EE407 Process Control
Experiment 5

1. Reverting back to actual temperature,

$$T(x,t) = T_e + (M - T_e)erfc(\frac{x}{2\sqrt{\alpha t}})\ for\ x > 0\ and\ t > 0$$

For our case;

$$T(L,t) = T_e + (M - T_e)erfc(\frac{L}{2\sqrt{\alpha t}})$$

with given constants and $T_e = 0$,for convenience, and $M = 1$ for unit step input; The response can be seen at *Figure 1*. Source code for this part can be examined at **Appendix A**.

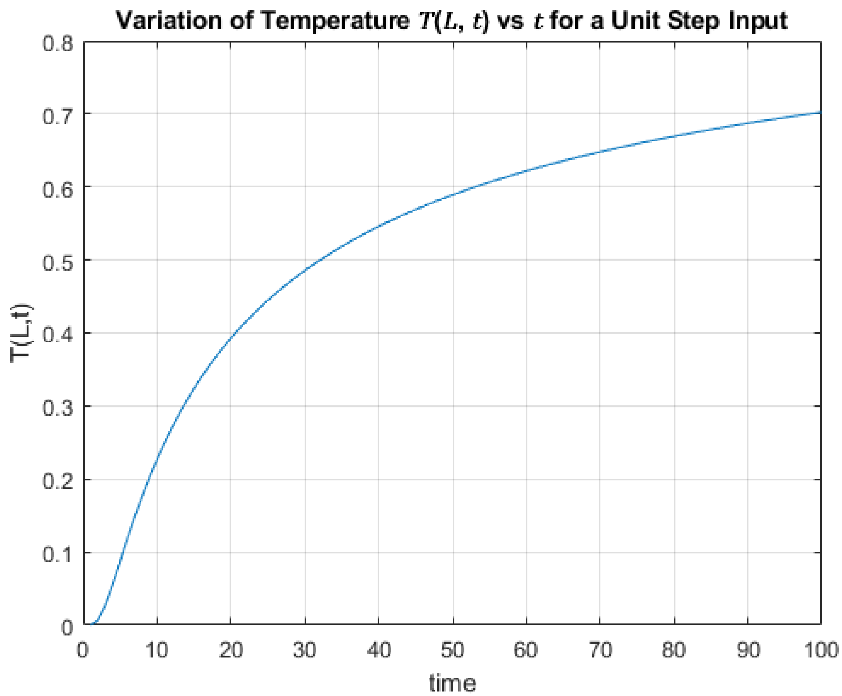

Figure 1: Variations of Temperature T(L,t) vs time

2. In general,

$$\frac{1}{\alpha}\frac{dT_r(t)}{dt} = \frac{T_{r-1}(t) - 2T_r(t) + T_{r+1}(t)}{(\Delta x)^2}\ forr = 1, 2, ...n$$

T_{n+1} can be assumed to be equal to T_n for n equations to solve for unknowns.

- for n=1

$$\frac{1}{\alpha}\frac{dT_1(t)}{dt} = \frac{T_0(t) - 2T_1(t) + T_2(t)}{(\Delta x)^2}$$

where T_o is the input and $T_1 = T_2$ assumption can be made.
Taking Laplace transforms of both sides, it can be found that;

$$\left(\frac{(\Delta x)^2}{\alpha}s + 1\right)T_1(s) = T_0(s)$$

$$\frac{T_1(s)}{T_0(s)} = \frac{1}{\frac{(\Delta x)^2}{\alpha}s + 1}$$

it can be seen that the time constant of this equation is

$$\tau_1 = \frac{(\Delta x)^2}{\alpha} = 29.2002\ seconds$$

where $\Delta x = L = 5\ cm$.
Let us now find the circuit equivalent of this approximation. The circuit at at *Figure 2* seems a good fit. Let us now analyse this circuit and find its time constant. It is desired that the time constant of this circuit should at lest 10^{-5} times of the approximation model.

