Homework #5

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Conthrol Theory. Group 2
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April 2020

Intro and overview

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Generated variant is: **G**
(A)prove that it is possible to design state observer of the linearizedsystem
Ok we have the following system which is derived from previous assignment.

$$\delta\dot{z} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & mg/M & 0 & 0 \\ 0 & (M+m)g/M & 0 & 0 \end{bmatrix} \delta z + \begin{bmatrix} 0 \\ 0 \\ 1/M \\ 1/(Ml) \end{bmatrix} \delta u \tag{1}$$

$$\delta y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \delta z \tag{2}$$

By assigning $M = 11.6$, $m = 2.7$, $l = 0.57$ and $g = 9.81$ according to provided variant we will derive the following:

$$\delta\dot{z} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2.28 & 0 & 0 \\ 0 & 12.09 & 0 & 0 \end{bmatrix} \delta z + \begin{bmatrix} 0 \\ 0 \\ 0.08 \\ 0.15 \end{bmatrix} \delta u \tag{3}$$

$$\delta y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \delta z \tag{4}$$

Lets substitute x for δz and y for δy and u for δu to make it simple And we get **state estimate:**

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) = A\hat{x} + Bu + LC(x - \hat{x})$$

output estimate:

$$\hat{y} = C\hat{x}$$

state error:

$$\dot{e_x} = \dot{x} - \dot{\hat{x}} = (A - LC)e_x$$

output error:

$$e_y = y - \hat{y} = Ce_x$$

We know that system is observable if matrix $[C, CA, ...CA^{n-1}]^T$ is full rank(n).

$$O_m = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ ... & ... & ... & ... \\ ... & ... & ... & ... \end{bmatrix} \tag{5}$$

The matrix started with 4 independent columns which already says that it is rank is 4, so we can conclude that system is **observable**

(B) for open loop state observer, is the error dynamics stable?

It has been provided that the observer canonical and controller canonical forms are duals:

$$A_{observer} = A^T_{controller}$$
$$B_{observer} = C^T_{controller}$$
$$C_{observer} = B^T_{controller}$$

And as A has eigenvalues $\lambda_1 = \frac{\sqrt{1209}}{10}$, $\lambda_2 = -\frac{\sqrt{1209}}{10}$ and $\lambda_{3,4} = \frac{\sqrt{1209}}{10}$ we can conclude that the dynamicare are **not stable**

(C) design Luenberger observer for linearized system using both pole placement and LQR methods

If we choose matrix L so that A - LC have only negative eigenvalues then state error converges.

By transposing A and C and using "place" function from matlab we get our L matrix

$$L = \begin{bmatrix} 7 & 0 \\ 0 & 3 \\ 12 & 1.96 \\ 0 & 13.8 \end{bmatrix} \tag{6}$$

$$A - LC = \begin{bmatrix} -7 & 0 & 1 & 0 \\ 0 & -3 & 0 & 1 \\ -12 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix} \tag{7}$$

$$\dot{\hat{x}} = \begin{bmatrix} -7 & 0 & 1 & 0 \\ 0 & -3 & 0 & 1 \\ -12 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ 0 \\ 0.08 \\ 0.15 \end{bmatrix} u + \begin{bmatrix} 7 & 0 \\ 0 & 3 \\ 12 & 1.96 \\ 0 & 13.8 \end{bmatrix} (y - \hat{y}) \tag{8}$$

As task asks the L matrix can be obtaine through the LQR method insted of pole placement using MATLABs lqr function with A^T and C^T as parameters:

$$L = \begin{bmatrix} 1.89 & 0.80 \\ 0.80 & 5.10 \\ 1.60 & 3.47 \\ 2.10 & 12.81 \end{bmatrix} \tag{9}$$

$$A - LC = \begin{bmatrix} -1.89 & -0.80 & 1 & 0 \\ -0.80 & -5.10 & 0 & 1 \\ -1.60 & -1.51 & 0 & 0 \\ -2.10 & -6.27 & 0 & 0 \end{bmatrix} \tag{10}$$

$$\dot{\hat{x}} = \begin{bmatrix} -1.89 & -0.80 & 1 & 0 \\ -0.80 & -5.10 & 0 & 1 \\ -1.60 & -1.51 & 0 & 0 \\ -2.10 & -6.27 & 0 & 0 \end{bmatrix} \tag{11}$$

$$\hat{x} + \begin{bmatrix} 0 \\ 0 \\ 0.08 \\ 0.15 \end{bmatrix} u + \begin{bmatrix} 7 & 0 \\ 0 & 3 \\ 12 & 1.96 \\ 0 & 13.8 \end{bmatrix} (y - \hat{y}) \tag{12}$$

