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AN ANALYTICAL MODEL FOR THE PLATEAU STAGE OF TYPE II SUPERNOVAE

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ABSTRACT

This paper presents an analytical model for hydrogen envelope cooling in Type II supernovae during the plateau phase. Observable features of SN IIp light curves, such as the duration of the plateau, t_p, and its V magnitude, are expressed through such basic parameters as the energy of explosion and the mass, and the initial radius of the envelope. The relationships derived are in excellent agreement with similar ones obtained from fits to a grid of hydrodynamical calculations. The duration of the plateau turns out to be most sensitive to the envelope mass, and much less to the explosion energy and the initial radius. This conclusion provides support for the results of the previous analytical models of SN IIp's. We present relationships, allowing observers to estimate the basic SN parameters from observational data. For the "average" Type IIp supernova our estimate gives results close to those obtained with the fitting of hydrodynamical models.

Subject headings: stars: interiors — supernovae: general

1. INTRODUCTION

Analytic models of supernovae (SNs) explosions are useful for providing quick estimates of explosion energy, mass, and initial radius of the exploding star from the shape and magnitude of the SN light curves without performing complicated hydrodynamical computations. Such estimates may be used as a preliminary study of recently observed SN events; the parameters obtained can be substituted afterward into more complicated computations to get a detailed description of the supernova. The estimates obtained also connect the observed variety of SN light curves with the variety of the SN progenitors, which is interesting for the study of the evolution of stars.

In this paper we describe the plateau phase of a Type II SN using a "two-zone" model that has previously provided good agreement with observational data for SN 1987A in Arnett & Fu (1989) and Imshennik & Popov (1992). The SN envelope is considered to be a homologously expanding sphere of hydrogen that is uniform in density. The strong nonlinear dependence of the optical opacity κ_t on temperature T is approximated by a step function: $\kappa_t = 0$, when $T < T_{\rm ion}$ and $\kappa_t = \kappa = \text{const}$ when $T > T_{\rm ion}$. Here κ is approximated as consisting solely of electron scattering, which for a pure hydrogen gas gives 0.4 $\text{cm}^2\ \text{g}^{-1}$. The presence of He and heavier elements will lead to smaller values. The hydrogen recombination temperature $T_{\rm ion}$ coincides with the temperature of transparency (Imshennik & Nadyozhin 1989), which is equal to ≈ 5000 K.

The region of the opacity jump (zero width in the model described here) coincides with the photosphere and divides the envelope into two regions: opaque and transparent. It was also considered as the wavefront of cooling and recombination (WCR) in Imshennik & Popov (1992). We may compare the results of this approach to the results of the WCR model, suggested by Chugai (1991), and to observational data which give the changes of photospheric radius with time.

Hydrodynamical calculations (Grasberg, Imshennik, & Nadyozhin 1971; Falk & Arnett 1977; Weaver & Woosley 1980; Litvinova & Nadyozhin 1985) show that characteristics of the plateau in SN IIp's are determined mostly by the thermal energy, which is input into the envelope by the explosion, while the influence of the radioactive heating (the decay sequence $^{56}\text{Ni} \rightarrow {}^{56}\text{Co} \rightarrow {}^{56}\text{Fe}$) is small and can be neglected in approximate estimates. Then under several assumptions, that are given below, we obtain analytically the photospheric radius, light curves and main characteristics of the plateau of an SN IIp.

2. MAIN ASSUMPTIONS AND DIFFERENTIAL EQUATIONS

Using the mass m as a Lagrangean coordinate for a shell of a radius r and assuming spherical symmetry, we may write the first law of thermodynamics as

$$\frac{\partial e}{\partial t} + P \frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) = \epsilon - \frac{\partial L}{\partial m}, \tag{1}$$

where e is the thermal energy per gram, P is the pressure, ρ is the density, ϵ is the rate of heating per gram per second, and the luminosity L is

$$L = -\frac{4\pi r^2 c}{3\kappa\rho} \frac{\partial(aT^4)}{\partial r}, \tag{2}$$

where κ is the Rosseland mean opacity. The envelope energy density is radiation dominated, hence $e = aT^4/\rho$, $P = aT^4/3$. Since the supernova envelope expands homologously, the distance of a Lagrangean particle from the center may be written as

$$r = xR(t), \tag{3}$$

where x is the dimensionless radius and $R(t)$ is the scaling factor; the fluid velocity will be

