EE402 Discrete Time Systems
MP-5

1. To easy the design process, the Tustin transform will be used throughout this mini-project.

```
%T=0.1
%z1 = (1+T*s)/(1-T*s)
%G_z_T= (0.047*z1+0.044)/(z1^2-1.8*z1+0.82) % Find Tustin Tr.
G_z_T= d2c(G_z,'tustin')
bode(G_z_T)

hold off
figure()
bode(G_z_T)
grid on

%% for PM 25-30
```

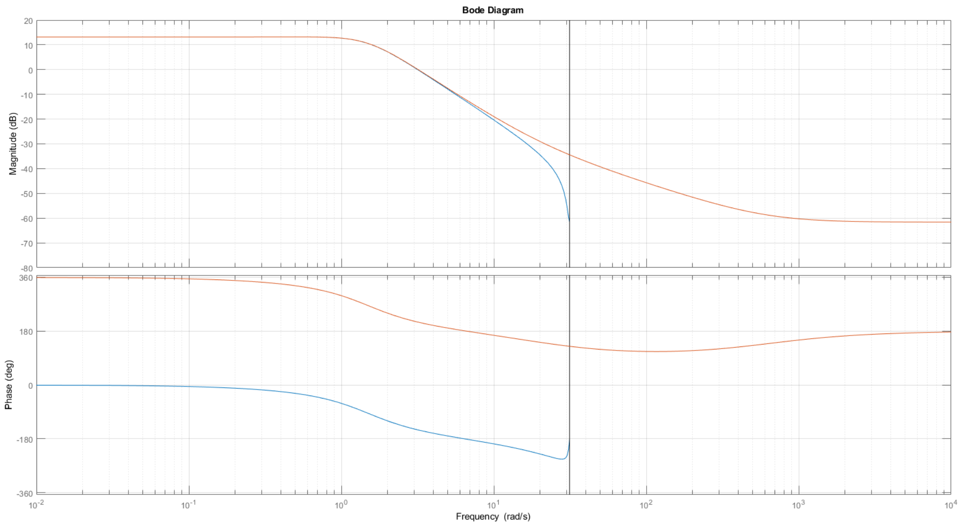

Figure 1: Bode Plot of Discrete TF and of its Tustin Transform

Let us now find the uncompensated phase margin for T=1.

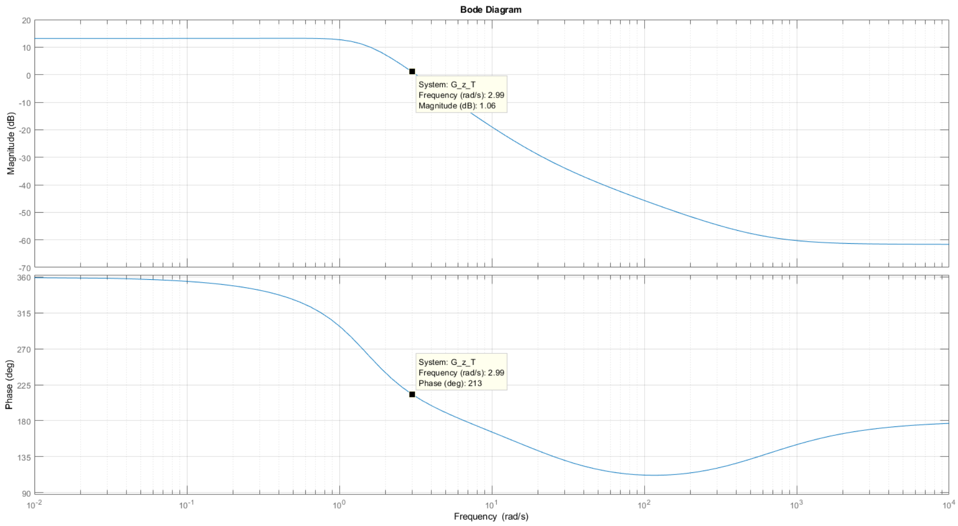

Figure 2: -

As can be seen from the *Figure 2* that the $PM \approx 29$.
for desired $PM = [10, 15]$

$$\Delta PM \approx [-19, -14]$$

For negative phase margin, let us use the following compensator type

$$G_C(s) = \frac{1}{K_{Lead}\dfrac{T_L as + 1}{Tl/as + 1}}$$

After trying couple of compensator bode plot for T=1 and varying a's. $a = 1.6$ seems like a good choice. The bode plot for a compensator with $a = 1.6$ can be seen at *Figure 3*

