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A two-parameter luminosity correction for Type Ia supernovae*

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Abstract. If, in addition to the usual luminosity correction described by the light curve slope parameter b, we introduce a color correction parameter R, then an extraordinarily good fit is realized for all the 29 distant Type Ia supernovae of the recent Calán/Tololo supernova survey. The reduced χ^2 found is much lower than can be expected. All of the intrinsic dispersion is thereby removed from the data sample, leaving perfectly standardized candles insofar as one can measure with present techniques. The best-fit solution has a mysteriously low value for R (referred to the B-band) of about 2 and a value of b ≈ 0.5 which is smaller than previously reported without the color correction. These parameters lead, using pre-Hipparcos distances for Cepheid calibrations, to essentially the same value for the Hubble constant. Our preliminary value, subject to further investigation of Cepheid-calibrated supernovae using this same color correction, is $H_0 = 60 \pm 6$ kms^{-1}Mpc^{-1}.

Key words: supernovae: general – cosmology: observations – distance scale

1. Introduction

It is generally accepted that Type Ia supernovae (SNe Ia) are imperfect standard candles. The available data on well-observed distant ($0.01 < z < 0.1$) SNe Ia display, without corrections, an rms dispersion in luminosity of about 0.35 magnitudes, which is considerably in excess of measurement and distance uncertainties. In extreme cases, nearby SNe Ia are said to differ by more than 2.5 magnitudes (Phillips, 1993). Thus their usefulness as standard candles in establishing the cosmological expansion parameters has been questioned. However, ways have been found to remove much of the dispersion in selected samples by adjusting the luminosity of each supernova in accordance with its measured decay rate (Phillips, 1993, Hamuy et al., 1995, 1996). Standardized in this way, a collection of 29 distant SNe Ia carefully measured by modern photometric methods and reduced in a uniform and consistent way by the Calán/Tololo supernova survey (Hamuy et al., 1996), when pruned of three exhibiting excessive reddening, displays a dispersion nearly, but not quite, compatible with the measurement and distance uncertainties alone (Tripp, 1997).

In this paper I show that all 29 of these supernovae, regardless of color, can be standardized by introducing an additional adjustment to the luminosity dependent on the measured color of the supernova. The resulting fit of the corrected data displays no intrinsic dispersion in excess of measurement uncertainties and in fact has a confidence level much higher than expected, suggesting that the quoted measurement errors of the individual SNe may have been overestimated or that the errors may be correlated.

2. Method

The procedure followed is essentially the same as that used by Tripp (1997) to investigate H_0, q_0, and b. That analysis used the Hamuy et al. (1996) data and six more distant ($z > 0.3$) SNe Ia of Perlmutter et al. (1997). The Hubble constant H_0 is calculated for each supernova using the expression

$$\log H_0 = \frac{M_B - m_B + 52.38}{5} + \log \frac{1 - q_0 + q_0 z - (1 - q_0)\sqrt{1 + 2q_0 z}}{q_0^2} \quad (1)$$

Here m_B is the apparent blue magnitude and M_B the absolute blue magnitude at maximum light, while q_0 is the deceleration constant and z is the measured redshift.

Following the evidence of Phillips (1993) and Hamuy et al. (1995) that supernovae whose light curves fall more rapidly have lower peak luminosities, the absolute magnitudes M_B of the supernovae have been modified by adding a term b Δm_{15}. The measured quantity Δm_{15} is the decline in magnitude during the first 15 days beyond maximum blue light and b is a decline-rate parameter to be varied in a least squares fit of all the supernovae. To accommodate different amounts of reddening observed in the 29 supernovae of Hamuy et al. (1996), arising either from an intrinsically reddened supernova or from intervening dust in the parent galaxy, we introduce, in the same spirit as the parameter b, another phenomenological parameter R in the form R (B − V).