$$v(r, t) = xv_{\rm sc}, \tag{4}$$

with the scaling velocity $v_{\rm sc} = dR/dt$ being constant, so

$$R(t) = R_0 + v_{\rm sc}\, t. \tag{5}$$

We assume a uniform density profile

$$\rho(x, t) = \rho_0 \frac{R_0^3}{R^3(t)} \tag{6}$$

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and approximate opacity by a step function of temperature, assuming that recombination of hydrogen occurs at a certain temperature $T_{\rm ion}$:

$$\kappa_t(x, t) = \begin{cases} \kappa\,, & T \geq T_{\rm ion}\,, \\ 0\,, & T < T_{\rm ion}\,. \end{cases} \quad (7)$$

So we have a "two-zone" model, with an opaque inner part of the envelope and a transparent outer part. The boundary between them is the recombination front with radius R_i (dimensionless radius x_i). The radial temperature profile in the opaque region may be approximated by a spherical Bessel function, similarly to Arnett (1980) and Arnett & Fu (1989):

$$T^4(x, t) = T_0^4 \frac{R_0^4}{R^4(t)} \varphi(t) \frac{\sin(\beta(t)x)}{\beta(t)x}\,. \quad (8)$$

Here $\beta(t) \equiv \pi$ in models without a recombination front and with "radiative-zero" boundary conditions (Arnett 1980); time-dependent $\beta(t)$ is necessary in the case where a recombination front is moving through the envelope.

Under these assumptions the bolometric luminosity at $x = x_i$ will be

$$L(x_i, t) = \frac{4\pi ac T_0^4 R_0 \varphi(t) x_i(t)}{3\kappa\rho_0} \times \left\{\frac{\sin(\beta(t)x_i(t))}{\beta(t)x_i(t)} - \cos[\beta(t)x_i(t)]\right\}; \quad (9)$$

in this model it is equal to the observed bolometric luminosity of the supernova, and the flux from the photosphere passes the transparent part of the envelope without interaction. For brevity define

$$\omega(t) = \beta(t)x_i(t)\,. \quad (10)$$

Below we consider the case where the influence of radioactive heating is negligible (this corresponds to $\epsilon = 0$ in eq. [1]). Then integration of equation (1) by $dm = 4\pi\rho r^2 dr$ over the opaque region $0 < r < R_i$, using equations (3)–(10), gives

$$\frac{(\sin\omega - \omega\cos\omega)}{\beta^3}\frac{d\varphi}{dt} + \frac{\varphi d\beta}{\beta^4 dt}[(\omega^2 - 3)\sin\omega + 3\omega\cos\omega] = -\frac{cR(t)}{3\kappa\rho_0 R_0^3}\varphi x_i\left(\frac{\sin\omega}{\omega} - \cos\omega\right). \quad (11)$$

We see that in order to determine the luminosity from equation (9) and other observable features of SNs, one must calculate the unknown functions $\varphi(t)$, $x_i(t)$, and $\beta(t)$ or $\omega(t)$, but there is still only one equation (11) for them. Two more equations can be obtained from the boundary conditions. For example, if we consider a model with no recombination front (when surface temperature is higher than $T_{\rm ion}$) we have $x_i \equiv 1$. If we also use the "radiative-zero" boundary condition, implying $\beta(t) \equiv \pi$ (see eq. [8]), the necessary relations are set. This model is physically identical to the one by Arnett (1980).

In the case of the recombination front, the physically consistent boundary conditions are (see Imshennik & Popov 1992) as follows:

1. The temperature at $x = x_i$ is equal to $T_{\rm ion}$.
2. The bolometric luminosity at $x = x_i$ is equal to the luminosity of a blackbody with radius R_i and surface wavelength $T_{\rm ion}$ (so the effective temperature is $(2)^{4/2} T_{\rm ion}$).

This model gave a good fit to observations of SN 1987A with a reasonable choice of explosion parameters (Imshennik & Popov 1992). Instead of the boundary condition (1) (above), we can set $\omega(t) \equiv \pi$, so that $\beta(t) = \pi/x_i$. This approximation is also implicitly used in Arnett & Fu (1989). Formally, this corresponds to zero surface temperature (see eq. [8]). But if we also use the boundary condition (2), the results will be very close to the results of the self-consistent model, described above (Imshennik & Popov 1992), and can fit observations of SN 1987A as well, but the equations are much simpler. We assume this simplification below.

