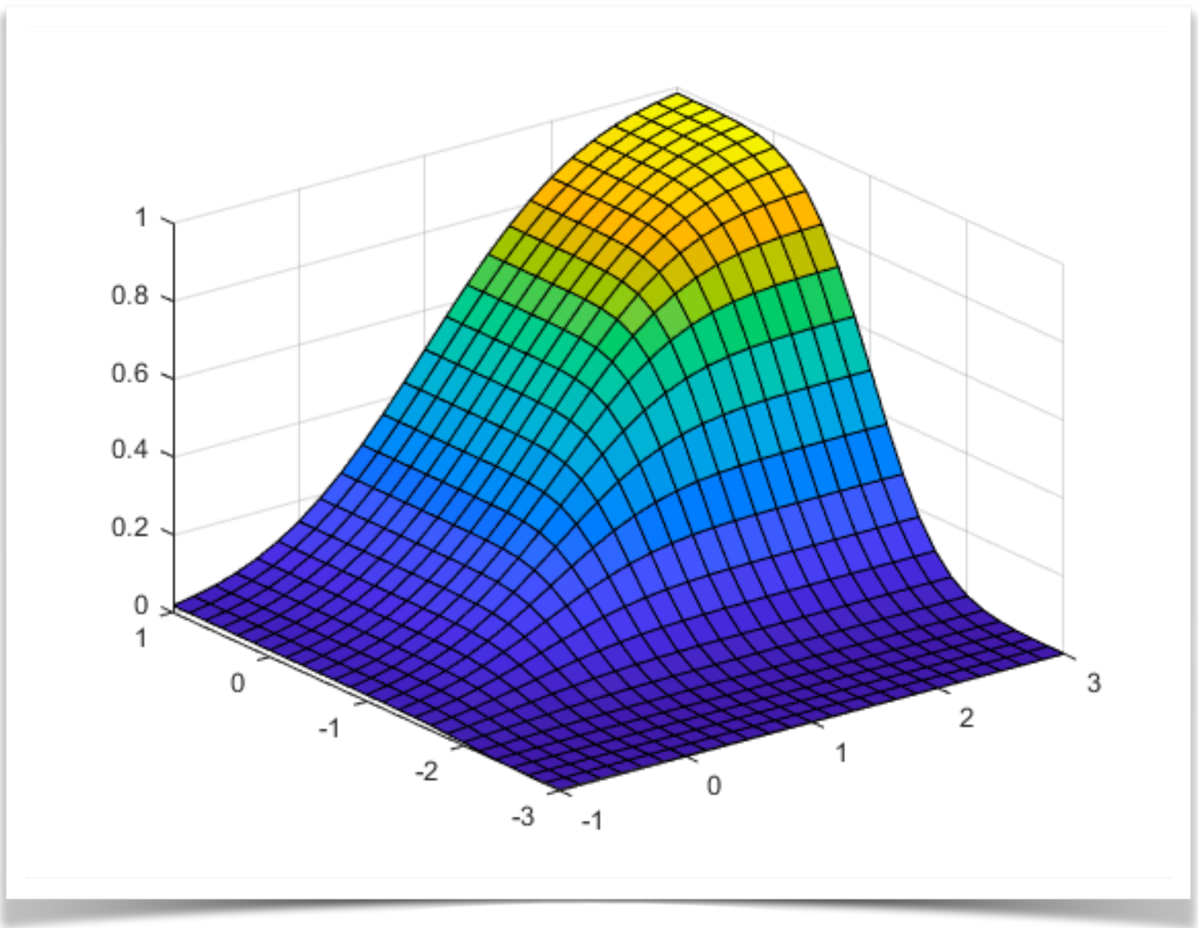




Copulas

$F(x_1, \ldots, x_n) = \text{Prob}\{X_1 \leq x_1, \ldots, X_n \leq x_n\}$

If $(X_1, \ldots, X_n)$ is uniformly distributed



$$F : [0,1]^n \to [0,1]$$

Sklar's theorem

Given $F(x_1, \ldots, x_n)$, with marginals $F_i(x_i)$

$\left(F_1^{-1}(x_1), \ldots, F_n^{-1}(x_n)\right)$ is marginally Uniformly distributed

Copula

$$C : [0,1]^n \rightarrow [0,1]$$

Recipe for Disaster: The Formula That Killed Wall Street

In the mid-'80s, Wall Street turned to the quants—brainy financial engineers—to invent new ways to boost profits. Their methods for minting money worked brilliantly... until one of them devastated the global economy.

