

EE402 Mini Project 2

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Due: 20-Mar-2018, @9:40 AM (Beginning of Class)

Important: In this mini project, there are four problems. In problems 2 and 3, you need to perform some computations in MATLAB and plot some results. In problem 4, you will need to build a Simulink model and perform some tests. You should provide all of your source codes, Simulink models, and results with your submission.

You are also supposed to submit your computational solutions (everything except your hand written solutions) though the ODTUclass page of the course. Deadline of the online submission will be 6:00 PM (18:00) same day of the in-class submission.

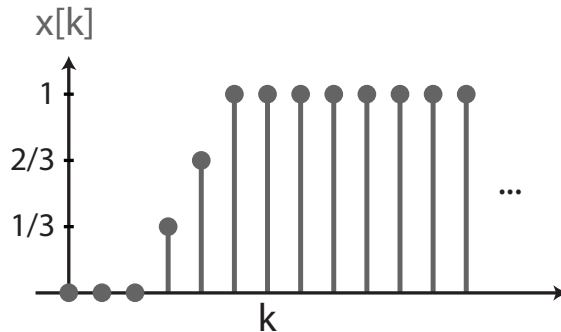
1. (25 Points) Find the Z-transform of the following discrete-time signals (given that $x[k] = 0$ for $k < 0$ for all parts.) or continuous time transfer functions ($\mathcal{Z}\{X(s)\}$).

(a) $x[k] = 9k 2^{k-1} - 2^k + 3$

(b) $x[k] = \sum_{h=0}^k a^h$, where a is a constant

(c) $x[k] = k(k-1)a^{k-2}$ (Answer is already given in the tables of the textbook. Your goal is to derive the answer in this question)

(d) $x[k]$ given by the following graph



(e) $X(s) = \frac{1-e^{Ts}}{s} \frac{1}{(s+a)^2}$

2. (30 Points) In this problem your goal is taking the inverse Z-transform of the following z-domain function using 4 different methods and comparing all solutions in MATLAB.

$$X(z) = \frac{z^{-1} (0.5 - z^{-1})}{(1 - 0.5z^{-1})(1 - 0.8z^{-1})^2}$$

- (a) Find a closed form expression of the inverse Z-transform of $X(z)$ using partial fraction expansion.

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- (b) In this part, your goal is to find the inverse Z transform by long division method. However, unlike the example in the class (or examples in textbook) you will not perform the long-division by hand. Instead, you are supposed to write a MATLAB code that performs the long-division iteratively and finds the inverse Z-transform of $X(z)$ until $k = 20$.
- (c) Inside the Lecture Notes 3, we covered a method of inverse Z-transform which treats the $X(z)$ as a “system”, then transforms this synthetic system to the difference equation form, and finally the inverse Z transform is found by “simulating” this new difference equation form.
- Following the procedure detailed in the Lecture notes 3, find the necessary difference equation form.
 - Write a custom code that “simulates” the difference equation (with proper input) until $k = 20$. The output of this simulation should give $x[k] = \mathcal{Z}^{-1}\{X(z)\}$
- (d) MATLAB has built-in toolboxes (Control System Toolbox, Signal Processing Toolbox, Symbolic Math toolbox, etc.) that can compute the inverse Z-transform of a function using different approaches (One of them is explained in the textbook). Compute/find the inverse Z-transform of $X(z)$ using one of the built-in methods of MATLAB.
- (e) (25 Points) In the parts a-d of this problem, you solved/computed the inverse Z-transform of the same $X(z)$. In MATLAB, plot each result on the same figure, by using *subplot* feature in order to compare the results. Use same axis limits for all sub-plots. Please appropriately label each plot and clearly indicate the type of the method. I recommend you to solve the MATLAB solutions of all parts on the same m-file.
3. In this problem, your goal is to solve the following difference equation using 3 different methods.

$$y[k] - 3y[k-1] + 2y[k-2] = x[k], \quad y[k] = 0, \quad \text{for } k < 0$$

$$x[k] = 5 \cdot 2^k + 2 \cdot 3^k, \quad x[k] = 0, \quad \text{for } k < 0$$

- (a) Solve the difference equation in time domain (express the solution as sum of two parts: homogeneous and particular) and find a closed form expression.
- (b) Solve the difference equation using Z-transform and find a closed form expression.
- (c) Simulate the difference equation in MATLAB until $k = 20$. You can either implement your own custom code, or use one of the built-in features of MATLAB.
- (d) Plot the $y[k]$ in MATLAB for the solutions of all parts and compare the results.
4. (25 Points) Consider the following transfer function of a discrete time controller/filter.

$$G(z) = \frac{0.02z^{-1} + 0.03z^{-2} + 0.02z^{-3}}{1 - 3z^{-1} + 4z^{-2} - 2z^{-3} + 0.5z^{-4}}$$

- Find a minimal realization of the discrete time transfer function.
- In Simulink, build the block diagram realization (use *delay block* and other necessary elements) and connect a sinusoidal input source to the input terminal and connect a scope to the output terminal.
- On the same model page, implement the transfer function now using the *Discrete Filter* block and connect the input of this block to the same input source, and connect the output to the same scope.
- Simulate the Simulink model(s) and compare the outputs.
- Perform the simulation with different input frequencies and comment on the behavior of the frequency response of the model(s).