[O III]/[N II] as an abundance indicator at high redshift

Max Pettini^{1★} and Bernard E. J. Pagel²

¹Institute of Astronomy, Madingley Road, Cambridge CB3 0HA

Accepted 2004 January 5. Received 2003 December 10

ABSTRACT

Among 'empirical' methods of estimating oxygen abundances in extragalactic H II regions, the use of the ratio of nebular lines of [O III] and [N II], first introduced by Alloin et al., is reappraised with modern calibration data and shown to have certain advantages over $R_{23} \equiv ([O II] + [O III])/H\beta$ and $N2 \equiv [N II] \lambda 6583/H\alpha$, particularly when applied to star-forming galaxies at high redshifts.

Key words: ISM: abundances – H II regions – galaxies: abundances.

1 INTRODUCTION

Extragalactic H II regions are central to the study of chemical abundances in the Universe (e.g. Garnett 2004). Accurate abundance measurements require the determination of the electron temperature, which in turn is usually obtained from the ratios of auroral to nebular line intensities, such as $[O III] \lambda 4363/\lambda 5007$. This is often referred to as the 'direct' $T_{\rm e}$ method. A well-known difficulty, however, arises from the fact that, as the metallicity increases, the electron temperature decreases (as the cooling is via metal lines) and the auroral lines eventually become too faint to measure. Detailed modelling can overcome this problem. For example, Kewley & Dopita (2002) have recently used a combination of stellar population synthesis and photoionization models to develop a set of ionization parameters and abundance diagnostics based only on the strong optical emission lines. Their 'optimal' method uses ratios of [NII], [OII], [OIII], [SII], [SIII] and the Balmer lines, which is the full complement of strong nebular lines accessible from the ground. They also recommended procedures for the derivation of abundances in cases where only a subset of these lines is available.

The advent of 8–10 m class telescopes has made it possible to extend observations of these lines to galaxies at high redshifts (e.g. Teplitz et al. 2000; Pettini et al. 2001; Kobulnicky et al. 2003; Lilly, Carollo & Stockton 2003; Steidel et al. 2004). At high redshift, however, the study of nebular emission lines encounters new obstacles. The first and obvious difficulty is that, given the much larger distances involved, the line fluxes are reduced dramatically. The problem is compounded by the fact that the lines are redshifted into the near-infrared wavelength region, where the sky background is orders of magnitude higher than in the optical. Secondly, at a given redshift only a subset of the strong lines will fall within an atmospheric window and thus be accessible from the ground. Even at particularly favourable redshifts, such as $z \simeq 2.3$ which shifts [O II], [O III] and H β , and H α and [N II] respectively into the middle range

There is thus an incentive to explore simpler abundance indicators based on only a few emission lines – preferably close in wavelength - which, while admittedly less accurate than the full treatments mentioned above, may still be adequate to characterize the chemical enrichment of distant galaxies. The idea of such indicators goes back to a fundamental paper by Searle (1971) who, noting some measurements of nebular [O III] lines in M33 by Aller (1942), himself found a systematic increase in $[O III]/H\beta$ and a corresponding decrease in $[NII]/H\alpha$ with galactocentric distance as a generic property of giant H II regions in late-type spiral galaxies. Searle pointed out that these trends were most likely due to radial abundance gradients, thereby raising the possibility of using these line ratios more generally as indicators of abundance. Such indicators can be calibrated against more secure abundance determinations, based on auroral line measurements or on detailed photoionization models, to establish their typical accuracy; for this reason they are often referred to as 'empirical' methods. The first exploration of such a method was by Jensen, Strom & Strom (1976) who considered the ratio $[O III]/H\beta$. The subsequent extension of this idea by Pagel et al. (1979), who included [O II], led to the most widely used abundance indicator, the R_{23} index, which relates the abundance of oxygen to the ratio of ([O II] + [O III]) to H β . In the last 25 yr there have been many efforts to refine the calibration of R_{23} . The most successful are the calibration by McGaugh (1991) which is based on photoionization models, and the empirical one by Pilyugin (2003); both improve the accuracy by making use of the [O III]/[O II] ratio as an ionization parameter. Even so, systematic differences of up to 0.5 dex remain between the oxygen abundances derived from the R_{23} index and those deduced from the 'direct' T_e method (e.g. Skillman, Côté & Miller 2003; Kennicutt, Bresolin & Garnett 2003). When compounded with the well-known double-valued behaviour of (O/H) versus R_{23} , this can

²Astronomy Centre, School of Science & Technology, Sussex University, Brighton BN1 9QH

of the J, H and K bands, it is not possible to record all these lines in a single exposure. Rather, different spectrograph settings are normally required, and this can easily introduce additional errors in the relative flux calibrations. For all of these reasons, there are practical limitations to the accuracy with which emission line ratios can be measured in high-redshift objects.

