Homework #2

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Conthrol Theory. Group 2
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Intro and overview

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Generated variant is: **B**

Problem 2

(A) Find total transfer function
Lets, define Z as intermediate signal. Then,

$$Y(s) = Z(s)W_2W_3$$

$$Z(s) = X(s)W_1 - W_4\frac{1}{W_3}Y(s)$$

Now, we substitute plug right-hand-side of Z to Y

$$Y(s) = W_2W_3\big[W_1X(s) - W_4\frac{1}{W_3}Y(s)\big]$$

$$Y(s) = W_1W_2W_3X(s) - W_2W_4Y(s)$$

$$Y(s)(1 + W_2W_4) = W_1W_2W_3X(s)$$

Divide, both side to $X(s)$

$$\frac{Y(s)}{X(s)} = \frac{W_1W_2W_3}{1 + W_2W_4}$$

Now, we assign given values

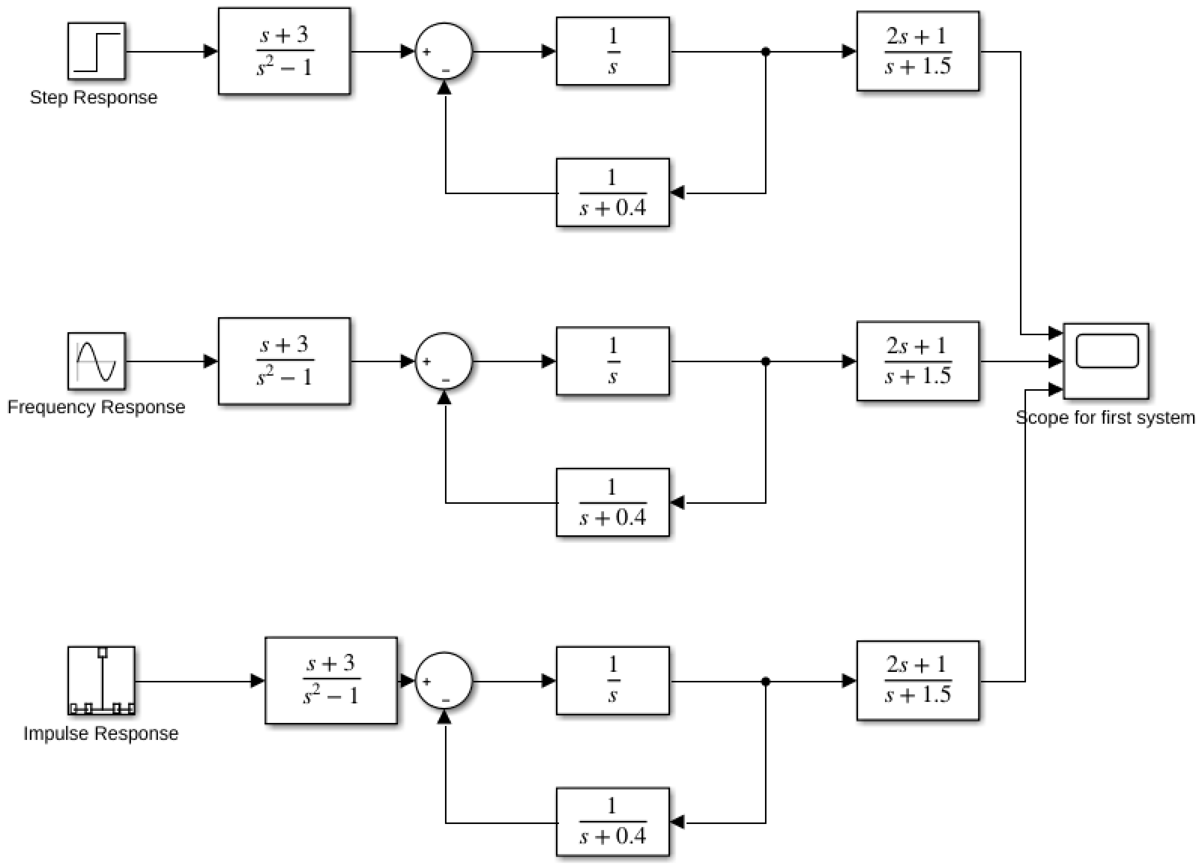
$$W_1 = \frac{s+3}{s^2-1}, W_2 = \frac{1}{s}, W_3 = \frac{2s+1}{s+1.5}, W_4 = \frac{1}{s+0.4}$$

Which gives final result:

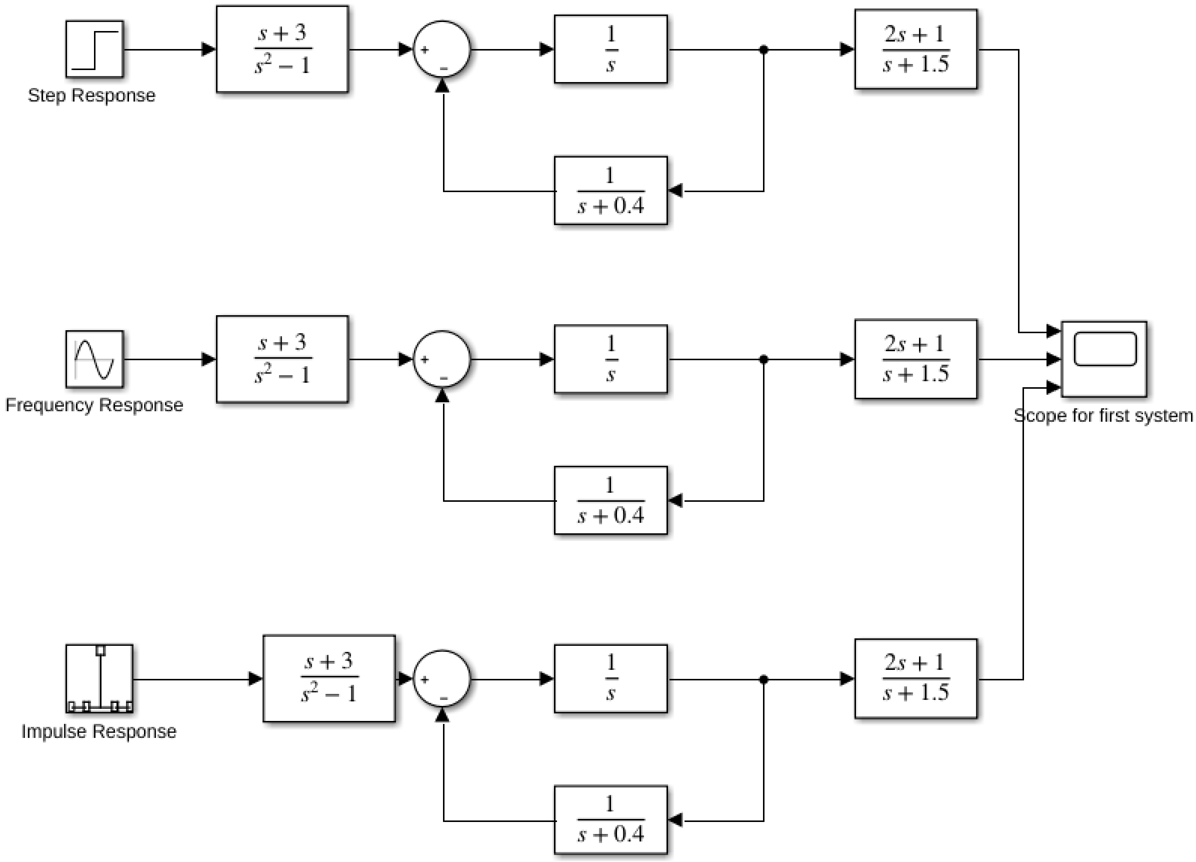
$$\frac{Y(s)}{X(s)} = \frac{2s^3 + 7.8s^2 + 5.8s + 1.2}{s^5 + 1.9s^4 + 0.6s^3 - 0.4s^2 - 1.6s - 1.5}$$

