

## EE402 Discrete Time Systems

### MP-5

1. To ease the design process, the Tustin transform will be used throughout this mini-project.

```

1 %T=0.1
2 %z1 = (1+T*s)/(1-T*s)
3 %G_z_T= (0.047*z1+0.044)/(z1^2-1.8*z1+0.82) % Find Tustin
  Tr.
4 G_z_T= d2c(G_z, 'tustin')
5 bode(G_z_T)
6
7 hold off
8 figure()
9 bode(G_z_T)
10 grid on
11
12 %% for PM 25-30

```

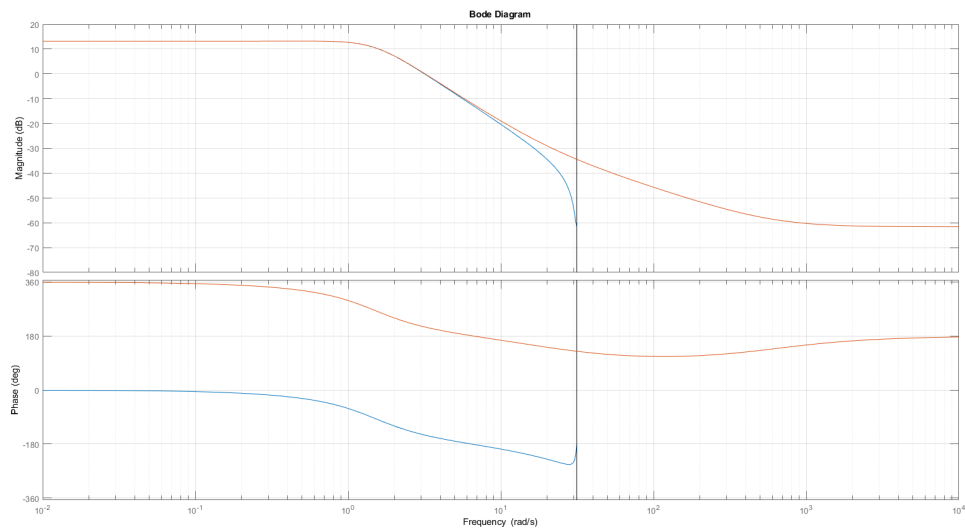


Figure 1: Bode Plot of Discrete TF and of its Tustin Transform

Let us now find the uncompensated phase margin for  $T=1$ .



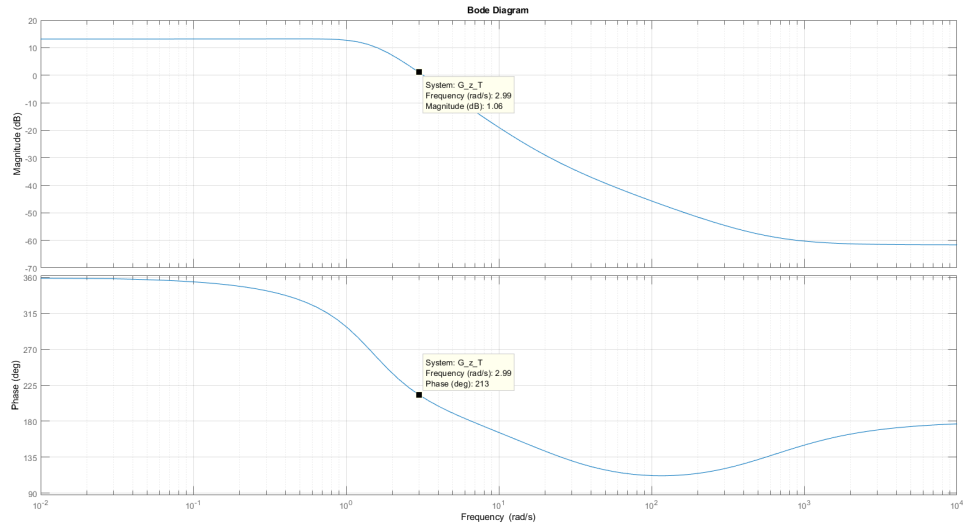


Figure 2: -

As can be seen from the *Figure 2* that the  $PM \approx 29$ .  
for desired  $PM = [10, 15]$

$$\Delta PM \approx [-19, -14]$$

For negative phase margin, let us use the following compensator type

$$G_C(s) = \frac{1}{K_{Lead} \frac{T_L a s + 1}{T_L / a s + 1}}$$

After trying couple of compensator bode plot for  $T=1$  and varying  $a$ 's.  $a = 1.6$  seems like a good choice. The bode plot for a compensator with  $a = 1.6$  can be seen at *Figure 3*