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Here B − V is the difference between the apparent B and V magnitudes at their respective maximum light as measured by Hamuy et al.(1996) and R (B − V) (called R_B (B − V) in the conventional terminology of dust absorption) incorporates both the intrinsic dependence of M_B on B − V as well as the B-band extinction due to dust. The modified value of M_B for any supernova with a measured value of Δm_{15} and B − V is then written as:

$$M_B = -19.48 + b\,(\Delta m_{15} - 1.05) + R\,(B - V). \tag{2}$$

The value −19.48 is the mean absolute magnitude of seven Cepheid-calibrated supernovae[1] and 1.05 is an average decline rate as used previously (Tripp, 1997). These are preliminary values pending further study of the nearby Cepheid-calibrated supernovae in the light of this new color correction (Tripp & Branch, in preparation). Any revision will have absolutely no effect on the quality of the fits nor on the best-fit values of the two correction parameters b and R obtained in this paper. However, it will directly affect H_0 through the relation $\delta H_0 = 28(H_0/60)\delta M_B$) obtained from Eq. (1).

Fig. 1 is a plot of the B − V color vs. Δm_{15} for the 29 Calán/Tololo supernovae. Apart from the one highly reddened supernova, there appears to be only a relatively weak dependence of color on Δm_{15}, so we may anticipate that the color correction is not strongly correlated with the Δm_{15} correction.

Knowing M_B, m_B, and z, we use Eqs. (1) and (2) to evaluate H_0 for each supernova, with b, R, and q_0 as parameters. The uncertainty in H_0 for each supernova is obtained by combining in quadrature the quoted errors δm_B, $\delta\Delta m_{15}$, and δ(B − V) with an uncertainty in the luminosity distance due to a possible peculiar motion δv = 400 km/s of the host galaxy with respect to the Hubble flow. Thus

$$\delta H_0 = H_0\sqrt{\left(\frac{\ln 10}{5}\right)^2\left[\delta m_B^2 + (b\delta\Delta m_{15})^2 + (R\delta(B-V))^2\right] + \left[\left(\frac{1}{z} + \frac{1-q_0}{2}\right)\delta z\right]^2} \tag{3}$$

The uncertainty in absolute magnitude calibration is common to all the supernovae so is not, at this point, included. A weighted average of the 29 values of $H_0 \pm \delta H_0$ is calculated. In this way a least-squares value for H_0 along with its uncertainty and a χ^2 for the fit is obtained with b and R as parameters. Since it has little effect on this discussion, we fix the deceleration parameter q_0 = 0.385 as determined by the previous fit (Tripp, 1997) of the Hamuy et al. (1996) data combined with six more distant cosmological SNe Ia of Perlmutter et al. (1997) that are much more sensitive to q_0. (We do not include these cosmological SNe Ia in the present analysis since their B − V colors were not measured.) The parameters b and R are then varied to search for a χ^2 minimum.

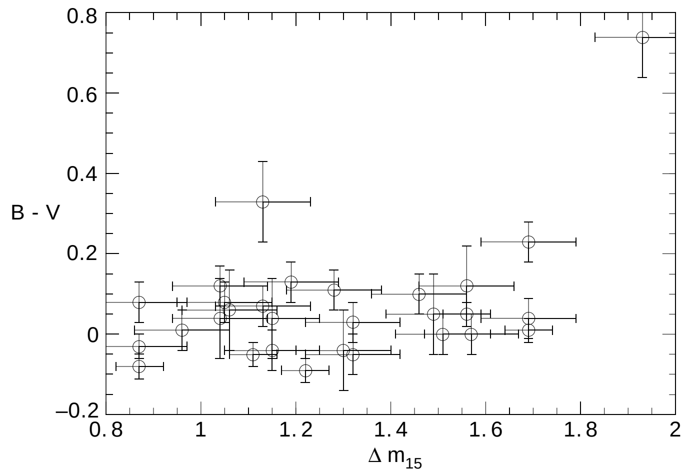


Fig. 1. Plot of the B − V color vs. Δm_{15} for the 29 Calán/Tololo SNe. Previous fits (Hamuy et al., 1996, Tripp, 1997) had excluded from their analyses the three highly reddened supernovae with values of B − V > 0.2

