Differential Equations Assignment Report

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Variant: #25

O.D.E. and initial conditions: $\begin{cases} y' = xy^2 - 3xy \\ y(0) = 2 \\ x \in [0, 6.4] \end{cases}$

Content:

1. Introduction

This project is created to demonstrate the usage and precision of such numerical methods like Euler, Improved Euler and Runge-Kuffa methods in plotting the graph of ordinary differential equations. Here I try to plot the graph of the function represented by the function $y^{'} = xy^2 - 3xy$on [0, 6,4] range and the initial value $y(0) = 2$. For such purpose, I implemented a GUI application plotting the functions and their errors w.r.t. the exact solution. Additionally, there is an opportunity to change the initial conditions of the o.d.e. and show the other graph. More information about the application is given in section 3.

2. Analytical Solution

$$y^{'} = xy^2 - 3xy$$

$$This\ is\ a\ Bernoulli\ equation$$

$$y^{'} + 3xy = xy^2 \mid : y^2$$

$$\frac{y'}{y^2} + \frac{3x}{y} = x$$

$$Let^{'}s\ use\ substitution$$

$$z = \frac{1}{y}, \qquad z^{'} = -\frac{y'}{y^2}$$

$$-z^{'} + 3xz = x$$

Method of variation of parameter

$$-z' + 3xz = 0$$

$$\frac{dz}{z} = 3xdx$$

$$\int \frac{dz}{z} = \int 3xdx$$

$$\log|z| = \frac{3}{2}x^2 + \log u$$

$$z = ue^{\frac{3}{2}x^2}$$

$$z' = u'e^{\frac{3}{2}x^2} + 3xe^{\frac{3}{2}x^2}u$$

$$-u'e^{\frac{3}{2}x^2} - 3xe^{\frac{3}{2}x^2}u + 3xe^{\frac{3}{2}x^2}u = x$$

$$u' = -\frac{x}{e^{\frac{3}{2}x^2}}$$

$$u = -\int \frac{xdx}{e^{\frac{3}{2}x^2}} = \left| \begin{aligned} v &= \frac{3}{2}x^2 \\ dv &= 3xdx \end{aligned} \right| = -\frac{1}{3}\int \frac{dv}{e^v} = \frac{1}{3e^v} + C$$

$$= \frac{1}{3e^{\frac{3}{2}x^2}} + C$$

$$z = \frac{1}{3} + Ce^{\frac{3}{2}x^2}$$

$$y = \frac{1}{\frac{1}{3} + Ce^{\frac{3}{2}x^2}}, \qquad y(0) = 2$$

$$2 = \frac{1}{\frac{1}{3} + C}, \qquad C = \frac{1}{6}$$

Solution:

$$y = \frac{1}{\frac{1}{3} + \frac{1}{6}e^{\frac{3}{2}x^2}} = \frac{6}{2 + e^{\frac{3}{2}x^2}}$$

3. Information about the application

This is a GUI application created on Python with the use of matplotlib, numpy, pandas, tkinter and pandastable libraries. Using it you can check the graph of function for some specific differential equation. In this project the equation is y'=xy^2-3xy (variant #25).

In order to run the application, you should activate the virtual environment located in the venv directory:

```
$ source venv/bin/activate
```