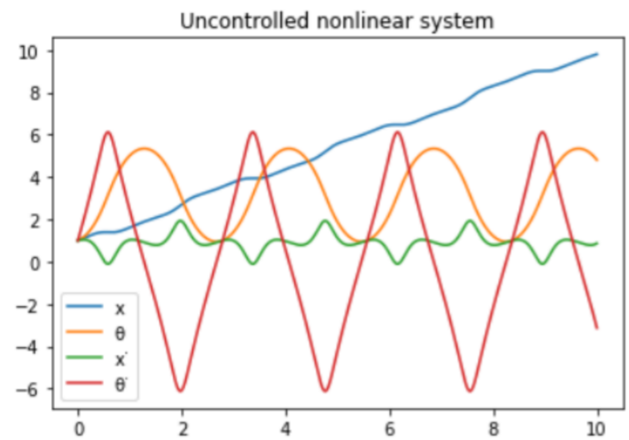
(D) design state feedback controller for linearized system
State feedback controller is defined as $u = -Kx$, we can find K to be [-12.23 246.09 -25.48 75.48]

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2.28 & 0 & 0 \\ 0 & 12.09 & 0 & 0 \end{bmatrix} x - \begin{bmatrix} 0 \\ 0 \\ 0.08 \\ 0.15 \end{bmatrix} \begin{bmatrix} -12.23 & 246.09 & -25.48 & 75.48 \end{bmatrix} x \tag{13}$$

This is **stable** because its all eigenvalues have negative real part
simulate nonlinear system with Luenberger observer

$$\dot{z} = \begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{\theta} \\ \frac{2.7(g\cos(\theta) - 0.57\theta^2)\sin(\theta)}{11.6 + 2.7\sin^2(\theta)} \\ \frac{(14.1g - 1.5\dot{\theta}^2\cos\theta)\sin(\theta)}{0.57(11.6 + 2.7\sin^2(\theta)} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{11.6 + 2.7\sin^2(\theta)} \\ \frac{\cos(\theta)}{0.57(11.6 + 2.7\sin^2(\theta)} \end{bmatrix} u \tag{14}$$

First, to see how that system looks like, it was simulated with a step function for u from 0 to 1 starting at initial conditions 1,1,1,1:

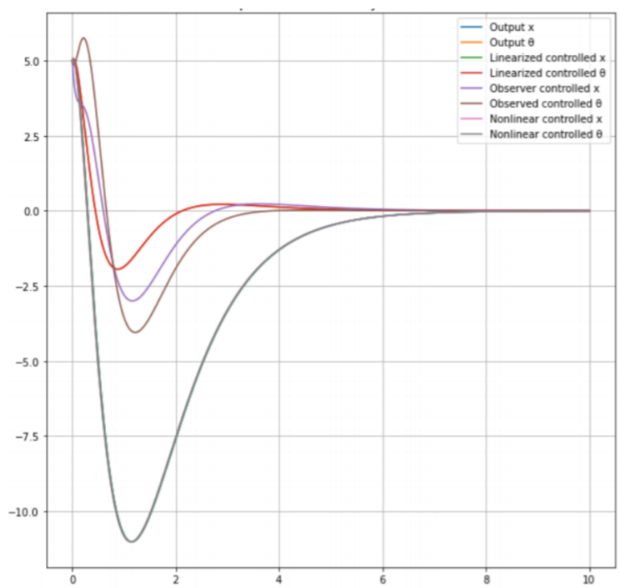


It is clearly unstable. To control it, we use the Luenberger observer to observe the linearized version of the system and use the same controller on the original nonlinear system.
Let's pick the Luenberger's observer designed using pole placement and plugin $y = Cx$:

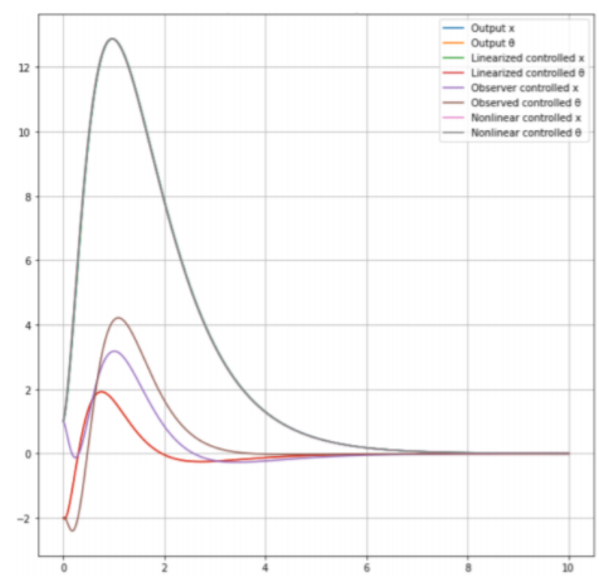
$$\dot{\hat{x}} = \begin{bmatrix} -7 & 0 & 1 & 0 \\ 0 & -3 & 0 & 1 \\ -12 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ 0 \\ 0.08 \\ 0.15 \end{bmatrix} u + \begin{bmatrix} 7 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 12 & 1.96 & 0 & 0 \\ 0 & 13.8 & 0 & 0 \end{bmatrix} (x - \hat{x}) \tag{15}$$

To apply the state feedback controller, we can use the matrices obtained in the previous step to define u = [12.23 246.09 25.48 75.48] $\hat{x}$. We also use the state feedback controlled linearized system in the last step to obtain the state x and then we can get $\hat{x}$ and compare it to the original nonlinear system, using the same u as its control to show that the estimation should approach the original nonlinear system. In the end, we can see that they all approach 0.