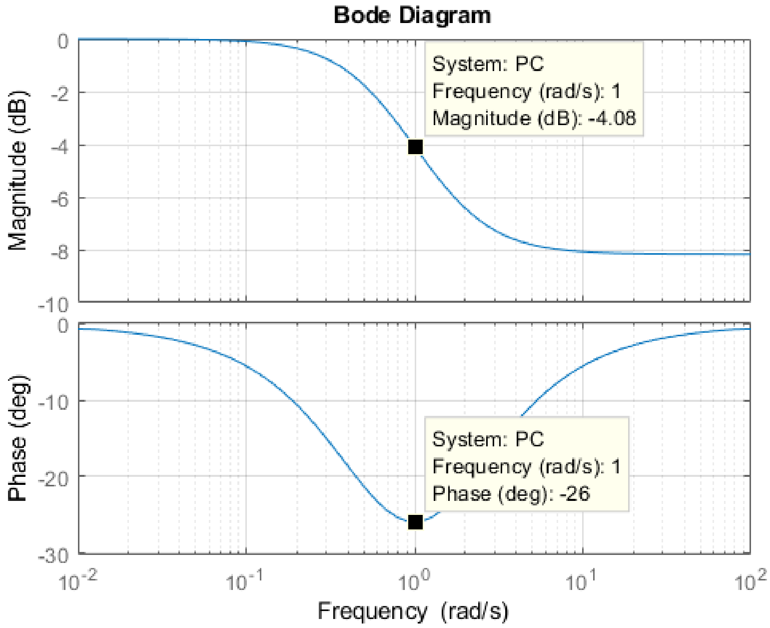

Figure 3: Bode plot for a compensator with $a = 1.6$

To find the proper T_L value, the bode plot of the uncompensated system was investigated. At a frequency of $w_x = 4\ rad/sec$, the system has a gain of

$$20log_{10}(1/a) = -4.08$$

Thus;

$$T_L = 1/w_x = 1/4 = 0.25$$

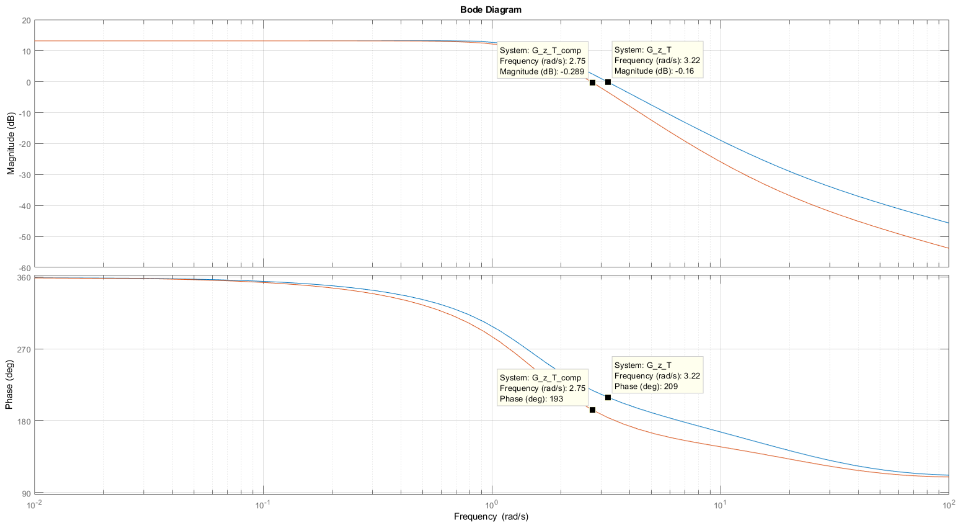

Figure 4: Bode Plots of compensated and uncompensated system

From the bode plots at *Figure 3*, it can be observed that th new compensated system has a phase margin of 13 degree which is desired region.