3. Results

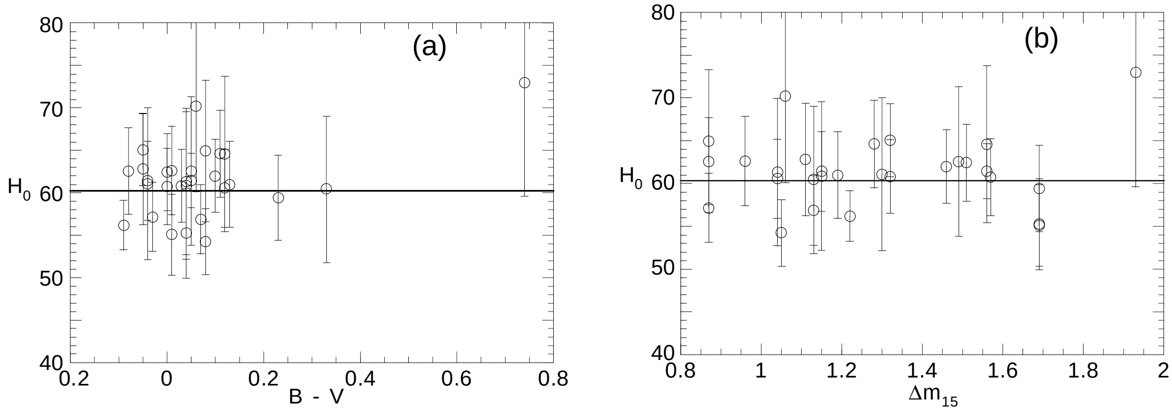
Using tabulated data (Hamuy, 1996) from the 29 Calán/Tololo supernovae and keeping q_0 fixed at 0.385, we search the space of b and R for the value of H_0 yielding the lowest χ^2. This best fit results in an unusually small reduced χ^2 of 0.51 to be compared to 0.97 expected for 29 data points and 3 parameters (b, R, and H_0), leading to an overly high confidence level (CL) of 0.98. For this minimum in χ^2, b = 0.52, R = 2.09 and H_0 = 60.1. Fig. 2 displays H_0 vs. B − V, Δm_{15}, and z for the above fit. All show that H_0 = 60.1 describes the data very well over the full ranges of these quantities. Fig. 2c, shown for completeness, is the analogue of the conventional Hubble diagram but with a clearer display of the total error for each supernova.

Thus, by introducing one additional parameter R, we have achieved a very substantial reduction in χ^2 compared to our

[1] The mean absolute magnitude of −19.48 is the pre-Hipparcos value, based on a distance modulus to the Large Magellanic Cloud (LMC) of μ = 18.50 mag. Hipparcos parallax measurements have so far lead to conflicting results for this distance modulus. Feast & Walker (1997) report a value of 18.70 ± 0.10, while a reassessment by Madore & Freedman (1997) finds no significant change. The ring around SN 1987A, located in the LMC, provides a direct measurement of the distance, independent of the Cepheid-based distance, and is generally in better agreement with the pre-Hipparcos value. Using this method Panagia et al. (1996) find the distance to SN 1987A to be 51.5 ± 1.2 kpc while Panagia et al. (1991) estimate the center of the LMC to be nearer to us by 1.1 kpc. Thus their distance to the LMC center becomes 50.4 kpc, yielding a distance modulus μ = 18.51 ± 0.05. Using the same circumstellar ring, Gould (1995) finds after placing SN 1987A 0.5 kpc in front of the center of the LMC, a distance modulus to the center of the LMC of μ < 18.37 ± 0.04. Meanwhile, better evidence for a metallicity dependence to the Cepheid period-luminosity relation coming from the EROS microlensing survey of Cepheids in the Large and Small Magellanic Clouds (Beaulieu et al., 1997) suggests a small reduction in the absolute magnitude of some of the calibrating Type Ia supernovae. In the absence of a more definitive analysis of the distance modulus and metallicity dependence incorporating these recent developments, we retain this pre-Hipparcos average value for the absolute magnitude of SNe Ia.

