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1) a. $y(t) = (\sin(t))^3$

① Memoryless ✓ The output $y(t)$ depends only the current values of t / does not depend on the past or future.

② Not Linear : $u(t) \longrightarrow \boxed{} \longrightarrow y(t) = (\sin(t))^3$

$a u(t) \longrightarrow \boxed{} \longrightarrow y(t) = (\sin(t))^3 \neq \alpha y(t)$

The output does not depend on the input.

③ Time Varying : A shift in the input does not create any shift at the output.

④ Causal ✓ : Because it is memoryless

⑤ Infinite Dimensional : There is no ODE in u,y that models the system

b. $y(t) = \int_0^t \tau u(\tau) d\tau$

① Has memory: The output depends on the past values of the input.

② Linear ✓ : $u(t) \longrightarrow \boxed{} \longrightarrow y_1(t)$

$u_2(t) \longrightarrow \boxed{} \longrightarrow y_2(t)$

$\alpha u_1(t) + \beta u_2(t) \longrightarrow \boxed{} \longrightarrow \int_0^t (\alpha u_1(\tau) + \beta u_2(\tau)) \tau d\tau$

$= \underbrace{\int_0^t \tau \alpha u_1(\tau) d\tau}_{} + \underbrace{\int_0^t \tau \beta u_2(\tau) d\tau}_{}$

$= \alpha y_1(t) + \beta y_2(t)$

③ Time - Varying ✓ $\underset{\parallel}{u(t-T_0)} \longrightarrow \boxed{} \longrightarrow \int_0^t \tau u_x(\tau) d\tau$
$u_x(t)$

$= \int_0^t (\tau) u(\tau - T_0) d\tau$

$= \int_{-T_0}^{t-T_0} (\tau + T_0) u(\tau) d\tau \neq y(t-T_0)$

④ Causal ✓: The output does not depend on the fut-④ ⑦
values of the input / only past & present

⑤ Finite Dimensional: $\dot{y} = tu(t)$ (ODE representation)

c. $y(t) = 2u(t) + 10$

① Memoryless ✓ The output at any time instant depends
only on the value of the input at that
particular instant.

② Not - Linear ✓ $u_1(t) \longrightarrow \boxed{} \longrightarrow y_1(t) = 2u_1(t) + 10$

$u_2(t) \longrightarrow \boxed{} \longrightarrow y_2(t) = 2u_2(t) + 10$

$\alpha u_1(t) + \beta u_2(t) \longrightarrow \boxed{} \longrightarrow 2\left(\alpha u_1(t) + \beta u_2(t)\right) + 10$

$= 2\alpha u_1(t) + 2\beta u_2(t) + 10$

$\neq \alpha y_1(t) + \beta y_2(t)$

③ Time-Invariant ✓ $\underbrace{u(t-T_0)}_{U_x(t)} \longrightarrow \boxed{} \longrightarrow 2u_x(t) + 10$

$= 2u(t-T_0) + 10$

$y(t-T_0) = 2u(t-T_0) + 10$ ✓

④ Causal ✓: Since the system is memoryless.

⑤ Finite Dimensional : $\dfrac{dy}{dt} = 2\dfrac{du}{dt}$ (ODE)

d. $y(t) = \cos(t) u(t)$

① Memoryless ✓ $y(t_0)$ only depends on $u(t_0)$ for all $t \in \mathbb{R}$.

② Linear ✓: $(S_c(\alpha u_1 + \beta u_2))(t) \overset{?}{=} \alpha(S_c u_1)(t) + \beta(S_c u_2)(t)$

$\cos(t)[\alpha u_1(t) + \beta u_2(t)] \overset{✓}{=} \alpha \cos(t) u_1(t) + \beta \cos(t) u_2(t)$

③ Time - Varying ✓: $\sigma_T u(t) = u(t-T)$

$(S_c \sigma_T u)(t) = \cos(t) u(t-T) \neq y(t-T) = \cos(t-T) u(t-T)$

④ Causal : Since it is memoryless.