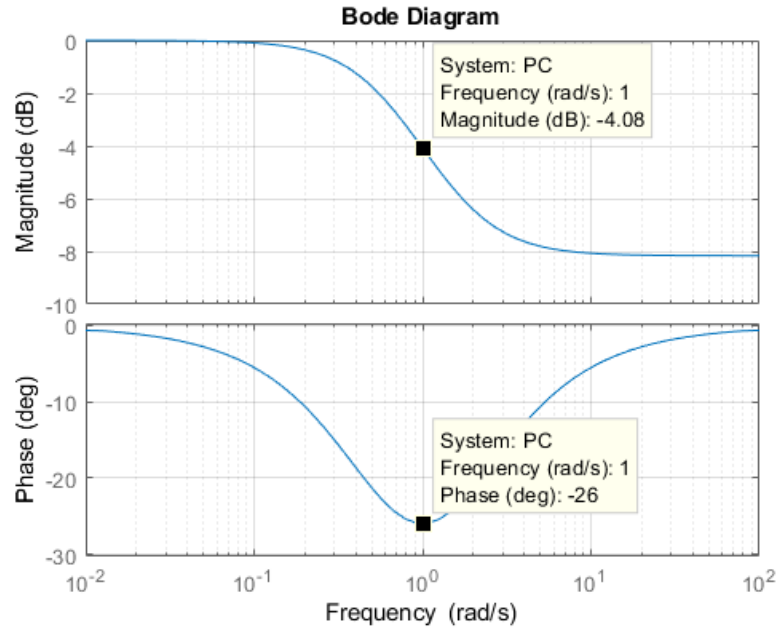


Figure 3: Bode plot for a compensator with  $a = 1.6$

To find the proper  $T_L$  value, the bode plot of the uncompensated system was investigated. At a frequency of  $w_x = 4 \text{ rad/sec}$ , the system has a gain of

$$20\log_{10}(1/a) = -4.08$$

Thus;

$$T_L = 1/w_x = 1/4 = 0.25$$



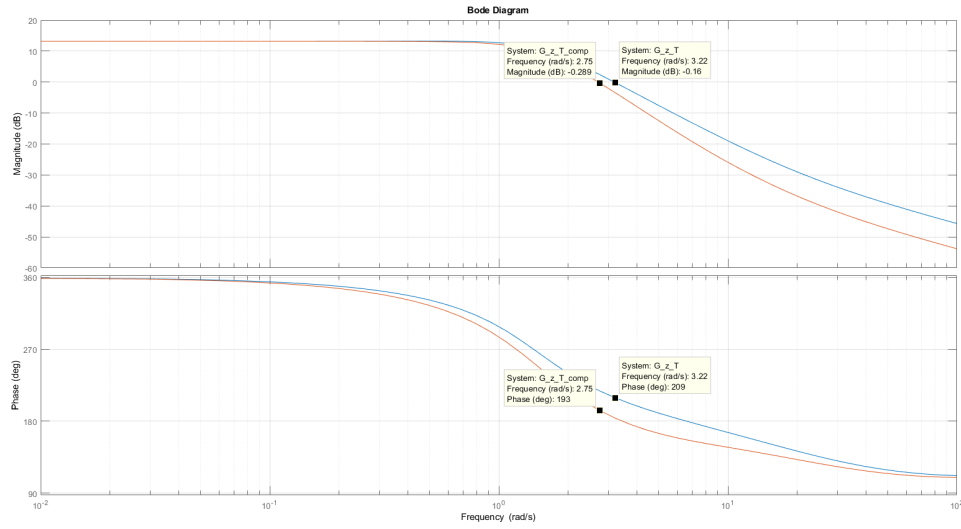


Figure 4: Bode Plots of compensated and uncompensated system

From the bode plots at *Figure 3*, it can be observed that the new compensated system has a phase margin of 13 degree which is desired region.