Table 1. Quantities associated with the best fits to all 29 and to a color-selected sample of 26 SNe when the parameters b or R or both are set to zero or are free parameters. In the latter case, results of three treatments of the measured uncertainties are listed.

Uncertainties	full	full	full	full	δm_B, δv only	scale=.55
Supernovae	29	29	29	29	29	29
b	0	0	1.09	0.52	0.44	0.48
R	0	2.8	0	2.09	1.99	2.08
H_0	53.9	57.1	60.9	60.1	59.7	59.9
reduced χ^2	4.81	0.95	2.16	0.51	0.94	0.96
CL	7×10^{-16}	0.54	0.0005	0.98	0.55	0.52
disp.(mag)	0.35	0.24	0.25	0.15	0.14	0.15
Supernovae	26	26	26	26	26	26
b	0	0	0.8	0.53	0.44	0.49
R	0	3.67	0	2.44	2.06	2.3
H_0	55.3	57.2	60.2	60.2	59.8	60.1
reduced χ^2	2.77	0.93	1.63	0.52	1.02	1.02
CL	5×10^{-6}	0.56	0.027	0.97	0.44	0.44
disp.(mag)	0.25	0.23	0.17	0.14	0.13	0.13



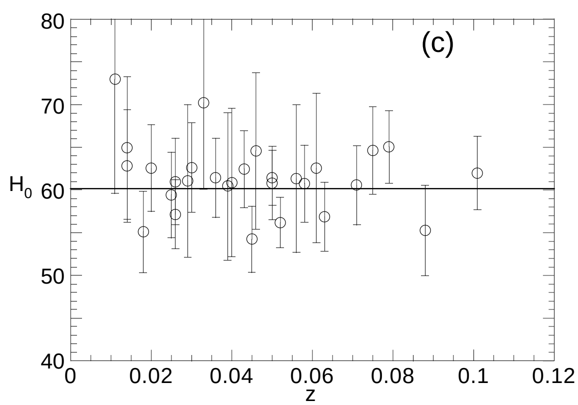


Fig. 2. a Plot of H_0 vs. the $B - V$ color, **b** H_0 vs. Δm_{15}, and **c** H_0 vs. z for the best fit of 29 Calán/Tololo SNe for a Δm_{15} slope parameter b = 0.52 and a $B - V$ color parameter R = 2.09. A best-fit value of $H_0 = 60.1$ passes within 1.5 σ of all 29 supernovae, indicating that the fit is much better than can be expected with these errors. Quantitatively, the reduced $\chi^2 = 0.51$, leading to a confidence level CL = 0.98.

previous analysis. There, without a color correction, the best fit obtained when the smaller sample of 26 Calán/Tololo SNe that passed the color cut $B - V < 0.2$ plus the six cosmological SNe yielded a large reduced $\chi^2 = 1.47$ with a low confidence level of CL = 0.05. Since these two studies use somewhat different sets of data, a closer comparison would be to the best fit of 29 Calán/Tololo supernovae alone which yields for R set to zero, b = 1.09 and CL = 5×10^{-4} while a fit to 26 Calán/Tololo supernovae passing the color cut, yields b = 0.80 and CL = 0.027 for R = 0.

It could be argued that the low χ^2 merely reflects the introduction of yet another uncertainty $\delta(B - V)$ multiplied by R in Eq. (3). However a convincing argument that this is not the case can be made by arbitrarily setting both the measurement uncertainties $\delta(B - V)$ and $\delta\Delta m_{15}$ to zero. When this is done and the χ^2 minimized, the reduced χ^2 becomes 0.94 (CL = 0.55), i.e.