⑤ Finite Dimensional : $\dot{y} = \sin(t)\, \dot{u}$

e. $y(t) = u(t-T)$

① Linear : $(Sc(\alpha u_1 + \beta u_2))(t) \stackrel{?}{=} \alpha(Scu_1)(t) + \beta(Scu_2)(t)$

$\alpha u_1(t-T) + \beta u_2(t-T) = \alpha u_1(t-T) + \beta u_2(t-T)$

② Has memory : The output depens on the past values
of the input provided that $T > 0$.

③ Time-Invariant : $(Sc\,\sigma_{T_0}u)(t) = u(t-T_0-T)$

$y(t-T_0) = u(t-T-T_0)$ $\nearrow$ the same

④ Causal : Provided that $T \geqslant 0$, the output is dependent
on the past values of the input.

⑤ Infinite Dimensional : There is no ODE representation
of the system.

f. $y[n] = u[k-n]$

① Linear : $(Sc(\alpha u_1 + \beta u_2))[n] \stackrel{?}{=} \alpha(Sc\,u_1)[n] + \beta(Sc\,u_2)[n]$

$\alpha u_1[k-n] + \beta u_2[k-n] \stackrel{\checkmark}{=} \alpha u_1[k-n] + \beta u_2[k-n]$

② Has memory : If $k > 0 \Rightarrow y[0] = y[k-0] = y[k]$

does not depend on
the current instant
but future

③ Time-Varying : $u_x[n] = u[n-N_0] \longrightarrow \boxed{} \longrightarrow u_x[k-n]$

$= u[-n-N_0+k]$

$y[n-N_0] = u[k-n+N_0]$

$y[n-N_0] \neq u[-n-N_0+k]$

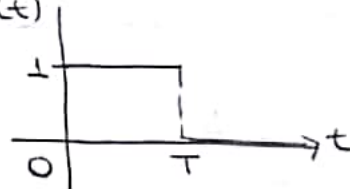④ Not causal ✓: For $k \geq 0$, the output depends on values of the input

⑤ Finite Dimensional ✓: There is an ODE modelling the system.

9. $x(t) \longrightarrow \boxed{\dfrac{1 - e^{-Ts}}{s}} \longrightarrow y(t)$

① Linear ✓: We cannot model a nonlinear system using transfer functions.

② Has Memory ✓: $H(s) = \dfrac{1}{s} - \dfrac{e^{-Ts}}{s} \longrightarrow h(t) = u(t) - u(t-T)$

$\underset{\{unit\text{-}step\}}{}$

for $T \geq 0 \Rightarrow h(t)$

$Y(s) = \dfrac{X(s)}{s} - \dfrac{e^{-Ts}}{s} X(s)$

$(\mathcal{L}^{-1}$

$y(t) = \displaystyle\int_{-\infty}^{t} x(\tau)\,d\tau - \int_{-\infty}^{t} x(\tau - T)\,d\tau$

$= \displaystyle\int_{-\infty}^{t} x(\tau)\,d\tau - \int_{-\infty}^{t-T} x(\tau)\,d\tau$

$y(t) = \displaystyle\int_{t-T}^{t} x(\tau)\,d\tau \Rightarrow$ depends on past values

③ Time - Invariant ✓: System has to be LTI in order to be represented by a transfer function

④ Causal ✓: Depends on the past & current values of the input

⑤ Infinite Dimensional ✓: There is no ODE modelling of the system.

