EE402 Discrete Time Systems
MP-4

1.

$$G_{OL} = K_p \frac{0.1275(z+1)}{z^2 - 2.255z + 1}$$

$$G_{OL} = K_p \frac{0.1275(z+1)}{(z-1.648)(z-0.606)}$$

Let us find break in/away points

$$\frac{d}{d\sigma}\frac{1}{G_{OL}(\sigma)} = \sigma^2 + 2\sigma - 3.25 = 0$$

$$\sigma_{1,2} = 1,06 \ \& \ 3,06$$

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z=tf([1 0],[1],0.1)

Ga=(z+1)/(z^2-2.255*z+1)

rlocus(Ga)
```

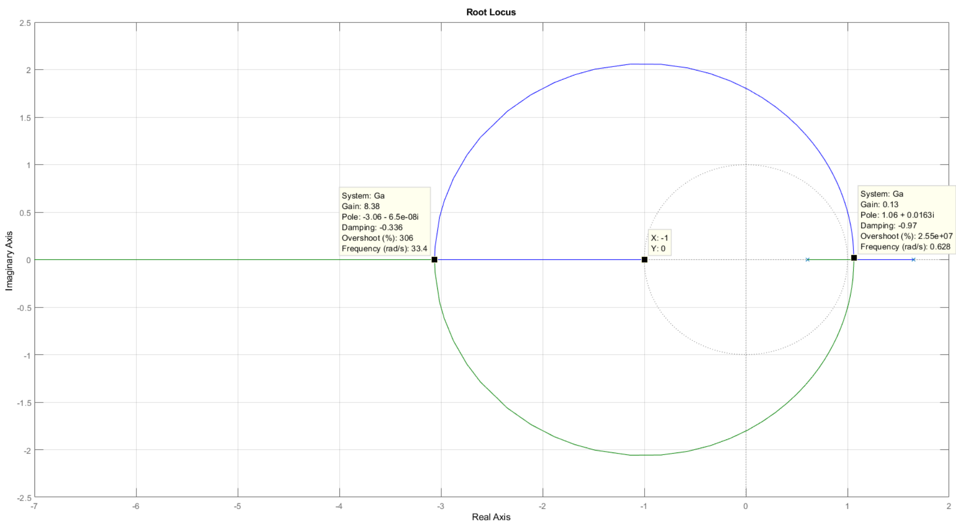

Figure 1: -

2.

$$G_{OL} = K_p \frac{0.1275(z+1)}{z^2 - 1.215z - 0.04}$$

$$G_{OL} = K_p \frac{0.1275(z+1)}{(z-1.247)(z+0.032)}$$

Let us find break in/away points

$$\frac{d}{d\sigma}\frac{1}{G_{OL}(\sigma)} = \sigma^2 + 2\sigma - 1.275 = 0$$

$$\sigma_{1,2} = 0,47 \;\&\; -2,474$$

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z=tf([1  0],[1],0.1)

Gb=(z+1)/(z^2-1.215*z-0.04)

rlocus(Gb)
```

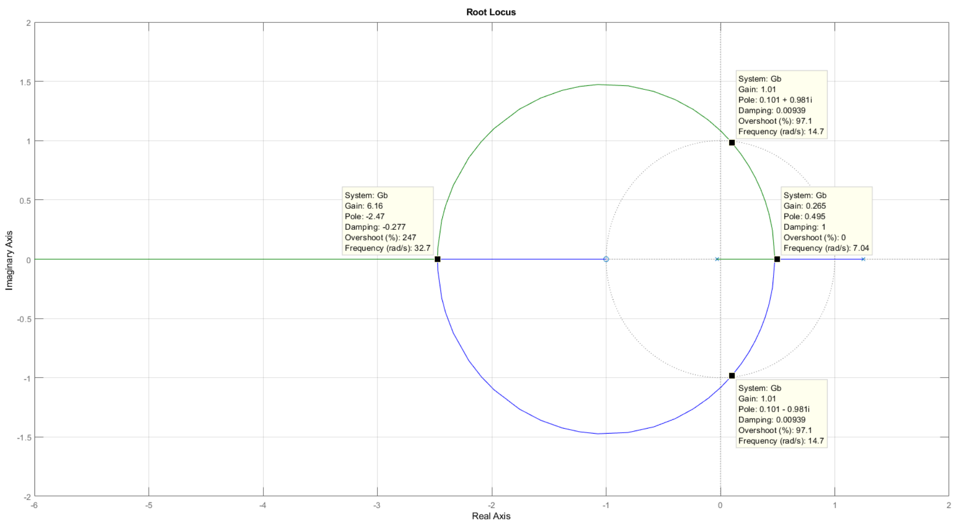


Figure 2: -

3.

