İCLAL SATICI
2034437

18/01/2019
EE402

NP7

Q1) $w^T G = \lambda w^T$

G $w^T G H = \lambda w^T H = 0$ $w^T GH = 0$ similarly $w^T A^k B = 0$, so $w^T R = 0$

i.e the system is unreachable

Q2) $Gv = \lambda v$ lets take P is invertible matrix colums of right eigenvector G
D is Diagonal matrix with elements equal to eigenvalue of G

$G = P^{-1} D P$

observability matrix $Q = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^n \end{bmatrix}$ if $Cv = 0$ for any eigen vector of G $\Rightarrow rank(Q) < n$
Then the system is not observable.

Q3 okada

Q4) $x[k+1] = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} x[k] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \underline{u[k]} = \begin{bmatrix} 0 & 1 \\ 1+\alpha & -1+\alpha \end{bmatrix} x[k]$

$\alpha[1 \ \ 1] x[k]$

$\det \begin{bmatrix} \lambda & -1 \\ -1-\lambda & \lambda+1-\alpha \end{bmatrix} = \lambda^2 + \lambda(1-\alpha) - (1-\alpha)$ $\lambda_i = -1 \rightarrow -1 > 0$ X not possible proof

$\lambda_i = 1 \rightarrow \alpha < \frac{1}{2}$

b) $\hat{G} \begin{bmatrix} 0 & 1 \\ 1+\alpha & -1+\alpha \end{bmatrix}$

$\alpha = 1$
$x[1] = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} x[0]$

$\alpha = 1$
$x[2] = \begin{bmatrix} 0 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} x[0] = \begin{bmatrix} 0 & -2 \\ 0 & 0 \end{bmatrix} x[0]$

$x[3] = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 0 & -2 \\ 0 & 0 \end{bmatrix} = \underline{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}} x[0]$

proven

w 3'üncü soyfada 4b görevin

1

3) $G=\begin{bmatrix}0.5 & 0 & 0\\ 0 & 0.5 & 1\\ 0 & 0 & 0.5\end{bmatrix}$ $H=\begin{bmatrix}a\\ b\\ c\end{bmatrix}$ $M=[G^2H|GH|H]$, check whether $\det M \overset{?}{=} 0$

$GH=\begin{bmatrix}0.5a\\ 0.5b+c\\ 0.5c\end{bmatrix}$ $G^2=\begin{bmatrix}0.5 & 0 & 0\\ 0 & 0.5 & 1\\ 0 & 0 & 0.5\end{bmatrix}\begin{bmatrix}0.5 & 0 & 0\\ 0 & 0.5 & 1\\ 0 & 0 & 0.5\end{bmatrix}=\begin{bmatrix}0.25 & 0 & 0\\ 0 & 0.25 & 1\\ 0 & 0 & 0.25\end{bmatrix}$

$G^2H=\begin{bmatrix}0.25a\\ 0.25b+c\\ 0.25c\end{bmatrix}$ $M=\begin{bmatrix}0.25a & 0.5a & a\\ 0.25b+c & 0.5b+c & b\\ 0.25c & 0.5c & c\end{bmatrix}$

$\det M=0.25a\begin{vmatrix}0.5b+c & b\\ 0.5c & c\end{vmatrix}-0.5a\begin{vmatrix}0.25b+c & b\\ 0.25c & c\end{vmatrix}+a\begin{vmatrix}0.25b+c & 0.5b+c\\ 0.25c & 0.5c\end{vmatrix}$

$=0.25a(\cancel{0.5bc}+c^2-\cancel{0.5bc})-0.5a(0.25bc+c^2-\cancel{0.25bc})+a(\underbrace{0.125bc+0.5c^2-0.125bc-0.25c^2}_{0.25c^2})$

$=ac^2(\underbrace{0.25-0.5+0.25}_{0})=0$ the determinant is 0 and does not depend on a,b,c

so the system is not reachable.