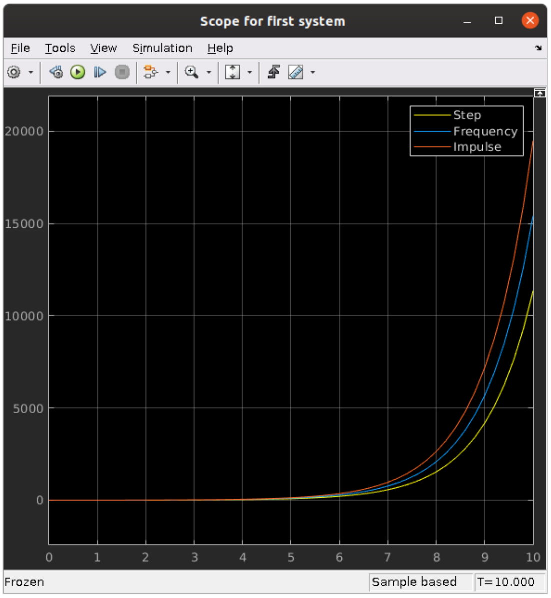
(B) Simulink schemas and plots

The simulink schema for first system

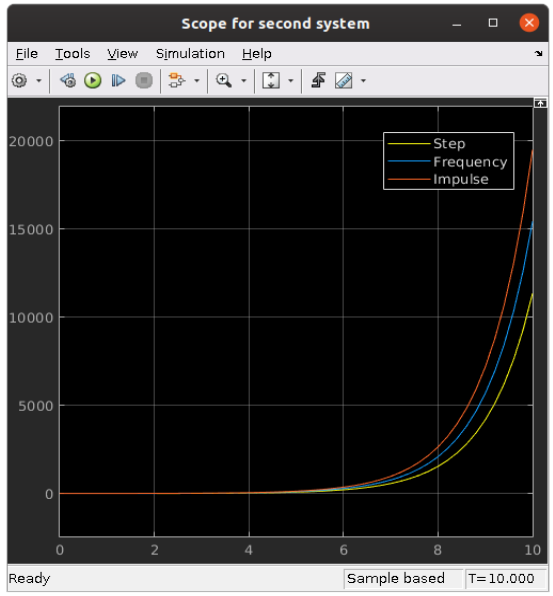


The simulink schema for second system:





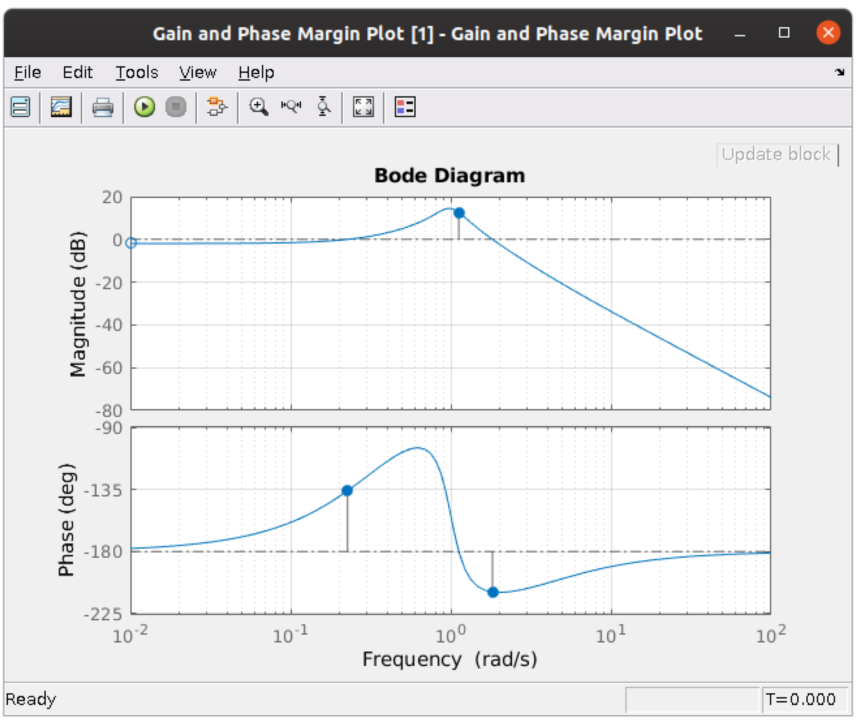
(a) Scope for first system



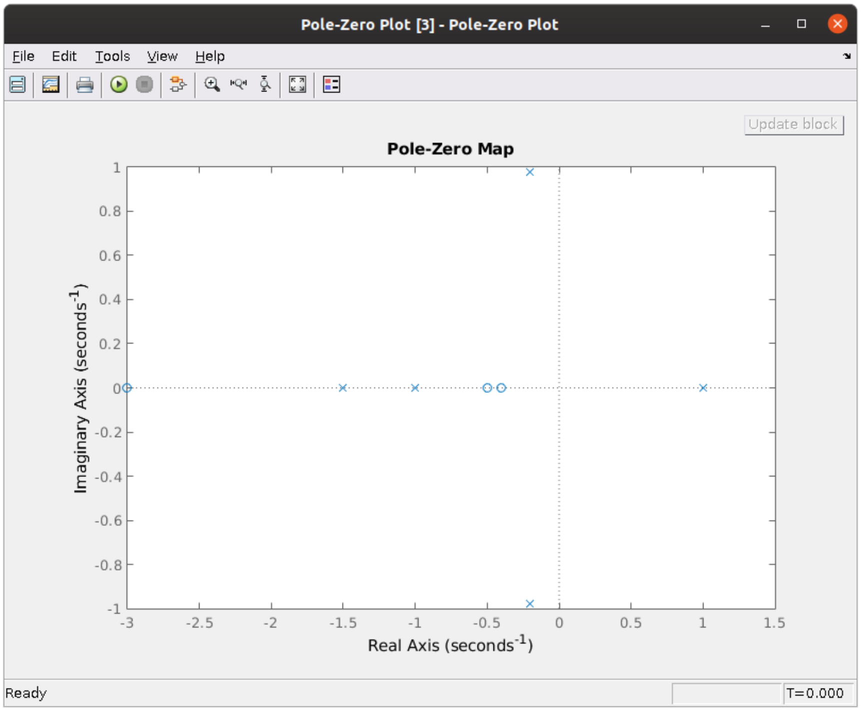
(b) Scope for second system

Figure 1: Scope of Step, Frequency and Impulse responses for each of systems

(C) Generate Bode and Pole-zero map plots The frequency input was chosen to generate following plots.



From the above plot we can conclude that system is **unstable** because there is negative phase margin.



And Pole-Zero plot for the same input, we can see that not all poles and zeros are in left half-plane, so system is unstable.

Problem 3

Find total transfer function of the closed-loop system

The total transfer function is the sum of all transfer functions. Let Φ_g be TF for input $g(t)$ and Φ_f for input $f(t)$

$$\Phi_g(s) = \frac{X}{G} = \frac{W(s)}{1+W(s)}$$

$$\Phi_f(s) = \frac{X}{F} = \frac{M(s)}{1+W(s)}$$

$$X = \Phi_g G + \Phi_g F = \frac{W(s)}{1+W(s)} G + \frac{M(s)}{1+W(s)} F$$

Now, we plug M and W

$$X = \begin{bmatrix} \frac{s^2+4s+1}{3s^2+9s+1} & \frac{6s^2+15s}{3s^3-26s-3} \end{bmatrix} \begin{bmatrix} G \\ F \end{bmatrix} \tag{1}$$

Problem 4

Find transfer function of the system
The variant provides SS matrices:

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 3 & 0 \end{bmatrix} \tag{2}$$

According to lecture slides we derived formula to convert SS to TF :

$$W(p) = C(Ip - A)^{-1}B$$

After substitutions of matrices we have:

$$C(Ip - A)^{-1} = \begin{bmatrix} p-1 & 1 \\ -2 & p-1 \end{bmatrix}^{-1} = \frac{1}{(p-1)^2 + 2} \begin{bmatrix} 3p-3 & -3 \end{bmatrix} \tag{3}$$

$$C(Ip - A)^{-1}B = \frac{1}{(p-1)^2 + 2} \begin{bmatrix} 3p-3 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{6p-9}{p^2 - 2p + 3} \tag{4}$$

Finally, we have transfer function

$$W(p) = \frac{6p-9}{p^2 - 2p + 3}$$

Problem 5

Find transfer functions of the system.
This task is quite similar to previous one only with one difference of nonzero D matrix.
The variant provides matrices:

$$A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} -1 & 4 \end{bmatrix}, D = \begin{bmatrix} 2 & 1 \end{bmatrix} \tag{5}$$

According to lecture slides we can calculate transfer function of SS with following formula:

$$W(p) = C(Ip - A)^{-1}B + D$$

After substitutions:

$$C(Ip - A)^{-1} = \begin{bmatrix} -1 & 4 \end{bmatrix} \begin{bmatrix} p-1 & 2 \\ -2 & p+1 \end{bmatrix}^{-1} = \frac{1}{p^2 + 3} \begin{bmatrix} 7-p & 4p-2 \end{bmatrix} \tag{6}$$

$$C(Ip - A)^{-1}B = \frac{1}{p^2 + 3} \begin{bmatrix} 7-p & 4p-2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 2 & 0 \end{bmatrix} = \frac{1}{p^2 + 3} \begin{bmatrix} 2(3p+5) & -3(p-7) \end{bmatrix} \tag{7}$$

And we add D:

$$C(Ip - A)^{-1}B + D = \begin{bmatrix} \frac{2(3p+5)}{p^2+3} & \frac{-3(p-7)}{p^2+3} \end{bmatrix} + \begin{bmatrix} 2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2p^2+6p+16}{p^2+3} & \frac{p^2-3p+24}{p^2+3} \end{bmatrix} \tag{8}$$