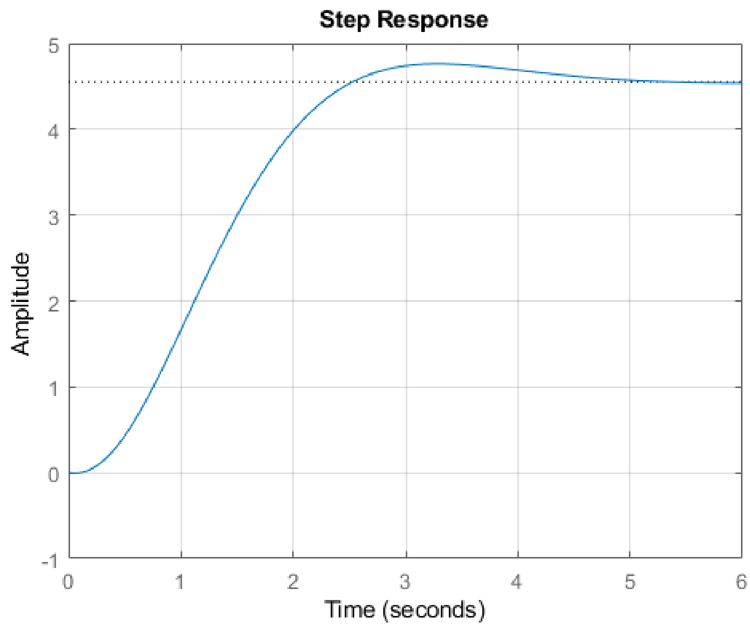

Figure 5: Step Response of the compensated system

Thus, the compensator can be designed to be as

$$G_C(s) = \frac{1}{K_{Lead}\frac{T_L a s + 1}{Tl/as + 1}}$$
$$K_{Lead} = 1$$
$$TL = 0.25$$
$$a = 1.6$$