Substituting $\beta = \pi/x_i$ into equation (9) and setting the luminosity from equation (9) to be equal to $2\pi acR^2(t)x_i^2(t)T_{\rm ion}^4$, we can express $\varphi(t)$ as

$$\varphi(t) = \frac{3\kappa\rho_0 R^2(t) T_{\rm ion}^4}{2R_0 T_0^4} x_i\,.$$

Substituting this relation into equation (11) and taking into account that $\omega \equiv \pi$, $\beta = \pi/x_i$, we finally get

$$\frac{dx_i}{dt} = -\frac{R(t)}{4x_i R_0 t_d} - \frac{x_i R_0}{2t_e R(t)}\,. \quad (12)$$

The characteristic time scales are the diffusion time $t_d = 9\kappa M/(4\pi^3 c R_0)$ and the expansion time $t_e = R_0/v_{\rm sc}$. The envelope kinetic energy is $E_{\rm kin} \approx E$, the total energy of the SN explosion. We can estimate the scale velocity in the case of uniform density as

$$v_{\rm sc} = (10E/3M)^{1/2}\,. \quad (13)$$

During the homologous expansion, approximately $R(t) = v_{\rm sc} t$, and we can rewrite (12) as

$$\frac{dx_i}{dt} = -\frac{t}{4x_i t_d t_e} - \frac{x_i}{2t}\,. \quad (14)$$

As a parameter, equation (14) includes the characteristic time of changing of SN magnitude in models without WCR (Arnett 1980):

$$t_a = \sqrt{2t_d t_e}\,.$$

In our models recombination begins at the moment, t_i, when the surface temperature of SN reaches $T_{\rm ion}$. The value of t_i will be determined below from the models without WCR.

Equation (14) with initial condition $x_i(t_i) = 1$ has the analytical solution

$$x_i^2(t) = \frac{t_i}{t}\left(1 + \frac{t_i^2}{3t_a^2}\right) - \frac{t^2}{3t_a^2}\,, \quad (15)$$

so for the square of the radius of photosphere we have

$$R_i^2(t) = v_{\rm sc}^2\left[t_i t\left(1 + \frac{t_i^2}{3t_a^2}\right) - \frac{t^4}{3t_a^2}\right], \quad (16)$$

and for the luminosity of SN

$$L_{\rm bol}(t) = 8\pi\sigma_{\rm SB} T_{\rm ion}^4 v_{\rm sc}^2\left[t_i t\left(1 + \frac{t_i^2}{3t_a^2}\right) - \frac{t^4}{3t_a^2}\right], \quad (17)$$

where σ_{SB} is the Stefan-Boltzmann constant. The maximum of the bolometric luminosity will occur at

$$t_m = \left[\frac{3}{4}\, t_i t_a^2 \left(1 + \frac{t_i^2}{3t_a^2}\right)\right]^{1/3} . \tag{18}$$

We can estimate the duration of the plateau, t_p, by setting $R_i(t_p) = 0$ in equation (16) and solving for t_p:

$$t_p = \left[3t_i t_a^2 \left(1 + \frac{t_i^2}{3t_a^2}\right)\right]^{1/3} = 4^{1/3} t_m . \tag{19}$$

Note that the analytic estimate, given by Chugai (1991), which uses a different method to describe WCR, led to the relation $t_p = 5^{1/4} t_m$. We see that both methods give similar results. Also from equation (18) and (16) it is possible to determine the maximum radius of the photosphere:

$$R_m = v_{sc}\, t_m \left(\frac{t_m}{t_a}\right) . \tag{20}$$

At $t < t_i$, the surface temperature of the envelope is higher than T_{ion}. In this case, consequently, we can use the constant opacity models of Arnett (1980) to estimate t_i. If we neglect the radioactive heating, then for $t < t_i$

$$L_{bol}(t) = \frac{E_{th}(0)}{t_d}\, e^{-t^2/t_a^2} \tag{21}$$

[here $E_{th}(0) \approx E/2$ is the total thermal energy of the envelope just after propagating of the shock wave]. Note that bolometric light curve (21), a "parabola," does not reveal the plateau stage and especially an abrupt decrease after it, so WCR must be considered in analytical models of SN IIp's. At the moment t_i, the bolometric luminosity from equation (21) becomes equal to the luminosity of a blackbody with radius $R(t_i) = v_{sc}\, t_i$ and surface temperature T_{ion}:

$$\frac{E_{th}(0)}{t_d}\, e^{-t_i^2/t_a^2} = 8\pi\sigma_{SB}\, v_{sc}^2\, t_i^2\, T_{ion}^4 . \tag{22}$$

