EE402 Mini Project 7

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1. (10 Points) *Modal reachability test* states that A DT system represented in state space form with G is the system matrix and H is the input matrix is un-reachable if $w^T H = 0$ for a left eigenvector of G. A left eigenvalue, eigenvector pair of G is defined as

$$w^T G = \lambda w^T \quad , \quad w \in \mathbb{R}^n$$

In this context, show that if $w^T H = 0$ for a left eigenvector of G, then the system is un-reachable.

2. (10 Points) *Modal observability test* states that A DT system represented in state space form with G is the system matrix and C is the output matrix is un-observable if $Cv = 0$ for a (right) eigenvector of G. A right eigenvalue, eigenvector pair of G is defined as

$$Gv = \lambda v \quad , \quad v \in \mathbb{R}^n$$

In this context, show that if $Cv = 0$ for a left eigenvector of G, then the system is un-observable.

3. (10 Points) Given a SISO DT dynamical system

$$x[k+1] = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} x[k] + Hu[k]$$
$$y[k] = Cx[k]$$

   (a) If possible find an input vector $H \in \mathbb{R}^n$, such that the system is fully reachable. If it is not possible to find such an H, then show it why?

   (b) If possible find an output row-vector $C \in \mathbb{R}^{1\times n}$, such that the system is fully observable. If it is not possible to find such an C, then show it why?

4. (25 Points) Consider the discrete-time plant given by

$$x[k+1] = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} x[k] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u[k]$$
$$y[k] = \begin{bmatrix} 1 & 1 \end{bmatrix} x[k]$$

(a) Assume that we "close the loop" around the plant using constant output feedback control law

$$u[k] = \alpha y[k]$$

Show that the closed-loop system can not be stabilized regardless of the choice of α.

The moral here is that the system cannot be stabilized using static output feedback gain.

(b) Now we will consider a time-dependent output feedback control policy. The control policy is given by

$$u[k] = \alpha[k] y[k] \quad \text{where}$$
$$\alpha[k] = \begin{cases} -1 & k \text{ is even} \\ 3 & k \text{ is odd} \end{cases}$$

Show that the state trajectories corresponding to any initial condition return to the origin in at most 4 time steps.

5. (25 Points) Consider the following DT state evolution equation

$$x[k+1] = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix} x[k] + \begin{bmatrix} 1 - \sqrt{3}/2 \\ 1/2 \end{bmatrix} u[k]$$

(a) Design a state-feedback controller, $u[k] = -K^* x[k]$ such that the closed-loop behavior shows a dead-beat response. Then, use Matlab to compute and plot the response of each of the state variables from $k = 0$ to $k = 10$ assuming $x[0] = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$.

(b) Now suppose that there is an inevitable delay such that the control action effects the system after one sample delay

$$u[k] = -K^* x[k-1]$$

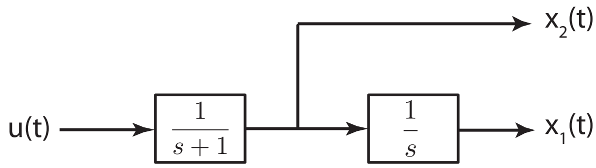
i. Find a state-space model for the closed-loop system in this case (*Hint: controller now has a memory*). Use the same gain you computed in previous part.

ii. Compute the eigenvalues of this new closed-loop system.

iii. Determine if this new system is Reachable or not.

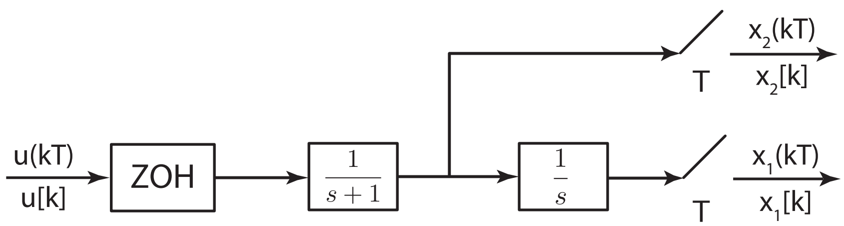
iv. Agin, use Matlab to compute and plot the response of each of the state variables (for this case) from $k = 0$ to $k = 10$ assuming $x[0] = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$. Comment on the result.

6. (70 Points) In this problem you will going to analyze many different aspects of state-space design and analysis of DT control systems using the CT plant, which is illustrated below.