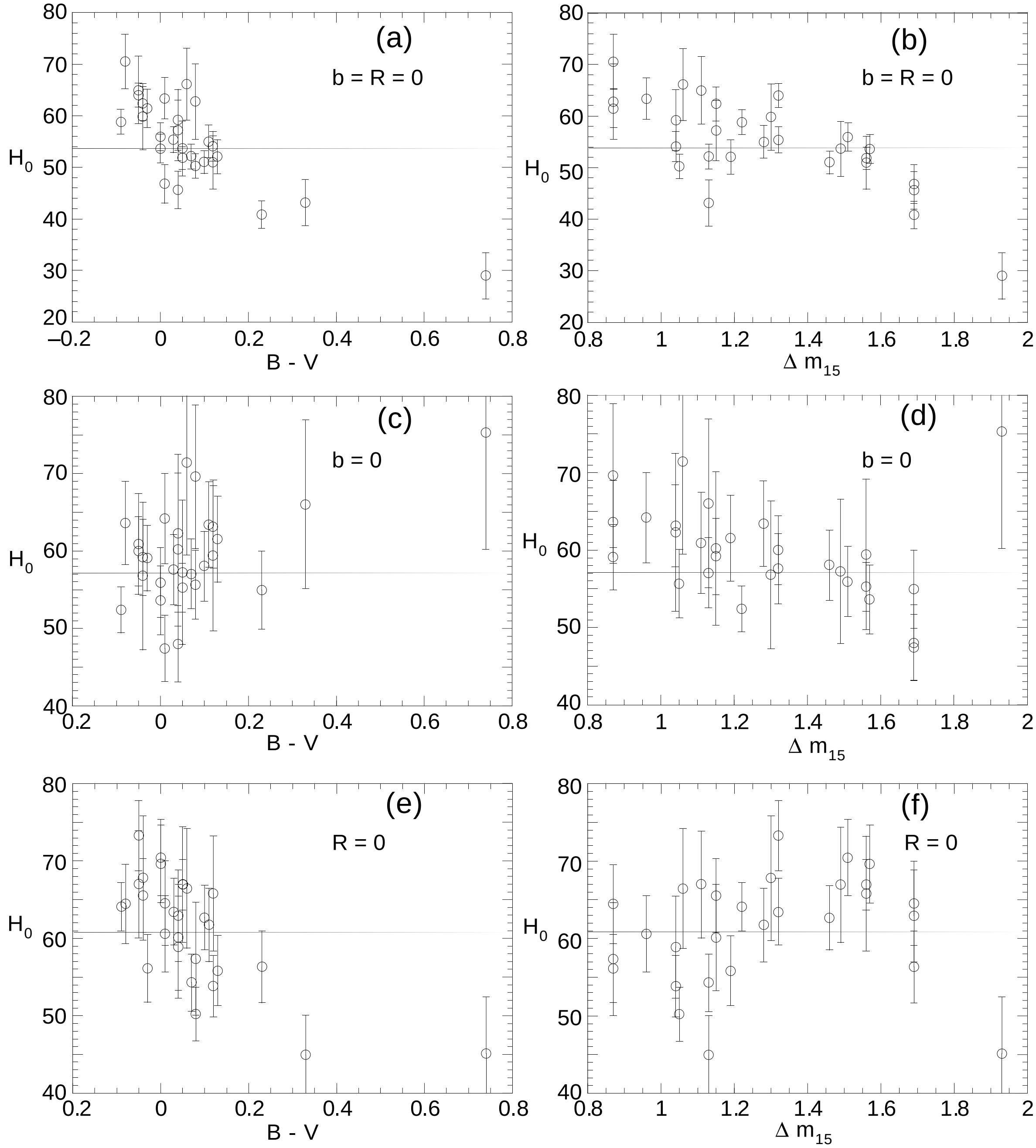


Fig. 3a–f. Examples of unacceptable fits that occur without b and/or R corrections. Plot of H_0 vs. the $B-V$ color and H_0 vs. Δm_{15} for the best-fit value of H_0 for the 29 Calán/Tololo SNe: **a,b** without corrections (b = 0 and R = 0) where H_0 = 53.9, **c,d** with no Δm_{15} correction (b = 0, R = 2.8) where H_0 = 57.1, **e,f** with no color correction (R = 0, b = 1.09) where H_0 = 60.9.

a highly probable representation of the data using for measurement uncertainties only the apparent magnitude uncertainties δmb and the assumed peculiar velocity $\delta v = 400\ \mathrm{km/s}$, which is generally considered a minimal estimate of peculiar velocity. Thus the introduction of a second parameter R has indeed improved the fit and is fully justified. As an alternative (and more realistic) way to obtain a more likely confidence level of about 0.5, all measurement errors of the 29 Calán/Tololo SNe could be scaled by a factor of 0.55.