2. for $PM = [25, 30]$

$$\Delta PM \approx [-4, 1]$$

Following the similar steps, choosing $a = 1.1$ seems good,

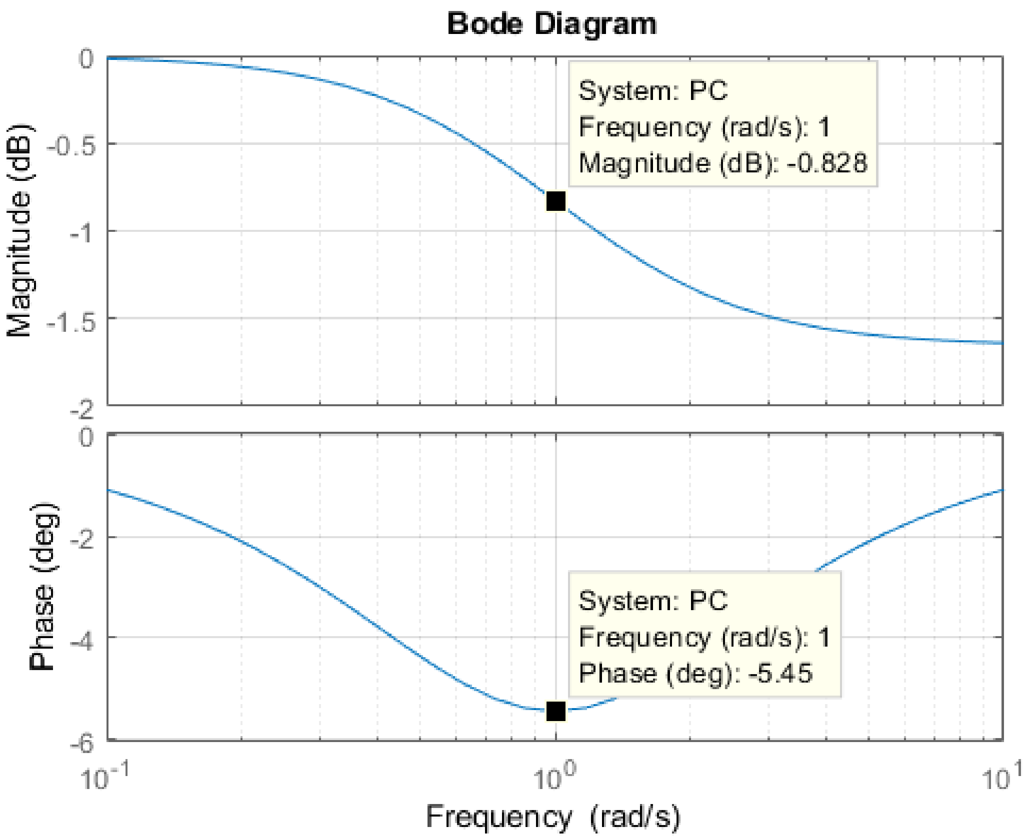

Figure 6: Bode plot for a compensator with $a = 1.1$

$$T_L = 1/2.82. = 0.35$$

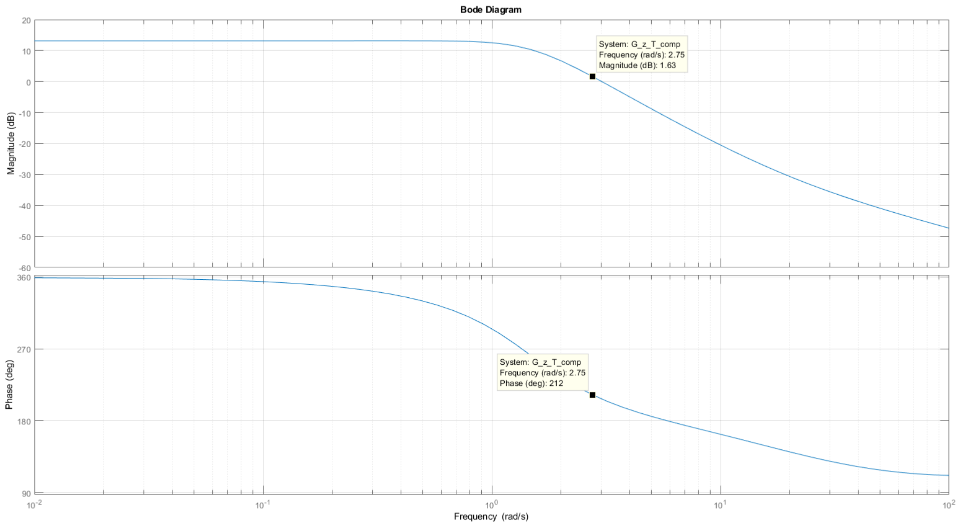

Figure 7: Bode Plots of compensated system

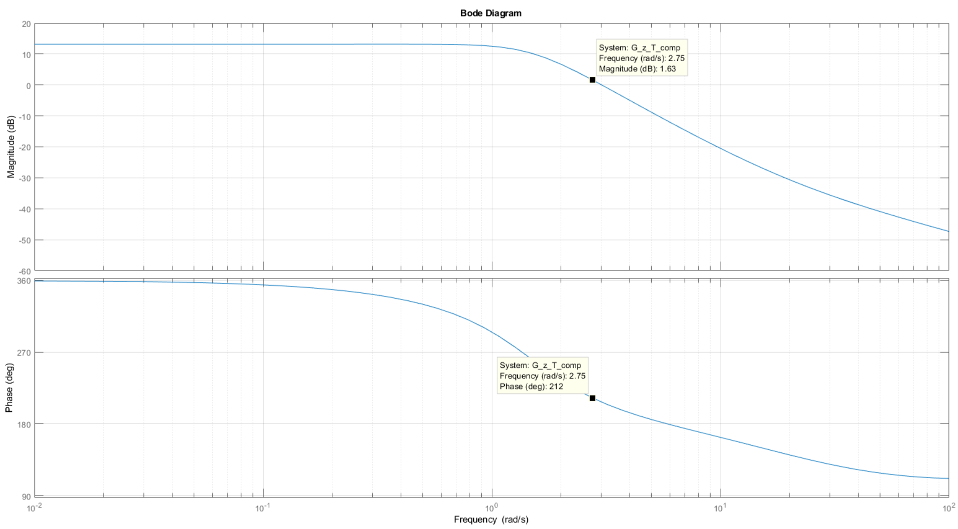

Figure 8: Bode Plots of compensated and uncompensated system

From *Figures 7 and 8*, the new phase margin for compansated system can be approximately found to be as 28 degree degree.