For initial conditions [5 5 5 5]:

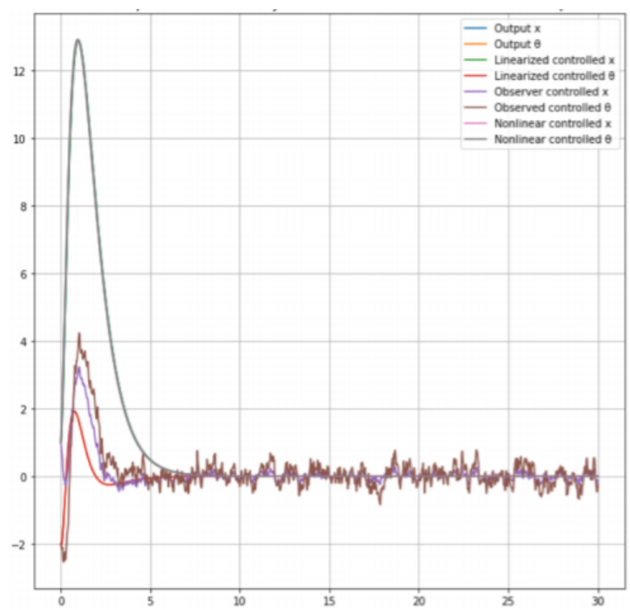


For initial conditions [1 -2 5 -3]:



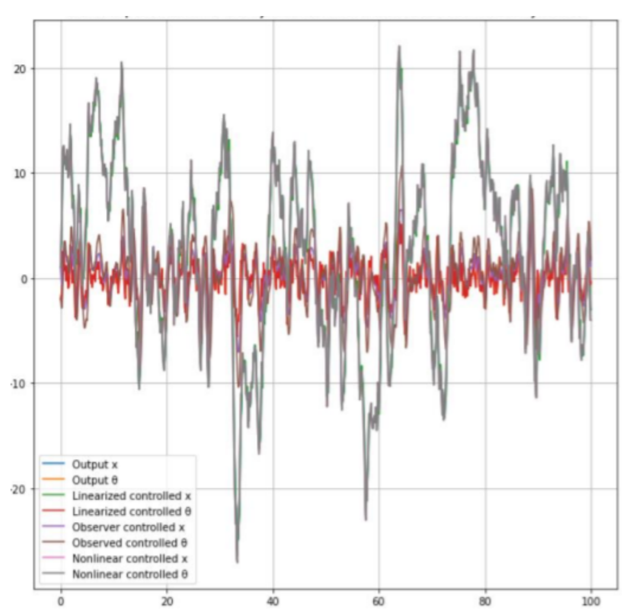
(F) add white gaussian noise to the output

The noise affects the speed at which the system approaches stability, but it doesn't affect the fact it eventually reaches it. Additionally, since the noise is added to the output, it affects only the observer and not the regular linearized system nor the nonlinear system that are not affected by the system's output.



(G) add white gaussian noise to the output

Since the noise is added to the linearized system dynamics, it affects the controller as it uses the state of that linearized system. Therefore, the errors build up and the system keeps oscillating, seemingly to never reach stability.



(J) using KF function implement LQG controller

To implement a Linear-Quadratic-Gaussian controller, we need to design an observer with the following dynamics:

$$\dot{\hat{x}} = A(t)\hat{x}(t) + B(t)u(t) + L(t)(y(t) - C(t)\hat{x}(t)))$$

, where $L(t)$ is the Kalman gain of the associated Kalman filter computed using $A(t), C(t)$ the noise intensity matrices $V(t)$ and $W(t)$ and $E[x(0)x^T(0)]$ through Ricatti differential equation:

$$\dot{P(t)} = A(t)P(t) + P(t)A^T(t) - P(t)C^T(t)W^{-1}(t)C(t)P(t) + V(t)$$
$$P(0) = E[x(0)x^T(0)]$$
$$L(t) = P(t)C^T(t)W^{-1}(t)$$

The controller can then be chosen as $u(t) - K(t)\hat{x}(t)$ where $K(t)$ is feedback gain determined using

$$-\dot{S}(t) = A^T(t)S(t) + S(t)A(t) - S(t)B(t)R^{-1}(t)B^T(t)S(t) + Q(t)$$
$$S(t) = F$$
$$K(t) = R^{-1}(t)B^T(t)S(t)$$