Easter egg hunt:
discover as many errors as you can in this article

In the mid-'80s, Wall Street turned to the quants—brainy financial engineers—to invent new ways to boost profits. Their methods for minting money worked brilliantly... until one of them devastated the global economy. ■ JIM KRANTZ / INDEX STOCK IMAGERY, INC. / GALLERY STOCK

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UN Sustainability Goals

1	NO POVERTY
2	ZERO HUNGER
3	GOOD HEALTH AND WELL-BEING
4	QUALITY EDUCATION
5	GENDER EQUALITY
6	CLEAN WATER AND SANITATION
7	AFFORDABLE AND CLEAN ENERGY
8	DECENT WORK AND ECONOMIC GROWTH
9	INDUSTRY, INNOVATION AND INFRASTRUCTURE
10	REDUCED INEQUALITIES
11	SUSTAINABLE CITIES AND COMMUNITIES
12	RESPONSIBLE CONSUMPTION AND PRODUCTION
13	CLIMATE ACTION
14	LIFE BELOW WATER
15	LIFE ON LAND
16	PEACE, JUSTICE AND STRONG INSTITUTIONS
17	PARTNERSHIPS FOR THE GOALS

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The genesis of ESG

E pillar

Its use of or dependence on fossil fuels
Its use or management of water and other resources
Pollution levels
Climate change
Hazardous materials and their disposal
Carbon footprint and whether it uses renewable energy

S pillar

Employment equality and gender diversity
Product safety concerns and liability
Employee health and safety
Training and development
Animal testing
Stance on physical and mental health-related issues
Supply chain transparency
Human rights
Privacy issues

G pillar

Compensation of employees and board executives
Board and company diversity
Tax strategy and accounting standards
Bribery and corruption
Fraud
Ethics and values
Transparency and anti-corruption
Shareholder rights

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The Origin: Socially responsible investing

May 24, 1990

Harvard and CUNY Shedding Stocks in Tobacco

By TAMAR LEWIN

Harvard University and the City University of New York have decided to eliminate stocks of tobacco companies from their investment portfolios, in what may be harbingers of a new tactic to highlight the dangers of smoking.

The action by Harvard was disclosed by President Derek Bok in a letter dated May 18 to three students at the university's public health school. He said Harvard had decided on divestiture in September and completed the stock sale in March.

And on Monday the City University of New York's board of trustees voted 9 to 2 to divest itself of tobacco stock representing about $3.5 million of the university's $60 million portfolio.