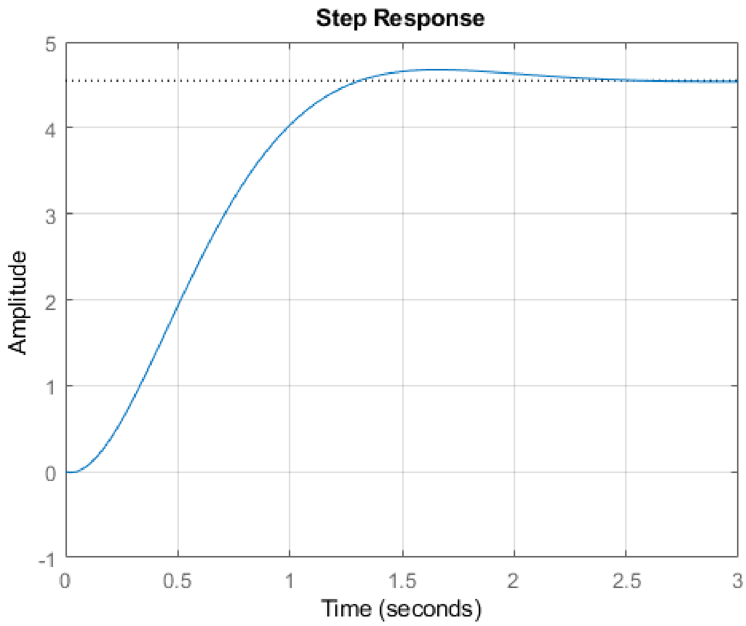

Figure 9: Step Response of the compensated system

Thus, the compensator can be designed to be as

$$G_C(s) = 1\frac{1}{K_{Lead}\frac{T_L as + 1}{Tl/as + 1}}$$

$$K_{Lead} = 1$$

$$TL = 0.35$$

$$a = 1.1$$

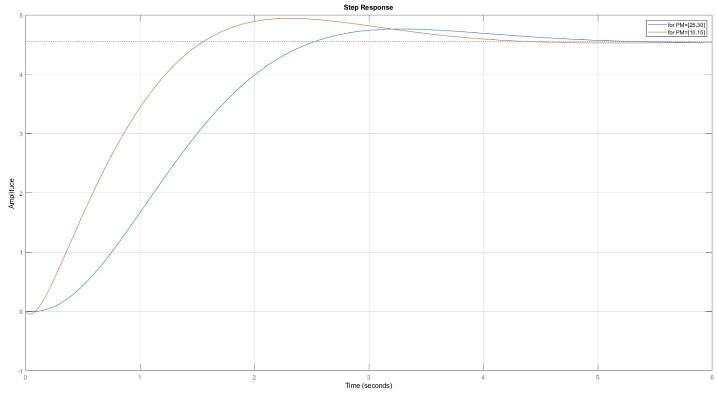

Figure 10: Difference in Step Responses of both compensated systems

As the phase margin is decreased the settling time is increased, the overshoot decreased. The steady state error stayed same.

