EE402 Mini Project I

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1) a. $y(t) = (\sin(t))^3$

① Memoryless: The output y(t) depends only the current values of t / does not depend on the past or future

② Not Linear : $u(t) \longrightarrow \boxed{} \longrightarrow y(t) = (\sin(t))^3$

$\alpha u(t) \longrightarrow \boxed{} \longrightarrow y(t) = (\sin(t))^3 \neq \alpha y(t)$

The output does not depend on the input.

③ Time Varying : A shift in the input does not create any shift at the output.

④ Causal : Because it is memoryless

⑤ Infinite ~~Dimensional~~ Dimensional : There is no ODE in u,y that models the system

b. $y(t) = \int_0^t \tau u(\tau) d\tau$

① Has Memory: The output depends on the past values of the input.

② Linear : $u_1(t) \longrightarrow \boxed{} \longrightarrow y_1(t)$

$u_2(t) \longrightarrow \boxed{} \longrightarrow y_2(t)$

$\alpha u_1(t) + \beta u_2(t) \longrightarrow \boxed{} \longrightarrow \int_0^t (\alpha u_1(\tau) + \beta u_2(\tau)) \tau d\tau$

$$= \underbrace{\int_0^t \tau \alpha u_1(\tau) d\tau}_{} + \underbrace{\int_0^t \tau \beta u_2(\tau) d\tau}_{}$$

$$= \alpha y_1(t) + \beta y_2(t)$$

③ Time -Varying : $\underset{u_x(t)}{\overset{\parallel}{u(t-T_0)}} \longrightarrow \boxed{} \longrightarrow \int_0^t \tau u_x(\tau) d\tau$

$$= \int_0^t (\tau) u(\tau - T_0) d\tau$$

$$= \int_{-T_0}^{t-T_0} (\tau + T_0) u(\tau) d\tau \neq y(t - T_0)$$

④ Causal: The output does not depend on the future values of the input / only past & present ①

⑤ ~~Finite~~ ~~Dimensional~~: $\dot{y} = tu(t)$ (ODE representation)

c. $y(t) = 2u(t) + 10$

① Memoryless: The output at any time instant depends only on the value of the input at that particular instant

② Not - Linear: $u_1(t) \longrightarrow \boxed{} \longrightarrow y_1(t) = 2u_1(t) + 10$

$u_2(t) \longrightarrow \boxed{} \longrightarrow y_2(t) = 2u_2(t) + 10$

$\alpha u_1(t) + \beta u_2(t) \longrightarrow \boxed{} \longrightarrow 2(\alpha u_1(t) + \beta u_2(t)) + 10$

$= 2\alpha u_1(t) + 2\beta u_2(t) + 10$

$\neq \alpha y_1(t) + \beta y_2(t)$

③ Time-Invariant: $\underbrace{u(t-T_0)}_{u_x(t)} \longrightarrow \boxed{} \longrightarrow 2u_x(t) + 10$

$= 2u(t-T_0) + 10$

$\parallel$

$y(t-T_0) = 2u(t-T_0) + 10$

④ Causal: Since the system is memoryless.

⑤ Finite Dimensional: $\frac{dy}{dt} = 2\frac{du}{dt}$ (ODE)

d. $y(t) = \cos(t)\,u(t)$

① Memoryless: $y(t_0)$ only depends on $u(t_0)$ for all $t \in \mathbb{R}$

② Linear: $(S_c(\alpha u_1 + \beta u_2))(t) \overset{?}{=} \alpha(S_c u_1)(t) + \beta(S_c u_2)(t)$

$\cos(t)[\alpha u_1(t) + \beta u_2(t)] = \alpha\cos(t)u_1(t) + \beta\cos(t)u_2(t)$

③ Time - Varying: $\sigma_T u(t) = u(t-T)$

$(S_c \sigma_T u)(t) = \cos(t)\,u(t-T) \neq y(t-T) = \cos(t-T).u(t-T)$

④ Causal: Since it is memoryless.

⑤ Finite Dimensional: $\dot{y} = \sin(t)\,\dot{u}$

e. $y(t) = u(t-T)$

① Linear: $(S_c(\alpha u_1 + \beta u_2))(t) \stackrel{?}{=} \alpha(S_c u_1)(t) + \beta(S_c u_2)(t)$

$\alpha u_1(t-T) + \beta u_2(t-T) = \alpha u_1(t-T) + \beta u_2(t-T)$

② Has memory: The output depens on the past values of the input provided that $T>0$.

③ Time-Invariant: $(S_c \sigma_{T_0} u)(t) = u(t-T_0-T)$

$y(t-T_0) = u(t-T-T_0)$ the same

④ Causal: Provided that $T \geq 0$, the output is dependent on the past values of the input.