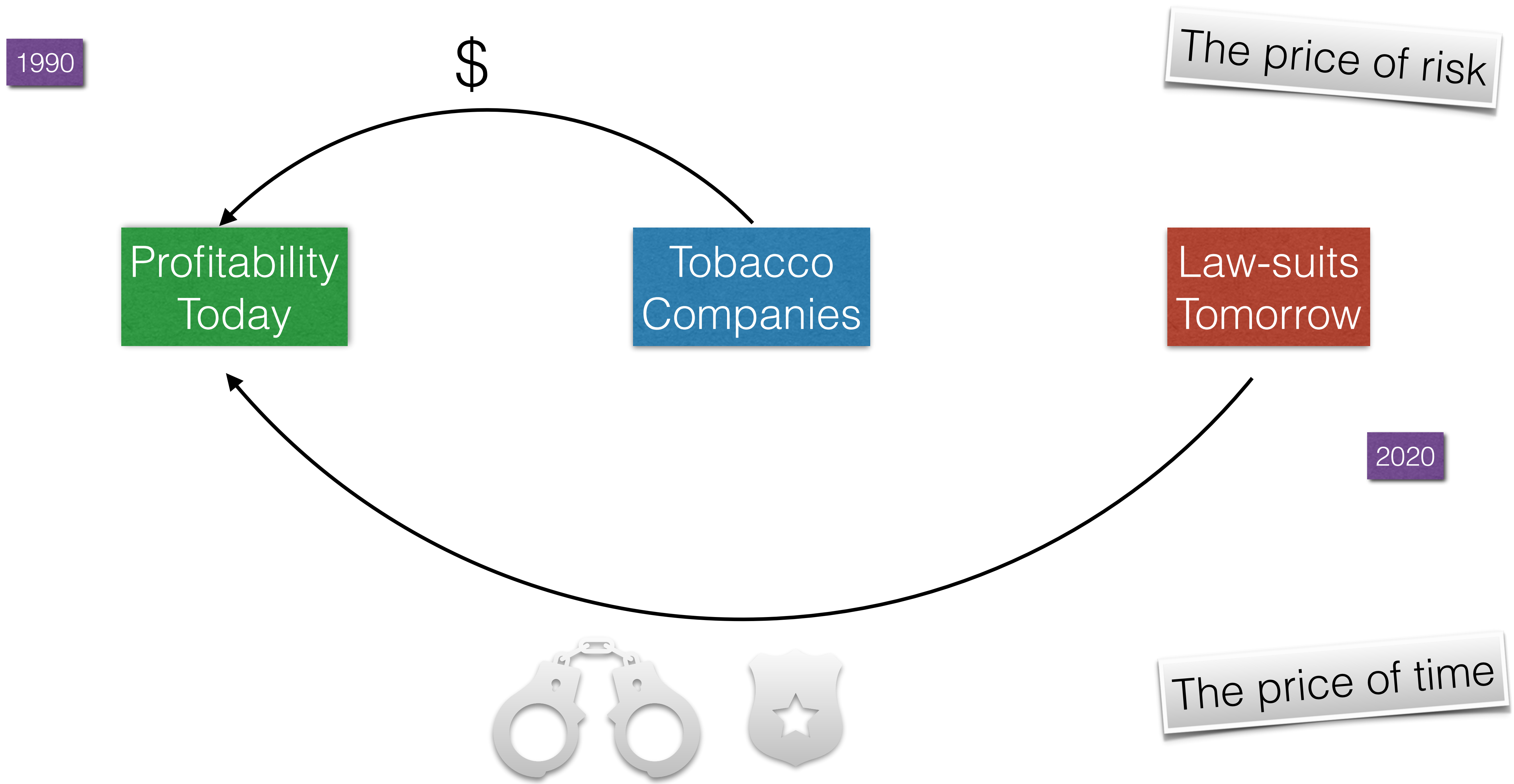
Harold Jacobs, a trustee at City University, said, "We didn't know anything about Harvard doing it, and this was the first meeting where tobacco divestiture appeared on the agenda, but our vice chairman, Edith Everett, argued very persuasively that cigarettes are unhealthy and we should not hold tobacco stock, so we passed it."

Ms. Everett and her husband, Henry, have long been active in opposing the tobacco industry, and they serve on the board of trustees of the Tobacco Divestment Project, a Boston-based nonprofit group formed earlier this month.

That group and others working against cigarette smoking say the two universities' moves may be the start of a far more widespread divestiture campaign, including a proposal in the Massachusetts Legislature to require state pension funds to sell their tobacco stocks. By some estimates, more than 60 percent of tobacco stock over all is held by institutional investors, primarily banks, pension funds and insurance companies.

"Harvard and CUNY are the two great victories so far, but this is just the beginning," said Brad Krever, ex-

Continued on Page B8, Column 3


1990
$
The price of risk
Profitability Today
Tobacco Companies
Law-suits Tomorrow
2020
The price of time