$$f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau) \, g(t-\tau) \, d\tau$$

$$(f(t) * g(t)) * h(t) = \int_{-\infty}^{\infty} (f(\tau) * g(\tau)) \, h(t-\tau) \, d\tau$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(\tilde{\tau}) \, g(\tau - \tilde{\tau}) \, d\tilde{\tau} \right] h(t-\tau) \, d\tau$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\tilde{\tau}) \, g(\tau - \tilde{\tau}) \, h(t-\tau) \, d\tilde{\tau} \, d\tau$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\tilde{\tau}) \, g(\tau - \tilde{\tau}) \, h(t-\tau) \, d\tau \, d\tilde{\tau}$$

$$= \int_{-\infty}^{\infty} f(\tilde{\tau}) \left[\int_{-\infty}^{\infty} g(\tau - \tilde{\tau}) \, h(t-\tau) \, d\tau \right] d\tilde{\tau}$$

$$\underbrace{\underbrace{\int_{-\infty}^{\infty} g(\tau + \tilde{\tau} - \tilde{\tau}) \, h(t - \tau - \tilde{\tau}) \, d\tau}_{}}_{\int_{-\infty}^{\infty} g(\tau) \, h((t - \tilde{\tau}) - \tau) \, d\tau}$$

$$= \int_{-\infty}^{\infty} f(\tilde{\tau}) \, (g*h)(t - \tilde{\tau}) \, d\tilde{\tau} \qquad \text{let } \tilde{\tau} = \tau$$

$$= \int_{-\infty}^{\infty} f(\tau) \, (g*h)(t - \tau) \, d\tau$$

$$= f(t) * \left(g(t) * h(t) \right) \; //$$

b. $f(t-\tau) = f(t) * \delta(t-\tau)$

$$f(t) * \delta(t-\tau) = \int_{-\infty}^{\infty} f(\bar{\tau}) \cdot \delta(t - \tau - \bar{\tau}) \, d\bar{\tau}$$

$$= \int_{-\infty}^{\infty} f(t-\tau) \cdot \delta(t - \tau - \bar{\tau}) \, d\bar{\tau}$$

$$= f(t-\tau) \; //$$

c. $\mathcal{L}\{f * g\} = \mathcal{L}\{f\}\mathcal{L}\{g\}$

$\mathcal{L}\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$

$\mathcal{L}\{f(t) * g(t)\} = \int_{-\infty}^{\infty} (f(t) * g(t))\, e^{-st}\, dt$

$\quad = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau \right) e^{-st}\, dt$

$\quad = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, e^{-st}\, dt\, d\tau$

$\quad = \int_{-\infty}^{\infty} f(\tau) \int_{-\infty}^{\infty} g(t-\tau)\, e^{-st}\, dt\, d\tau$

$\qquad\qquad\qquad\qquad \text{let } \tilde{\tau} = t - \tau$

$\quad = \int_{-\infty}^{\infty} f(\tau) \int_{-\infty}^{\infty} g(\tilde{\tau})\, e^{-s(\tilde{\tau}+\tau)}\, d\tilde{\tau}\, d\tau$

$\quad = \int_{-\infty}^{\infty} f(\tau)\, e^{-s\tau} \int_{-\infty}^{\infty} g(\tilde{\tau})\, e^{-s\tilde{\tau}}\, d\tilde{\tau}\, d\tau$

$\quad = \underbrace{\int_{-\infty}^{\infty} f(\tau)\, e^{-s\tau}\, d\tau}_{\mathcal{L}\{f(t)\}} \cdot \underbrace{\int_{-\infty}^{\infty} g(\tilde{\tau})\, e^{-s\tilde{\tau}}\, d\tilde{\tau}}_{\mathcal{L}\{g(t)\}}$

$\quad = \quad \mathcal{L}\{f(t)\} \quad \cdot \quad \mathcal{L}\{g(t)\} \quad \text{//}$

d. $\mathcal{L}\{f + g\} = \mathcal{L}\{f\} + \mathcal{L}\{g\}$

$\mathcal{L}\{f(t) + g(t)\} = \int_{-\infty}^{\infty} (f(t) + g(t))\, e^{-st}\, dt$