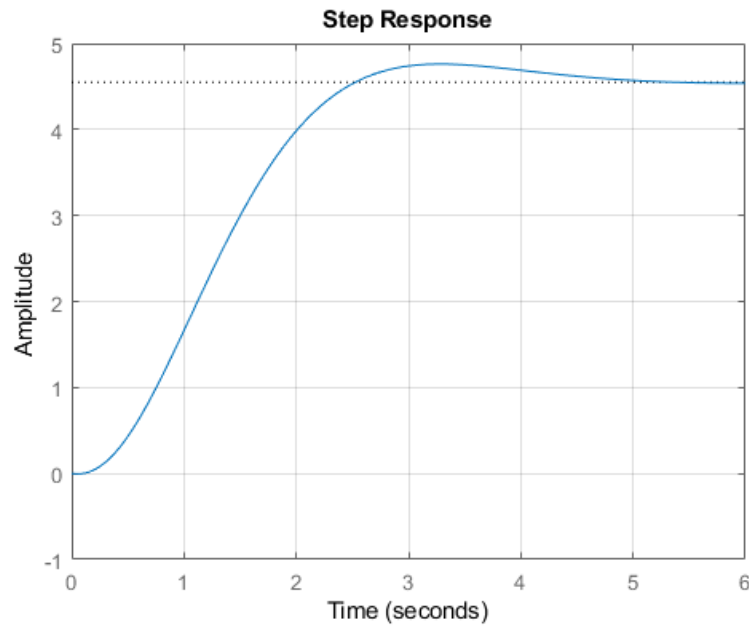


Figure 5: Step Response of the compensated system

Thus, the compensator can be designed to be as



$$G_C(s) = \frac{1}{K_{Lead} \frac{T_L a s + 1}{T_L / a s + 1}}$$

$$K_{Lead} = 1$$

$$T_L = 0.25$$

$$a = 1.6$$

2. for  $PM = [25, 30]$

$$\Delta PM \approx [-4, 1]$$

Following the similar steps, choosing  $a = 1.1$  seems good,

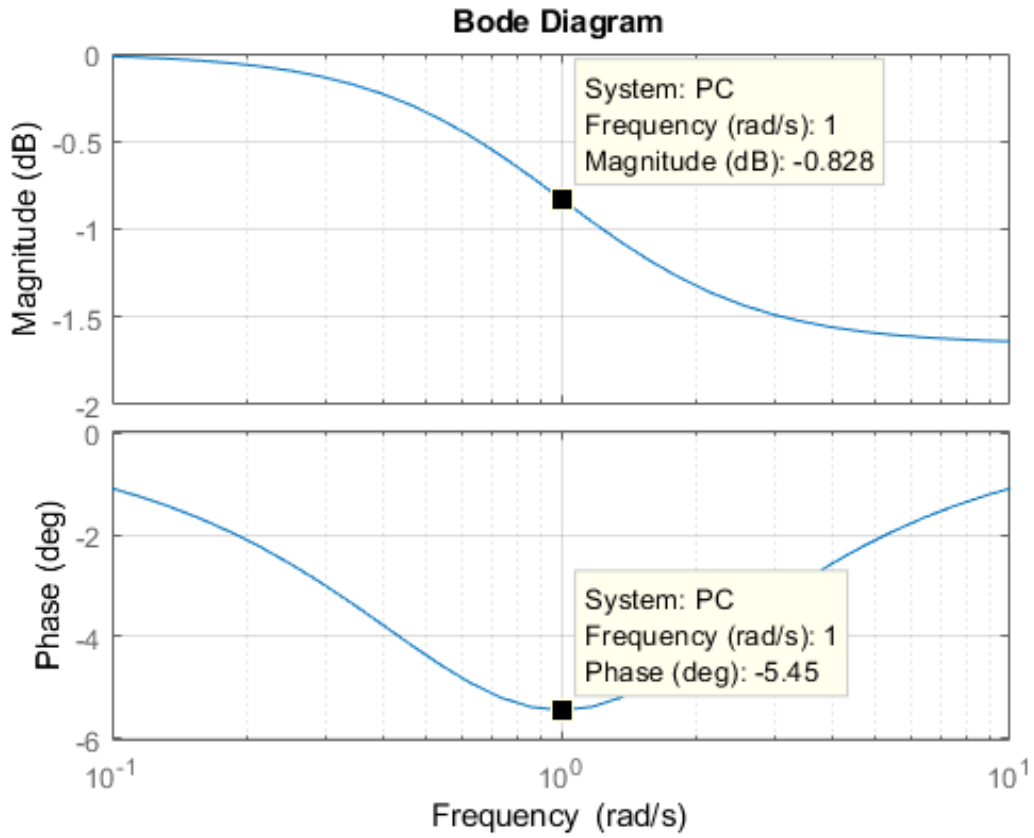