$$G_{OL} = \frac{2(0.1275)(z+1))}{(z^2+2.255z+1) + K_D(0.52)(z-1)}$$

characteristic equation

$$1 + G_{OL} = 0$$

or

$$(z^2 + 2.255z + 1) + K_D(0.52)(z-1) + 2(0.1275)(z+1) = 0$$

$$1 + K_d \frac{(0.52)(z-1)}{(z^2 + 2.255z + 1) + 2(0.1275)(z+1)} = 0$$

$$\hat{G}_{OL} = \frac{(0.52)(z-1)}{(z^2 + 2.255z + 1) + 2(0.1275)(z+1)}$$

$$\hat{G}_{OL} = \frac{0.52(z-1}{z^2 - 2z + 1.255} = \frac{0.52(z-1))}{(z-1-0.5j)(z-1+0.5j)}$$

Let us find break in/away points

$$\frac{d}{d\sigma}\frac{1}{G_{OL}(\sigma)} = z^2 - z + 0.745 = 0$$

$$\sigma_{1,2} = 0,49 \ \& \ 1.5$$

since $\sigma = 1.5$ is out of root locus, it is unnecessary

```
z=tf([1  0],[1],0.1)

Gc=(z-1)/(z^2-2*z+1.255)

rlocus(Gc)
```

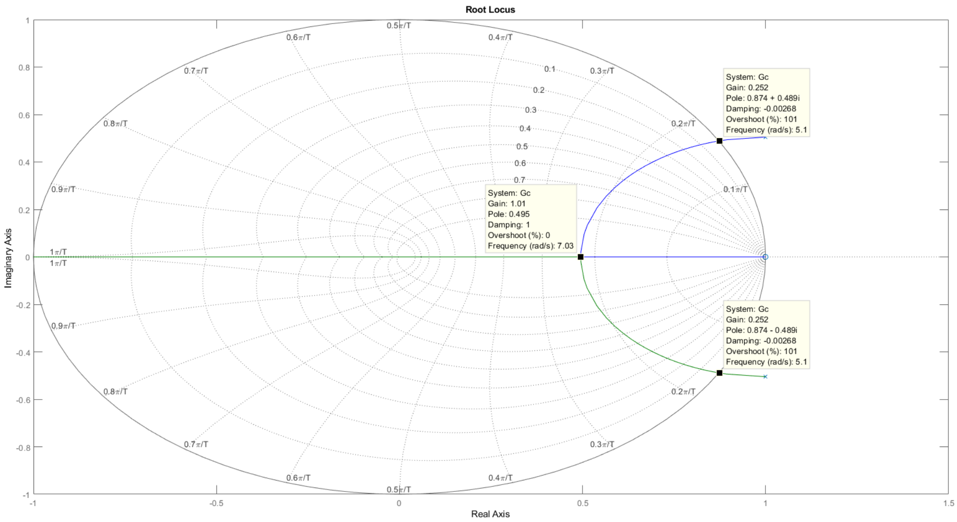


Figure 3: -

4. (a) -

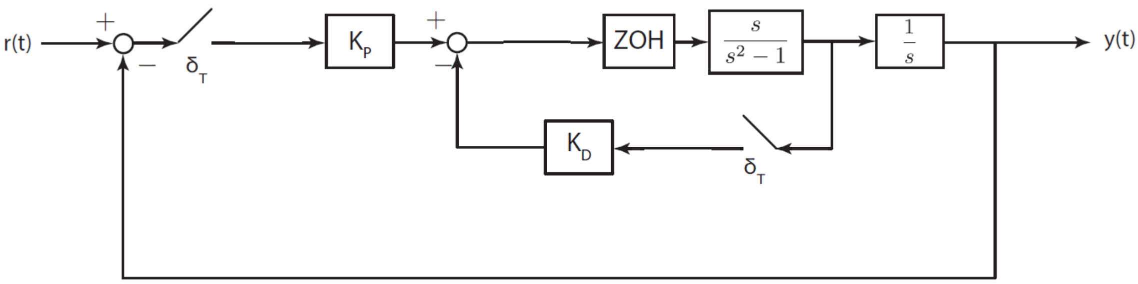


Figure 4: -

$$e_{ss} = \lim_{z \to 1} \frac{1 + K_D G_x(z)}{1 + K_D G_x(z) + K_p G_y(z)} = \frac{1}{1 - K_p}$$

