

EE407 Process Control HW 1

1. We will analyse the system shown in the *Figure 1*.
 - (a) To write the SS model of the system, let us begin with writing fundamental equation describing the system.

$$F_{Net} = m\ddot{x} = F - b\dot{x} - kx$$

Choosing $\underline{x} = [x \ \dot{x}]^T$ and $\underline{y} = [1 \ 0]\underline{x}$, we can build our Space-State Model for the system as

$$\dot{\underline{x}} = A\underline{x} + B u \ \& \ \underline{y} = C\underline{x} + D u$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u$$

where u is the input force F.

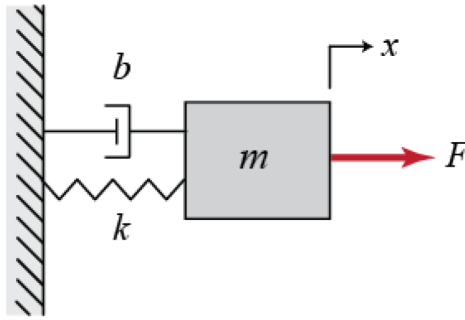


Figure 1: Mass Spring Damper System

- (b) Simulink Model for the Mass Spring Damper System can be seen at *Figure 2*

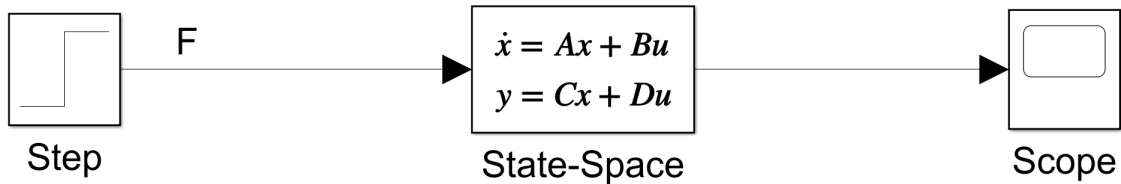


Figure 2: Simulink Model for the Mass Spring Damper System

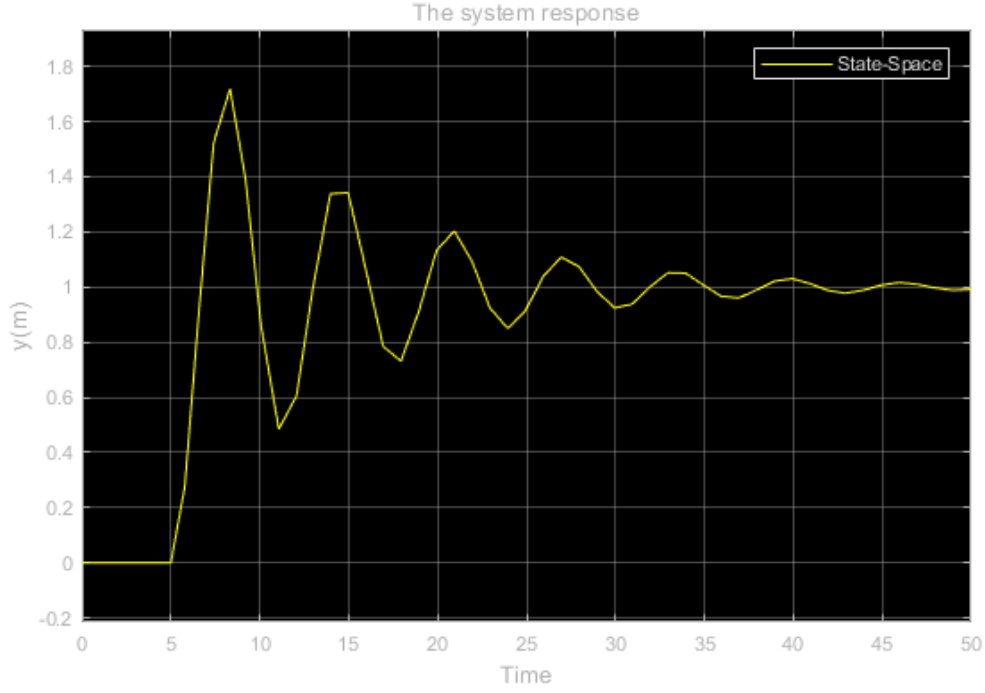


Figure 3: The System Response for MSD as $m = 1\text{kg}$, $b = 0.2\text{Ns/m}$, $k = 1\text{N/m}$

