Halil TEMURTAŞ / 2094522
Pair
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EE407 Process Control
Experiment 1

1. Assume the thermocouple is in a liquid at a temperature $T_initial$ and in equilibrium. Assume also that, at $t = 0$ the temperature of liquid is increased to T_final at instant time. Let $T_f(t)$ represents the temperature of fluid at time t and $T(t)$ represents the temperature of the thermocouple.

$$\dot{Q} \;:\; Heat\ Transfer\ Rate$$

$$h \;:\; Heat\ Transfer\ Coefficient$$

$$A \;:\; Area\ of\ Junction$$

$$\dot{Q} = hA(T_f(t) - T(t))$$

$$Q = hA(\Delta T_f(t) - \Delta T(t))$$

$$m \;:\; mass\ of\ junction$$

$$C \;:\; Spesific\ Heat\ of\ Junction$$

$$Q = mC\frac{d\Delta T(t)}{dt} = hA(\Delta T_f(t) - \Delta T(t))$$

$$\frac{mC}{hA}\frac{d\Delta T(t)}{dt} + \Delta T(t) = \Delta T_f(t)$$

Taking the Laplace transform of both sides;

$$\left(\frac{mC}{hA}s + 1\right)\Delta T(s) = \Delta T_f(s)$$

Let us define $\tau_1 = \frac{mC}{hA}$. Thus the equations becomes;

$$(\tau_1 s + 1)\,\Delta T(s) = \Delta T_f(s)$$

Therefore, the transfer function can be written as;

$$G(s) = \frac{\Delta T(s)}{\Delta T_f(s)} = \frac{1}{\tau_1 s}$$

$$K \;: Seeback\ Coefficient$$

$$V = K(V - V_{REF})$$

$$\boxed{G_1(s) = \frac{V(s)}{T_1(s)} = \frac{K}{\tau_1 s}}$$

2.
$$\boxed{G_2(s) = G_1(s)G'(s) = \frac{K}{\tau_1 s + 1}\frac{1}{\tau_2 s + 1}}$$

3. Assuming $K = 1$ and given that $\tau_1 = 1$ sec and $\tau_2 = 10$ sec

$$G_1(s) = \frac{1}{s+1}$$

$$G_2(s) = \frac{1}{(s+1)}\frac{1}{(10s+1)} = \frac{1}{10s^2 + 11s + 1}$$

Step responses of bare(G_1) and sheath(G_2) thermocouples and the zero time constant approximation for sheath thermocouple(G_3) can be seen at *Figure 1*.