3. T=0.5

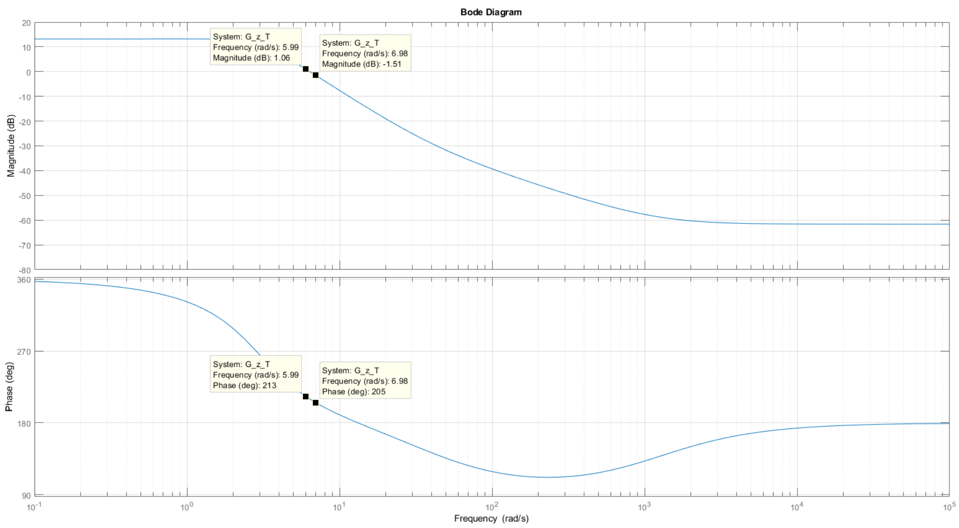

Figure 11: -

$$PM \approx 30$$

for $PM = [10, 15]$

$$\Delta PM \approx [-20, -15]$$

Choose $a = 1.55$ seems good,

$$T_L = 1/4 = 0.24$$

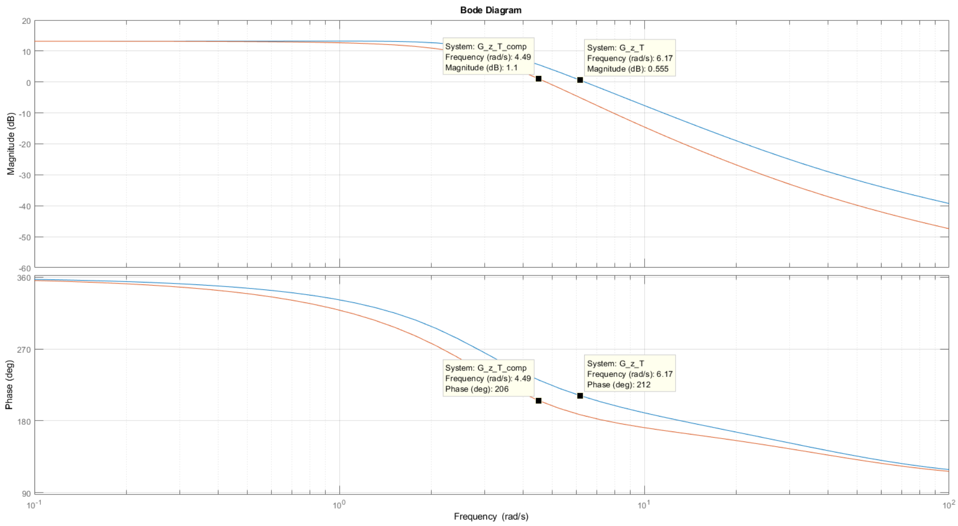

Figure 12: Bode Plots of compensated and uncompensated system

New phase margin is 14 degree

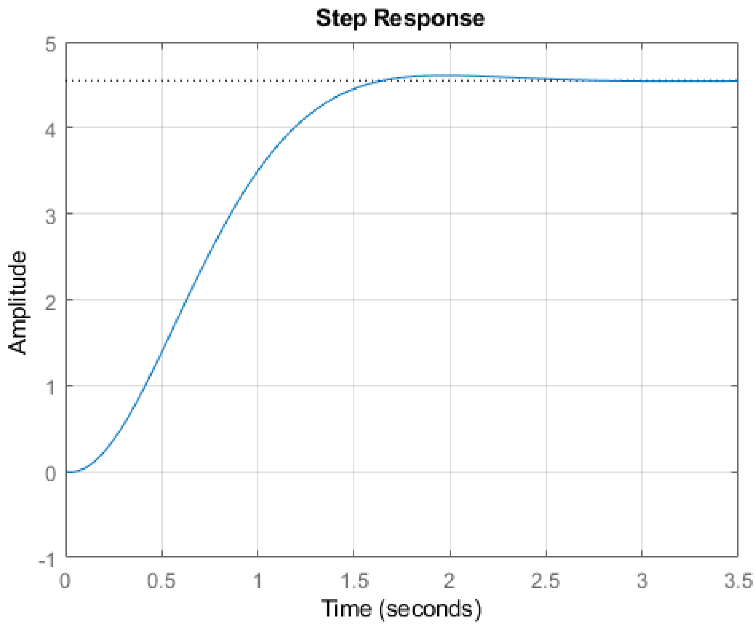

Figure 13: Step Response of the compensated system

$$G_C(s) = 1\frac{1}{K_{Lead}\frac{T_L as+1}{Tl/as+1}}$$

$$K_{Lead} = 1$$

$$TL = 0.24$$

$$a = 1.55$$

4. for $PM = [25, 30]$

$$\Delta PM \approx [-5, 0]$$

Choose $a = 1.14$ seems good,

$$T_L = 0.37$$

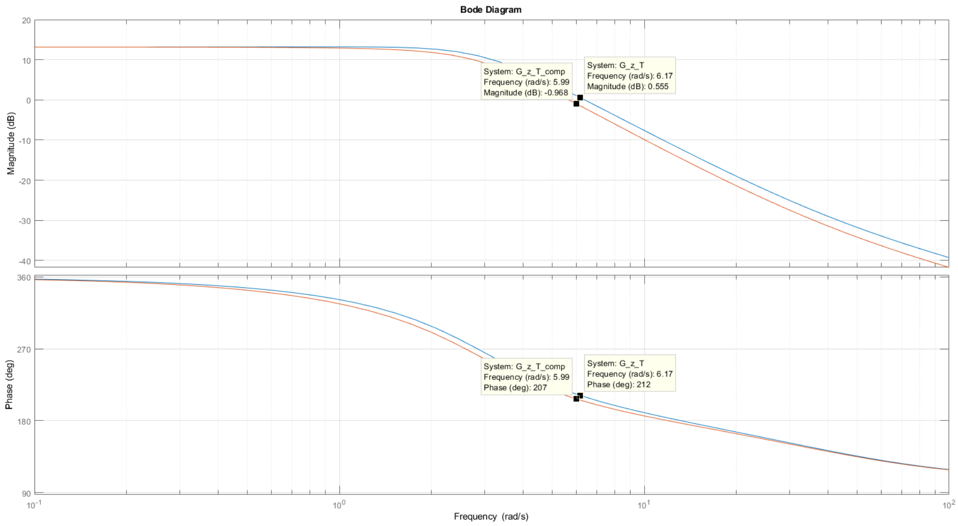

Figure 14: Bode Plots of compensated and uncompensated system

New phase margin is approximately 29 degree degree

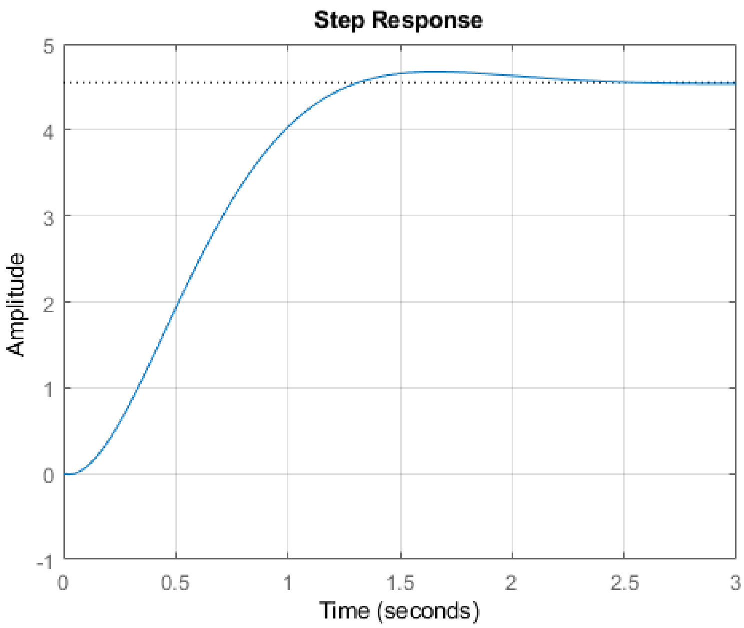

Figure 15: Step Response of the compensated system

$$G_C(s) = 1\frac{1}{K_{Lead}\dfrac{T_L as + 1}{Tl/as + 1}}$$

$$K_{Lead} = 1$$

$$TL = 0.37$$

$$a = 1.14$$

5. -

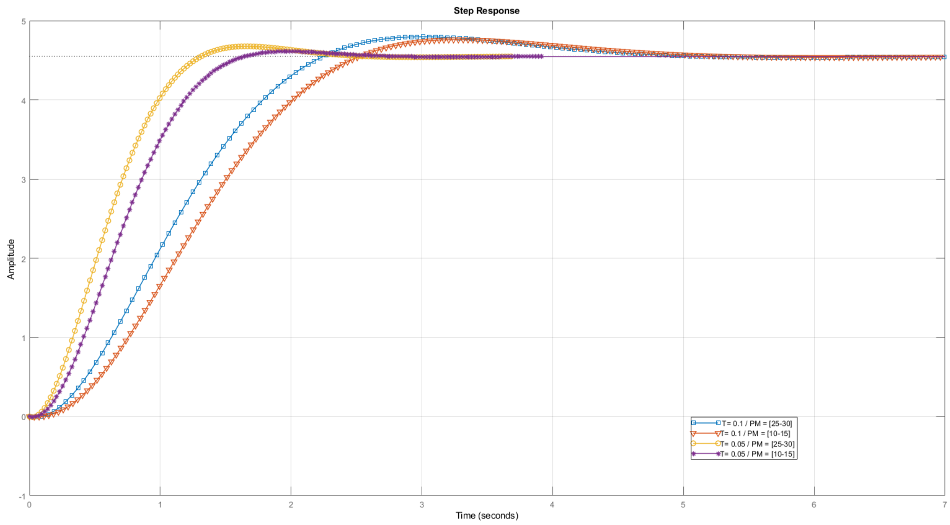

Figure 16: Difference in Step Responses of both compensated systems