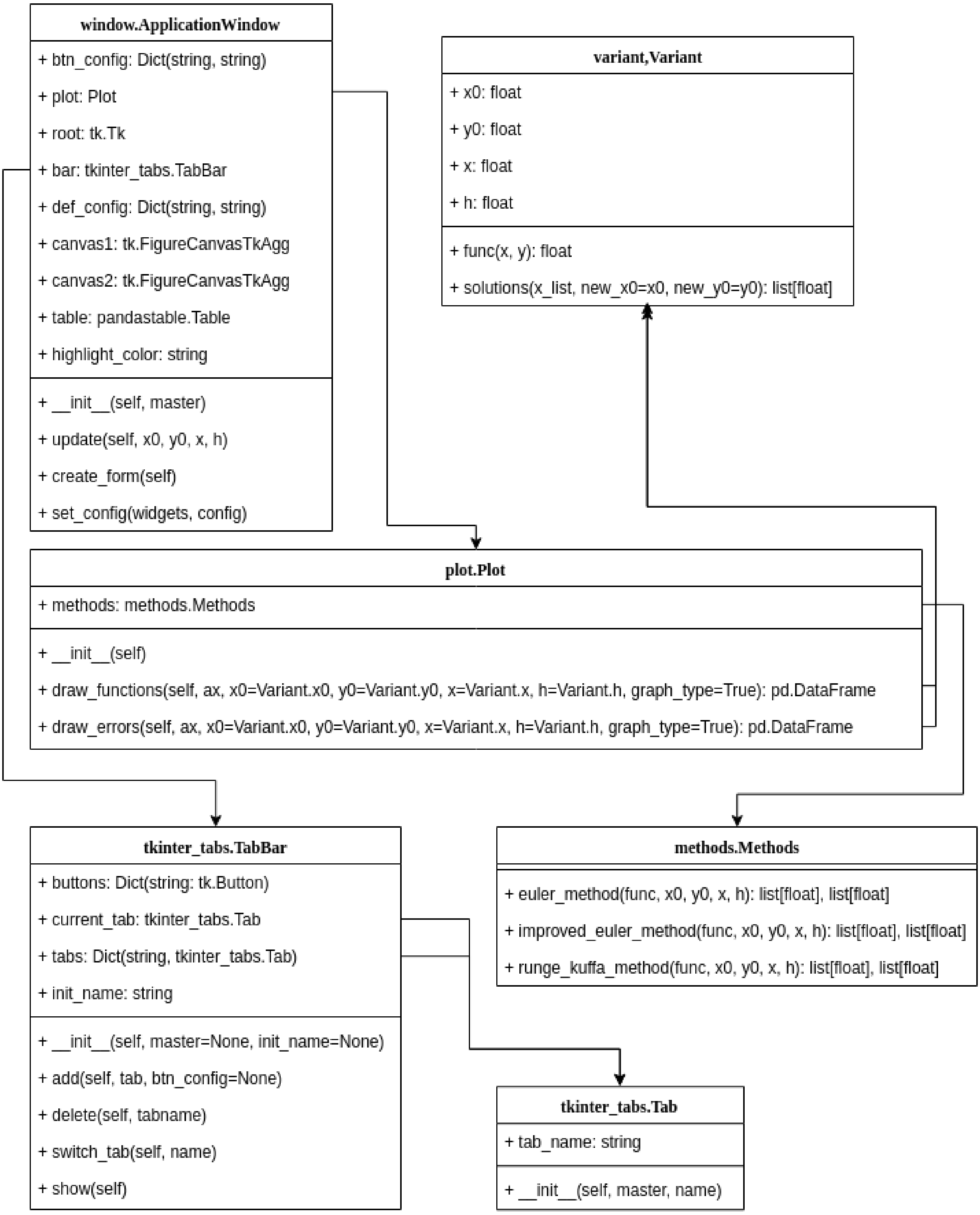
You don't need to install any packages. Everything that is needed is already in the virtual environment.