⑤ Infinite Dimensional: There is no ODE representation of the system

f. $y[n] = u[k-n]$

① Linear: $(S_c(\alpha u_1 + \beta u_2)[n] \stackrel{?}{=} \alpha(S_c u_1)[n] + \beta(S_c u_2)[n]$

$\alpha u_1[k-n] + \beta u_2[k-n] \stackrel{\checkmark}{=} \alpha u_1[k-n] + \beta u_2[k-n]$

② Has memory: If $k>0 \Rightarrow$ $y[0] = y[k-0] = y[k]$

does not depend on the current instant but future

③ Time-Varying: $u_x[n] = u[n-N_0] \longrightarrow \boxed{\quad} \longrightarrow u_x[k-n]$

$= u[-n-N_0+k]$

$y[n-N_0] = u[k-n+N_0]$

$y[n-N_0] \neq u[-n-N_0+k]$

④ Not causal: For $k \geq 0$, the output depends on values of the input

⑤ Finite Dimensional: There is an ODE modelling the system.

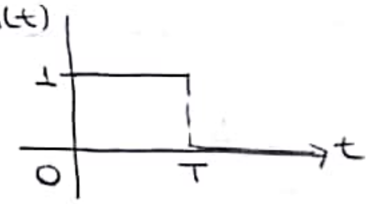
9. $x(t) \longrightarrow \boxed{\dfrac{1-e^{-Ts}}{s}} \longrightarrow y(t)$

① Linear: We cannot model a nonlinear system using transfer functions.

② Has Memory: $H(s) = \dfrac{1}{s} - \dfrac{e^{-Ts}}{s} \longrightarrow h(t) = u(t) - u(t-T)$

{unit-step}

for $T \geq 0 \Rightarrow$ $h(t)$



$$Y(s) = \frac{X(s)}{s} - \frac{e^{-Ts}}{s} X(s)$$

$\mathcal{L}^{-1}$

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau - \int_{-\infty}^{t} x(\tau - T)\, d\tau$$

$$= \int_{-\infty}^{t} x(\tau)\, d\tau - \int_{-\infty}^{t-T} x(\tau)\, d\tau$$

$$y(t) = \int_{t-T}^{t} x(\tau)\, d\tau \Rightarrow \text{depends on past values}$$

③ Time-Invariant: System has to be LTI in order to be represented by a transfer function

④ Causal: Depends on the past & current values of the input

⑤ Infinite Dimensional: There is no ODE modelling of the system.

$$f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau$$

$$(f(t) * g(t)) * h(t) = \int_{-\infty}^{\infty} (f(\tau) * g(\tau))\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(\tilde{\tau})\, g(\tau-\tilde{\tau})\, d\tilde{\tau} \right] h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\tilde{\tau})\, g(\tau-\tilde{\tau})\, h(t-\tau)\, d\tilde{\tau}\, d\tau$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\tilde{\tau})\, g(\tau-\tilde{\tau})\, h(t-\tau)\, d\tau\, d\tilde{\tau}$$

$$= \int_{-\infty}^{\infty} f(\tilde{\tau}) \underbrace{\left[\int_{-\infty}^{\infty} g(\tau-\tilde{\tau})\, h(t-\tau)\, d\tau \right]}_{\underbrace{\int_{-\infty}^{\infty} g(\tau+\tilde{\tau}-\tilde{\tau})\, h(t-\tau-\tilde{\tau})\, d\tau}_{\int_{-\infty}^{\infty} g(\tau)\, h((t-\tilde{\tau})-\tau)\, d\tau}} d\tilde{\tau}$$

$$= \int_{-\infty}^{\infty} f(\tilde{\tau})\, (g * h)(t-\tilde{\tau})\, d\tilde{\tau} \qquad \text{let } \tilde{\tau} = \tau$$

$$= \int_{-\infty}^{\infty} f(\tau)\, (g * h)(t-\tau)\, d\tau$$

$$= f(t) * (g(t) * h(t)) \;//$$

b. $f(t-\tau) = f(t) * \delta(t-\tau)$

$$f(t) * \delta(t-\tau) = \int_{-\infty}^{\infty} f(\bar{\tau}) \cdot \delta(t-\tau-\bar{\tau})\, d\bar{\tau}$$

$$= \int_{-\infty}^{\infty} f(t-\tau) \cdot \delta(t-\tau-\bar{\tau})\, d\bar{\tau}$$

$$= f(t-\tau) \;//$$

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4. $[Y(s) - U(s)]M(s) = Y(s)$