$C=[a \quad b \quad c]$ $O=\begin{bmatrix}C\\ CG\\ CG^2\end{bmatrix}$ → check det O

$CG=[a \quad b \quad c]\begin{bmatrix}0.5 & 0 & 0\\ 0 & 0.5 & 1\\ 0 & 0 & 0.5\end{bmatrix}=[0.5a \quad 0.5b \quad b+0.5c]$

$CG^2=[a \quad b \quad c]\begin{bmatrix}0.25 & 0 & 0\\ 0 & 0.25 & 1\\ 0 & 0 & 0.25\end{bmatrix}=[0.25a \quad 0.25b \quad b+0.25c]$

$O=\begin{bmatrix}a & b & c\\ 0.5a & 0.5b & b+0.5c\\ 0.25a & 0.25b & b+0.25c\end{bmatrix}$ → $\det O=a\begin{vmatrix}0.5b & b+0.5c\\ 0.25b & b+0.25c\end{vmatrix}-b\begin{vmatrix}0.5a & b+0.5c\\ 0.25a & b+0.25c\end{vmatrix}+c\begin{vmatrix}0.5a & 0.5b\\ 0.25a & 0.25b\end{vmatrix}$

$=a(0.5b^2+0.125bc-0.25b^2-0.125bc)-b(0.5ab+0.125ac-0.25ab-0.125ac)+c(\underbrace{0.125ab-0.125ab}_{0})$

$=ab^2(0.25-0.25)=0$ → again $\det O=0$ independently a,b,c, so the system can not be observable. I also see that M first and third row are $0.25a \quad 0.5a \quad a$, $0.25c \quad 0.5c \quad c$ and because of them the det is directly 0. And for O, the same case happens because of first and second column.

⇐ Q4 page 2

(2)

a)

$u(k) \mapsto$ ... $x(1) = \begin{bmatrix} x_1(1) \\ x_2(1) \end{bmatrix}$ $y(t)=x(t)$ $(C=I)$

b-Agosni Analyze $x[0]=\begin{bmatrix} a \\ b \end{bmatrix}$ $y[0]=[a+b]$ $u[0]=[-a-b]$

$x[1]=\underbrace{\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}}_{\begin{bmatrix} b \\ a-b \end{bmatrix}}+\underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}[-a-b]}_{\begin{bmatrix} 0 \\ -a-b \end{bmatrix}}=\begin{bmatrix} b \\ -2b \end{bmatrix}$ $y[1]=[-b]$ $u[1]=[-3b]$

$x[2]=\underbrace{\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} b \\ -2b \end{bmatrix}}_{\begin{bmatrix} -2b \\ 3b \end{bmatrix}}+\underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}[-3b]}_{\begin{bmatrix} 0 \\ -3b \end{bmatrix}}=\begin{bmatrix} -2b \\ 0 \end{bmatrix}$ $y[2]=[-2b]$ $u[2]=[2b]$

$x[3]=\underbrace{\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -2b \\ 0 \end{bmatrix}}_{\begin{bmatrix} 0 \\ -2b \end{bmatrix}}+\underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}[2b]}_{\begin{bmatrix} 0 \\ 2b \end{bmatrix}}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $y[3]=[0]$ $u[3]=0$

I show that any initial condition return to the origin.

$x[k+1]=\begin{bmatrix} -3/4 & 7/4 \\ -1/4 & -3/4 \end{bmatrix}x[k]+\begin{bmatrix} -1 \\ 1 \end{bmatrix}u[k]$

5) $p^*(z)=z^2$ $K=[k_1 \ \ k_2]$

$x[k+1]=Gx[k]+Hu[k]$ $u \to -Kx[k]$

$x[k+1]=\underbrace{(G-HK)}_{\hat{G}}x[k]$

$\det(zI-(G-HK))$

$G-HK=\begin{bmatrix} -3/4 & 7/4 \\ -1/4 & -3/4 \end{bmatrix}-\underbrace{\begin{bmatrix} -1 \\ 1 \end{bmatrix}[k_1 \ \ k_2]}_{\begin{bmatrix} -k_1 & -k_2 \\ k_1 & k_2 \end{bmatrix}}$

$\det(zI-(G-HK))=\begin{bmatrix} z+\frac{3}{4}-k_1 & -\frac{7}{4}-k_2 \\ \frac{1}{4}+k_1 & z+\frac{3}{4}+k_2 \end{bmatrix}$