Now you can run the application:

```
$ python main.py
```

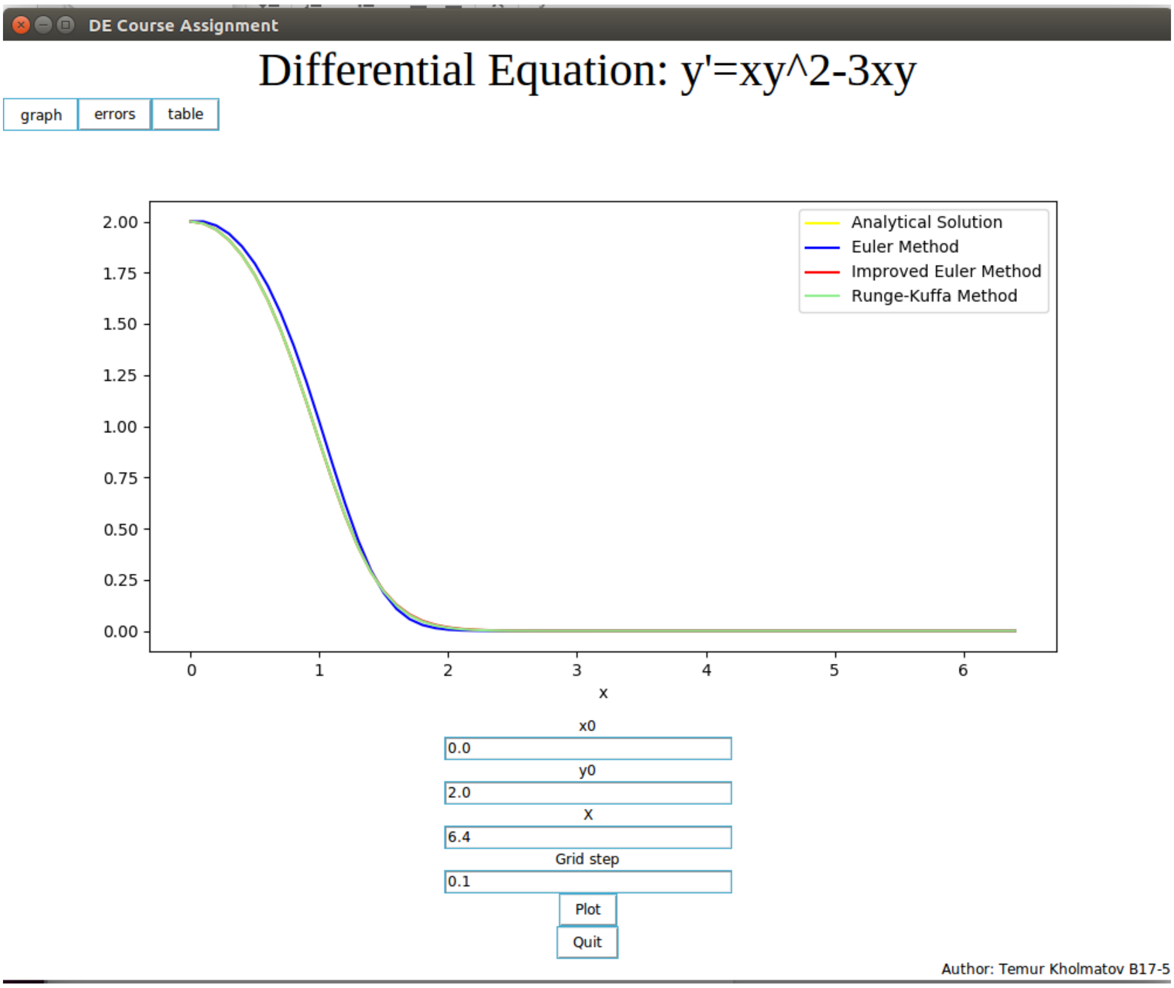
The values of initial conditions are changeable through the form in the window of app. However, to change the differential equation and the function of the analytical solution, you need to change the code.