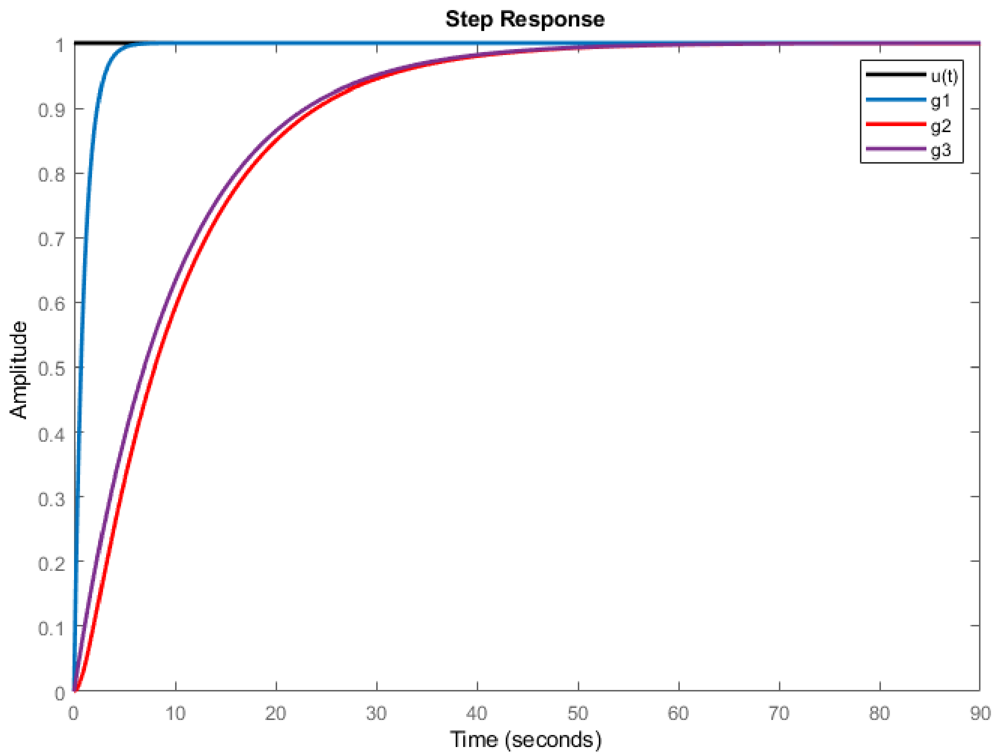

Figure 1: Step Responses of Bare(G_1) and Sheath(G_2) Thermocouples

4. It stated that that the time constant of the bare thermocouple is usually much smaller than the one caused by sheathing. Therefore, one may approximate the overall transfer function as a first order one as can be interpreted from the *Figure 1*.

$$G(s) = \frac{K}{1+\tau_2 s}$$

It is also given that, the voltage generated across wires would be

$$V(t) = \Delta V \left(1 - e^{-\dfrac{t}{\tau_2}}\right) + V_0$$

$$V(\tau_2) = \Delta V \left(1 - e^{-\dfrac{\tau_2}{\tau_2}}\right) + V_0 = \Delta V \left(1 - e^{-1}\right) + V_0$$

$$\boxed{V(\tau_2) = 0.632\Delta V + V_0}$$

$$ln\left(1 - \frac{V(t) - V_0}{\Delta V}\right) = -\frac{t}{\tau_2}$$

$$\boxed{\tau_2 = \frac{-\,t}{ln\left(1 - \dfrac{V(t) - V_0}{\Delta V}\right)}}$$

