EE 407
Homework 4

Due 23:55, 25.12.2017

1 Introduction

In this homework, you will be evaluating several Padé approximations, followed by model-based design methods. Distributed parameter systems as well as feedforward/cascade controllers will also be examined.

2 Questions

1. **Padé Approximation**

   In mathematics, a Padé approximant is the "best" approximation of a function by a rational function of given order. This definition is obtained from Wikipedia article titled Padé approximant. Finding the approximation requires you to first find the Taylor series expansion of any given function around a point x_0 and then equate the coefficients of Padé polynomial to that of the Taylor series. Now, as an example, let's prove the equation given in lectures. Let $m > 0$ denote the order of the numerator and $n \geq 1$ denote the order of the denominator of the approximant $R_{[m/n]}$ (that is, $R_{[m/n]} = \frac{\sum_{j=0}^{m} a_j s^j}{1+\sum_{j=1}^{n} b_j s^j}$). For a 1 by 1 approximant $R_{[1/1]}$, using the first $m + n + 1 = 3$ coefficients from Taylor series expansion around $s = 0$,

$$
\begin{aligned}
&e^{-\theta s} = 1 - \theta s + \frac{\theta^2 s^2}{2!} - \ldots \cong \frac{a_0 + a_1 s}{1 + b_1 s} \\
&a_0 + a_1 s = (1 + b_1 s)(1 - \theta s + \frac{\theta^2}{2} s^2) + \text{ higher order terms of order greater than } s^2 \\
&a_0 = 1, a_1 = b_1 - \theta, 0 = \frac{\theta^2}{2} - b_1 \theta \\
&\implies b_1 = \theta/2 \text{ and } a_1 = -\theta/2
\end{aligned}
\tag{1}
$$

   (a) Calculate the $R_{[2/2]}$ Padé approximation of $e^{-\theta s}$ around $s = 0$ using $m+n+1$ coefficients of the Taylor series expansion around $s = 0$.

   (b) Calculate the $R_{[0/1]}$ and $R_{[0/2]}$ Padé approximations of $e^{-\theta s}$ as well.

   (c) Plot the magnitude and phase responses of $R_{[0/1]}$, $R_{[0/2]}$, $R_{[1/1]}$ and $R_{[2/2]}$ for a fixed $\theta = 1$(sec). Compare them to those of the e^{-s}. Justify which one is expected to represent the original system better?

(d) Plot the step responses of those five systems. Does your justification hold from the previous step? How do the systems with non-minimum phase zeros behave arount $t = 0$?

2. **Model-Based Design Methods**

You have learnt two model based controller design methods in your classes. Both methods are closely related and may lead to the same controller parameters if design parameters are specified consistently. In the first part of this problem, you are required to find the IMC controller parameters for two systems as a verification of DS PID parameters. In the second part, you are required to design a PID controller using the DS method and observe the behavior of the plant under uncertainties.

(a) Derive the PI controller parameters using Internal Model Control design method for the following plant. Assume that $r = 1$.

$$\tilde{G}_p(s) = K_p \frac{1}{\tau_p s + 1} \tag{2}$$

(b) Derive the PID controller parameters using Internal Model Control design method for the following plant. Assume that $r = 1$.

$$\tilde{G}_p(s) = K_p \frac{1}{(\tau_1 s + 1)(\tau_2 s + 1)} \tag{3}$$

(c) Derive the PID controller parameters using Internal Model Control design method for the following plant. Assume that $r = 1$.

$$\tilde{G}_p(s) = K_p \frac{(-\beta s + 1)}{(\tau_1 s + 1)(\tau_2 s + 1)}, \beta > 0$$

Now, consider the following process.

$$\tilde{G}_p(s) = \frac{K_p}{(10s + 1)(5s + 1)} \tag{4}$$

where $K = 1$. Find the PID controller parameters using Direct Synthesis mehod with $\tau_c = 5$ (min).

(d) Simulate the system for the perfect model.

(e) Suppose that K changes unexpectedly from 1 to $1 + \alpha$. Find the range of α for which the closed loop system is stable.

(f) What is the limitting value of τ_c if you are given that $|\alpha| < 0.2$?

3. **Distributed Parameter Systems**

Consider the following normalized partial differential equation

$$\frac{\partial T}{\partial x} + \frac{\partial T}{\partial t} = 0 \tag{5}$$

subject to the following conditions.

$$T(x,0) = T_e \text{ for } 0 < x \leq 1 \text{ (initial condition)}$$
$$T(0,t) = V(t) + T_e \text{ for } t > 0 \text{ (boundary condition)}$$

Let $\eta(x,t) = T(x,t) - T_e$. Rewrite the differential equation & boundary conditions. Then use them to find T(x,t) using Laplace transform. Note that as the DE is normalized algebraic equations between x and t are allowed (e.g. x/t).