Figure 6: Bode plot for a compensator with  $a = 1.1$

$$T_L = 1/2.82. = 0.35$$



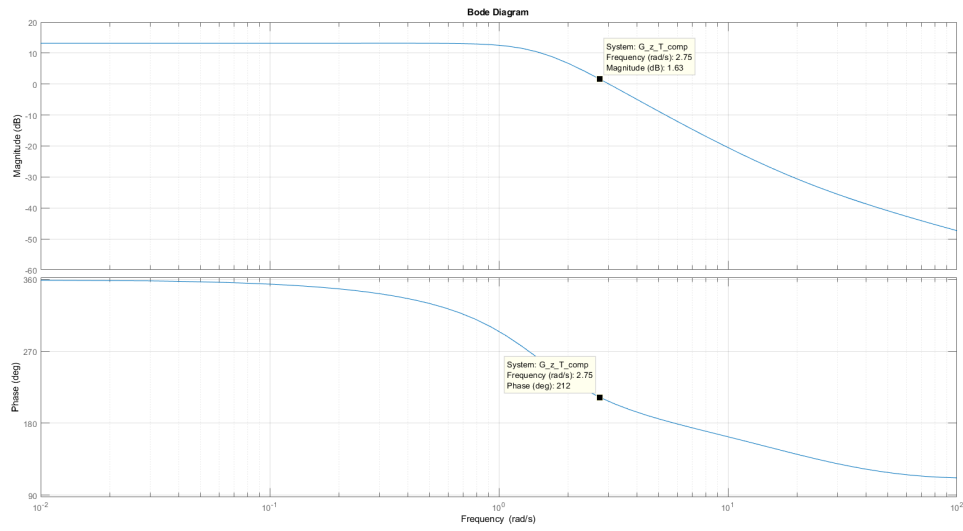


Figure 7: Bode Plots of compensated system

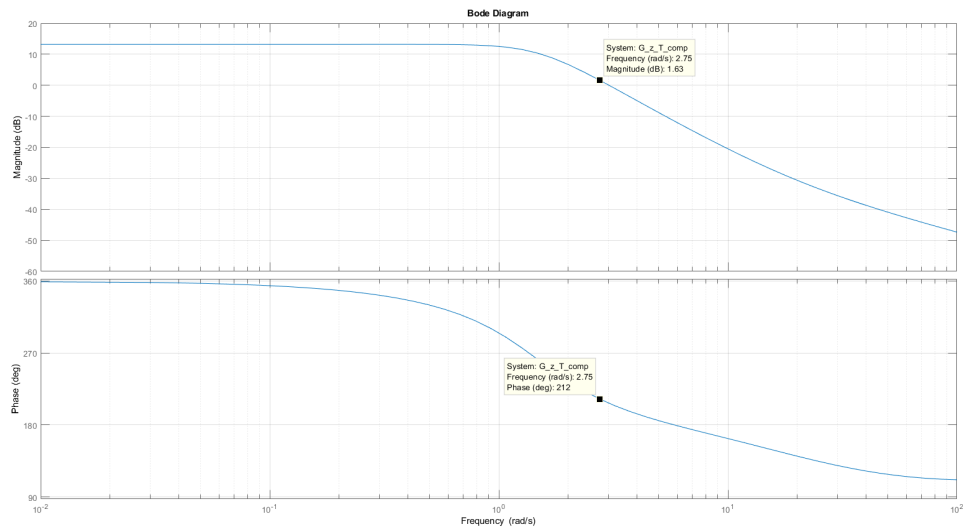


Figure 8: Bode Plots of compensated and uncompensated system

From *Figures 7 and 8*, the new phase margin for compensated system can be approximately found to be as 28 degree degree.