5. Pole-Zero plot for the bare(G_1) and sheath(G_2) thermocouple models can be seen at *Figure 2*

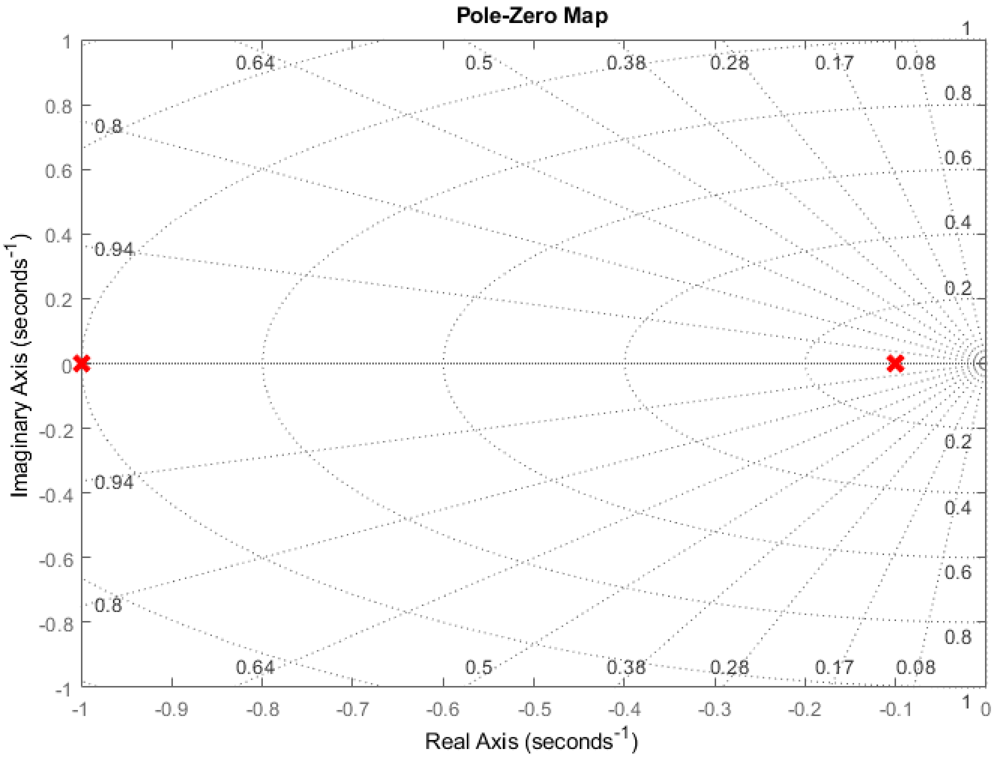

Figure 2: Pole-Zero plot for the Bare(G_1) and Sheath(G_2) Thermocouple Models

6.

$$v_i = v_c + v_o$$

$$v_c = v_i - v_o$$

$$i_c + i_{R_1} = i_{R_2}$$

$$I_c(s) = sCV_c(S) = sC(V_c(s)) = \frac{V_o(s)}{R_2} - \frac{V_c(s)}{R_1}$$

$$\left(sC + \frac{1}{R_1}\right)[Vi(s) - V_o(s)] = \frac{V_0}{R_2}$$

$$\vdots$$

$$G(s) = \frac{V_o(s)}{V_i(s)} = \frac{sCR_1R_2 + R_2}{sCR_1R_2 + R_1 + R_2} = \alpha\frac{1+\tau_c s}{1+\alpha\tau_c s}$$

Thus,

$$\boxed{\alpha = \frac{R_2}{R_1 + R_2}}, \quad \boxed{\tau_c = cR_1}$$

It can be seen from the above equation that $0 < \alpha < 1$ according to resistor values. Therefore, it can be concluded that the compensator is indeed a **Lead Compensator**.

7. With $\alpha = 0.5$ and $\tau_c = \tau_2 = 10$ *seconds*

$$G_{ovll}(s) = \frac{1}{(10s+1)} \frac{1+10s}{2(1+5s)}$$

The step response of overall system can be seen at *Figure 3*.

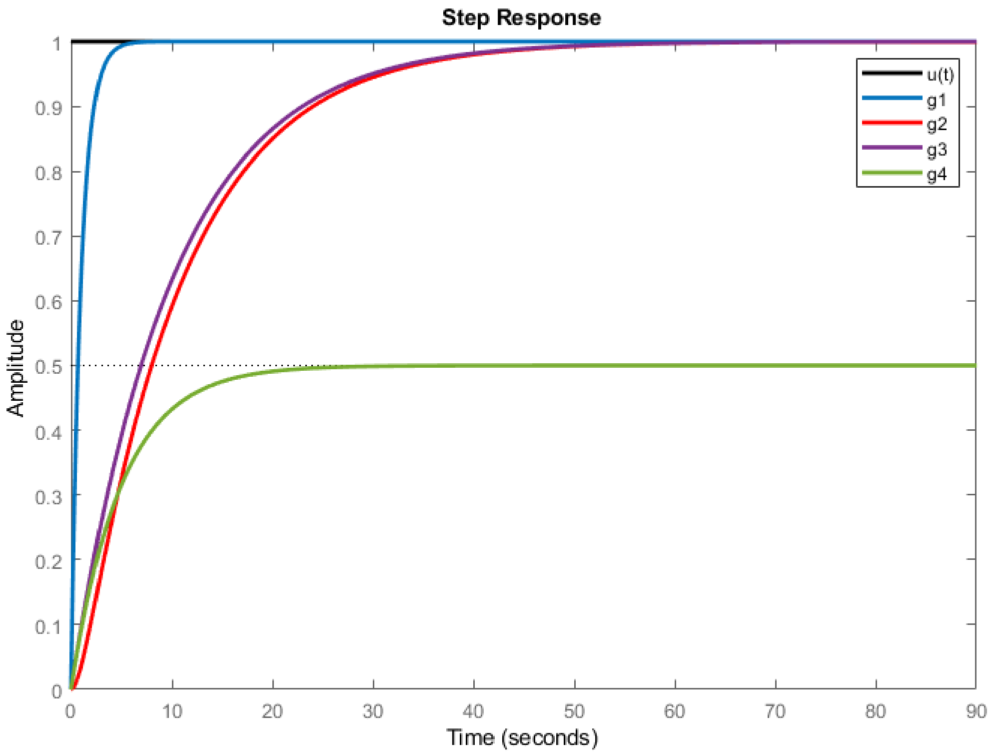

Figure 3: Step Responses of Bare(G_1) and Sheath(G_2) Thermocouples and Sheath Thermocouple with Compensator(G_4