$=(z+\frac{3}{4}-k_1)(z+\frac{3}{4}+k_2)-\left(\frac{1}{4}+k_1\right)\left(-\frac{7}{4}-k_2\right)$

$=z^2+z\left(\frac{3}{2}-k_1+k_2\right)+\frac{9}{16}+\frac{3}{4}k_2-\frac{3}{4}k_1\cancel{-k_1k_2}+\frac{7}{16}+k_2\frac{1}{4}+k_1\frac{7}{4}\cancel{+k_1k_2}$

match the eq

$=z^2+z\left(\frac{3}{2}-k_1+k_2\right)+1+k_2+k_1$ $\doteq z^2$

$-k_1+k_2=-\frac{3}{2}$ $k_2+k_1=-1$

$k_1-k_2=\frac{3}{2}$

$k_1+k_2=-1$

$2k_1=\frac{1}{2}$

$\boxed{k_1=\frac{1}{4} \quad k_2=-\frac{5}{4}}$

$\hat{G}=\begin{bmatrix} -3/4 & 7/4 \\ -1/4 & -3/4 \end{bmatrix}-\underbrace{\begin{bmatrix} -1 \\ 1 \end{bmatrix}[1/4 \ \ -5/4]}_{\begin{bmatrix} -1/4 & 5/4 \\ 1/4 & -5/4 \end{bmatrix}}=\begin{bmatrix} -1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$

so $x[0]=\begin{bmatrix} 4 \\ 0 \end{bmatrix}$

$x[1]=\begin{bmatrix} -1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \end{bmatrix}=\begin{bmatrix} -2 \\ -2 \end{bmatrix}$

$\hat{G}^2=\begin{bmatrix} -1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}\begin{bmatrix} -1/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$x[2]=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x[3]=$ "

b) i) $u[k] = x_3[k] = -K^* x[k-1]$

$x_3[k+1] = -K^* x[k] = -k_1 x_1[k] - k_2 x_2[k]$

$$x[k+1] = \begin{bmatrix} -\frac{3}{4} & \frac{7}{4} & -1 \\ -\frac{1}{4} & -\frac{3}{4} & 1 \\ -\frac{1}{4} & \frac{5}{4} & 0 \end{bmatrix} x[k] \qquad x[k] = \begin{bmatrix} x_1[k] \\ x_2[k] \\ x_3[k] \end{bmatrix}$$

ii. Compute eigenvalues

$$\det\left(\begin{bmatrix} \lambda + \frac{3}{4} & -\frac{7}{4} & 1 \\ \frac{1}{4} & \lambda + \frac{3}{4} & -1 \\ \frac{1}{4} & -\frac{5}{4} & \lambda \end{bmatrix}\right) = 0$$

$$\left(\lambda + \frac{3}{4}\right)\left(\underbrace{\lambda\left(\lambda + \frac{3}{4}\right) - \frac{5}{4}}_{\lambda^2 + \frac{3}{4}\lambda - \frac{5}{4}}\right) + \frac{7}{4}\left(\frac{1}{4}\cdot\lambda + \frac{1}{4}\right) + \left(-\frac{5}{16} - \left(\frac{\lambda}{4} + \frac{3}{16}\right)\right)$$

$$= \lambda^3 + \frac{3}{4}\lambda^2 - \frac{5}{4}\lambda + \frac{3}{4}\lambda^2 + \frac{9}{16}\lambda - \frac{15}{16} + \frac{7}{16}\lambda + \frac{7}{16} - \frac{1}{2} - \frac{\lambda}{4}$$

$$= \lambda^3 + \frac{3}{2}\lambda^2 - \frac{1}{2}\lambda - 1 \longrightarrow \text{using matlab}$$