4. UML diagram of the application

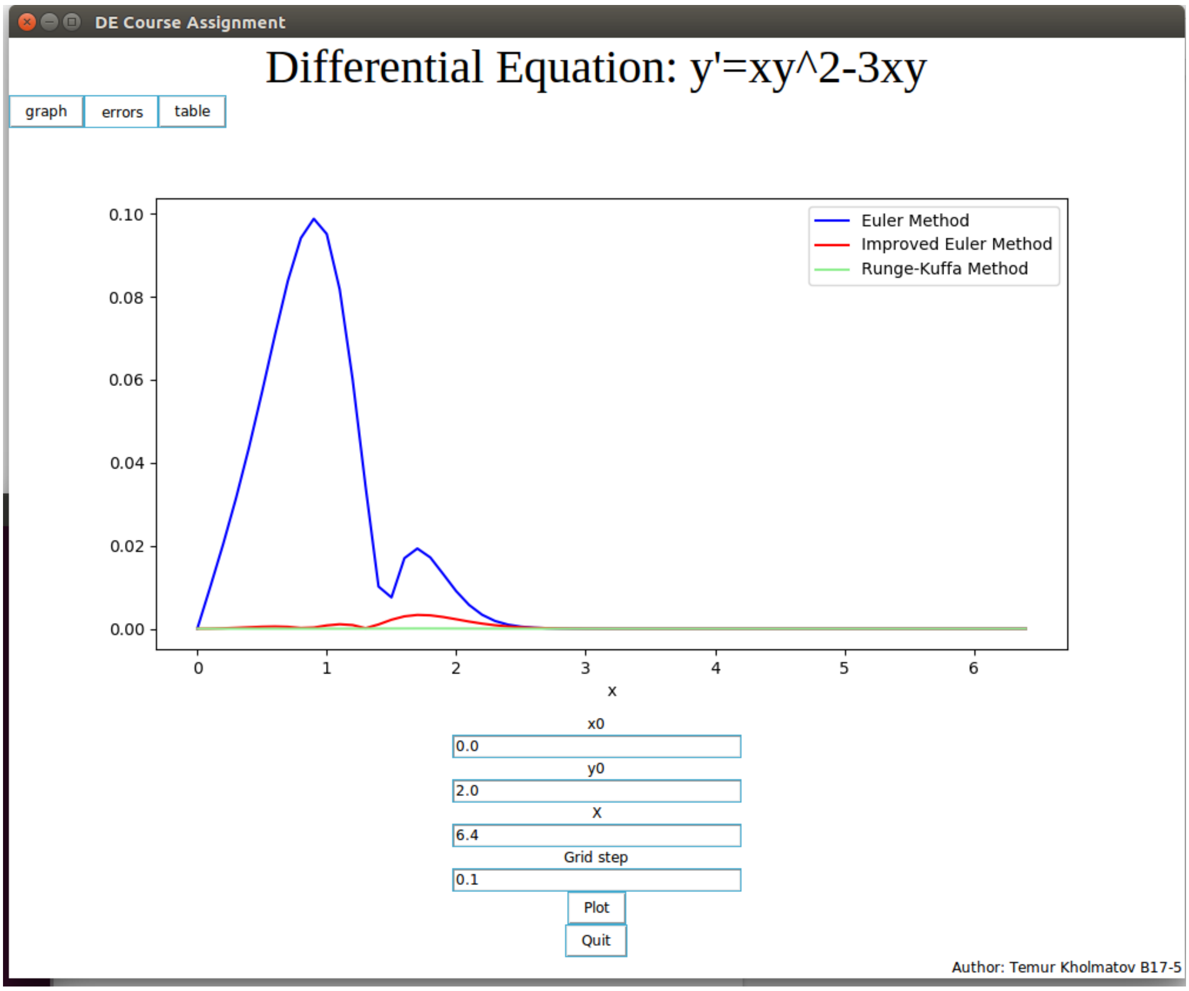


5. Screenshots of the results

The graph of the function using different numerical methods and the function of the analytical solution



The graph of the function errors of different numerical methods comparing to the analytical solution



The table with the results of different numerical methods and the analytical solution