$\quad = \int_{-\infty}^{\infty} (f(t)e^{-st} + g(t)e^{-st})\, dt$

$\quad = \underbrace{\int_{-\infty}^{\infty} f(t)e^{-st}\, dt}_{\mathcal{L}\{f(t)\}} + \underbrace{\int_{-\infty}^{\infty} g(t)e^{-st}\, dt}_{\mathcal{L}\{g(t)\}}$

$\quad = \quad \mathcal{L}\{f(t)\} \quad + \quad \mathcal{L}\{g(t)\} \quad \text{//}$

$\frac{Y(s)}{U(s)} = H_x(s) = ?$
$\qquad\qquad y(t) = \int_{-\infty}^{\infty} h_x(t-\tau) u(\tau) d\tau$

We know $\qquad y(t) = \int_{t-T}^{t} h(t-\tau) u(\tau) d\tau$

$$= \int_{t-T}^{t} (t-\tau) u(\tau) d\tau$$

Let $\tilde{\tau} = t - \tau$

$$= -\int_{T}^{0} \tilde{\tau} \, u(t-\tilde{\tau}) d\tilde{\tau}$$

$$= \int_{0}^{T} \tilde{\tau} \, u(t-\tilde{\tau}) d\tilde{\tau}$$

this has to be true for all $u(t)$

let $\qquad u(t) = 1 \quad$ for $\; t \geqslant 0$
$\qquad\qquad u(t-\tilde{\tau}) = 1 \quad$ for $\; t \geqslant \tilde{\tau}$

$$= \int_{t}^{T} \tilde{\tau} \cdot 1 d\tilde{\tau} = \frac{\tilde{\tau}^2}{2} \Big|_{t}^{T} = \frac{T^2}{2} - \frac{t^2}{2}$$

Then for this input, $y(t) = \frac{T^2}{2} - \frac{t^2}{2}$

$Y(s) = \frac{T^2}{2s} - \frac{1}{s^3} = \frac{T^2 s^2 - 2}{2 s^3}$

$U(s) = \frac{1}{s}$

$\qquad\qquad \frac{Y(s)}{U(s)} = \frac{T^2}{2} - \frac{1}{s^2} = \frac{T^2 s^2 - 2}{2 s^2}$ //

4. $[Y(s) - U(s)]m(s) = Y(s)$

$U(s) = [R(s) - Y(s)]C(s)$

$[Y(s) - R(s)C(s) + Y(s)C(s)]m(s) = Y(s)$

$Y(s)[m(s) + C(s)m(s) - 1] = R(s)C(s)m(s)$

$T(s) = \dfrac{Y(s)}{R(s)} = \dfrac{C(s)m(s)}{m(s)C(s) + m(s) - 1}$

$= \dfrac{\dfrac{k}{s+1} \cdot \dfrac{1}{s-a}}{\dfrac{k}{(s+1)(s-a)} + \dfrac{1}{s-a} - 1} = \dfrac{k}{k + s+1 - (s+1)(s-a)}$

$= \dfrac{k}{-s^2 + as - s + a + s + 1 + k} = \dfrac{k}{-s^2 + as + a + 1 + k}$

Routh Array

s^2	-1	$a+1+k$
s^1	a	0
1		

System is unstable since $a > 0$

there is no k making the system stable.

5. a. $\sin\theta = \theta$ $d\cos\theta \doteq d$ {small angle approx.}

$\ddot{\theta} - \dfrac{g}{L}\theta = \dfrac{1}{mL^2}d\, u(t)$

b. Taking Laplace Transform {zero initial conditions}

$s^2\theta(s) - \dfrac{g}{L}\theta(s) = \dfrac{1}{mL^2}d\, U(s)$ $\dfrac{\theta(s)}{U(s)} = \dfrac{\dfrac{d}{mL^2}}{s^2 - \dfrac{g}{L}} = \dfrac{d}{(s^2 L - g)mL}$