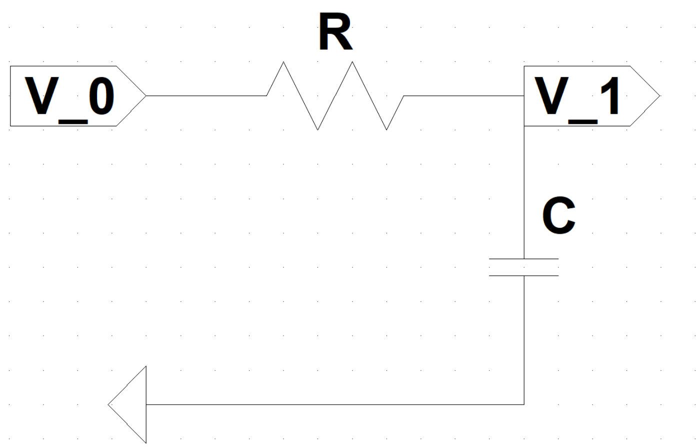

Figure 2: Circuit Equivalent of Lumped Approximation Model for n=1

$$\frac{V_1(s)}{V_0(s)} = \frac{1}{1 + (RC)s}$$

$$\tau_c = RC = 10^{-5}\tau_1 = 2.92\ 10^{-4}$$

Assuming $C = 1\ \mu F$

R value can be found to be as 292.002 Ω

- for n=2;

$$\frac{1}{\alpha}\frac{dT_1(t)}{dt} = \frac{T_0(t) - 2T_1(t) + T_2(t)}{(\Delta x)^2}$$
$$\frac{1}{\alpha}\frac{dT_2(t)}{dt} = \frac{T_1(t) - 2T_2(t) + T_3(t)}{(\Delta x)^2}$$

where T_o is the input and $T_2 = T_3$ assumption can be made.

Taking Laplace transform of both sides

Assume from now on $A = \frac{(\Delta x)^2}{\alpha}$

$$(As + 2)T_1(s) = T_0(s) + T_2(s)$$

$$(As + 1)T_2(s) = T_1(s)$$

$$[(As + 2)(As + 1) - 1]T_2(s) = T_0(s)$$

$$\frac{T_2(s)}{T_0(s)} = \frac{1}{A^2s^2 + 3As + 1} = \frac{1}{\tau_2^2 s^2 + 2\xi\tau_2 + 1}$$

with $\Delta x = L/2 = 2.5cm$

$$\tau_2 = A = \frac{(\Delta x)^2}{\alpha} = 7.3001\ secs$$

Desired time constant for circuit equivalent then $\tau_{c2} = 10^{-5}\ \tau_2 = 7.3\ 10^{-5}\ secs$

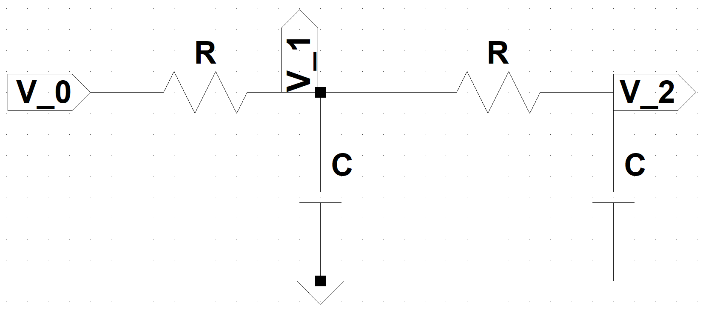

Figure 3: Circuit Equivalent of Lumped Approximation Model for n=2

The transfer equation for the circuit at *Figure 3* can be found as

$$\frac{V_2(s)}{V_1(s)} = \frac{1}{(RC)^2 s^2 + 3(RC)s + 1} == \frac{1}{\tau_{c2}^2 s^2 + 2\xi\tau_{c2} + 1}$$

$$\tau_{c2} = RC = 7.3\ 10^{-5}\ secs$$

Assuming $C = 1\ \mu F$
R value can be found to be as 73.0005 Ω

- for n, the circuit model at *Figure 8* can be used

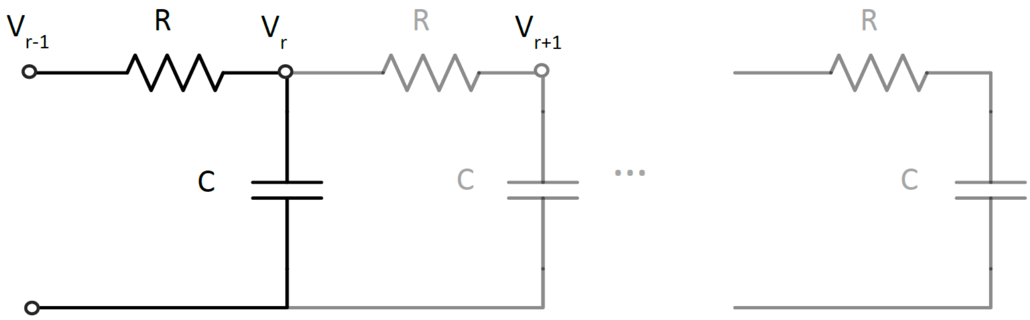

Figure 4: Simulation of the lumped approximate model

The pattern can be observed from the first two equations that found earlier for $n = 1, 2$