- (c) For the following subsections, the simulations are for the model when the applied force is a unit step function starting at $t = 5$ sec, i.e., $u(t - 5)$.
- The spring force is proportional to the displacement of the mass, x with the direction of opposite to the F . Therefore, when the spring constant decreased, the displacement of x is increased. The figures are consistent with these, *Figure 4* has small k value and it reaches far than *Figure 3*.
 - It is known that $F_{net} = ma = m\ddot{x}$, then mass and acceleration that is related to position are oppositely proportional, so when mass increased, the output will be decreased. The *Figure 3* and *Figure 5* are expected.

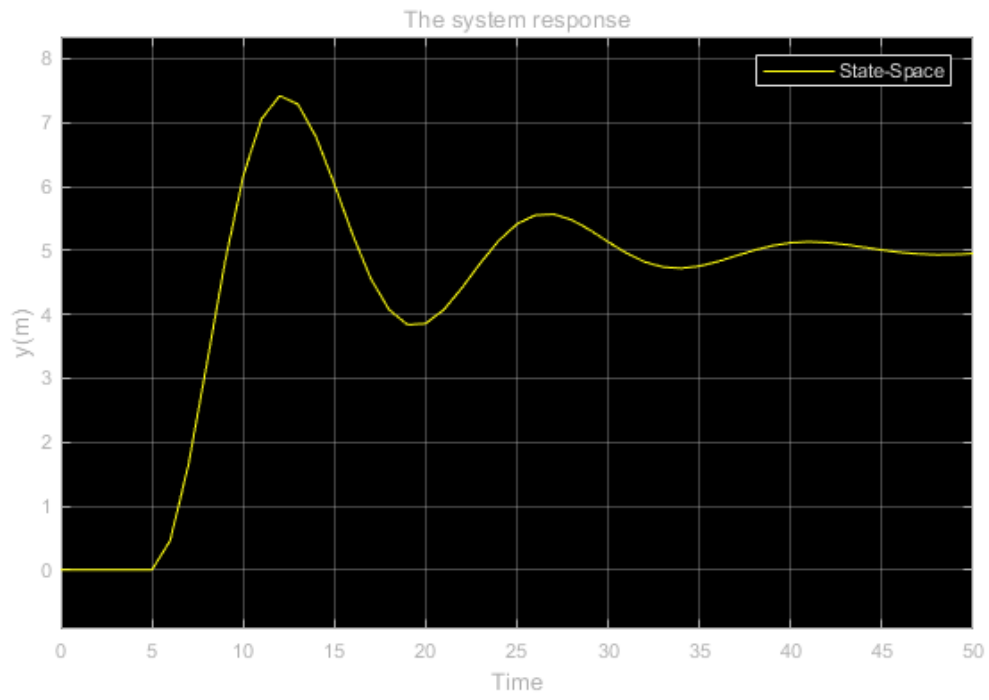


Figure 4: The System Response for MSD as $m = 1\text{kg}$, $b = 0.2\text{Ns/m}$, $k = 0.2\text{N/m}$