$U(s) = [R(s) - Y(s)]C(s)$

$[Y(s) - R(s)C(s) + Y(s)C(s)]M(s) = Y(s)$

$Y(s)[M(s) + C(s)M(s) - 1] = R(s)C(s)M(s)$

$$T(s) = \frac{Y(s)}{R(s)} = \frac{C(s)M(s)}{M(s)C(s) + M(s) - 1}$$

$$= \frac{\frac{K}{s+1} \cdot \frac{1}{s-a}}{\frac{K}{(s+1)(s-a)} + \frac{1}{s-a} - 1} = \frac{K}{K + s + 1 - (s+1)(s-a)}$$

$$= \frac{K}{-s^2 + as - \cancel{s} + a + \cancel{s} + 1 + k} = \frac{K}{-s^2 + as + a + 1 + k}$$

Routh Array

s^2	-1	$a+1+k$
s^1	a	0
1		

system is unstable since $a > 0$
there is no K making the system stable.

5. a. $\sin\theta = \theta$ $\quad d\cos\theta \doteq d$ $\quad$ {small angle approx.}

$$\ddot{\theta} - \frac{g}{L}\theta = \frac{1}{mL^2} d\,u(t)$$

b. Taking Laplace Transform {zero initial conditions}

$$s^2\theta(s) - \frac{g}{L}\theta(s) = \frac{1}{mL^2} d\,U(s) \qquad \frac{\theta(s)}{U(s)} = \frac{\frac{d}{mL^2}}{s^2 - \frac{g}{L}} = \frac{d}{(s^2L - g)mL}$$

$$= \frac{d}{s^2 mL^2 - gmL}$$

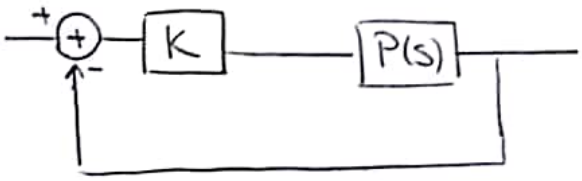
poles: $s^2 mL^2 - gmL = 0$

$s^2 L - g = 0$

$s^2 = \frac{g}{L}$ $\quad s = \pm\sqrt{\frac{g}{L}}$

Unstable ⇒ pole on the positive real part

c. $$P(s) = \frac{0{,}3924^2}{s^2(0{,}3924)^2 - 9.81 \times 0{,}3924} = \frac{0{,}3924}{s^2 \times 0{,}3924 - 9{,}81} = \frac{1}{s^2 - 25}$$

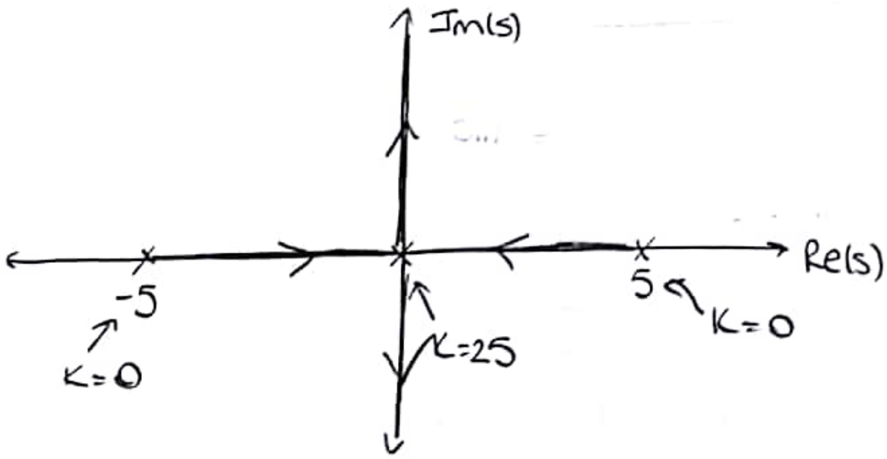


$$T(s) = \frac{KP(s)}{KP(s) + 1}$$

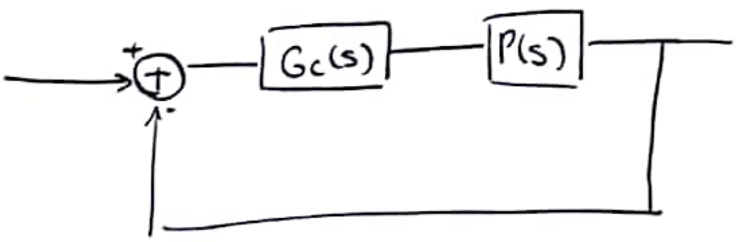
$$= \frac{K}{K + s^2 - 25}$$

Char. eq. ⇒ $s^2 + (K - 25) = 0$ $\quad s_{1,2} = \pm\sqrt{(25-K)}$

$= \pm j\sqrt{K-25}$ for $K > 25$



d.