	x	Analytical Solution	Euler Method	Improved Euler Method	Runge-Kuffa Method
1	0.0	2.0	2.0	2.0	2.0
2	0.1	1.989975125780137	2.0	1.99	1.98997517
3	0.2	1.9596081988013834	1.98	1.95970529	1.95960837
4	0.3	1.908071175970593	1.939608	1.90828031	1.90807159
5	0.4	1.8341617297510011	1.87790566	1.83450563	1.83416251
6	0.5	1.736617918821381	1.79361816	1.73708578	1.73661924
7	0.6	1.6146364155184167	1.68542874	1.61516242	1.61463845
8	0.7	1.468614966607913	1.55249177	1.46906256	1.46861787
9	0.8	1.3010396574400718	1.39518465	1.30121436	1.3010435
10	0.9	1.1172572540974886	1.21606355	1.11697573	1.11726201
11	1.0	0.9256846378913173	1.02081934	0.92490197	0.92569044
12	1.1	0.7370032989770018	0.81878075	0.73593589	0.73701096
13	1.2	0.5622643238269259	0.62232732	0.56139255	0.56227592
14	1.3	0.41049693063788184	0.44476444	0.41038346	0.41051566
15	1.4	0.28686384047890406	0.29702231	0.28788077	0.28689272
16	1.5	0.19215813057674871	0.18462405	0.19431424	0.1921981
17	1.6	0.12364639540917512	0.10665614	0.12662004	0.12369537
18	1.7	0.07660393423818114	0.05728128	0.07991966	0.07665741
19	1.8	0.04579306649837732	0.02862562	0.0490078	0.04584581
20	1.9	0.02646056291030139	0.01331528	0.02927198	0.0265082
21	2.0	0.014799146228146511	0.00575926	0.0170667	0.01483903
22	2.1	0.00801919549119668	0.00231034	0.00973098	0.00805044
23	2.2	0.004212724213146776	0.00085595	0.00543476	0.0042358
24	2.3	0.0021464150255688865	0.00029118	0.00297768	0.00216257
25	2.4	0.0010609460785248467	9.029e-05	0.00160284	0.00107172
26	2.5	0.0005088230965246155	2.528e-05	0.0008489	0.00051569
27	2.6	0.00023679412499200223	6.32e-06	0.00044302	0.00024098
28	2.7	0.00010693641239060816	1.39e-06	0.00022818	0.00010939
29	2.8	4.686421630501705e-05	2.6e-07	0.00011617	4.824e-05
30	2.9	1.993074239366642e-05	4e-08	5.856e-05	2.068e-05
31	3.0	8.225731964020555e-06	1e-08	2.928e-05	8.62e-06
32	3.1	3.294547828703518e-06	0.0	1.454e-05	3.49e-06
33	3.2	1.2805242960448408e-06	0.0	7.19e-06	1.38e-06
34	3.3	4.830041014218233e-07	0.0	3.55e-06	5.3e-07
35	3.4	1.76801077959469e-07	0.0	1.75e-06	2e-07
36	3.5	6.280440945367003e-08	-0.0	8.6e-07	7e-08
37	3.6	2.1650429637886215e-08	0.0	4.3e-07	3e-08
38	3.7	7.242926256111083e-09	-0.0	2.1e-07	1e-08
39	3.8	2.3514338837750537e-09	0.0	1.1e-07	0.0
40	3.9	7.408369680111272e-10	-0.0	5e-08	0.0
41	4.0	2.265080726396442e-10	0.0	3e-08	0.0
42	4.1	6.720721347419445e-11	-0.0	1e-08	0.0
43	4.2	1.9351706081006698e-11	0.0	1e-08	0.0
44	4.3	5.407465966976939e-12	-0.0	0.0	0.0
45	4.4	1.4663562544398593e-12	0.0	0.0	0.0
46	4.5	3.85883636487071e-13	-0.0	0.0	0.0
47	4.6	9.854722393881577e-14	0.0	0.0	0.0
48	4.7	2.4423257734318323e-14	-0.0	0.0	0.0
49	4.8	5.874000184222209e-15	0.0	0.0	0.0
50	4.9	1.3709937805528883e-15	-0.0	0.0	0.0
51	5.0	3.105333003481127e-16	0.0	0.0	0.0
52	5.1	6.825776108430344e-17	-0.0	0.0	0.0
53	5.2	1.4560190742379427e-17	0.0	0.0	0.0
54	5.3	3.0140694521507523e-18	-0.0	0.0	0.0
55	5.4	6.054950707110851e-19	0.0	0.0	0.0
56	5.5	1.180426998450933e-19	-0.0	0.0	0.0
57	5.6	2.233257605366299e-20	0.0	0.0	0.0
58	5.7	4.1002437456553694e-21	-0.0	0.0	0.0
59	5.8	7.305529411761561e-22	0.0	0.0	0.0
60	5.9	1.2631789440687133e-22	-0.0	0.0	0.0
61	6.0	2.11957714332049e-23	0.0	0.0	0.0
62	6.1	3.451475086899807e-24	-0.0	0.0	0.0
63	6.2	5.45420458149956e-25	0.0	0.0	0.0
64	6.3	8.364293627613338e-26	-0.0	0.0	0.0
65	6.4	1.244796281328927e-26	0.0	0.0	0.0

6. Conclusion

According to the results, the most accurate numerical method is the Runge-Kuffa method whereas the most inaccurate one is the Euler method. The function does not have any points of discontinuity and is decreasing on the range [x0, X].