$$\Delta x = \frac{L}{n}$$

$$\alpha = \frac{\lambda}{\rho c}$$

$$\tau_n = A = \frac{(\Delta x)^2}{\alpha}$$

$$\tau_{cn} = RC = \tau_n 10^{-5}$$

- for n=3

$$\Delta x = \frac{L}{3}$$

$$\tau_3 = A = 3.2445\ secs$$

$$\tau_{c3} = RC = \tau_3 10^{-5} = 3.2445\ 10^{-5}\ secs$$

Assuming $C = 1\ \mu F$
R value can be found to be as 32.4447 Ω

- for n=5

$$\Delta x = \frac{L}{5}$$

$$\tau_5 = A = 1.1680\ secs$$

$$\tau_{c5} = RC = \tau_5 10^{-5} = 1.1680\ 10^{-5}\ secs$$

Assuming $C = 1\ \mu F$
R value can be found to be as 11.6801 Ω

3. The responses can be examined below

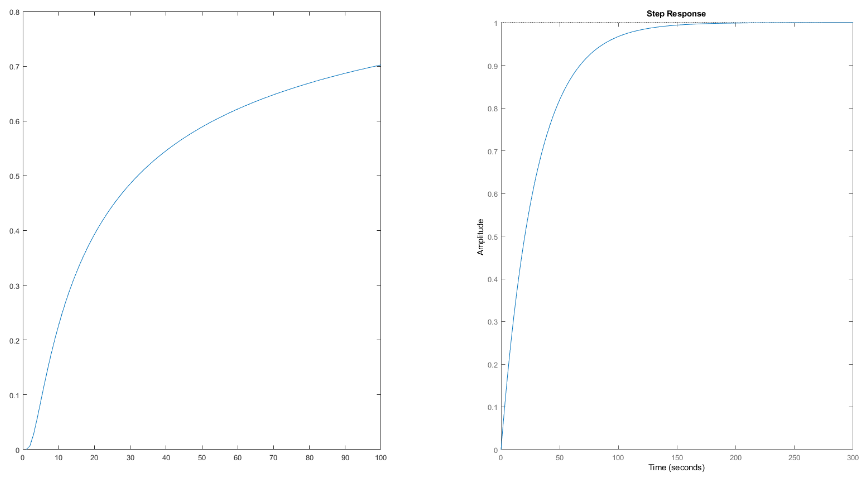

Figure 5: Step response of System and steps response pf its Lumped Parameter approximation for n=1

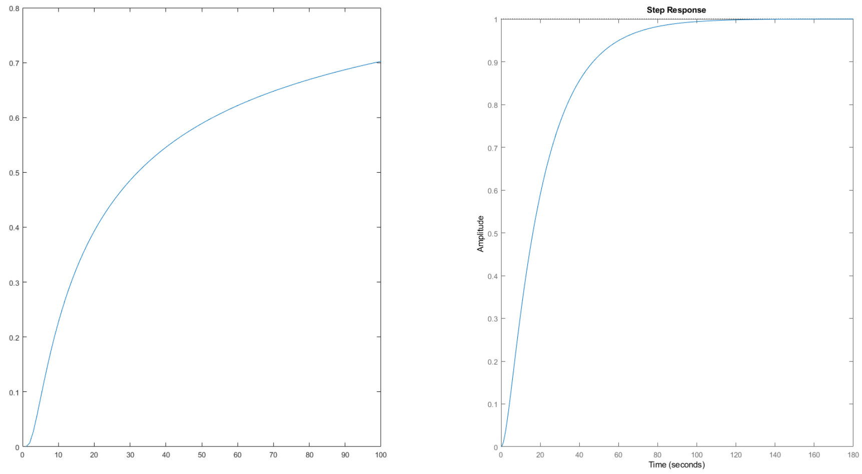

Figure 6: Step response of System and steps response pf its Lumped Parameter approximation for n=2