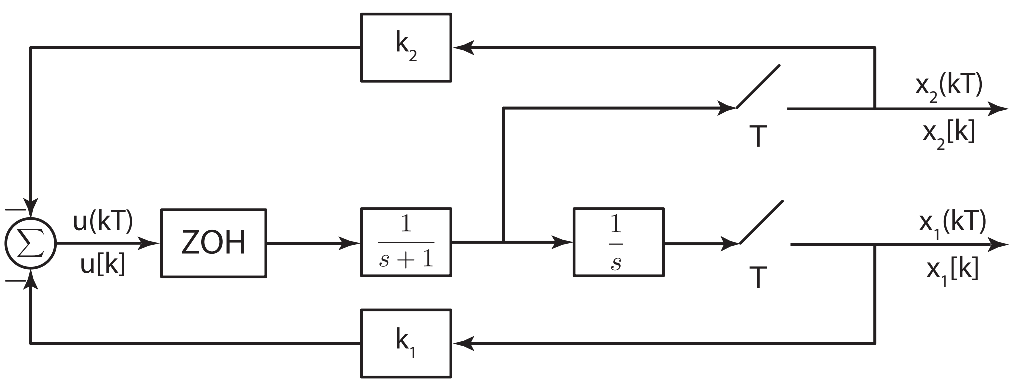
(a) Derive a state-space representation for the given CT LTI system. Use $\mathbf{x}(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$ as the state vector, and output equation is simply given as $\mathbf{y}(t) = \mathbf{x}(t)$ (i.e. $C = I$).

(b) In this part you will analyze the following digital (discretized) control system. Given that $T = 1\ s.$, derive a discrete-time state-space representation, where the input, state-vector, and output vector are defined as $u[k]$, $\mathbf{x}[k] = \begin{bmatrix} x_1[k] & x_2[k] \end{bmatrix}^T$, and $\mathbf{y}[k] = \mathbf{x}[k]$, respectively.



(c) In this part you will analyze the digital system that is controlled via a state-feedback control law, as illustrated in the Figure below. Choose a (k_1, k_2) pair such that closed-loop system rejects all initial conditions in finite-time (i.e. dead-beat behavior).

Then, simulate the closed-loop system in Simulink (or MATLAB), starting from different initial conditions. (*Hint: It is possible to assign initial conditions in Simulink's transfer function blocks*).



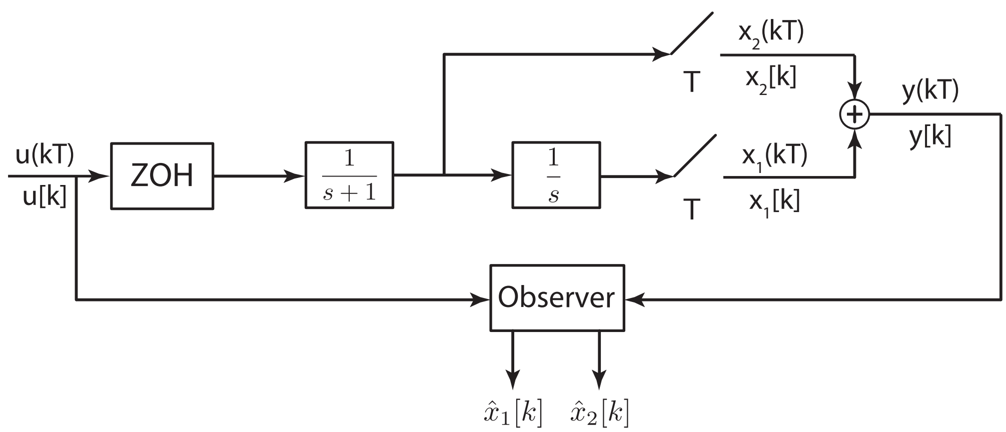
(d) In this part the goal is to design and implement a discrete-time observer for the same system. The block-diagram topology is given below. In this case, we don't have direct access to the system states, but we measure $y[k] = x_1[k] + x_2[k]$.

Design a dead-beat observer such that given the inputs $u[k]$ and $y[k]$, observer outputs the estimated states $\hat{\mathbf{x}}[k] = \begin{bmatrix} \hat{x}_1[k] & \hat{x}_2[k] \end{bmatrix}^T$.

Simulate the block-diagram topology given below in Simulink (or MATLAB) and plots the trajectories of the actual & estimated states, as well as the difference between actual and estimated variables.

In these simulations assume that $\begin{bmatrix} \hat{x}_1[k] & \hat{x}_2[k] \end{bmatrix}^T = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, however start the initial conditions of actual states from an arbitrary point. The input $u[k]$ of the plant is the unit-step input.

Hint: In Simulink you can implement a DT State-Space Model.



(e) Now, in this part you will combine the state-feedback controller and the observer that you had designed. The combined topology is provided in the Figure below.

Derive the state-space model of the combined system where the state-feedback controller uses the estimated states to produce the input. Compute the eigenvalues of this closed-loop system.

Simulate the closed-loop system in Simulink (or MATLAB) and provide the state-trajectories of actual states as well as estimated states.

For estimated states, assume that initial conditions are zero. For the actual states, start from an arbitrary initial condition.