EE402 Mini Project 6

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1. In this problem, we will investigate the matrix exponential. Consider $A, A_i, Pin\mathbb{R}^{nn}$

   (a) Show that when $A_1 A_2 = A_2 A_1$
   $$e^{A_1 t} e^{A_2 t} = e^{(A_1 + A_2)t}$$

   (b) Show that when $\det(P) \neq 0$
   $$e^{\left(P^{-1}AP\right)t} = P^{-1} e^{At} P$$

   (c) Show that when $\det(A) \neq 0$
   $$\left(\int_0^T e^{A\lambda} d\lambda \right) = A^{-1}\left(e^{AT} - I\right) = \left(e^{AT} - I\right) A^{-1}$$

   (d) Given that $(\lambda \ , \ \nu)$ is an eigenvalue and eigenvector pair of A. Based on this information, derive the associated eigenvalue and eigenvector pair of e^{At}.
   You are supposed to derive the result, thus don't just type the answer.

   (e) Compute e^{At} for the following matrix
   $$A = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$$
   Hint: Your solution should be in terms of sinusoidal and exponential functions of ωt and σt.

   (f) Compute e^{At} for the following matrix
   $$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
   without using the Laplace transform domain solution method.

   (g) Does there exist a A matrix such that
   $$e^{At} = \begin{bmatrix} e^{t} & e^{t} \\ 0 & e^{-t} \end{bmatrix}$$

2. Consider the following CT state-space representation

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

(a) Based on the procedures detailed in the lecture notes, discretize this state-space formulation under ZOH operation at the input and uniform ideal sampling at the states and compute the DT state-space representation and associated matrixes.

$$x[k+1] = Gx[k] + Hu[k]$$

T should exist symbolically in your matrices.

(b) Now approximate e^{AT} using the first order approximation given below

$$e^{AT} \approx I + AT$$

and using this approximation compute the approximated discretie time state-space equation

$$x[k+1] \approx \tilde{G}x[k] + \tilde{H}u[k]$$

(c) Compute (G, H) and $(\tilde{G}, \tilde{H})$ for different values of T, compre the results, and comment on them.

3. Stability of CT and DT dynamical systems

(a) Consider the DT system

$$x[k+1] = \begin{bmatrix} 0 & 1 \\ \alpha/2 & \alpha - 1/2 \end{bmatrix} x[k] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u[k]$$
$$y[k] = \begin{bmatrix} -2 & 1 \end{bmatrix} u[k]$$

i. For what values of parameter α is the system asymptotically stable?

ii. For what values of parameter α is the system BIBO stable?

(b) Consider the CT system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ \alpha/2 & \alpha - 1/2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} -2 & 1 \end{bmatrix} u(t)$$

i. For what values of parameter α is the system asymptotically stable?

ii. For what values of parameter α is the system BIBO stable?