Homework #2

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Intro and overview

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Generated variant is: **B**

Problem 2

(A) Find total transfer function
Lets, define Z as intermediate signal. Then,

$$Y(s) = Z(s)W_2W_3$$

$$Z(s) = X(s)W_1 - W_4\frac{1}{W_3}Y(s)$$

Now, we substitute plug right-hand-side of Z to Y

$$Y(s) = W_2W_3\big[W_1X(s) - W_4\frac{1}{W_3}Y(s)\big]$$

$$Y(s) = W_1W_2W_3X(s) - W_2W_4Y(s)$$
$$Y(s)(1 + W_2W_4) = W_1W_2W_3X(s)$$

Divide, both side to $X(s)$

$$\frac{Y(s)}{X(s)} = \frac{W_1W_2W_3}{1 + W_2W_4}$$

Now, we assign given values

$$W_1 = \frac{s+3}{s^2-1}, W_2 = \frac{1}{s}, W_3 = \frac{2s+1}{s+1.5}, W_4 = \frac{1}{s+0.4}$$

Which gives final result:

$$\frac{Y(s)}{X(s)} = \frac{(s+3)(2s+1)(s+0.4)}{(s^2-1)(s+1.5)(s^2+0.4s+1)}$$

Problem 4

Find transfer function of the system
The variant provides SS matrices:

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 3 & 0 \end{bmatrix} \tag{1}$$

According to lecture slides we derived formula to convert SS to TF :

$$W(p) = C(Ip - A)^{-1}B$$

After substitutions of matrices we have:

$$C(Ip - A)^{-1} = \begin{bmatrix} p-1 & 1 \\ -2 & p-1 \end{bmatrix}^{-1} = \frac{1}{(p-1)^2 + 2} \begin{bmatrix} 3p-3 & -3 \end{bmatrix} \tag{2}$$

$$C(Ip - A)^{-1}B = \frac{1}{(p-1)^2 + 2} \begin{bmatrix} 3p-3 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \frac{6p-9}{p^2 - 2p + 3} \tag{3}$$

Finally, we have transfer function

$$W(p) = \frac{6p-9}{p^2 - 2p + 3}$$

Problem 5

Find transfer functions of the system.
This task is quite similar to previous one only with one difference of nonzero D matrix.
The variant provides matrices:

$$A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}, C = \begin{bmatrix} -1 & 4 \end{bmatrix}, D = \begin{bmatrix} 2 & 1 \end{bmatrix} \tag{4}$$

According to lecture slides we can calculate transfer function of SS with following formula:

$$W(p) = C(Ip - A)^{-1}B + D$$

After substitutions:

$$C(Ip - A)^{-1} = \begin{bmatrix} -1 & 4 \end{bmatrix} \begin{bmatrix} p-1 & 2 \\ -2 & p+1 \end{bmatrix}^{-1} = \frac{1}{p^2 + 3} \begin{bmatrix} 7-p & 4p-2 \end{bmatrix} \tag{5}$$

$$C(Ip - A)^{-1}B = \frac{1}{p^2 + 3} \begin{bmatrix} 7-p & 4p-2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 2 & 0 \end{bmatrix} = \frac{1}{p^2 + 3} \begin{bmatrix} 2(3p+5) & -3(p-7) \end{bmatrix} \tag{6}$$

And we add D:

$$C(Ip - A)^{-1}B + D = \begin{bmatrix} \frac{2(3p+5)}{p^2+3} & \frac{-3(p-7)}{p^2+3} \end{bmatrix} + \begin{bmatrix} 2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2(3p+5)}{p^2+3} & \frac{-3(p-7)}{p^2+3} \end{bmatrix} \tag{7}$$

Finally, we have transfer function

$$W(p) = \begin{bmatrix} \frac{2(3p+5)}{p^2+3} & \frac{-3(p-7)}{p^2+3} \end{bmatrix}$$