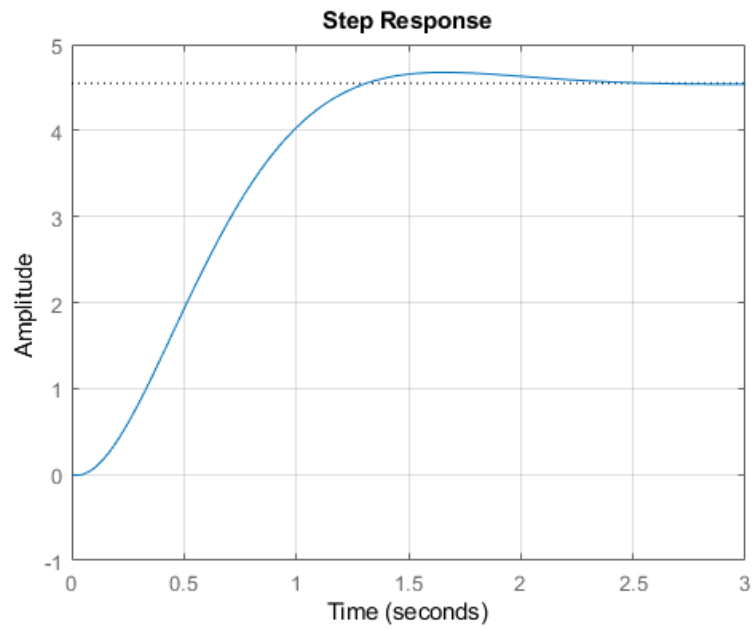


Figure 9: Step Response of the compensated system

Thus, the compensator can be designed to be as

$$G_C(s) = 1 \frac{1}{K_{Lead} \frac{T_L a s + 1}{T l / a s + 1}}$$

$$K_{Lead} = 1$$

$$T L = 0.35$$

$$a = 1.1$$



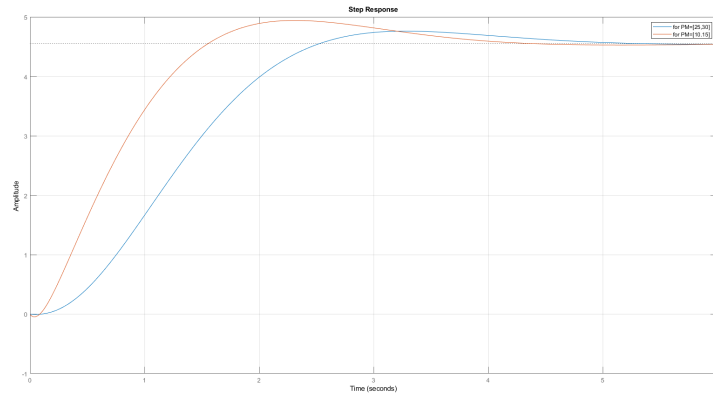


Figure 10: Difference in Step Responses of both compensated systems

As the phase margin is decreased the settling time is increased, the overshoot decreased. The steady state error stayed same.