$$T(s) = \frac{G_c(s)\,P(s)}{G_c(s)P(s) + 1}$$

$$= \frac{G_c(s)}{G_c(s) + s^2 - 25}$$

$$T(s) = \frac{G_c(s)}{G_c(s) + s^2 - 25}$$

Let $G_c(s) = As$

Routh - Array

s^2	1	-25
s	A	0
1	$25A$	

All must be positive so choose $A = 2$

$$\boxed{G_c(s) = 2 \cdot s}$$

$\frac{Y(s)}{U(s)} = H_x(s) = ?$ $\qquad y(t) = \int_{-\infty}^{\infty} h_x(t-\tau)\, u(\tau)\, d\tau$

We know $\quad y(t) = \int_{t-T}^{t} h(t-\tau)\, u(\tau)\, d\tau$

$$= \int_{t-T}^{t} (t-\tau)\, u(\tau)\, d\tau$$

Let $\tilde{\tau} = t-\tau$

$$= -\int_{T}^{0} \tilde{\tau}\, u(t-\tilde{\tau})\, d\tilde{\tau}$$

$$= \int_{0}^{T} \tilde{\tau}\, u(t-\tilde{\tau})\, d\tilde{\tau}$$

this has to be true for all $u(t)$

let $u(t) = 1$ for $t \geq 0$
$u(t-\tilde{\tau}) = 1$ for $t \geq \tilde{\tau}$

$$= \int_{t}^{T} \tilde{\tau} \cdot 1\, d\tilde{\tau} = \left.\frac{\tilde{\tau}^2}{2}\right|_t^T = \frac{T^2}{2} - \frac{t^2}{2}$$

Then for this input, $y(t) = \frac{T^2}{2} - \frac{t^2}{2}$

$$Y(s) = \frac{T^2}{2s} - \frac{1}{s^3} = \frac{T^2 s^2 - 2}{2s^3}$$

$U(s) = \frac{1}{s}$ $\qquad \frac{Y(s)}{U(s)} = \frac{T^2}{2} - \frac{1}{s^2} = \frac{T^2 s^2 - 2}{2s^2}$ //

α

c. $\mathcal{L}\{f * g\} = \mathcal{L}\{f\}\mathcal{L}\{g\}$

$$\mathcal{L}\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-st} dt$$

$$\mathcal{L}\{f(t) * g(t)\} = \int_{-\infty}^{\infty} (f(t) * g(t))\, e^{-st} dt$$

$$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f(\tau)\, g(t-\tau) d\tau \right) e^{-st} dt$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, e^{-st} dt\, d\tau$$

$$= \int_{-\infty}^{\infty} f(\tau) \int_{-\infty} g(t-\tau)\, e^{-st} dt\, d\tau$$

let $\tilde{\tau} = t - \tau$

$$= \int_{-\infty} f(\tau) \int_{-\infty}^{\infty} g(\tilde{\tau})\, e^{-s(\tilde{\tau}+\tau)}\, d\tilde{\tau}\, d\tau$$

$$= \int_{-\infty}^{\infty} f(\tau)\, e^{-s\tau} \int_{-\infty}^{\infty} g(\tilde{\tau})\, e^{-s\tilde{\tau}} d\tilde{\tau}\, d\tau$$

$$= \underbrace{\int_{-\infty}^{\infty} f(\tau)\, e^{-s\tau} d\tau}_{} \cdot \underbrace{\int_{-\infty}^{\infty} g(\tilde{\tau})\, e^{-s\tilde{\tau}} d\tilde{\tau}}_{}$$

$$= \mathcal{L}\{f(t)\} \cdot \mathcal{L}\{g(t)\} \;//$$

d. $\mathcal{L}\{f + g\} = \mathcal{L}\{f\} + \mathcal{L}\{g\}$

$$\mathcal{L}\{f(t) + g(t)\} = \int_{-\infty}^{\infty} (f(t) + g(t))\, e^{-st} dt$$

$$= \int_{-\infty}^{\infty} (f(t)e^{-st} + g(t)e^{-st}) dt$$

$$= \underbrace{\int_{-\infty}^{\infty} f(t)e^{-st} dt}_{} + \underbrace{\int_{-\infty}^{\infty} g(t)\, e^{-st} dt}_{}$$

$$= \mathcal{L}\{f(t) + \mathcal{L}\{g(t)\} \;//$$