As the phase margin is decreased the settling time is increased, the overshoot decreased. The steady state error stayed same. As the sampling time is increased from $T = 0.05$ to $T = 0.1$, the settling time again increased and the overshoot is decreased. The steady state error is not changed.

Appendices

A Source Code For Matlab Parts

```
%% - T=0.1
hold off

z = tf ([1 0],[1],0.1)

G_z= (0.047*z+0.044)/(z^2-1.8*z+0.82)
bode(G_z)
grid on
hold on
%T=0.1
%z1 = (1+T*s)/(1-T*s)
%G_z_T= (0.047*z1+0.044)/(z1^2-1.8*z1+0.82) % Find Tustin Tr.
G_z_T= d2c(G_z,'tustin')
bode(G_z_T)

hold off
figure()
bode(G_z_T)
grid on

%% for PM 25-30
hold off

a=1.1
K=1
T_l=0.35
PC = K*(T_l*a*s+1)/(T_l/a*s+1)
PC=1/PC
bode(PC)
grid on
hold off
G_z_T_comp=G_z_T*PC
bode(G_z_T,{0,100})
grid on
hold on
bode(G_z_T_comp,{0,100})
```

```
hold off
figure()
step(G_z_T_comp)
grid on

%% for PM 10-15
hold off

a=1.6
K=1
T_l=0.25
PC = K*(T_l*a*s+1)/(T_l/a*s+1)
PC=1/PC
bode(PC)
grid on
hold off
G_z_T_comp=G_z_T*PC
bode(G_z_T,{0,100})
grid on
hold on
bode(G_z_T_comp,{0,100})

hold off
figure()