8. With $\tau_c = 12$ *seconds*

$$G_5(s) = \frac{1}{(10s+1)} \frac{1+12s}{2(1+6s)}$$

With $\tau_c = 8$ *seconds*

$$G_6(s) = \frac{1}{(10s+1)}\frac{1+8s}{2(1+4s)}$$

The step responses were exactly the same for the plot limits of MATLAB, the response can be seen at *Figure 4*.

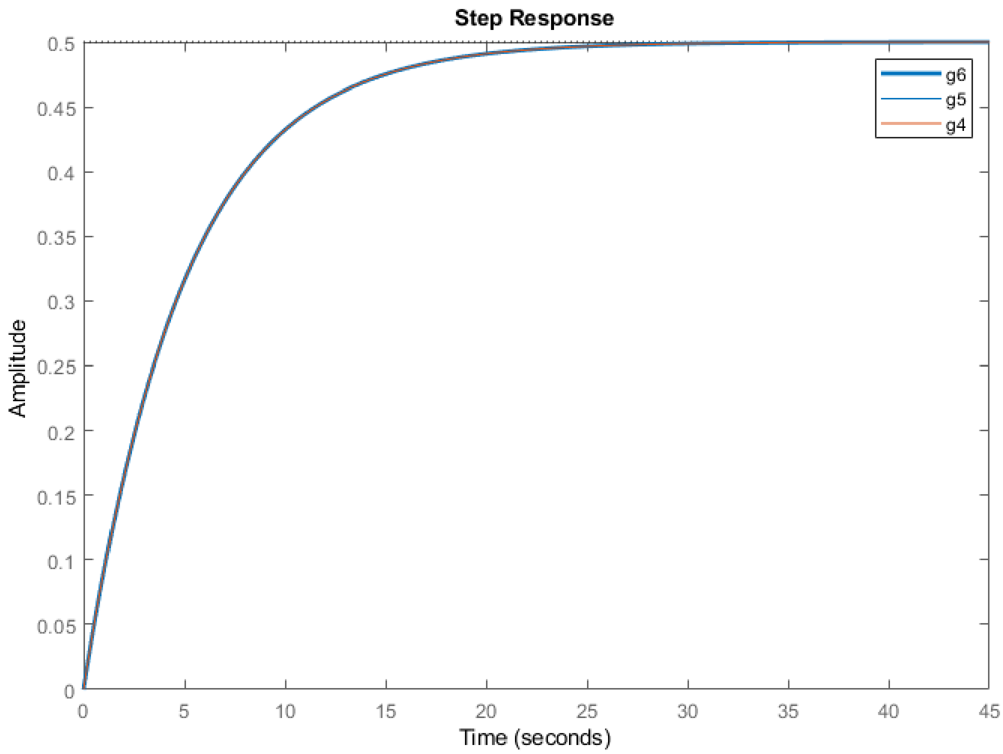

Figure 4: Step Responses of Sheath Thermocouple with Compensator Parameters(G_4

Appendices

A Source Code for Matlab Part

```
u=tf(1)

g1=tf([1],[1  1])

g2=tf([1],[10  11  1])
```

```
g3=tf([1],[10 1])

g4=tf([10 1],[100 30 2])

g5=tf([12 1],[120 34 2])

g6=tf([8 1],[80 26 2])

step(u)

hold on

step(g1)

hold on

step(g2)

hold on
step(g3)

hold on
step(g4)

hold on
step(g5)

hold on
step(g6)

%% -
hold off
%% -
pzmap(g1)
grid on
hold on
pzmap(g2)
grid on
```