This provides the needed equation to determine $s_i \equiv t_i^2/t_a^2$:

$$\frac{e^{-s_i}}{s_i} = \Lambda , \tag{23}$$

where

$$\Lambda = \frac{2\sigma_{SB}\, 81\sqrt{10}}{\pi^5 c^2 \sqrt{3}}\, \frac{\kappa^2 M^{3/2} T_{ion}^4}{E^{1/2} R_0} \tag{24}$$

[we expressed t_a, t_d, t_h, v_{sc}, $E_{th}(0)$ through E, M, R_0, and κ].

Litvinova & Nadyozhin (1985) showed that observational data on SN 1969L (Kirshner & Kwan 1974; Ciatti, Rosino, & Bertola 1971) are in good agreement with theoretical models if the explosion parameters are close to $E = 10^{51}$ ergs, $M = 10\, M_\odot$, $R_0 = 500\, R_\odot$. We substitute the Thompson opacity for the solar chemical composition, so $\kappa = 0.34\ \mathrm{cm^2\,g^{-1}}$. In hydrodynamical computations (Litvinova & Nadyozhin 1985; Weaver & Woosley 1980), the effective temperature, T_{eff}, on the plateau phase turns out to be equal approximately to 6000 K, which gives $T_{ion} = 6000/(2)^{4/2} \approx 5045$ K. Hence

$$\Lambda = 11.74\, \frac{\kappa_{,0.34}^2\, M_{,10}^{3/2}\, T_{ion,5054}^4}{E_{,51}^{1/2}\, R_{0,500}} .$$

For the range of parameters appropriate to SN IIp's, $\Lambda \gg 1$. Hence from equation (23) $s_i \approx 1/\Lambda$, so

$$t_i \approx t_a/\sqrt{\Lambda} \tag{25}$$

or

$$t_i \propto \frac{R_0^{1/2}}{\kappa^{1/2} T_{ion}^2} .$$

Note that when $\Lambda \gg 1$ we can neglect the term $t_i^2/3t_a^2 = 1/(3\Lambda)$ in equations (18) and (19). Then the duration of the plateau phase can be written as

$$t_p \approx 99\, \frac{\kappa_{0.34}^{1/6}\, M_{10}^{1/2} R_{0,500}^{1/6}}{E_{51}^{1/6} T_{ion,5054}^{2/3}}\ \text{days} . \tag{26}$$

From observations of SN 1969L the duration of the plateau can be estimated as $t_p \approx 80$–100 days (Ciatti et al. 1971; Kirshner & Kwan 1974), which is in a rather good agreement with the analytical estimate (26).

The condition $\Lambda \gg 1$ separates SNs with a plateau, where WCR determines the position of the photosphere, from SNs of other types. For example, for $E = 10^{51}$, $M = 10\, M_\odot$ only a model with a very extended envelope, $R_0 \gtrsim 5000\, R_\odot$, has $\Lambda \lesssim 1$. In this case numerical models reveal an absence of WCR, because in such an envelope the thermal wave after explosion in the center leaves the shock wave behind and radiates a large part of the total thermal energy from nonmoving layers (D. K. Nadyozhin, private communication). The observable features of such an event will significantly differ from a SN IIp.

On the other hand, for $E \gtrsim 10^{51}$ ergs a less massive SN ($M \approx 1\, M_\odot$), with $R_0 \gtrsim 100\, R_\odot$, will also have $\Lambda \lesssim 1$. Then, from equation (25), $t_i \gtrsim t_a$, and the light curve will be determined by radiative diffusion taking place before the onset of the WCR stage, which will be short ($t_p \approx t_i$) and not distinguished in observations. This is the case in SN IIL's, where there is no plateau phase. For SNs with compact progenitors ($R_0 \lesssim 100\, R_\odot$), the best known representative of which is SN 1987A, we conclude that the WCR stage occurs for all plausible values of E and M. It is important to remember, however, that in the models presented above radioactive heating has not been taken into account. The model will therefore need to be modified for SN Ia's, and for SN 1987A.