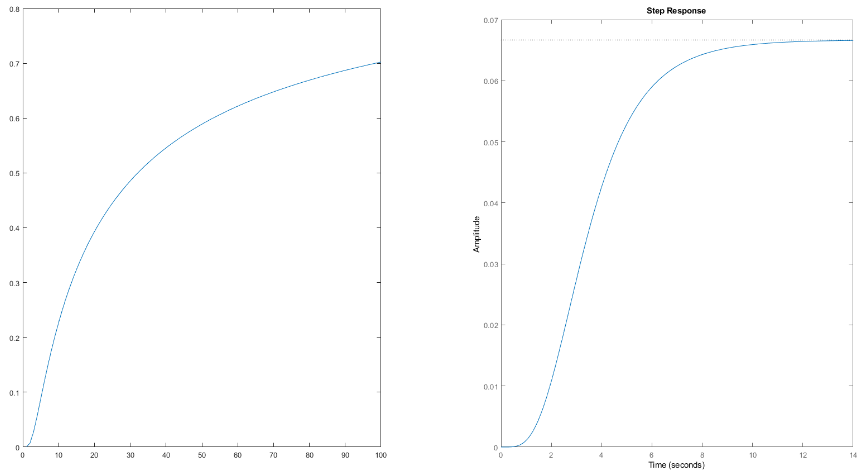

Figure 7: Step response of System and steps response pf its Lumped Parameter approximation for n=3

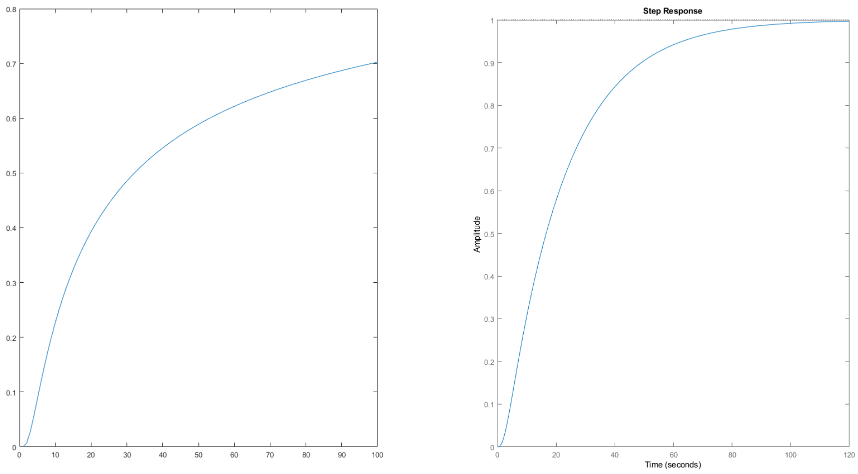

Figure 8: Step response of System and steps response pf its Lumped Parameter approximation for n=4

Appendices

A Source Code for Matlab Part

```
%% %%%%%%%%%% Q1 %%%%%%%%%%
%% Var Declr.

L=5
A=4.9
p=2.7
k=0.497
c=0.215
a=k/(p*c)
Te=0
M=1
%% Calculation of  the response
t=1:1:100
i=1
while (i < 101)
    T(i)=Te+(M-Te)*erfc(L/(2*sqrt(a*i)))
     i=i+1
end
```

```
%% Plotting the response
plot(t,T)
grid on
title('Variation of Temperature T(L,t) vs t for a Unit Step
   Input')
xlabel ('time')
ylabel ('T(L,t)')

%% %%%%%%%%% Q2 %%%%%%%%%
%% for n=1

tau1=(L^2)/(a)

taue=10^(-5)*tau1

C=10^-6

R=taue/C

%% for n=2
L2=L/2

tau2= (L2^2)/(a)

taue2=10^(-5)*tau2

C2=10^-6

R2=taue2/C2

%% for n=3
L3=L/3

tau3= (L3^2)/(a)

taue3=10^(-5)*tau3

C3=10^-6

R3=taue3/C3

%% for n=5
```

```
L5=L/5

tau5= (L5^2)/(a)

taue5=10^(-5)*tau5

C5=10^-6

R5=taue5/C2

%% %%%%%%%%%% Q2 %%%%%%%%%%
%% n=1

T1=tf([1],[tau1 1])

figure;
grid on
subplot(1,2,1)
plot(t,T)
subplot(1,2,2)
step(T1)

%% n=2

T2=tf([1],[tau2^2 3*tau2 1])

figure;
grid on
subplot(1,2,1)
plot(t,T)
subplot(1,2,2)
step(T2)

%% n=3

T3=tf([1],[tau3^3 5*tau3^2 7*tau3 1])

figure;
grid on
subplot(1,2,1)
plot(t,T)
subplot(1,2,2)
```

```
step(T3)

%% n=5
a=tf([tau5 2],[1])
b=tf([tau5 1],[1])
c= a^4*b-1

T5=1/c

figure;
grid on
subplot(1,2,1)
plot(t,T)
subplot(1,2,2)
step(T5)
```