^{*}E-mail: pettini@ast.cam.ac.uk

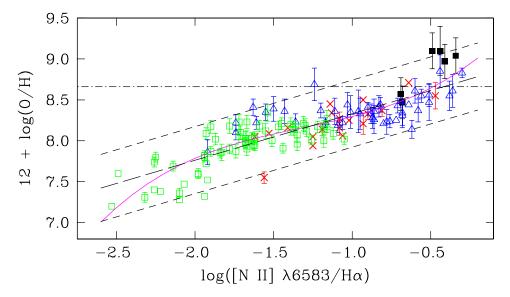


Figure 1. Oxygen abundance against the N2 index in extragalactic H II regions. The symbols have the following meanings. Filled squares – (O/H) from photoionization models by Díaz et al. (1991 – M 51) and Castellanos, Díaz & Terlevich (2002a,b – NGC 925 and NGC 1637). Crosses – M 101 (Kennicutt, Bresolin & Garnett 2003). Open triangles – spiral and irregular galaxies (Castellanos, Díaz & Terlevich 2002a; Garnett et al. 1997; González-Delgado et al. 1994, 1995; Kobulnicky & Skillman 1996; Mathis, Chu & Peterson 1985; Pagel, Edmunds & Smith 1980; Pastoriza et al. 1993; Shaver et al. 1983 [30 Dor and NGC 346]; Skillman et al. 2003; Vílchez et al. 1988). Open squares – blue compact galaxies (Campbell, Terlevich & Melnick 1986; Garnett 1990; Izotov, Thuan & Lipovetsky 1994; Kniazev et al. 2000; Kunth & Joubert 1985; Pagel et al. 1992; Skillman & Kennicutt 1993; Skillman et al. 1994; Thuan, Izotov & Lipovetsky 1995; van Zee 2000; Walsh & Roy 1989). Error bars in log (O/H) are only plotted when they are larger than the symbol to which they refer. The long-dashed line is the best-fitting linear relationship: $12 + \log (O/H) = 8.90 + 0.57 \times N2$. The short-dashed lines encompass 95 per cent of the measurements and correspond to a range in log (O/H) = ± 0.41 relative to this linear fit. The solid line is a cubic function of the form $12 + \log (O/H) = 9.37 + 2.03 \times N2 + 1.26 \times N2^2 + 0.32 \times N2^3$ which, however, gives only a slightly better fit to the data (95 per cent of the data points are within ± 0.38 of this line). The dot-dash horizontal line shows the solar oxygen abundance $12 + \log (O/H) = 8.66$ (Allende-Prieto, Lambert & Asplund 2001; Asplund et al. 2004).

lead to order of magnitude uncertainties in (O/H) in galaxies at $z\simeq 3$ (Pettini et al. 2001).

Partly to circumvent this latter problem, analogous $S_{23} \equiv ([S\, II] + [S\, III])/H\beta$ (Vílchez & Esteban 1996; Christensen, Petersen & Gammelgaard 1997; Díaz & Pérez-Montero 2000) and $S_{234} \equiv ([S\, II] + [S\, III] + [S\, IV])/H\beta$ (Oey et al. 2002) indices have been proposed. Although these sulphur-line based indices turn over at higher metallicities than R_{23} , the sulphur lines are weaker than their oxygen counterparts and their red and infrared rest wavelengths place them beyond the reach of ground-based observations at redshifts $z \gtrsim 1.5$.

More promising are methods which involve [N II] and H α . In this paper we reconsider the N2 index ($N2 \equiv \log\{[N II] \lambda 6583/H\alpha\}$) recently discussed by Denicoló, Terlevich & Terlevich (2002), and show how its accuracy can be further improved in the high abundance regime by comparison with $[O III] \lambda 5007/H\beta$.