3. RELATIONS BETWEEN SUPERNOVA PARAMETERS AND MAIN OBSERVABLE FEATURES OF THE PLATEAU PHASE

Litvinova & Nadyozhin (1985) calculated a wide network of SN models with different presupernova and explosion properties and determined numerical fits for the dependence of observable characteristics V and t_p on E, M, and R_0. The duration of the plateau was expressed as $t_p = R_0^\alpha M^\beta E^\gamma$. In Table 1 we compare values of α, β, and γ, given by their models, the model

TABLE 1

PARAMETERS OF THE DEPENDENCE OF THE PLATEAU DURATION ON INITIAL RADIUS, MASS, AND EXPLOSION ENERGY

Model	α	β	γ
Arnett 1980	0.0	0.75	−0.25
Chugai 1991	0.25	0.375	−0.125
Litvinova & Nadyozhin 1985	0.186	0.566	−0.191
Popov 1992	0.167	0.5	−0.167

by Arnett (1980) without WCR, an estimate by Chugai (1991), and our estimate (eq. [26]). Table 1 shows that the present estimate agrees very closely with the numerical results. A rather important conclusion derived from the numerical fits was that $\alpha \approx -\gamma$; now this result is confirmed analytically.

The bolometric light curve, calculated using equations (21) and (17), has a plateau, which is almost constant. It is possible to estimate $L_{\rm bol}$ on the plateau from its value at $t = t_m$ (we neglected in eq. [17] the term $t_i^2/3t_a^2$, as was discussed above)

$$L_{\rm bol}(t_m) = 8\pi\sigma_{\rm SB}\, T_{\rm ion}^4\, v_{\rm sc}^2 \left(\frac{3t_i}{4t_a}\right)^{4/3} t_a^2 \,,$$

which can be written as

$$L_{\rm bol}(\text{plateau}) \approx 1.64 \times 10^{42} \frac{R_{0,500}^{2/3}\, E_{51}^{5/6}\, T_{\rm ion,5054}^{4/3}}{M_{10}^{1/2}\, \kappa_{0.34}^{1/3}} \text{ ergs s}^{-1} \quad (27)$$

or

$$M_{\rm bol} = -16.85 - 1.67 \log R_{0,500} + 1.25 \log M_{10} - 2.08 \log E_{51} + 0.83 \log \kappa_{0.34} - 3.33 \log T_{\rm ion,5054} \,. \quad (28)$$

Since the effective temperature on the WCR stage is constant, the V magnitude on the plateau is unambiguously connected to $M_{\rm bol}$ and differs only by a constant bolometric correction. The effective temperature is $T_{\rm eff} \approx 6000$ K, so the bolometric correction is B.C. ≈ 0.1 (Allen 1973). The dependence of V on E, M, and R_0 will be the same as in equation (28). Expressing E in 10^{50} ergs, M in solar masses, and R_0 in solar radii and taking $\kappa = 0.34$ cm^2 g^{-1} and $T_{\rm ion} = 5054$ K, we obtain from the current model

$$V = -11.42 - 1.67 \log R + 1.25 \log M - 2.08 \log E \,, \quad (29)$$

$$\log t_p = 1.206 + 0.167 \log R + 0.5 \log M - 0.167 \log E \,. \quad (30)$$

Litvinova & Nadyozhin (1985) obtained

$$V = -11.307 - 1.80 \log R + 1.22 \log M - 2.34 \log E \,,$$

$$\log t_p = 1.047 + 0.186 \log R + 0.566 \log M - 0.191 \log E \,,$$

The discrepancy between both estimates is not large, especially in the range of parameters, typical to SN IIp's, if we take into account the fact that their numerical estimates were obtained with a fitting procedure which possessed an error comparable to the difference in coefficients.

Equations (29) and (30) can be inverted to obtain relations allowing us to determine E, M, and R_0 from observational data, which give t_p and V, but we need one more observable parameter. We approximately can set the photospheric velocity, $u_{\rm ph}$, on the early part of the plateau phase to be equal to our scaling velocity $v_{\rm sc}$, because at this moment the photosphere has already passed the most outer layers with very low density and high velocity and is approximately situated at the outer edge of the dense, homologously expanding layers. Then for $u_{\rm ph}$, expressed in 10^3 km s^{-1}, we have from equation (13)

$$\log u_{\rm ph} = 0.611 + 0.0 \log R - 0.5 \log M + 0.5 \log E \,; \quad (31)$$

the difference between this estimate and the similar one from Litvinova & Nadyozhin (1985)