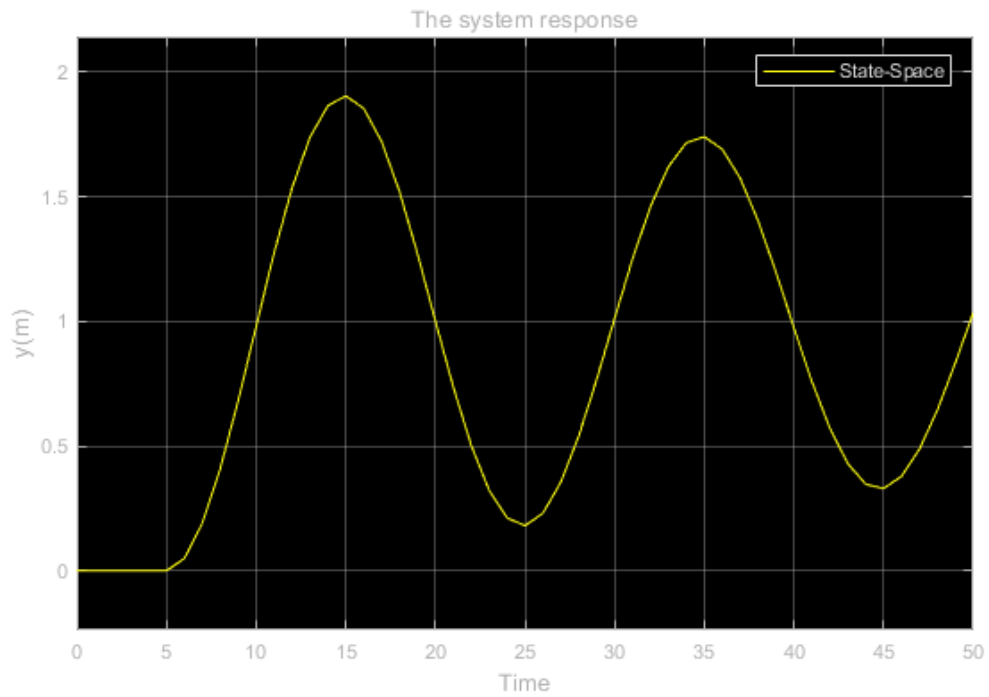


Figure 5: The System Response for MSD as $m = 10\text{kg}$, $b = 0.2\text{Ns/m}$, $k = 1\text{N/m}$

- iii. The viscous damping force is proportional to the velocity of the mass, $v = \dot{x}$ with the direction of opposite to the F . In this case, firstly this opposite direction is not so much because of the velocity is small and after some point this velocity value increases and effect the system with more opposite force. Therefore, *Figure 3* and *Figure 6* are expected.

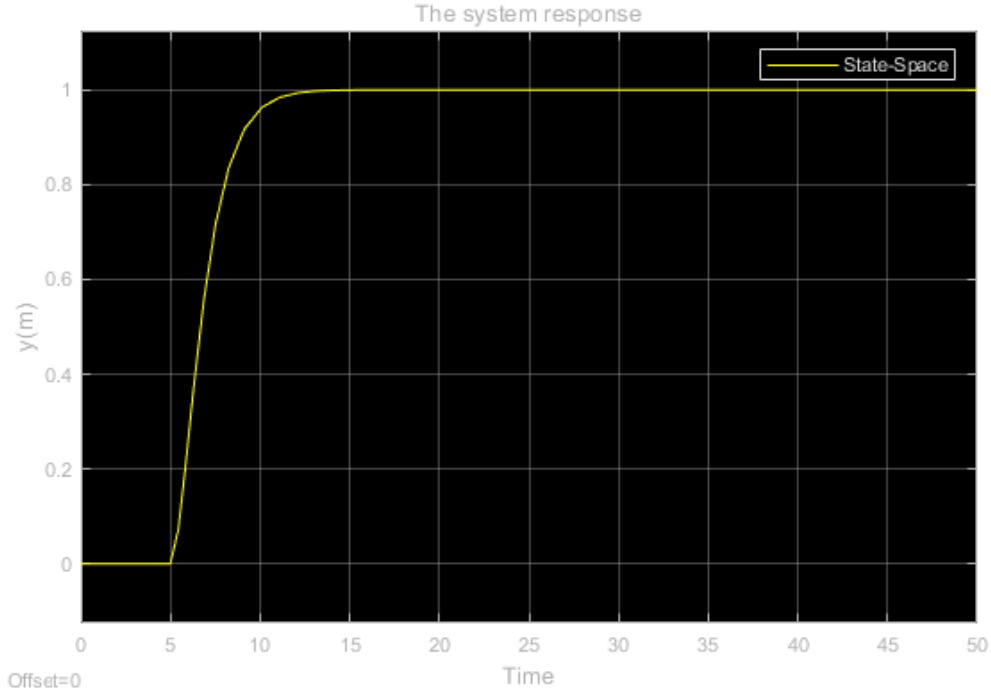


Figure 6: The System Response for MSD as $m = 1\text{kg}$, $b = 2\text{Ns/m}$, $k = 1\text{N/m}$