2 THE N2 INDEX

Following up on earlier work by Storchi-Bergmann, Calzetti & Kinney (1994) and by Raimann et al. (2000), Denicoló et al. (2002) have focused attention again on the N2 index, pointing out its usefulness in the search for low metallicity galaxies. The $[N\,II]/H\alpha$ ratio is highly sensitive to the 'metallicity', as measured by the oxygen abundance (O/H), through a combination of two effects. As (O/H) decreases below solar, there is a tendency for the ionization to increase (either from hardness of the ionizing spectrum or from the ionization parameter, or both), decreasing the ratio $[N\,II]/[N\,III]$, and, furthermore, the (N/O) ratio itself decreases at the high-abundance end because of the secondary nature of nitrogen (e.g. Henry,

Edmunds & Köppen 2000; Pettini et al. 2002a). From the ground, H α and [N II] can be followed all the way to redshift $z\simeq 2.5$ (the limit of the K band). With thousands of galaxies now known at z>1 (Madgwick et al. 2003; Steidel et al. 2004; Abraham et al., in preparation), the N2 index offers the means to determine metallicities in a wholesale manner over look-back times which span most of the age of the Universe. It is thus worthwhile reconsidering its accuracy.

Denicoló et al. (2002) compiled an extensive sample of nearby extragalactic H II regions which could be used to calibrate N2 versus (O/H). We have revised their data base by: (i) including only H II regions with values of (O/H) determined either via the $T_{\rm e}$ method or with detailed photoionization modelling, and excluding the untabulated estimates for H II regions from the catalogue of Terlevich et al. (1991), because for many of these (O/H) had been estimated from 'empirical' (i.e. R_{23} or S_{23}) indices; and (ii) updating it with recent measurements by Kennicutt et al. (2003) for H II regions in M 101 and by Skillman et al. (2003) for dwarf irregular galaxies in the Sculptor Group.

The total sample, shown in Fig. 1, consists of 137 extragalactic H II regions with well determined values of (O/H) and N2. In all but six cases (shown with filled symbols) the oxygen abundance was determined with the $T_{\rm e}$ method. References to the original works are given in the figure caption. It can be readily appreciated from Fig. 1 that, while N2 does indeed increase monotonically with $\log({\rm O/H})$, a linear relationship is only an approximate representation of the data, which rather tend to lie along an S-shaped curve. In particular, when $({\rm O/H}) \simeq 8.2 \pm 0.2$, the value of N2 can span one order of magnitude, from $N2 \simeq -1.8$ to $N2 \simeq -0.8$, whereas at solar metallicity and beyond [N II] tends to saturate (Baldwin, Phillips & Terlevich 1981) because it comes to dominate the cooling (Kewley

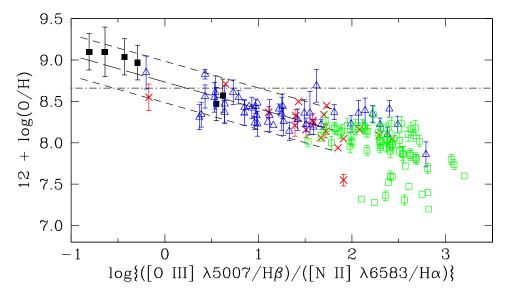


Figure 2. Oxygen abundance against the O3N2 index in extragalactic H II regions. The symbols have the same meaning as in Fig. 1. The long-dashed line is the best-fitting linear relationship, $12 + \log (O/H) = 8.73 - 0.32 \times O3N2$, valid when O3N2 < 1.9 The short-dashed lines encompass 95 per cent of the measurements which satisfy this condition and correspond to a range in $\log (O/H) = \pm 0.25$. The dot-dashed horizontal line shows the solar oxygen abundance $12 + \log (O/H) = 8.66$ (Allende-Prieto et al. 2001; Asplund et al. 2004).

& Dopita 2002). Forcing a linear fit to the sample, we obtain a line of best fit according to the relation:

$$12 + \log(O/H) = 8.90 + 0.57 \times N2 \tag{1}$$

shown with the long-dashed line in Fig. 1. Both the slope and the intercept are lower than the values of the fit by Denicoló et al. (2002) who proposed $12 + \log (O/H) = 9.12 + 0.73 \times N2$. While the formal statistical errors on the slope and intercept are small (0.03 and 0.04 respectively), of more interest is the dispersion of the points about the line of best fit in Fig. 1. Specifically, 95 per cent (68 per cent) of the measurements of log (O/H) lie within \pm 0.41 (\pm 0.18) of the line defined by equation (1); the 2 σ limits are shown as short-dashed lines in Fig. 1. Only a marginally better fit to the data is provided by a third-order polynomial of the form:

$$12 + \log (O/H) = 9.37 + 2.03 \times N2 + 1.26 \times N2^{2} + 0.32 \times N2^{3}$$
(2)

(valid in the range -2.5 < N2 < -0.3), with 95 per cent (68 per cent) of the measurements being within ± 0.38 (± 0.18) of the values given by equation (2). This cubic fit is indicated by the solid line in Fig. 1. We conclude that with the N2 calibrator it is possible to estimate the abundance of oxygen to within a factor of ~ 2.5 at the 95 per cent confidence level. This accuracy is comparable to that of the R_{23} method.