$$= \frac{d}{s^2 m L^2 - gmL}$$
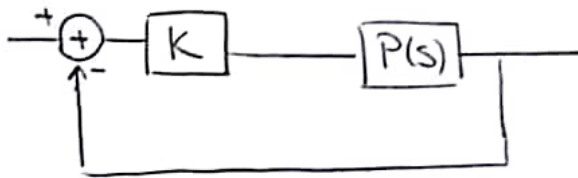
poles: $s^2 m L^2 - gmL = 0$

$$s^4 L - g = 0$$

$$s^2 = \frac{g}{L} \qquad s = \pm \sqrt{\frac{g}{L}}$$

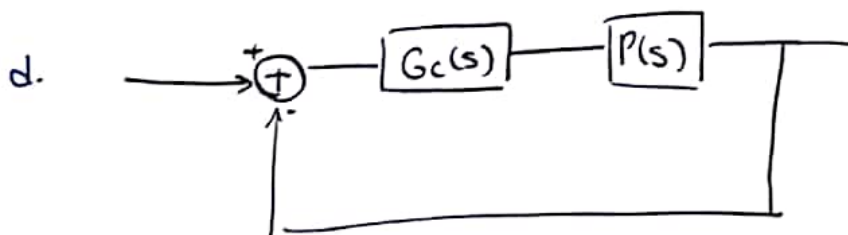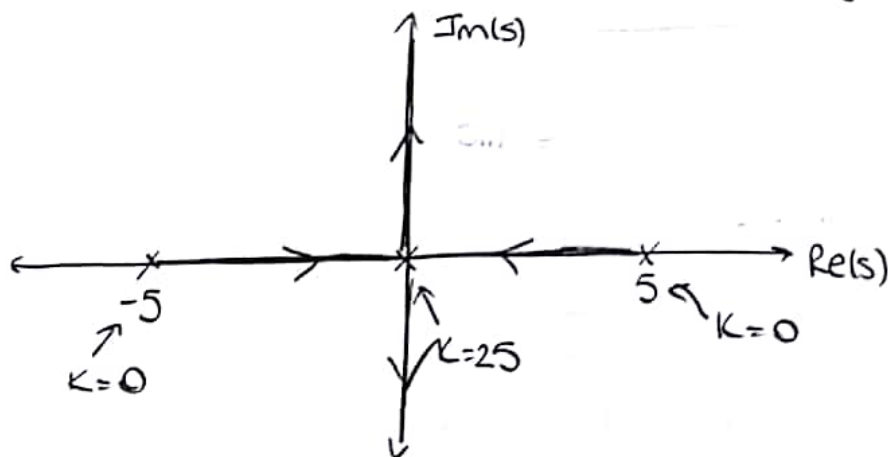Unstable $\Rightarrow$ pole on the positive real part

c. $P(s) = \dfrac{0,3924^2}{s^2(0,3924)^2 - 9.81 \times 0,3924} = \dfrac{0,3924}{s^2 \times 0,3924 - 9,81} = \dfrac{1}{s^2 - 25}$

$T(s) = \dfrac{K P(s)}{K P(s) + 1}$

$= \dfrac{K}{K + s^2 - 25}$

Char. eq. $\Rightarrow s^2 + (K - 25) = 0$

$s_{1,2} = \pm \sqrt{(25 - K)}$

$= \pm j \sqrt{K - 25}$ for $K > 25$

d.

$T(s) = \dfrac{G_c(s) P(s)}{G_c(s) P(s) + 1}$

$= \dfrac{G_c(s)}{G_c(s) + s^2 - 25}$

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$$T(s) = \frac{G_c(s)}{G_c(s) + s^2 - 2S}$$

Let $G_c(s) = As$

Routh - Array

s^2	1	$-2S$
S	A	0
1	$25A$	

$\underset{\smile}{}$ All must be positive so choose $A = 2$

$$\boxed{G_c(s) = 2 \cdot s}$$