Impact Investing → ESG → Sustainability

General awareness of the consequences of one's actions

Well-defined rules procedures in place known outcomes

Not YET

Comparative

BTI • NYSE

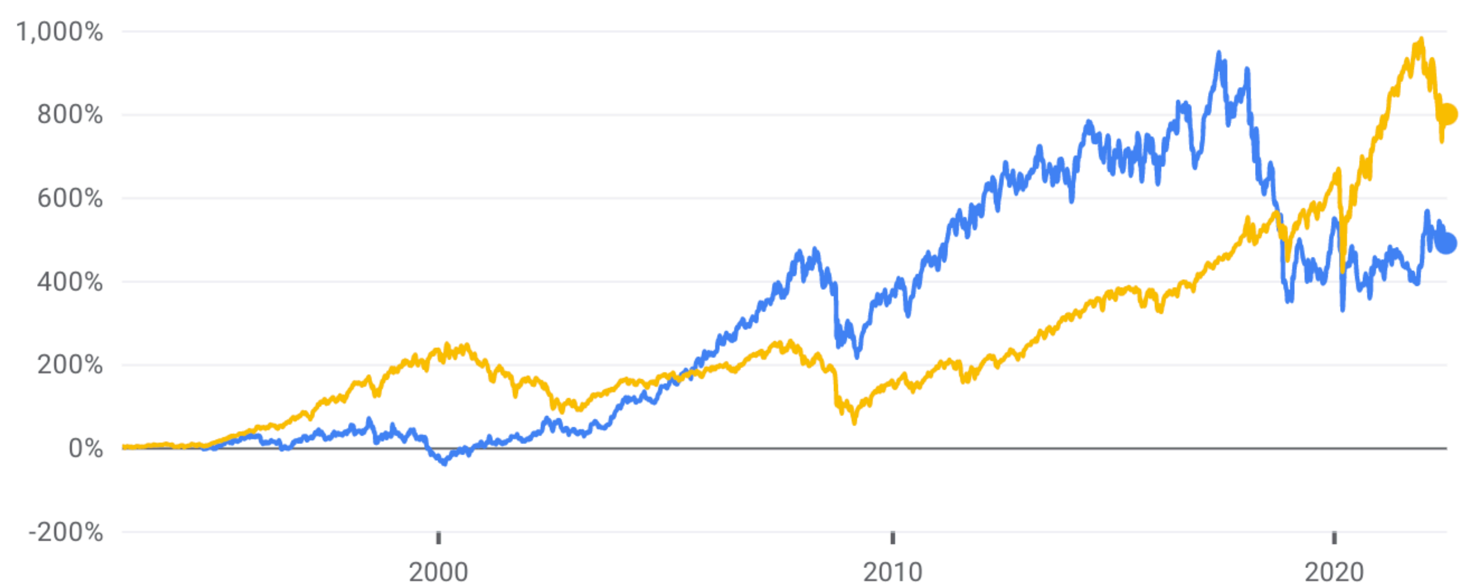
British American Tobacco PLC

$41.09 ↑489.53% +34.12 MAX

After Hours: $41.15 (↑0.15%) +0.060

Closed: Jul 22, 7:09:25 PM UTC-4 · USD · NYSE · Disclaimer

1D 5D 1M 6M YTD 1Y 5Y MAX



British American T...	$41.09	+$34.12	↑489.53%
SPDR S&P 500 ETF ...	$395.09	+$351.15	↑799.16%