$-1 \quad 0.7808 \quad -1.2808$

$$= (\lambda + 1)\left(\lambda^2 + \frac{1}{2}\lambda - 1\right)$$

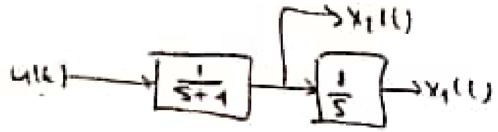
$\frac{1}{4} - 4 \cdot -1 = \frac{17}{4}$ $\qquad \frac{-\frac{1}{2} \pm \sqrt{\frac{17}{4}}}{2}$ → 0.7808, -1.2808

-1

-1

iii.

a)



$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}^T$ $\quad y(t)=x(t) \quad (C=I)$

[illegible] $\dot{x}(t)=Ax(t)+Bu(t)$
$y(t)=x(t)$

$X_2(s)=\frac{1}{s+1}U(s) \qquad X_1(s)=\frac{1}{s}X_2(s)$

$sX_2(s)=U(s)-X_2(s) \qquad sX_1(s)=X_2(s)$

$\downarrow \mathcal{L}^{-1}\{\} \qquad \downarrow \mathcal{L}^{-1}\{\}$

$\dot{x}_2(t)=u(t)-x_1(t) \qquad \dot{x}_1(t)=x_2(t)$

$$\dot{x}(t)=\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}}_{A} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{} u(t)$$

b) $T=0.5 \qquad x[k]=[x_1[k],\ x_2[k]]^T \qquad y[k]=x[k]$

$x[k+1]=Gx[k]+Hu[k]$

$G=e^{AT} \qquad H=\int_0^T e^{\lambda A}d\lambda \cdot B$

$$e^{At}=\mathcal{L}^{-1}\{[sI-A]^{-1}\}=\mathcal{L}^{-1}\left\{\begin{bmatrix} s & -1 \\ 0 & s+1 \end{bmatrix}^{-1}\right\}=\mathcal{L}^{-1}\left\{\frac{1}{s(s+1)}\begin{bmatrix} s+1 & 1 \\ 0 & s \end{bmatrix}\right\}$$

$\frac{1}{s}-\frac{1}{s+1}$

$$=\mathcal{L}^{-1}\left\{\begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+1)} \\ 0 & \frac{1}{s+1} \end{bmatrix}\right\}=\begin{bmatrix} 1 & 1-e^{-t} \\ 0 & e^{-t} \end{bmatrix} \qquad G=e^{AT}\big|_{0.5}=\begin{bmatrix} 1 & 1-e^{-0.5} \\ 0 & e^{-0.5} \end{bmatrix}$$

0.393

0.607

$$H=\int_0^T \begin{bmatrix} 1 & 1-e^{-\lambda} \\ 0 & e^{-\lambda} \end{bmatrix} d\lambda \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \int_0^T \begin{bmatrix} 1-e^{-\lambda} \\ e^{-\lambda} \end{bmatrix} d\lambda = \begin{bmatrix} \lambda+e^{-\lambda} \\ -e^{-\lambda} \end{bmatrix}\Bigg|_0^T = \begin{bmatrix} T+e^{-T}-1 \\ -e^{-T}+1 \end{bmatrix}$$

$(T=0.5)$
$$= \begin{bmatrix} 0.106 \\ 0.393 \end{bmatrix}$$

c) $p^*(z)=z^2 \qquad K=[k_1 \quad k_2]$

$x[k+1]=Ax[k]+B\underbrace{u[k]}_{-Kx[k]}=(A-BK)x[k]$

$$\begin{bmatrix} 1 & 0.393 \\ 0 & 0.607 \end{bmatrix} - \begin{bmatrix} 0.106 \\ 0.393 \end{bmatrix}[k_1 \quad k_2] = \begin{bmatrix} 1-0.106k_1 & 0.393-0.106k_1 \\ -0.393k_2 & 0.607-0.393k_2 \end{bmatrix}$$

$$\det(zI-A_{cl})=\begin{vmatrix} z-1+0.106k_1 & -0.393+0.106k_1 \\ 0.393k_2 & z-0.607+0.393k_2 \end{vmatrix} = z^2+z(-0.607+0.393k_2-1+0.106k_1)$$

$$[illegible](0.393))k_1k_2+(0.393)^2k_2-(0.393)\cancel{(0.106)k_1k_2}$$

Scanned by CamScanner