(d) d

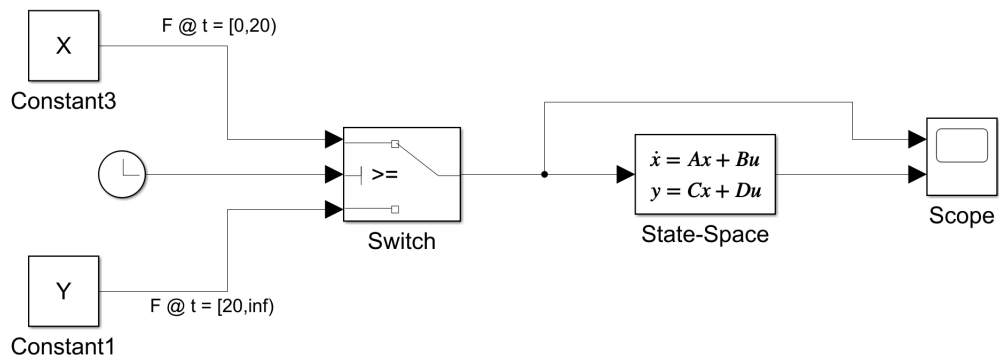


Figure 7: Simulink Model for the MSD with Varying Input Force

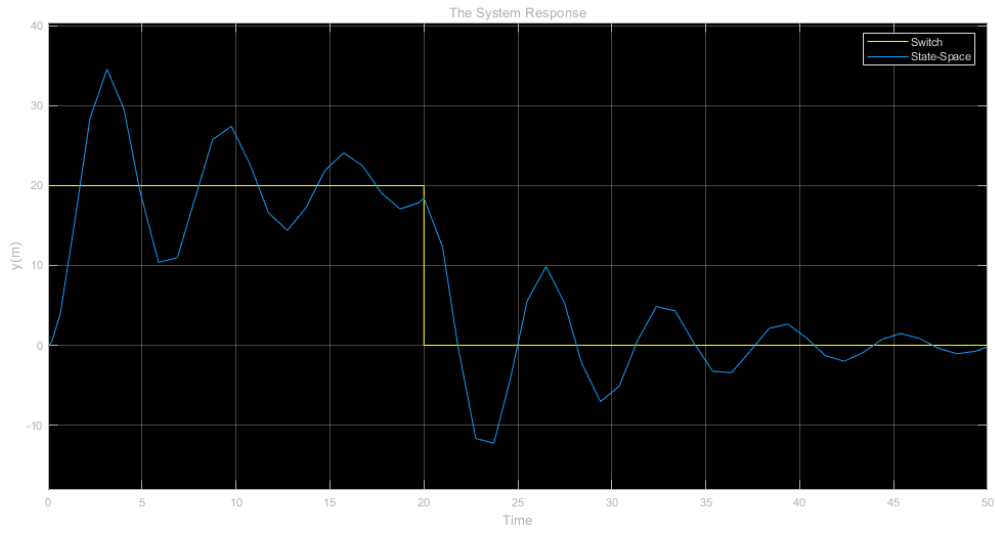


Figure 8: The System Response for MSD as the Input Changes at $t=20$ s

(e) e

(f) f

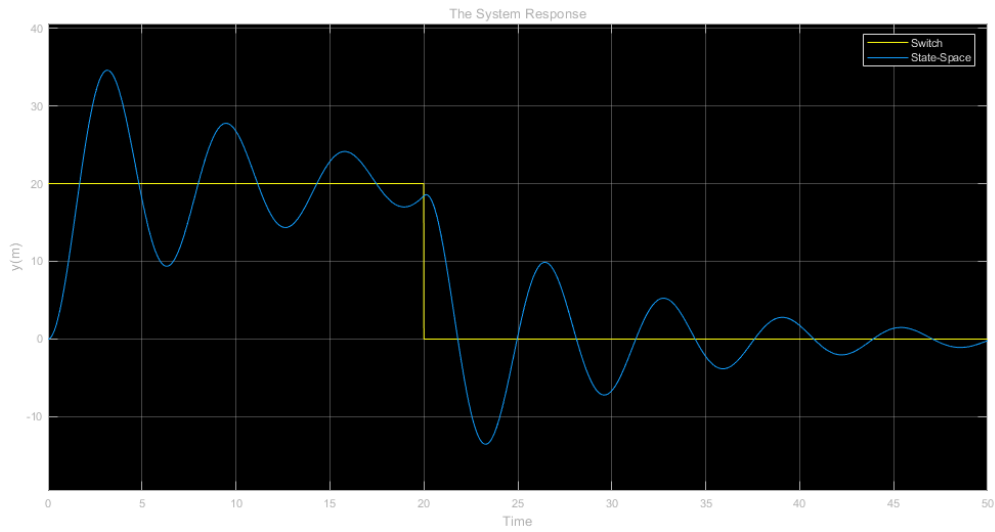