The above three fits with different treatment of errors, as well as others with b or R or both set to zero, are tabulated in Table 1. It is interesting to note that setting b = 0 yields an acceptable confidence level for all 29 SNe, while setting R = 0 does not. Thus the $B-V$ color correction appears to be more effective in standardizing SNe Ia than does the presently popular shape correction Δm_{15}, although both appear to be necessary as can be seen in Fig. 3. Fig. 3a,b shows the impact of setting both b and R to zero. A best-fit value of H_0 = 53.9 agrees with neither

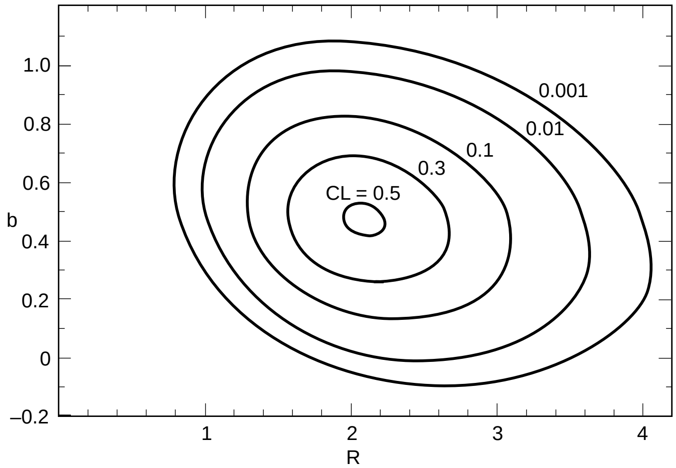


Fig. 4. Confidence level for 29 supernovae as a function of b and R using the measurement uncertainties reduced by a factor of 0.55. The uncertainties for b and R, including correlations, are δR = $\pm$0.38 and δb = $\pm$0.15.

plot; each displays a strong dependence of H_0 on B − V or Δm_{15}. Fig. 3c,d and Fig. 3e,f show the effect of setting b = 0 or R = 0 respectively. Here the b = 0 solution despite a satisfactory confidence level, nonetheless shows a downward trend in H_0 vs. Δm_{15}. Even more evident is the consequence of setting R = 0 which produces a visibly non-zero slope of H_0 vs. B − V. As can be seen from Fig. 3e, this problem persists even for the color-selected sample of 26 Calán/Tololo SNe previously investigated by Hamuy et al. (1996) and Tripp (1997). The best-fit values found when these 26 supernovae are fitted by themselves under the various conditions are also listed in Table 1.

Fig. 4 is a plot of confidence level (with measurement uncertainties scaled by 0.55) as a function of b and R, showing that there is some negative correlation between these parameters. Consequently by setting one parameter to zero as can be noted from Table 1, there is for the best-fit solution a compensatory increase in the other parameter. Further consequences of these omissions, as mentioned above and seen in Fig. 3, are less-than-satisfactory fits of H_0 vs. the neglected quantity.

Table 1 also lists under the various conditions the dispersion of the data points expressed in magnitudes. This is obtained from the standard deviation in the calculated values of H_0 divided by $H_0 \ln(10)/5$ in order to express it in magnitudes. From the table it can be seen that the uncorrected data (b = R = 0) has for 29 SNe a dispersion of 0.35 mag and for the color-selected sample of 26 SNe a dispersion of 0.25 mag. This is to be compared with typically 0.15 mag of dispersion for various b and R corrected good and overly good fits. Since the dispersion itself takes no account of measurements uncertainties and changes by only a modest amount in going from completely unacceptable fits to overly good fits, it is not a useful indication of the quality of the fit. A much more sensitive measure of whether a parametrization fits the data is obtained from the confidence level or, alternatively, reduced χ^2. However, even then, as noted above and seen in Fig. 3d, one should also inspect the fit to be reassured that an adequate χ^2 does not hide a trend in the data.