3 THE O3N2 INDEX

We now consider to what degree the accuracy in the determination of (O/H) can be improved by considering *two* ratios, $[N II]/H\alpha$ and $[O III]/H\beta$. Alloin et al. (1979) were the first to introduce the quantity $O3N2 \equiv \log \{([O III] \lambda 5007/H\beta)/([N II] \lambda 6583/M)\}$

 $H\alpha$) $\}$, but since then the O3N2 index has been comparatively neglected in nebular abundance studies. There is now a sufficient body of high quality data to reassess its merits. We expect the inclusion of [OIII] to be most useful in the high metallicity regime where [NII] saturates but the strength of [OIII] continues to decrease with increasing metallicity.

Fig. 2 shows how O3N2 varies with (O/H) for the 137 extragalactic H II regions in our sample. Clearly this method is of little use when $O3N2 \gtrsim 2$, but at lower values there appears to be a relatively tight, linear and steep relationship between O3N2 and log(O/H). A least-squares linear fit to the data in the range -1 < O3N2 < 1.9 yields the relation:

$$12 + \log(O/H) = 8.73 - 0.32 \times O3N2 \tag{3}$$

which is shown by the long-dashed line in Fig. 2, with 95 per cent (68 per cent) of the measurements within 0.25 (0.14) dex of the best fit line; the short-dashed lines in Fig. 2 show the 2σ limits. The caveats are that the statistics are somewhat limited, with only 65 out of the 137 H II regions in our sample satisfying the condition O3N2 < 1.9, and that the relationship defined by equation (3) relies heavily on the four data points with $12 + \log (O/H) \simeq 9.0$ deduced from photoionization models rather than the 'direct' T_e method. Nevertheless, in the high metallicity regime the O3N2 method appears promising and it would be very worthwhile extending the data base of nearby H II region measurements to improve its statistics.

4 CONCLUSIONS

In this Letter we have reassessed the usefulness of 'empirical' methods based on the $[N \, II]/H\alpha$ ratio for determining the oxygen abundance in H II regions, particularly with an eye to their application to the analysis of star-forming galaxies at high redshift. We have revised the extensive compilation prepared by Denicoló et al. (2002) to include only H II regions where the oxygen abundance is believed

¹ We performed an unweighted least-squares fit to the data because in compilations such as this the error estimates are highly heterogeneous. However, a weighted least-squares fit gives very similar results.

 $^{^2}$ This definition is slightly different from the original one proposed by Alloin et al. (1979) who included both [O III] doublet lines in the numerator of the first ratio.

to be known reliably, either because the electron temperature has been determined with high precision, or through detailed modelling of the ionization of the nebula. Our main conclusions are as follows:

- (i) The relationship between $N2 \equiv \log([N \text{ II}] \lambda 6583/\text{H}\alpha)$ and $\log(\text{O/H})$ is only approximately linear and yields (with the revised data set considered here) estimates of (O/H) which are only accurate to within \sim 0.4 dex at the 95 per cent confidence level. This level of precision is comparable to that of the more widely used R_{23} method.
- (ii) At moderate to high metallicities, greater than $\sim 1/4$ solar, the oxygen abundance can be deduced to within a factor of ~ 0.25 dex (again at the 95 per cent confidence level) if $O3N2 \equiv \log \{([O III] \lambda 5007/H\beta)/([N II] \lambda 6583/H\alpha)\}$ is lower than ~ 1.9 . The O3N2 index is particularly useful at solar and super-solar metallicities, where N2 saturates.

When applied to high-redshift galaxies, the N2 and O3N2 indices offer several advantages compared to the more familiar R_{23} method. First, the N-based indices have a monotonic behaviour with (O/H), overcoming the ambiguities introduced by the double-valued character of R_{23} . Secondly, both N_2 and O_3N_2 rely on ratios of emission lines which are close in wavelength, so that corrections for reddening, and accurate flux calibration of the spectra, are not necessary. Thirdly, both indices are highly sensitive to the oxygen abundance and are thus particularly well suited to the analysis of data of only moderate signal-to-noise ratio. Specifically, the slope of 0.57 of the relationship between $\log (O/H)$ and N2 implies that one only needs to measure the $[N II]/H\alpha$ ratio with an accuracy of about 25 per cent. Better precision would not improve the estimate of (O/H) because the random error would then be of minor importance relative to the systematic uncertainty of \pm 0.4 dex in the method. Similar considerations apply to the O3N2 index which is inherently more accurate in the high metallicity regime and varies even more steeply with log(O/H).

