EE402 Mini Project 6

M. Mert Ankarali*
Department of Electrical and Electronics Engineering
Middle East Technical University

Due: 14-Jan-2018, @10:30 AM

1. (30 Points)

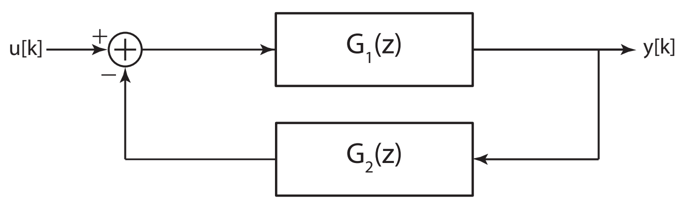
   (a) Find a minimal state-space realization for the following discrete time transfer function

   $$G_1(z) = \frac{z^2 + 0.5z}{z^3 - 2.2z^2 + 1.52z - 0.32}$$

   (b) Find a minimal state-space realization for the following discrete time transfer function

   $$G_2(z) = \frac{z^2 + 0.4z - 0.12}{z^2 + 0.6z - 0.4}$$

   (c) Using the answers of parts 1(a) and 1(b), find a minimal state-space realization for the following discrete-time closed-loop system.



2. (30 Points) In this problem, we will investigate the matrix exponential. Consider $A, P \in \mathbb{R}^{nn}$

   (a) Show that when $\det(P) \neq 0$

   $$e^{(P^{-1}AP)t} = P^{-1}e^{At}P$$

   (b) Show that when $\det(A) \neq 0$

   $$\left(\int_0^T e^{A\lambda} d\lambda \right) = A^{-1}\left(e^{AT} - I\right) = \left(e^{AT} - I\right)A^{-1}$$

   (c) Given that $(\lambda\ ,\ \nu)$ is an eigenvalue and eigenvector pair of A. Based on this information, derive the associated eigenvalue and eigenvector pair of e^{At}.

   You are supposed to derive the result, thus don't just type the answer.

(d) Compute e^{At} for the following matrix

$$A = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$$

Hint: Your solution should be in terms of sinusoidal and exponential functions of ωt and σt.

(e) Compute e^{At} for the following matrix

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

without using the Laplace transform domain solution method.

3. (30 Points) Consider the following CT state-space representation

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

(a) Based on the procedures detailed in the lecture notes, discretize this state-space formulation under ZOH operation at the input and uniform ideal sampling at the states and compute the DT state-space representation and associated matrixes.

$$x[k+1] = Gx[k] + Hu[k]$$

T should exist symbolically in your matrices.

(b) Now approximate e^{AT} using the first order approximation given below

$$e^{AT} \approx I + AT$$

and using this approximation compute the approximated discretie time state-space equation

$$x[k+1] \approx \tilde{G}x[k] + \tilde{H}u[k]$$

(c) Compute (G, H) and $(\tilde{G}, \tilde{H})$ for different values of T, compre the results, and comment on them.

4. (30 Points) Stability of CT and DT dynamical systems

(a) Consider the DT system

$$x[k+1] = \begin{bmatrix} 0 & 1 \\ \alpha & 2\alpha - 1/2 \end{bmatrix} x[k] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u[k]$$
$$y[k] = \begin{bmatrix} -2 & 1 \end{bmatrix} u[k]$$

i. For what values of parameter α is the system asymptotically stable?
ii. For what values of parameter α is the system BIBO stable?

(b) Consider the CT system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ \alpha & 2\alpha - 1/2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} -2 & 1 \end{bmatrix} u(t)$$

i. For what values of parameter α is the system asymptotically stable?
ii. For what values of parameter α is the system BIBO stable?