### 3. $T=0.5$

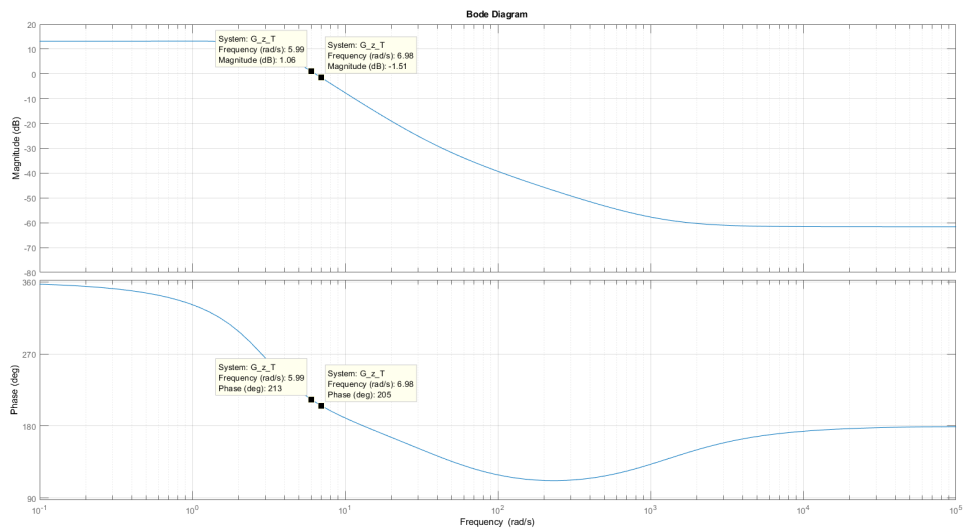


Figure 11: -

$$PM \approx 30$$

for  $PM = [10, 15]$





$$\Delta PM \approx [-20, -15]$$

Choose  $a = 1.55$  seems good,

$$T_L = 1/4 = 0.24$$

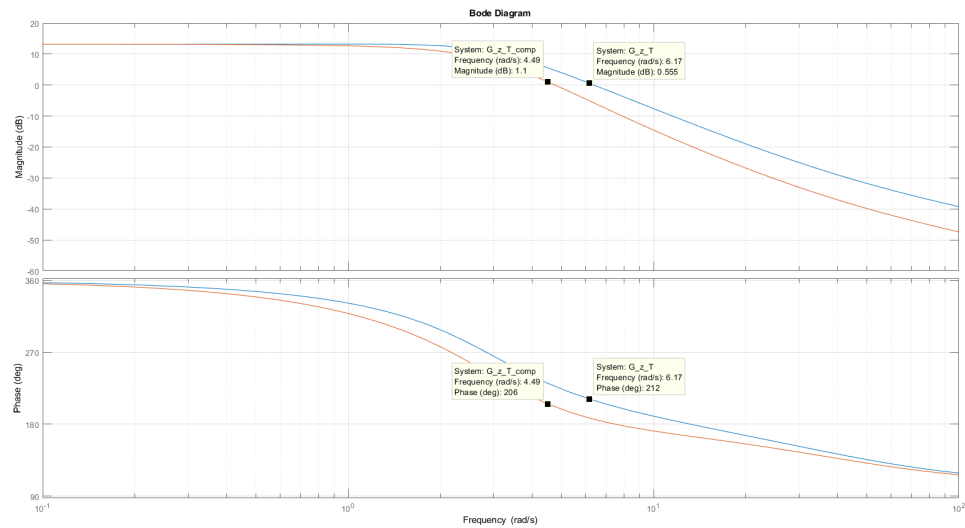


Figure 12: Bode Plots of compensated and uncompensated system

New phase margin is 14 degree



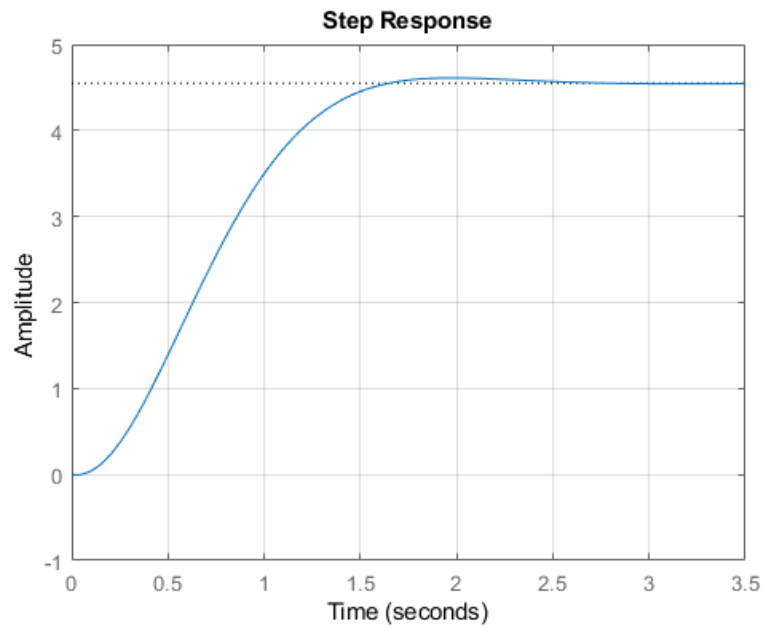


Figure 13: Step Response of the compensated system

$$G_C(s) = 1 \frac{1}{K_{Lead} \frac{T_L a s + 1}{T l / a s + 1}}$$

$$K_{Lead} = 1$$

$$T_L = 0.24$$

$$a = 1.55$$

4. for  $PM = [25, 30]$

$$\Delta PM \approx [-5, 0]$$

Choose  $a = 1.14$  seems good,

$$T_L = 0.37$$



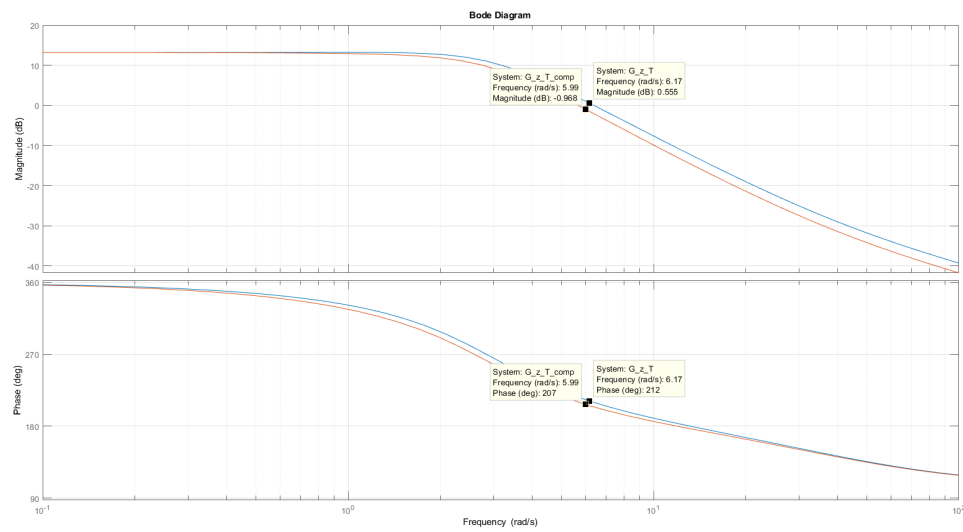