The high sensitivity of O3N2 to the oxygen abundance also helps to mitigate another complication which may arise preferentially at high redshift. The galaxies accessible to detailed spectroscopic studies at redshifts z = 1-3 support very high rates of star formation, one to two orders of magnitude higher than those seen in the local Universe (e.g. Steidel et al. 2003, 2004). There are also mounting indications that, even at these early epochs, such galaxies have already reached near-solar metallicities (e.g. Pettini 2004). When galactic chemical evolution proceeds at such a fast pace, the production of primary nitrogen from intermediate and low mass stars can lag behind that of oxygen from Type II supernovae, as found by Pettini et al. (2002b) in the z = 2.7276 Lyman break galaxy MS1512-cB58 (see also Matteucci & Pipino 2002). How far (N/O) is 'out of equilibrium' depends on the metallicity, the star formation rate, and the yet poorly constrained time-scale for the release of primary nitrogen (e.g. Meynet & Pettini 2003). However, even in cases where N is over-deficient relative to O by factors of 2-3, as in MS1512-cB58, the slope of -0.32 in the relation log (O/H) versus O3N2 limits the error in log (O/H) to $\lesssim 0.15$ (given that ([N II]/H α) varies linearly or slower with the nitrogen abundance in this regime).

Finally, it is encouraging that abundance calibrators developed from measurements in individual extragalactic H II regions seem to be fairly robust when applied to the integrated light of galaxies (Kobulnicky, Kennicutt & Pizagno 1999; Moustakas & Kennicutt, in preparation). Thus, we propose that, with the aid of the O3N2 index, it should be possible to measure the oxygen abundances of star forming galaxies at z > 1 to better than a factor of two with only a relatively modest investment in observing time.

ACKNOWLEDGMENTS

We are grateful to Chris Akerman, Mike Irwin and Samantha Rix for help with the production of the figures, and to Mike Dopita, Lisa Kewley and Chuck Steidel for valuable comments.

REFERENCES

Allende-Prieto C., Lambert D. L., Asplund M., 2001, ApJ, 556, L63 Aller L. H., 1942, ApJ, 95, 52

Alloin D., Collin-Souffrin S., Joly M., Vigroux L., 1979, A&A, 78, 200
 Asplund M., Grevesse N., Sauval A. J., Allende Prieto C., Kiselman D., 2004, A&A, in press (astro-ph/0312290)

Baldwin J., Phillips M. M., Terlevich R. J., 1981, PASP, 93, 5

Campbell A., Terlevich R., Melnick J., 1986, MNRAS, 223, 811

Castellanos M., Díaz A. I., Terlevich E., 2002a, MNRAS, 329, 315

Castellanos M., Díaz A. I., Terlevich E., 2002b, MNRAS, 337, 540

Christensen T., Petersen L., Gammelgaard P., 1997, A&A, 322, 41

Denicoló G., Terlevich R., Terlevich E., 2002, MNRAS, 330, 69

Díaz A. I., Pérez-Montero E., 2000, MNRAS, 312, 130

Díaz A. I., Terlevich E., Vílchez J. M., Pagel B. E. J., Edmunds M. G., 1991, MNRAS, 253, 245

Garnett D. R., 1990, ApJ, 363, 142

Garnett D. R., 2004, in Esteban C., Garcia López R., Herrero A., Sánchez F., eds, Cosmochemistry: The Melting Pot of Elements. Cambridge Univ. Press, Cambridge, in press (astro-ph/0211148)

Garnett D. R., Shields G. A., Skillman E. D., Sagan S. P., Dufour R. J., 1997, ApJ, 489, 63

González-Delgado R. M. et al., 1994, ApJ, 437, 239

González-Delgado R. M., Pérez E., Díaz A. I., García-Vargas M. L., Terlevich E., Vilchez J. M., 1995, ApJ, 439, 604

