Control Theory: HW2

Temurbek Hudzhaev

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Contents

1 Intro

d Temurbek Khujaev, t.xojayev@innopolis.university from G-2 and generated variant is: **B**

I did not make some tasks yet because of midterms and lots of deadlines, but I promise to finish homework PERFECTLY until friday so I ask you to not grade it yet

2 Transfer Functions.

$$\ddot{x} + 2\dot{x} - 3x = sin(4t)$$

A) Draw a schema in Simulink

1.png

Figure 1: Simulink Model

2.png

Figure 2: Plot

B) Draw a schema in Simulink (use transfer func block).

$$\frac{d}{dt} = p$$

$$p^2x + 2px - 3x = sin4t$$

$$(p^2 + 2p - 3)x = sin4t$$

$$(p^2 + 2p - 3)x = sin4t$$

$$x = \frac{1}{p^2 + 2p - 3} sin4t$$

$$W(p) = \frac{1}{p^2 + 2p - 3} sin4t$$

3.png

Figure 3: Simulink Model with Transfer Function

4.png

Figure 4: Plot

C) Solve diff equation with matlab function and draw a plot in matlab.

```
syms x(t)
Dx = diff(x);

ode = diff(x,t,2) == -2 * Dx + 3 * x + sin(4 * t);
cond1 = Dx(0) == 3;
cond2 = x(0) == 2;

conds = [cond1 cond2];
xSol(t) = dsolve(ode, conds);
xSol = simplify(xSol);

var = 0:0.05:15;
plot(var,xSol(var));
```

5.png

Figure 5: Plot

D) Solve diff equation with Laplace transform in matlab.

```
syms t s X

RHS = laplace(sin(4*t));

X1 = s * X - 2;
X2 = s * X1 - 3;

sols = solve(X2 + 2 * X1 - 3 * X - RHS, X);
solt = ilaplace(sols,s,t);


pretty(solt)

fplot(solt,[0,8])
grid on
```

6.png

Figure 6: Plot

3 Find State Space Model of the system.

$$\begin{cases} \ddot{x} = t + 3 \\ y = x + 2\dot{x} \end{cases} \tag{1}$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (t + 3) \tag{2}$$

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \tag{3}$$

4 Transfer function of the system.

The variant provides matrices:

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 3 & 0 \end{bmatrix} \tag{4}$$

According to lecture slides we derived formula to convert SS to TF :

$$W(p) = C(Ip - A)^{-1}B$$

After substitutions of matrices we have:

$$C(Ip - A)^{-1} = \begin{bmatrix} p-1 & 1 \\ -2 & p-1 \end{bmatrix}^{-1} = \frac{1}{(p-1)^2 + 2} \begin{bmatrix} 3p-3 & -3 \end{bmatrix} \tag{5}$$

$$C(Ip - A)^{-1}B = \frac{1}{(p-1)^2 + 2} \begin{bmatrix} 3p-3 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{6p-9}{p^2 - 2p + 3} \tag{6}$$

Finally, we have transfer function

$$W(p) = \frac{6p-9}{p^2 - 2p + 3}$$

5 Find the transfer function of the system

This task is quite similar to previous one only with one difference of nonzero D matrix.

The variant provides matrices:

$$A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} -1 & 4 \end{bmatrix}, D = \begin{bmatrix} 2 & 1 \end{bmatrix} \tag{7}$$

According to lecture slides we can calculate transfer function of SS with following formula:

$$W(p) = C(Ip - A)^{-1}B + D$$

After substitutions:

$$C(Ip - A)^{-1} = \begin{bmatrix} -1 & 4 \end{bmatrix} \begin{bmatrix} p-1 & 2 \\ -2 & p+1 \end{bmatrix}^{-1} = \frac{1}{p^2 + 3} \begin{bmatrix} 7-p & 4p-2 \end{bmatrix} \tag{8}$$

$$C(Ip - A)^{-1}B = \frac{1}{p^2 + 3} \begin{bmatrix} 7-p & 4p-2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 2 & 0 \end{bmatrix} = \frac{1}{p^2 + 3} \begin{bmatrix} 2(3p+5) & -3(p-7) \end{bmatrix} \tag{9}$$

And we add D:

$$C(Ip-A)^{-1}B+D=\begin{bmatrix}\frac{2(3p+5)}{p^2+3} & \frac{-3(p-7)}{p^2+3}\end{bmatrix}+\begin{bmatrix}2 & 1\end{bmatrix}=\begin{bmatrix}\frac{2(3p+5)}{p^2+3} & \frac{-3(p-7)}{p^2+3}\end{bmatrix} \tag{10}$$

Finally, we have transfer function

$$W(p)=\begin{bmatrix}\frac{2(3p+5)}{p^2+3} & \frac{-3(p-7)}{p^2+3}\end{bmatrix}$$

7.png

Figure 7: Plot of ODE and SS

We have eigenvalues $1 + 0 * i$ and $-3 + 0 * i$ which and because of eigenvalue with positive real part we can conclude that system is **not stable**. As the solution is exponential with the eigenvalues then the system will **diverge** as the power is positive