4. **Feedforward Control**

   For the feedforward control structure given in Fig. 1, let $G_p = \frac{1}{(2s+1)(3s+1)}$, $G_d = \frac{1}{s+1}$.

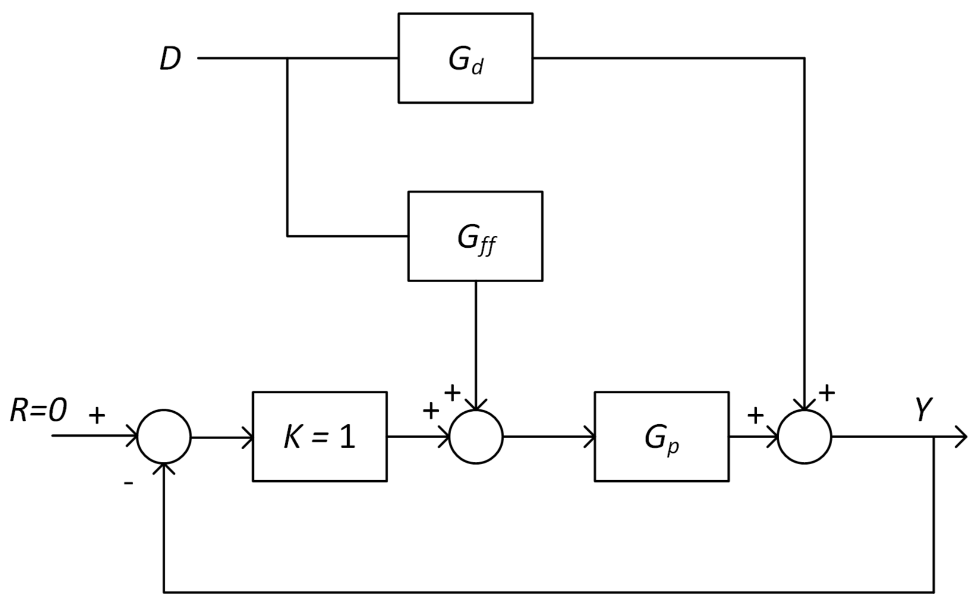

Figure 1

   (a) Find the ideal G_{ff} in order to have the transfer function $D(s)/Y(s) = 0$.

   (b) Discuss why the transfer function you found above is not physically realizable (an improper transfer function is unrealizable, but why?).

   (c) Approximate the numerator by a first order transfer function & find G_{ff}. Use the same approach as what you do when you approximate pure time delay as $e^{-\theta_p s} = 1 - \theta_p s + \frac{(\theta_p s)^2}{2!} - \ldots \cong 1 - \theta_p s$.

   (d) Simulate the system for a unit step disturbance with and without the feedforward controller. Comment on the effect of the feedforward controller.

5. **Cascade Control**

   Consider the cascade control structure given in Fig. 2. Let $G_v = \frac{5}{s+1}$, $G_p = \frac{4}{(4s+1)(2s+1)}$, $G_{c2} = K_{c2} = 4$, $G_{m1} = 0.05$ and $G_{m2} = 0.2$, where all time constants have the units of minutes.

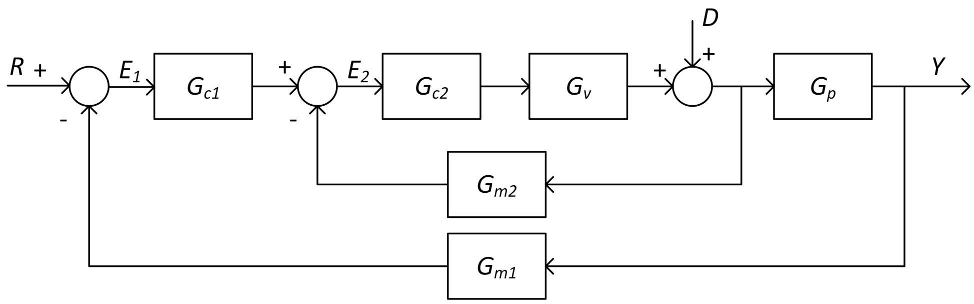

Figure 2

(a) Compare the open and closed loop time constants of the inner loop.

(b) Find the proportional only gain using Ziegler-Nichols continuous cycling method.

(c) Find the proportional only gain with the same method, but this time without the inner controller (that is, set $G_{m2} = 0$ and $K_{c2} = 1$).

(d) Find the steady state error values (E_1) for a unit step change in the disturbance D for both systems (with and without the inner controller).

(e) Simulate the systems to verify your results. You may assume that $R = 0$.

(f) Compare the two systems in terms of stability, disturbance rejection performance in the inner loop and speed.