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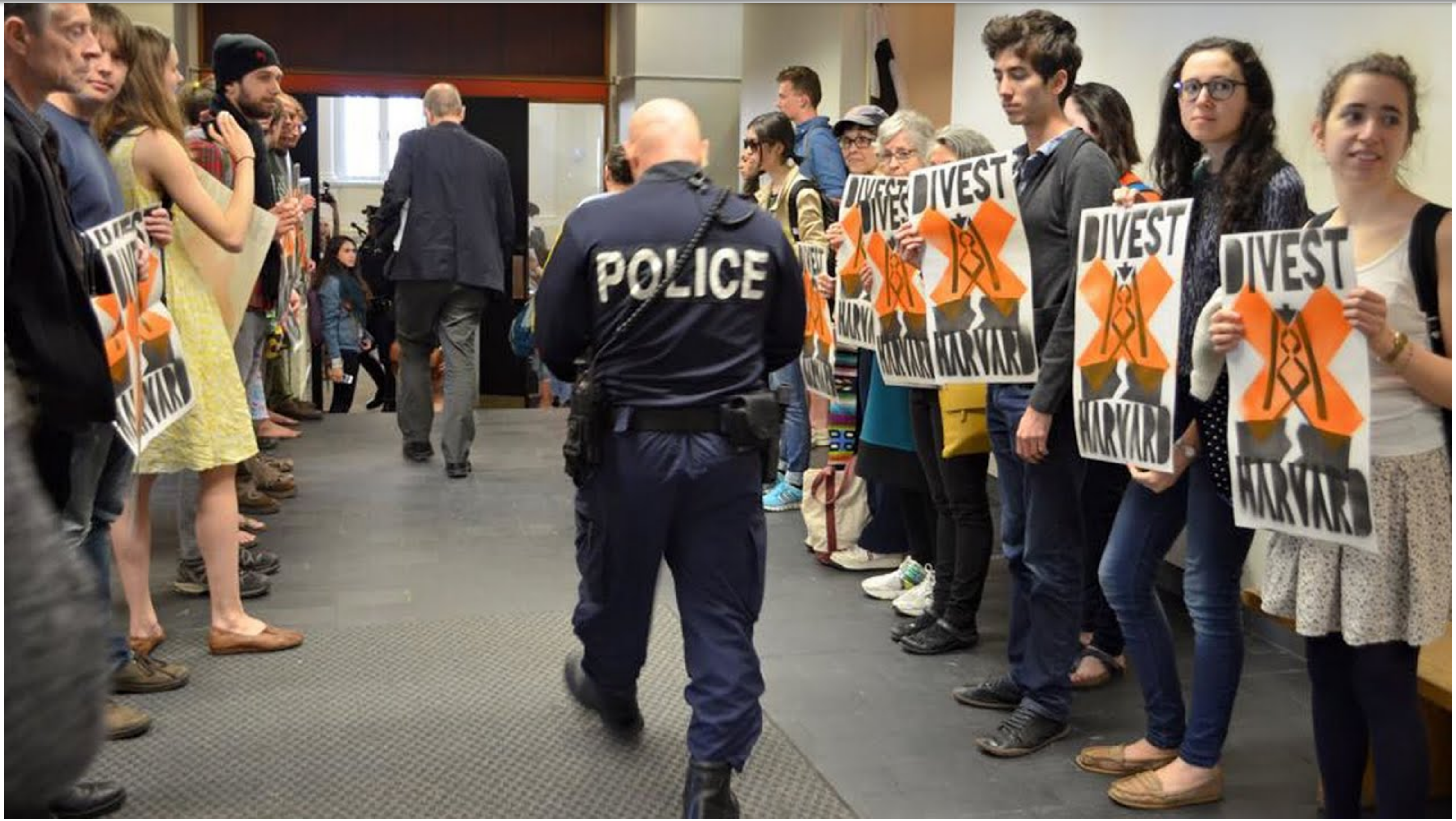
Universities & Fossil Fuels

To this date, the investment of Harvard Endowment and other universe endowments in Fossil Fuels is a contentious one



Impact investing

- Aims to generate specific beneficial social or environmental effects in addition to financial gains

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Sustainability: the new liability

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Environment

- Green Power stocks
- Water supply
- Renewable energy
- Pollution control
- Green transportation
- Waste reduction

Social

- Community impact
- Minorities

RiskLab
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Governance

- Principles
- Processes
- Ethics
- Regulation
- Compliance

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MSCI ESG Rating

- MSCI ESG Ratings aim to measure a company's management of financially relevant ESG risks and opportunities.

- It is a rules-based methodology to identify industry leaders and laggards according to their exposure to ESG risks and how well they manage those risks relative to peers.

- ESG Ratings range from leader (AAA, AA), average (A, BBB, BB) to laggard (B, CCC).