Figure 14: Bode Plots of compensated and uncompensated system

New phase margin is approximately 29 degree degree

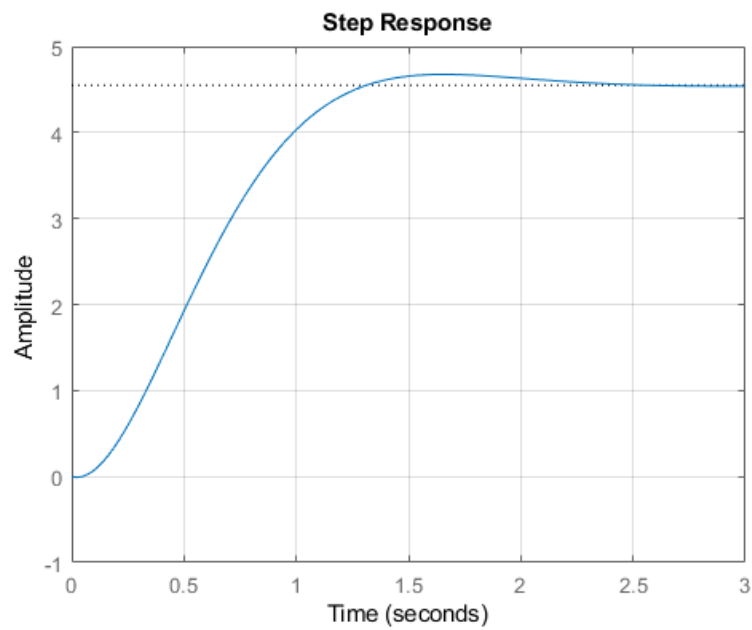


Figure 15: Step Response of the compensated system



$$G_C(s) = 1 \frac{1}{K_{Lead} \frac{T_L a s + 1}{T l / a s + 1}}$$

$$K_{Lead} = 1$$

$$T L = 0.37$$

$$a = 1.14$$

5. -

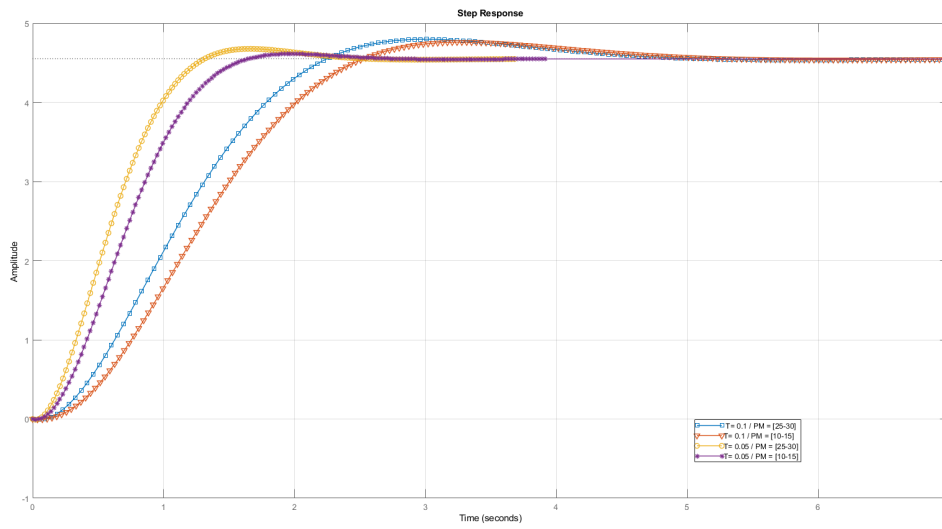


Figure 16: Difference in Step Responses of both compensated systems

As the phase margin is decreased the settling time is increased, the overshoot decreased. The steady state error stayed same. As the sampling time is increased from  $T = 0.05$  to  $T = 0.1$ , the settling time again increased and the overshoot is decreased. The steady state error is not changed.



