EE402 Mini Project 1

M. Mert Ankarali[*]
Department of Electrical and Electronics Engineering
Middle East Technical University

Due: 10-Oct-2018, @13:40 AM (Beginning of Class)

1. (20 Points) For each of the following systems with input u and output y, $t \geq 0$, determine whether the system is memoryless, linear, time-invariant, causal, finite-dimensional ? Justify your answers!

   (a) $y(t) = (\sin(t))^3$

   (b) $y(t) = \int_0^t \tau u(\tau) d\tau$

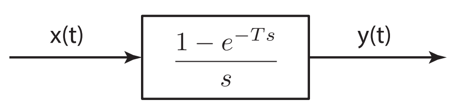
   (c) $y(t) = 2u(t) + 10$

   (d) $y(t) = \cos(t)u(t)$

   (e) $y(t) = u(t - T)$

   (f) $y[n] = u[k - n]$ (Discrete time version of the above system)

   (g) Now let's consider the following input–output dynamical system. The expression inside the block-diagram is the transfer funtion.



2. (20 Points) In this problem, we will review the basic properties of the convolution operation, denoted by $*$, as well as those of the Laplace transform, denoted by $\mathcal{L}$. Consider $f : \mathbb{R} \mapsto \mathbb{R}$, and $g : \mathbb{R} \mapsto \mathbb{R}$, and $h : \mathbb{R} \mapsto \mathbb{R}$.

   (a) Show that $*$ is *associative* that is $(f * g) * h = f * (g * h)$.

   (b) Show that $f(t - \tau) = f(t) * \delta(t - \tau), \tau \geq 0$. This property is referred to as the sifting property of the dirac delta function $\delta(t)$.

   (c) Show that $\mathcal{L}(f * g) = \mathcal{L}(f)\mathcal{L}(g)$.

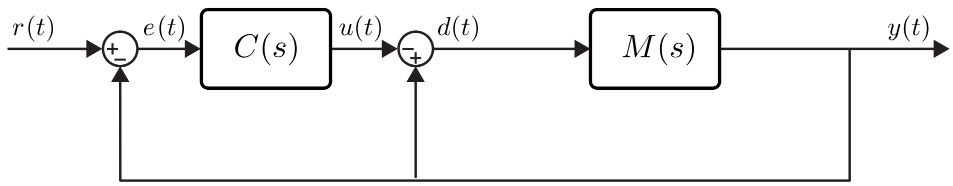
   (d) Show that $\mathcal{L}(f + g) = \mathcal{L}(f) + \mathcal{L}(g)$.

3. (15 Points) Compute $Y(s)/U(s)$ for the following system

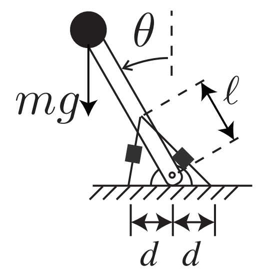
$$y(t) = \int_{t-T}^{t} h(t - \tau)u(\tau)d\tau$$

$$h(t) = \begin{cases} t \text{ if } t > 10 \\ 0 \text{ if } t \leq 0 \end{cases}$$

4. (15 Points) In this question you will analyze the the control system that is illustrated with the block diagram topology given below. Let's assume that $M(s) = \frac{1}{s-a}$, $a > 0$ and $C(s) = \frac{K}{s+1}$. Find the range of K such that the closed-loop system is stable.



5. (30 points) Consider an inverted pendulum of length L, with mass m, that is actuated by an agonist/antagonist linear actuator pair that attach a distance ℓ from the joint / pivot point. One can show



$$\ddot{\theta} - \frac{g}{L}\sin\theta = \frac{1}{mL^2}\tau(t), \tag{1}$$

where $\tau(t)$ is the *net moment* that results from forces applied by the linear actuators.

Suppose the left and right actuators produce linear contractile forces F_L and F_R, respectively. If we assume that $\ell \gg d$, we can have the following simplification:

$$\tau \approx (d\cos\theta)u(t) \tag{2}$$

where $u(t) = \Delta F(t) = F_L(t) - F_R(t)$, the difference between the forces applied by the muscles.

IMPORTANT: For the subsequent problems, use Eq. (2) for the torque unless you want a nightmare of a calculation.

(a) Combine Eq. (1) with (2), make a small-angle approximation to linearize the dynamics, and find a proper ODE that governs the linearized equations of motion.

(b) Compute the transfer function $P(s) = \Theta(s)/U(s)$. Call this the "plant". Find the poles. Is the system stable or unstable and why?

(c) Let

$$g = 9.81\ m/s^2\ ,\ L = 0.3924\ m\ ,\ M = 1\ kg\ ,\ d = 0.3924^2 m$$

Re-evaluate the transfer function using these quantities. Then, draw the root-locus of the plant by hand (based on rules covered in EE302) as well as in MATLAB. Decide if the system can can be controlled with a P controller or not.

(d) Design a "controller" ($G_c(s)$) so that the closed-loop "linear" system is stable and provide the transfer function of the closed-loop system. No other performance specification is given, just the stability condition.

(e) Draw the step and impulse response of the closed loop system using Control System Toolbox of MATLAB. *Hint: "step" and "impulse "commands.* By looking at these responses can you comment on the stability of the closed-loop system.

(f) Plot (in MATLAB) the bode diagrams/plots of the feedforward transfer function $G_c(s) * G(s)$ and find the Phase and Gain margin. Can you comment on the stability of the closed-loop system based on these margins.