step(G_z_T_comp)
grid on


%% - T=0.05
hold off

z = tf ([1 0],[1],0.05)

G_z= (0.047*z+0.044)/(z^2-1.8*z+0.82)
bode(G_z)
grid on
hold on
%T=0.1
%z1 = (1+T*s)/(1-T*s)
%G_z_T= (0.047*z1+0.044)/(z1^2-1.8*z1+0.82) % Find Tustin Tr.
G_z_T= d2c(G_z,'tustin')
bode(G_z_T)
```

```
hold off
figure()
bode(G_z_T)
grid on

%% for PM 35-30
hold off
s=tf([1 0],[1])
a=1.15
K=1
T_l=0.37
PC = K*(T_l*a*s+1)/(T_l/a*s+1)
PC=1/PC
bode(PC)
grid on
hold off
G_z_T_comp=G_z_T*PC
bode(G_z_T,{0,100})
grid on
hold on
bode(G_z_T_comp,{0,100})

hold off
figure()
step(G_z_T_comp)
grid on

%% for PM 10-15
hold off

a=1.6
K=1
T_l=0.25
PC = K*(T_l*a*s+1)/(T_l/a*s+1)
PC=1/PC
bode(PC)
grid on
hold off
G_z_T_comp=G_z_T*PC
bode(G_z_T,{0,100})
grid on
```

```
hold on
bode(G_z_T_comp,{0,100})

hold off
figure()
step(G_z_T_comp)
grid on





%% To choose desired a
s = tf([1 0],[1])
hold off
a=1.5
K=1
T_l=1
PC = K*(T_l*a*s+1)/(T_l/a*s+1)
bode(PC)
grid on
hold on
a=2 %% Choose a=1.8
K=1
T_l=1
PC = K*(T_l*a*s+1)/(T_l/a*s+1)
bode(PC)
hold on
a=1.1
K=1
T_l=1
PC = K*(T_l*a*s+1)/(T_l/a*s+1)
figure()
PC=1/PC
bode(PC)
grid on
```