(b) -

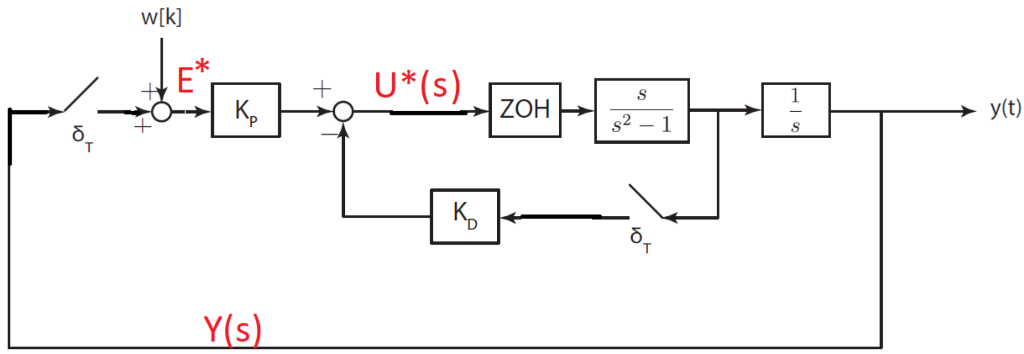


Figure 5: -

$$U*(s) = K_p E^*(s) - U^*(s) G_x^*(s) K_d$$

$$\frac{U^*(s)}{E^*(s)} = \frac{K_p}{1 + K_d G_x^*(s)}$$

$$E^*(s) = U^*(s) G_y(s) + W^*(s)$$

$$E^*(s) = \frac{K_p G_y(s)}{1 + K_d G_x^*(s)} E^*(s) + W^*(s)$$

$$\frac{E^*(s)}{W^*(s)} = \frac{1}{1 + \dfrac{K_p G_y(s)}{1 + K_d G_x^*(s)}}$$

$$\frac{Y^*(s)}{U^*(s)} = G_y(s)$$

$$\frac{Y^*(s)}{E^*(s)} = \frac{K_p G_y(s)}{1 + K_d G_x^*(s)}$$

$$\frac{Y^*(s)}{W^*(s)} = \frac{\dfrac{K_p G_y^*(s)}{1 + K_d G_x^*(s)}}{1 + \dfrac{K_p G_y^*(s)}{1 + K_d G_x^*(s)}} = \frac{K_p G_y^*(s)}{1 + K_D G_x^*(s) + K_p G_y^*(s)}$$

$$\frac{Y(z)}{W(z)} = \frac{K_p G_y(z)}{1 + K_D G_x(z) + K_p G_y(z)}$$

Remember that, the $G_x(z)$ and $G_Y(z)$ were found earlier (@ MP3) as

$$\boxed{G_y(z) \approx \frac{0.1275(z+1)}{z^2 - 2.255z + 1}}$$

$$\boxed{G_x(z) \approx \frac{0.52\,(z-1)}{z^2 - 2.255z + 1}}$$

$$y_s s = \lim_{z\to 1}(1 - z^{-1})W(z)\frac{K_p G_y(z)}{1 + K_D G_x(z) + K_p G_y(z)}$$

with $W(z) = \frac{1}{1-z^{-1}}$

$$y_{ss} = \frac{K_p \lim_{z\to 1} G_y(z)}{1 + K_D \lim_{z\to 1} G_x(z) + K_p \lim_{z\to 1} G_y(z)}$$