# Appendices

## A Source Code For Matlab Parts

```

1 %% - T=0.1
2 hold off
3
4 z = tf ([1 0],[1],0.1)
5
6 G_z= (0.047*z+0.044)/(z^2-1.8*z+0.82)
7 bode(G_z)
8 grid on
9 hold on
10 %T=0.1
11 %z1 = (1+T*s)/(1-T*s)
12 %G_z_T= (0.047*z1+0.044)/(z1^2-1.8*z1+0.82) % Find Tustin Tr.
13 G_z_T= d2c(G_z,'tustin')
14 bode(G_z_T)
15
16 hold off
17 figure()
18 bode(G_z_T)
19 grid on
20
21 %% for PM 25-30
22 hold off
23
24 a=1.1
25 K=1
26 T_l=0.35
27 PC = K*(T_l*a*s+1)/(T_l/a*s+1)
28 PC=1/PC
29 bode(PC)
30 grid on
31 hold off
32 G_z_T_comp=G_z_T*PC
33 bode(G_z_T,{0,100})
34 grid on
35 hold on
36 bode(G_z_T_comp,{0,100})
37

```



```

38 hold off
39 figure()
40 step(G_z_T_comp)
41 grid on
42
43 %% for PM 10-15
44 hold off
45
46 a=1.6
47 K=1
48 T_l=0.25
49 PC = K*(T_l*a*s+1)/(T_l/a*s+1)
50 PC=1/PC
51 bode(PC)
52 grid on
53 hold off
54 G_z_T_comp=G_z_T*PC
55 bode(G_z_T,{0,100})
56 grid on
57 hold on
58 bode(G_z_T_comp,{0,100})
59
60 hold off
61 figure()
62 step(G_z_T_comp)
63 grid on
64
65
66 %% - T=0.05
67 hold off
68
69 z = tf([1 0],[1],0.05)
70
71 G_z= (0.047*z+0.044)/(z^2-1.8*z+0.82)
72 bode(G_z)
73 grid on
74 hold on
75 %T=0.1
76 %z1 = (1+T*s)/(1-T*s)
77 %G_z_T= (0.047*z1+0.044)/(z1^2-1.8*z1+0.82) % Find Tustin Tr.
78 G_z_T= d2c(G_z,'tustin')
79 bode(G_z_T)

```



```

80
81 hold off
82 figure()
83 bode(G_z_T)
84 grid on
85
86 %% for PM 35–30
87 hold off
88 s=tf([1 0],[1])
89 a=1.15
90 K=1
91 T_l=0.37
92 PC = K*(T_l*a*s+1)/(T_l/a*s+1)
93 PC=1/PC
94 bode(PC)
95 grid on
96 hold off
97 G_z_T_comp=G_z_T*PC
98 bode(G_z_T,{0,100})
99 grid on
100 hold on
101 bode(G_z_T_comp,{0,100})
102
103 hold off
104 figure()
105 step(G_z_T_comp)
106 grid on
107
108 %% for PM 10–15
109 hold off
110
111 a=1.6
112 K=1
113 T_l=0.25
114 PC = K*(T_l*a*s+1)/(T_l/a*s+1)
115 PC=1/PC
116 bode(PC)
117 grid on
118 hold off
119 G_z_T_comp=G_z_T*PC
120 bode(G_z_T,{0,100})
121 grid on

```



```

122 hold on
123 bode(G_z_T_comp,{0,100})
124
125 hold off
126 figure()
127 step(G_z_T_comp)
128 grid on
129
130
131
132
133
134 %% To choose desired a
135 s = tf([1 0],[1])
136 hold off
137 a=1.5
138 K=1
139 T_l=1
140 PC = K*(T_l*a*s+1)/(T_l/a*s+1)
141 bode(PC)
142 grid on
143 hold on
144 a=2 %% Choose a=1.8
145 K=1
146 T_l=1
147 PC = K*(T_l*a*s+1)/(T_l/a*s+1)
148 bode(PC)
149 hold on
150 a=1.1
151 K=1
152 T_l=1
153 PC = K*(T_l*a*s+1)/(T_l/a*s+1)
154 figure()
155 PC=1/PC
156 bode(PC)
157 grid on

```