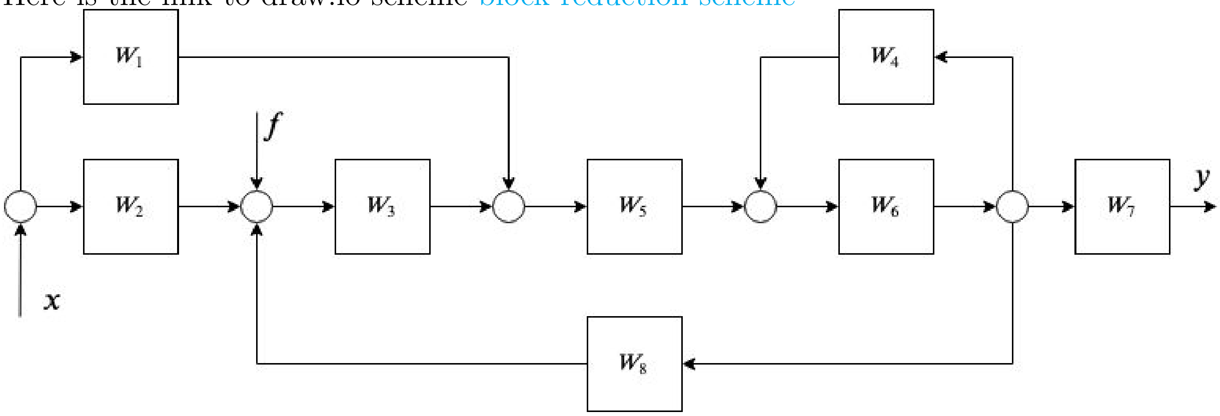
Finally, we have transfer function

$$W(p) = \begin{bmatrix} \frac{2p^2+6p+16}{p^2+3} & \frac{p^2-3p+24}{p^2+3} \end{bmatrix}$$

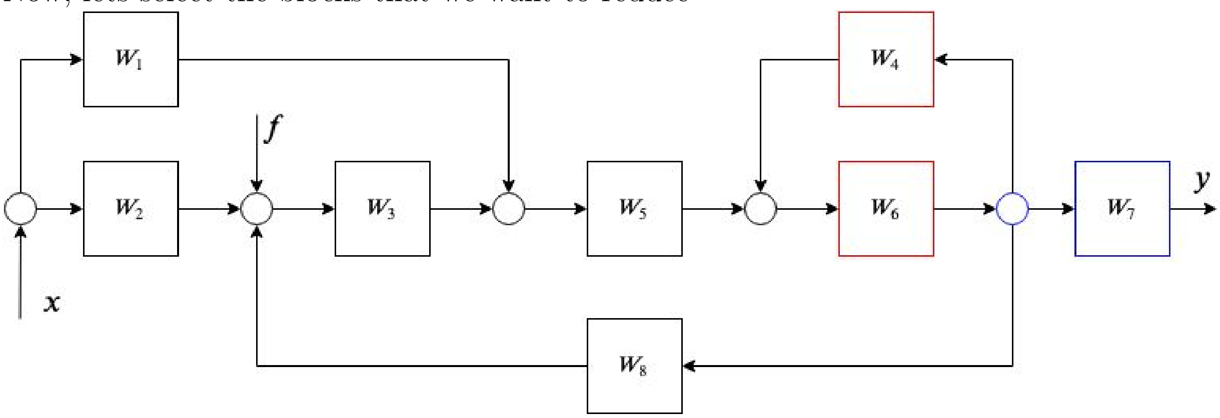
Problem 6

Simplify system step by step

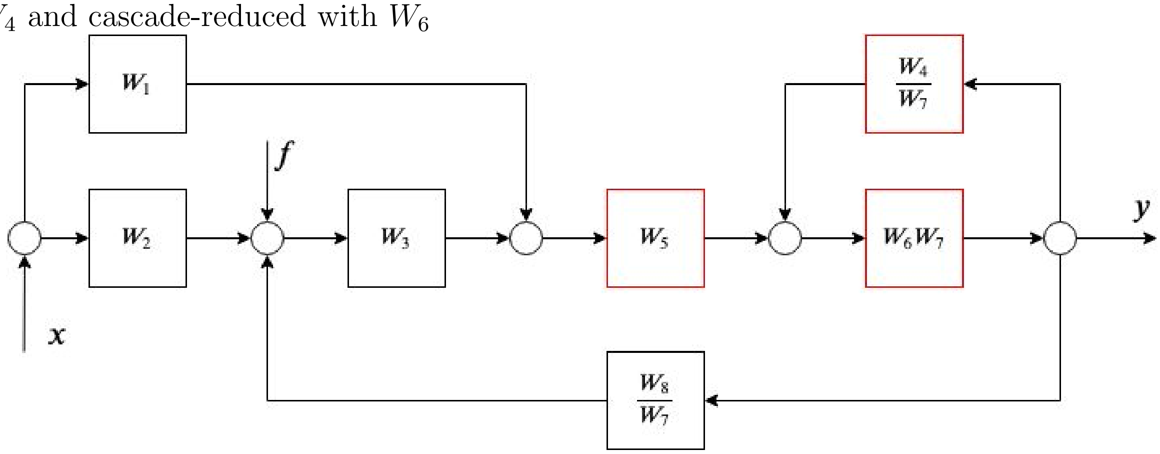
The task requires to to reduce multi input block diagram and calculate total transfer function. Here is the link to draw.io scheme block-reduction-scheme