4. Discussion

By introducing a B − V color correction, in addition to the usual shape correction Δm_{15}, we have obtained a very substantial improvement in the fit to 29 distant supernovae. The best-fit parameters are R = 2.09 $\pm$ 0.38 and b = 0.52 $\pm$ 0.15, to be compared to b $\approx$ 0.8 found previously for R = 0.

The preliminary value for the Hubble constant remains H_0 = 60, pending further investigation of the effect of the color correction on the mean absolute magnitude of the Cepheid-calibrated SNe. The uncertainty in H_0 comes from many sources but is dominated by the still-uncertain distance scale and metallicity dependence for the Cepheid calibration. Following Branch et al. (1996), we continue to assign $\pm$ 0.15 mag uncertainty to the Cepheid zero point. This contributes $\pm$ 4 to δH_0. The statistical uncertainty coming from the 29 Calán/Tololo supernovae along with the uncertainty in b, R, and q_0 combined in quadrature give $\pm$ 2. Apart from the Cepheid calibration, we assign, for the time being, another $\pm$ 0.15 mag uncertainty to the mean absolute magnitude of the seven calibrated SNe. Adding all these in quadrature yields an overall uncertainty $\delta H_0 = \pm\ 6$ kms^{-1} Mpc^{-1}.

The best fit is much too good, having a reduced $\chi^2 \approx 0.5$, and a confidence level CL = 0.98. This suggests that the measurement uncertainties of the Calán/Tololo supernovae may have been overestimated. Alternatively, as discussed previously (Tripp, 1997), there may be correlations between the errors δm_B, $\delta\Delta m_{15}$ and now δ(B − V) that are not calculated by the observers and which, if introduced, could reduce the overall uncertainty. In any case, reducing these uncertainties by 20% produces a more reasonable CL = 0.9, while a scale factor of 0.55 yields the most likely CL $\approx$ 0.5. If errors are indeed smaller than presently believed and if cosmological SNe Ia measurement uncertainties are found to be similarly overestimated, then smaller errors will lead to a smaller uncertainty in q_0. Also, by applying the same type of color correction to cosmological supernovae even without knowing whether reddening is intrinsic or due to dust, one will be able to completely standardize the light output of each explosion and thereby get substantially better values for q_0 and the mass density Ω_M of the universe.

That there should be a color dependence to the SNe Ia absolute magnitude is not surprising: Höflich & Khokhlov (1996), employing a range of acceptable carbon-oxygen white dwarf models of SNe Ia explosions, have demonstrated, assuming the correctness of the models, that the absolute magnitude should depend both on B − V and on Δm_{15}. These models are compared with data from 40 SNe Ia in Höflich et al. (1996). According to van den Bergh (1995), one expects from these models that the color dependence should yield an R value of about 4. By coincidence, this is quite similar to the dependence of reddening due to interstellar dust as measured within our Galaxy and presumed to be true in other regions of space as well. This reddening dependence as expected from dust outside of the Galaxy has recently been verified by Riess et al. (1996) using a multi-color analysis of extragalactic SNe Ia. Since our analysis combines both of these quite different color mechanisms into one phenomenological parameter R, we would have expected to find $R \approx 4$; instead we find R to be about 2. This puzzle has beset the study of nearby SNe Ia for many years and has been discussed at length in the review by Branch & Tammann (1992) of Type Ia supernovae as standard candles. Here it has resurfaced in an even more compelling form, coming as it now does from the high quality Calán/Tololo distant supernova data and in addition appearing to be in conflict with some of the favored theoretical models of Type Ia supernova explosions.

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Note added in Proof: In a multi-color analysis of SNeIa, Riess et al. (1996, ApJ 473, 88) use a parametrization which implicitly assumes a tight correlation between $B - V$ and Δm_{15} in the approximate form $B - V = 3.5\ (\Delta m_{15} - 1.1)$. This is very different from that observed in the data displayed in Fig. 1. They obtain a value of $R_V = 2.1$ corresponding to an $R_B = 3.1$, the value to be compared to our R = 2.09.