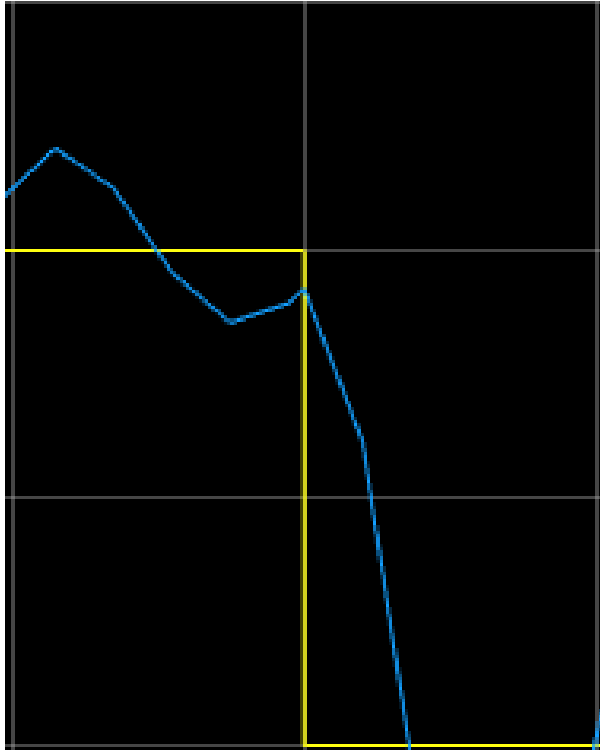
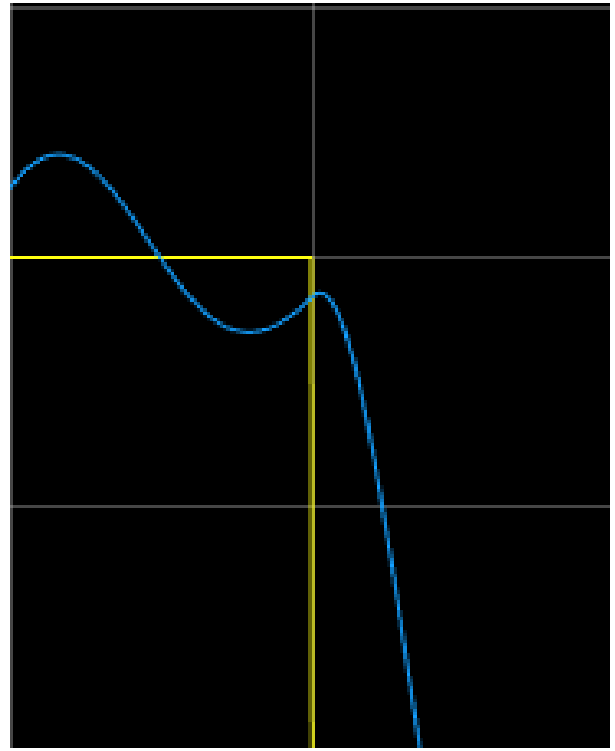


Figure 9: The System Response for MSD with Desired Parameters in Q1f



(a) Simulation with Variable Step



(b) Simulation with Fixed Step

Figure 10: Simulation with Variable and Fixed Step

2. 2. soru

- (a) a
- (b) b
- (c) c
- (d) d