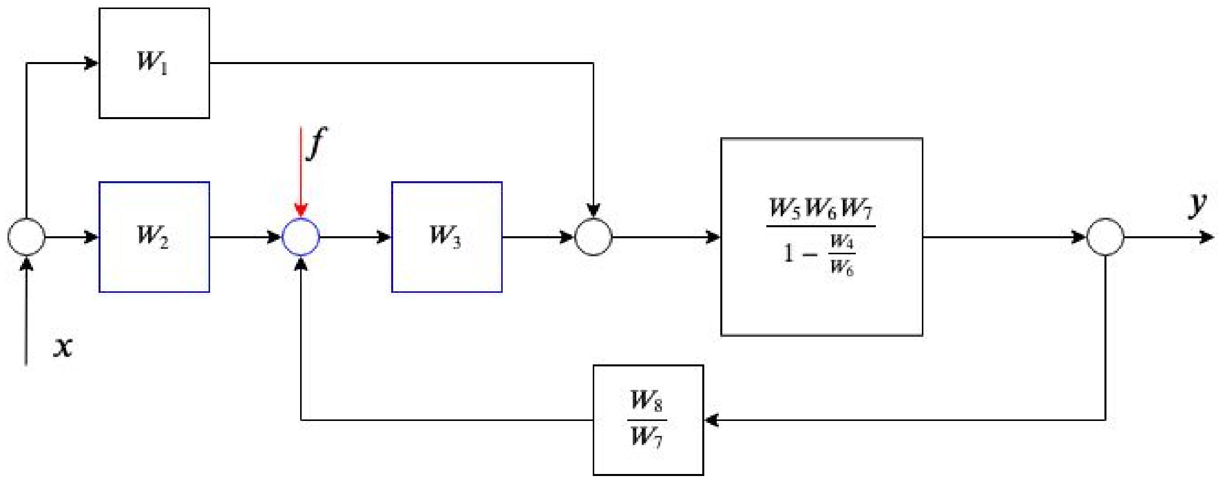
Now, lets select the blocks that we want to reduce



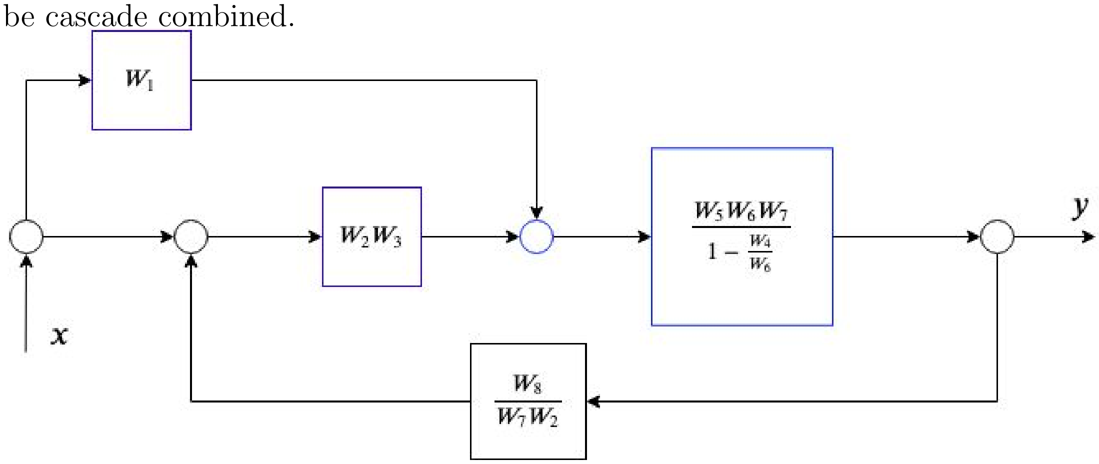
Here, two blue blocks from previous picture are transferred to before sum block, then divided W_4 and cascade-reduced with W_6