$$\lim_{z\to 1} G_x(z) = 0$$

$$\lim_{z\to 1} G_y(z) = \frac{0.1275(2)}{1 - 2.255 + 1} = -1$$

$$y_{ss} = \frac{K_p}{K_p - 1}$$

(c) -

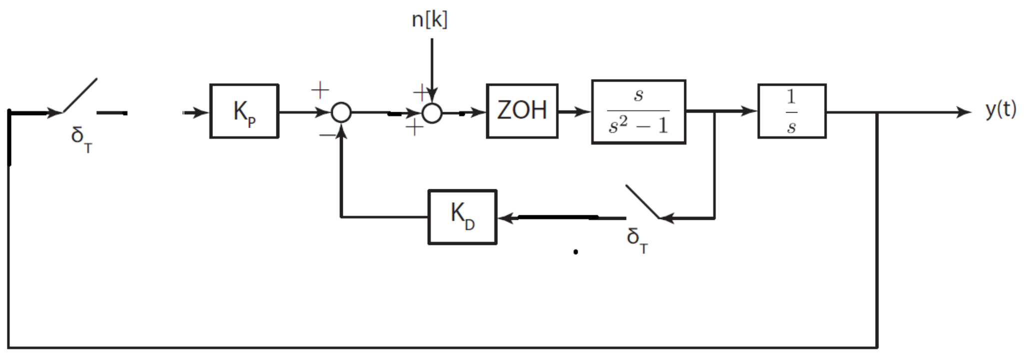


Figure 6: -

$$e_{ss} = \lim_{z\to 1}\frac{-K_D G_x(z)}{1 + K_D G_x(z) + K_p G_y(z)} = \frac{K_D}{1 - K_p}$$

(d) -

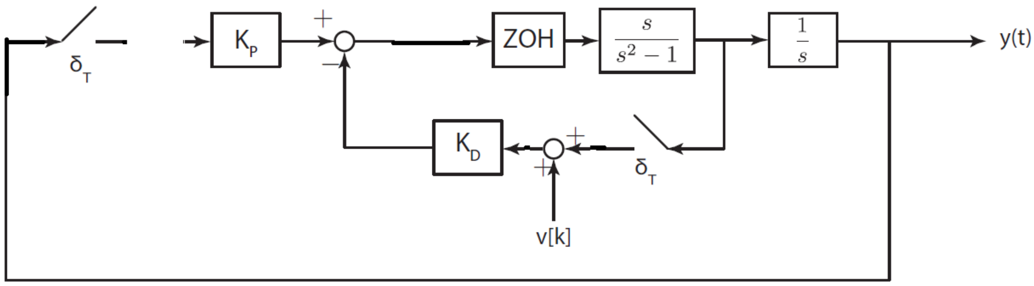
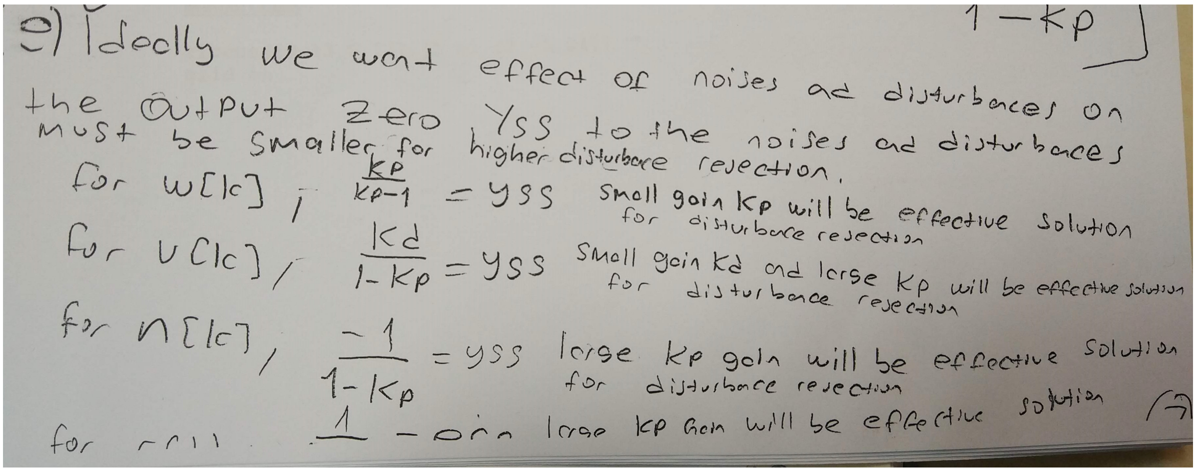


Figure 7: -

$$e_{ss} = \lim_{z \to 1} \frac{G_x(z)}{1 + K_D G_x(z) + K_p G_y(z)} = \frac{-1}{1 - K_p}$$

(e) -



1 − Kp

e) Ideally we want effect of noises and disturbances on the output zero y_{ss} to the noises and disturbances must be smaller for higher disturbance rejection.

for w[k]; $\frac{Kp}{Kp-1} = y_{ss}$ Small gain Kp will be effective solution for disturbance rejection

for v[k]; $\frac{Kd}{1-Kp} = y_{ss}$ Small gain Kd and large Kp will be effective solution for disturbance rejection

for n[k]; $\frac{-1}{1-Kp} = y_{ss}$ large Kp gain will be effective solution for disturbance rejection

for r[k] $\frac{1}{\ }$ ... large Kp gain will be effective solution

Figure 8: -