$$\log u_{\rm ph} = 0.729 - 0.0616 \log R - 0.476 \log M + 0.563 \log E$$

is also not large. Now, solving equations (29), (30), and (31), we find

$$\log E = 4.0 \log t_p + 0.4V + 5.0 \log u_{\rm ph} - 4.311 \,, \quad (32)$$

$$\log M = 4.0 \log t_p + 0.4V + 3.0 \log u_{\rm ph} - 2.089 \,, \quad (33)$$

$$\log R_0 = -2.0 \log t_p - 0.8V - 4.0 \log u_{\rm ph} - 4.278 \,, \quad (34)$$

where E is expressed in 10^{51} ergs, M in $M_\odot$, R_0 in $R_\odot$, and $u_{\rm ph}$ in 10^3 km s^{-1}.

Studies by Pskovskii (1978) and Tammann (1978) show that we can take as typical observed values $70^{\rm d}$ for t_p and $-17.5^{\rm mag}$ for absolute V magnitude for an "average" Type IIp SN. The investigations of SN spectra (Greenstein & Minkowski 1973; Kirshner et al. 1973; Kirshner & Kwan 1975) allow to estimate the typical velocity of a SN II envelope expansion as 5000–7000 km s^{-1}. Since $V_{\rm SC}$ is the velocity of the outer edge of the SN envelope in our model, the higher value 7000 km s^{-1} seems to be a more preferable estimate of it. Substitution into equations (32), (33), and (34) the values for an "average" SN IIp gives

$$E = 3.6 \times 10^{50} \text{ ergs}; \quad M = 2.44\, M; \quad R_0 = 1700\, R_\odot \text{ for } u_{\rm ph} = 5000 \text{ km s}^{-1}$$

and

$$E = 1.79 \times 10^{51} \text{ ergs}; \quad M = 6.71\, M_\odot; \quad R_0 = 443\, R_\odot \text{ for } u_{\rm ph} = 7000 \text{ km s}^{-1} \,,$$

where the second set of parameters seems to be more reliable, as mentioned above. The predicted mass and initial radius of a SN IIp progenitor (second set) are very close to similar values in Set A (Litvinova & Nadyozhin 1985). We may conclude that an "average" SN IIp is formed by an explosion of a red supergiant ($M \approx 4$–$8\, M_\odot$, $R_0 \approx 500$–$1500\, R_\odot$) with energy $(0.5$–$2) \times 10^{51}$ ergs.

4. CONCLUSIONS

The good agreement between predictions of the analytic model presented in this paper and results of numerical computations by Litvinova & Nadyozhin (1985) leads us to the conclusion that the complicated density profile, which was assumed to be constant in the presented work, and opacity behavior which was here approximated as a step function, do not overly influence the behavior of SN IIp light curves, and the constant density model can provide reliable estimates of mass, explosion energy, and initial radius for Type IIp supernovae. A similar conclusion was already reached by Arnett (1980), by Arnett & Fu (1989), and by Imshennik & Popov (1992). The model presented takes into account the wave of cooling and recombination as well as radiative diffusion inside it. This is as we believe, the reason that the current estimate is closer to results of hydrodynamical computations than the model with WCR but without radiative diffusion (Chugai 1991) and the model with radiative diffusion, but without WCR (Arnett 1980). The difference between the model presented with that Arnett & Fu (1989) is that we use the equality between the radiative flux through the photosphere and the luminosity of a blackbody with corresponding effective temperature as one of the conditions to determine the photospheric radius. Some conclusions of this work, such as the strong sensitivity of the plateau width on envelope mass, weaker sensitivity on the explosion energy and initial radius (that is, $\beta > \alpha$ and $\beta > |\gamma|$ in Table 1), qualitatively agree with the results of previous analytical studies of SN IIp's (Arnett 1980; Arnett & Fu 1989; Chugai 1991). The relations presented, connecting the duration and the magnitude of the plateau of SN IIp's and main explosion parameters, as well as the values of E, M, and R_0 for an "average" SN IIp, are close to the ones given by fitting grids of hydrodynamical models (Litvinova & Nadyozhin 1985), so this work provides additional analytical support for their fitting procedure. For SNs with more detailed observational data (e.g., SN 1987A) the analytical approach described reproduces the bolometric light curve (see Imshennik & Popov 1992) and allows us to determine the main SN parameters more definitely, than those for an "average" SN IIp.

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