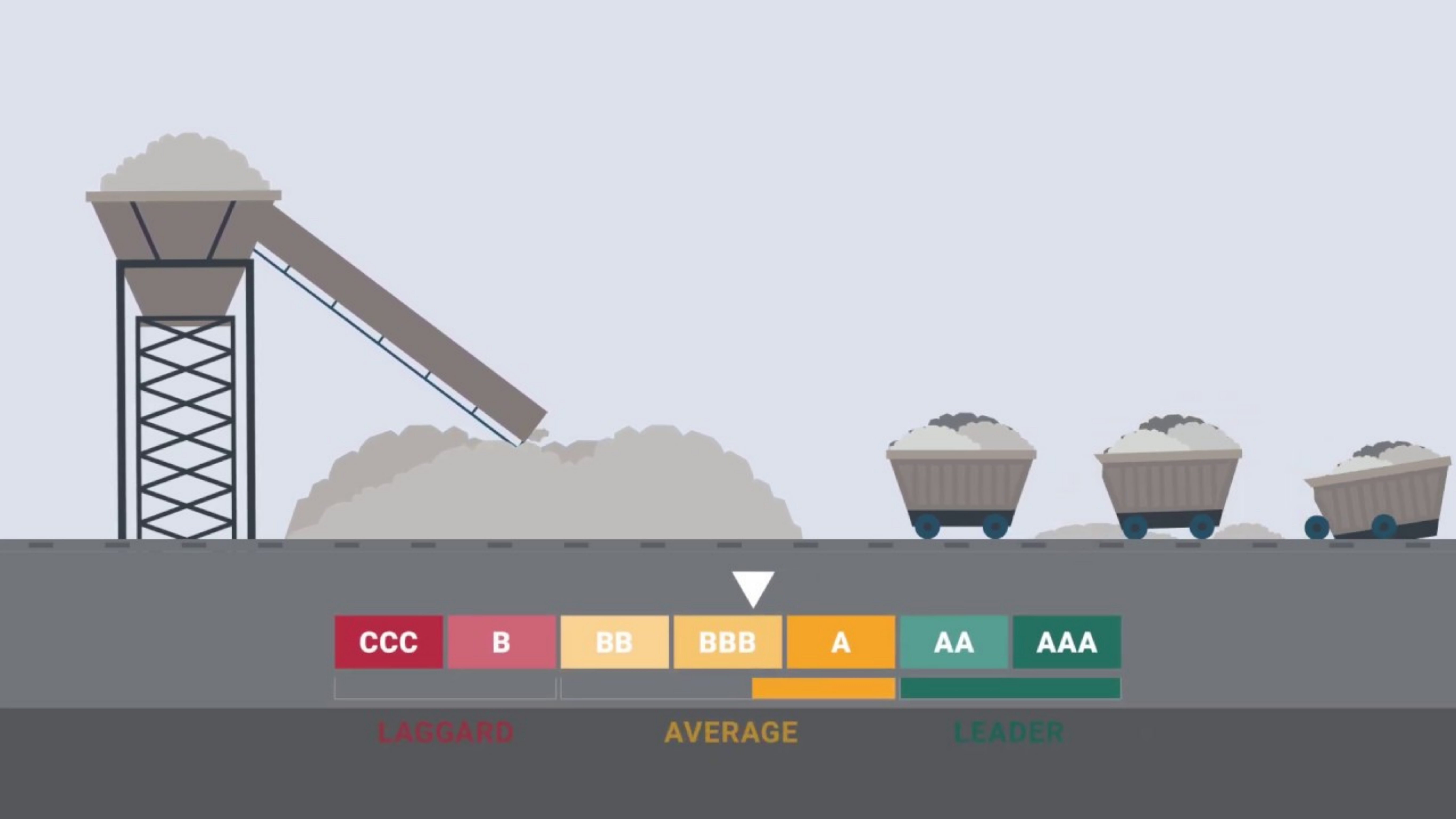
- They also rate equity and fixed income securities, loans, mutual funds, ETFs and countries.

Numerical ratings

Exhibit 8: Mapping the Industry Adjusted Company Score to Letter Ratings

Letter Rating	Leader/Laggard	Final Industry-Adjusted Company Score
AAA	Leader	8.571* - 10.0
AA	Leader	7.143 – 8.571
A	Average	5.714 – 7.143
BBB	Average	4.286 – 5.714
BB	Average	2.857 – 4.286
B	Laggard	1.429 – 2.857
CCC	Laggard	0.0 – 1.429

**Appearance of overlap in the score ranges is due to rounding imprecisions. The 0-to-10 scale is divided into seven equal parts, each corresponding to a letter rating.*


CCC
B
BB
BBB
A
AA
AAA
LAGGARD
AVERAGE
LEADER

Köszönöm תודה Спасибі

Thanks ありがとう

Gracias

ευχαριστώ 감사해요 cảm ơn

Kösz Dankon

Teşekkürler

Danke

Merci

Grazie

tack

rahmat

Takk

谢谢

ਧੰਨਵਾਦ

Gratias

dzięki

спасибо धन्यवादा ขอบใจ

شكرًا

asante dankie շնորհակալություն