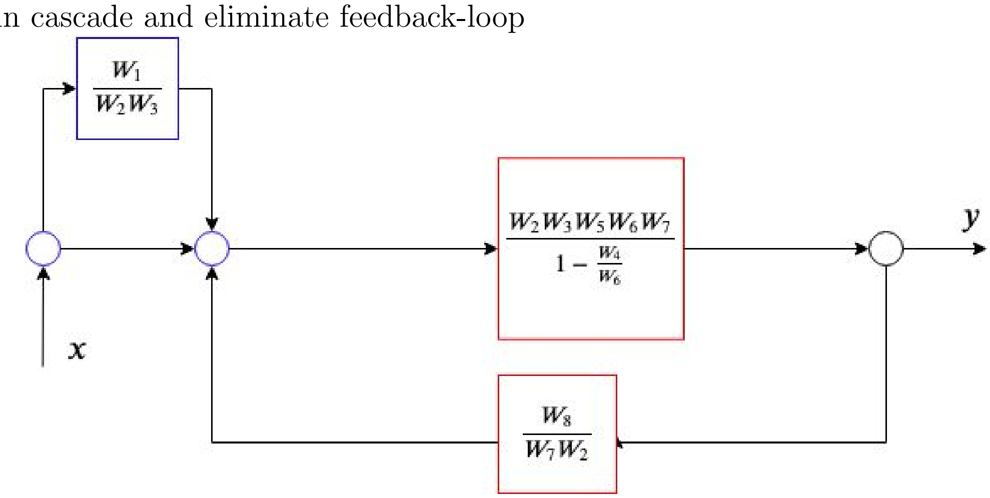
Now, we remove feedback-loop and multiply result with W_5



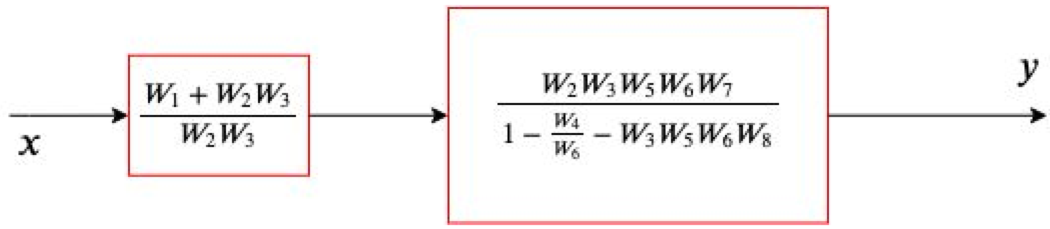
It is no more possible to reduce blocks, so we can substitute $f = 0$, and W_2 and W_3 blocks can be cascade combined.



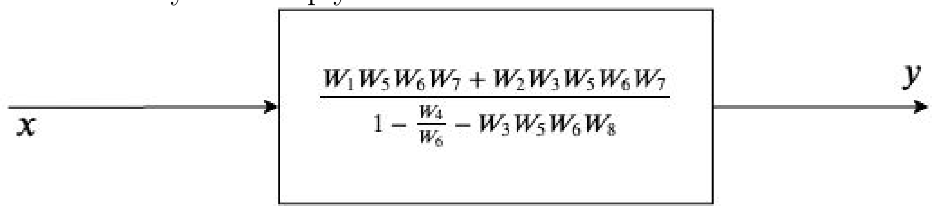
Now, there is sum block that hinders to eliminate feedback-loop, so we move sum block before W_2W_3 and combine with other sum block. After that it will be possible to combine blocks in cascade and eliminate feedback-loop



Now we can simplify sum block(blue) and feedback-loop(red)