Henry R. B. C., Edmunds M. G., Köppen J., 2000, ApJ, 541, 660

Izotov Y. I., Thuan T.-X., Lipovetsky V. A., 1994, ApJ, 435, 647

Jensen E. B., Strom K. M., Strom S. E., 1976, ApJ, 209, 748

Kennicutt R. C., Bresolin F., Garnett D. R., 2003, ApJ, 591, 801

Kewley L. J., Dopita M. A., 2002, ApJS, 142, 35

Kniazev A. Y. et al., 2000, A&A, 357, 101

Kobulnicky H. A., Skillman E. D., 1996, ApJ, 471, 211

Kobulnicky H. A., Kennicutt R. C., Pizagno J. L., 1999, ApJ, 514, 544

Kobulnicky H. A. et al., 2003, ApJ, 599, 1006

Kunth D., Joubert M., 1985, A&A, 142, 411

Lilly S. J., Carollo C. M., Stockton A. N., 2003, ApJ, 597, 730

Madgwick D. S. et al., 2003, ApJ, 599, 997

Mathis J. S., Chu Y.-S., Peterson D. E., 1985, ApJ, 292, 155

Matteucci F., Pipino A., 2002, ApJ, 569, L69

McGaugh S., 1991, ApJ, 380, 140

Meynet G., Pettini M., 2003, in Maeder A., Eenens P., eds, IAU Symp., 215, Stellar Rotation. Astron. Soc. Pac., San Francisco, in press (astro-ph/0301287)

Oey M. S., Shields J. C., Dopita M. A., Smith R. C., 2002, Rev. Mex. Astron. Astrofis. Ser. Conf., 12, 77

Pagel B. E. J., Edmunds M. G., Blackwell D. E., Chun M. S., Smith G., 1979, MNRAS, 189, 95

Pagel B. E. J., Edmunds M. G., Smith G., 1980, MNRAS, 193, 219

Pagel B. E. J., Simonson E. A., Terlevich R. J., Edmunds M. G., 1992, MNRAS, 255, 325

Pastoriza M. G., Dottori H. A., Terlevich E., Terlevich R., Díaz A. I., 1993, MNRAS, 260, 177

Pettini M., 2004, in Esteban C., Garcia López R., Herrero A., Sánchez F., eds, Cosmochemistry: The Melting Pot of Elements. Cambridge Univ. Press, Cambridge, in press (astro-ph/0303272)

Pettini M., Ellison S. L., Bergeron J., Petitjean P., 2002a, A&A, 391, 21

Pettini M., Rix S. A., Steidel C. C., Adelberger K. L., Hunt M. P., Shapley A. E., 2002b, ApJ, 569, 742

Pettini M., Shapley A. E., Steidel C. C., Cuby J., Dickinson M., Moorwood A. F. M., Adelberger K. L., Giavalisco M., 2001, ApJ, 554, 981 Pilyugin L., 2003, A&A, 399, 1003 Raimann D., Storchi-Bergmann T., Bica E., Melnick J., Schmitt H., 2000, MNRAS, 316, 559

Searle L., 1971, ApJ, 168, 327

Shaver P. A., McGee R. X., Newton L. M., Danks A. C., Pottasch S. R., 1983, MNRAS, 204, 53

Skillman E. D., Côté S., Miller B. W., 2003, AJ, 125, 610

Skillman E. D., Kennicutt R. C., 1993, ApJ, 411, 655

Skillman E. D., Terlevich R. J., Kennicutt R. C., Garnett D. R., Terlevich E., 1994, ApJ, 431, 172

Steidel C. C., Adelberger K. L., Shapley A. E., Pettini M., Dickinson M., Giavalisco M., 2003, ApJ, 592, 728

Steidel C. C., Shapley A. E., Pettini M., Adelberger K. L., Erb D. K., Reddy N. A., Hunt M. P., 2004, ApJ, in press (astro-ph/0407439)

Storchi-Bergmann T., Calzetti D., Kinney A. L., 1994, ApJ, 429, 572 Teplitz H. I. et al., 2000, ApJ, 533, L65

Terlevich R., Melnick J., Masegosa J., Moles M., Copetti M. V. F., 1991, A&AS, 91, 285

Thuan T. X., Izotov Y. I., Lipovetsky V. A., 1995, ApJ, 445, 108

van Zee L., 2000, ApJ, 543, L41

Vílchez J. M., Esteban C., 1996, MNRAS, 280, 720

Vílchez J. M., Pagel B. E. J., Díaz A. I., Terlevich E., Edmunds M. G., 1988, MNRAS, 235, 633

Walsh J. M., Roy J.-R., 1989, MNRAS, 239, 297

This paper has been typeset from a TeX/LATeX file prepared by the author.