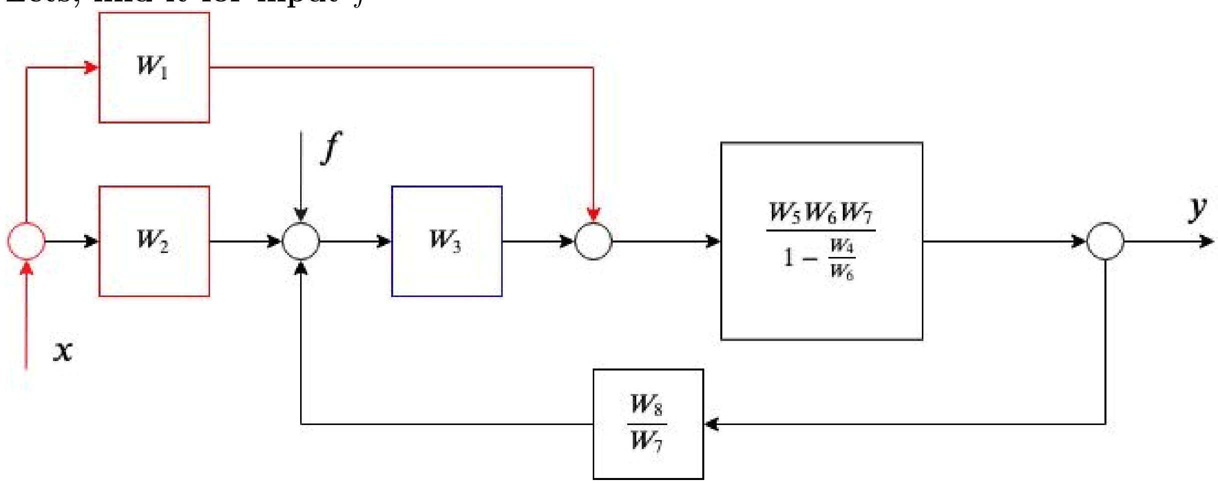


In this step we have only to multiply to transfer function

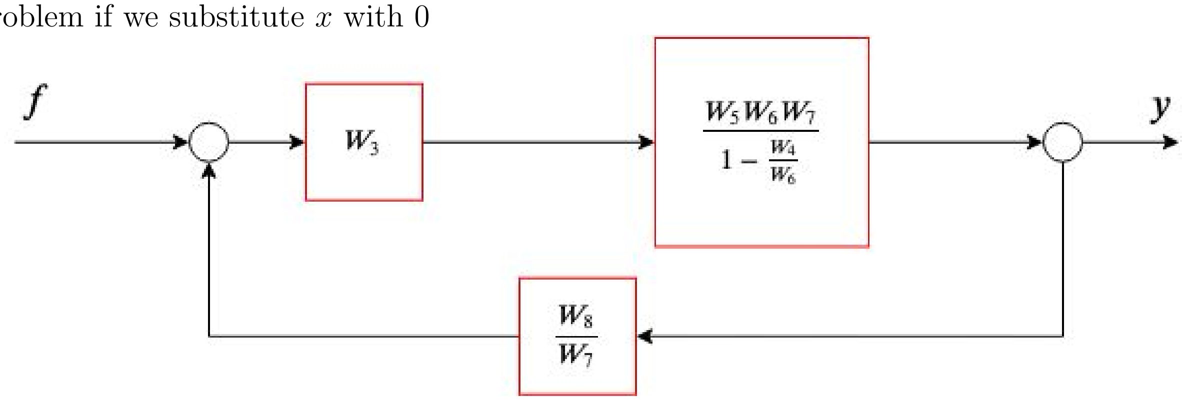


Now, we have transfer function Φ_x for input x,

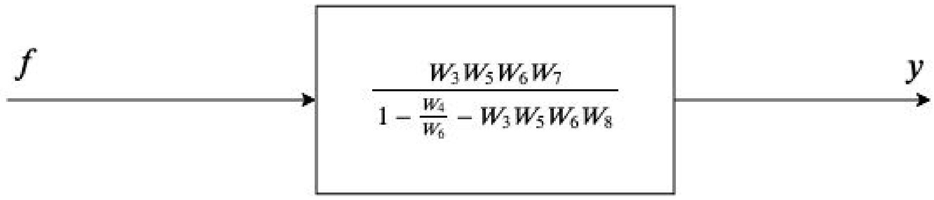
Lets, find it for input f



Back to simplified scheme with existing input f. Here we can remove red blocks without problem if we substitute x with 0



Now, we multiply W_3 with the block next to it, and eliminate feedback-loop and get Φ_f



After all, we have parallel transfer functions Φ_x and Φ_f.

$$\Phi = \begin{bmatrix} \Phi_x & \Phi_f \end{bmatrix} \tag{9}$$

Now, output is equal to transfer function multiplied to inputs

$$Y = \Phi \begin{bmatrix} X \\ F \end{bmatrix} \tag{10}$$

or

$$Y = \begin{bmatrix} \Phi_x & \Phi_f \end{bmatrix} \begin{bmatrix} X \\ F \end{bmatrix} \tag{11}$$

Finally, we have output equal to transfer function multiplied by inputs:

$$Y = \begin{bmatrix} \frac{W_1W_5W_6W_7+W_2W_3W_5W_6W_7}{1-\frac{W_4}{W_6}-W_3W_5W_6W_8} & \frac{W_3W_5W_6W_7}{1-\frac{W_4}{W_6}-W_3W_5W_6W_8} \end{bmatrix} \begin{bmatrix} X \\ F \end{bmatrix} \tag{12}$$