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EE407 Process Control
HW 2

1. (a) Conservation Law states that **Accumulation=(In flow)-(Out flow)**

$$\frac{dw}{dt} = q_i - q_o$$

$$q_o = Kw$$

$$\frac{d}{dt}(\frac{q_o}{K}) = q_i - q_o$$

$$\frac{d}{dt}(q_o) + Kq_o = Kq_i$$

$$sQ_o(s) + KQ_o(s) = KQ_i(s)$$

$$\boxed{H(S) = \frac{Q_o(s)}{Q_i(s)} = \frac{K}{K+s}}$$

(b) Given that the input $q_i(t)$ is unit step input, output can be calculated as follows;

$$Q_i(s) = \frac{1}{s}$$

$$Q_o(s) = H(s)Q_i(s) = \frac{1}{s}\frac{K}{K+s} = \frac{1}{s} - 1\frac{1}{K+s}$$

$$q_o(t) = \mathcal{L}^{-1}(Q_o(s)) = u(t) - e^(- tK)u(t)$$

$$\boxed{q_o(t) = (1 - e^{-tK})u(t)}$$

Using Matlab, the output can be observed as in *Figure 1*

(c) In this part, we are looking for $w_\gamma = w_{max} - w_{min}$

$$\frac{dw(t)}{dt} = q_i - q_o$$

$$q_o = Kw(t)$$

$$\frac{dw(t)}{dt} + Kw(t) = q_i$$

$$w(t) = w_p(t) + w_h(t)$$

For homogeneous solution, $y_h(t)$ will be in the form of $e^{\lambda t}$, where

$$\lambda + K = 0 \ , \ \boxed{\lambda = -K}$$

$$w_h(t) = Ae^{-tK}u(t)$$

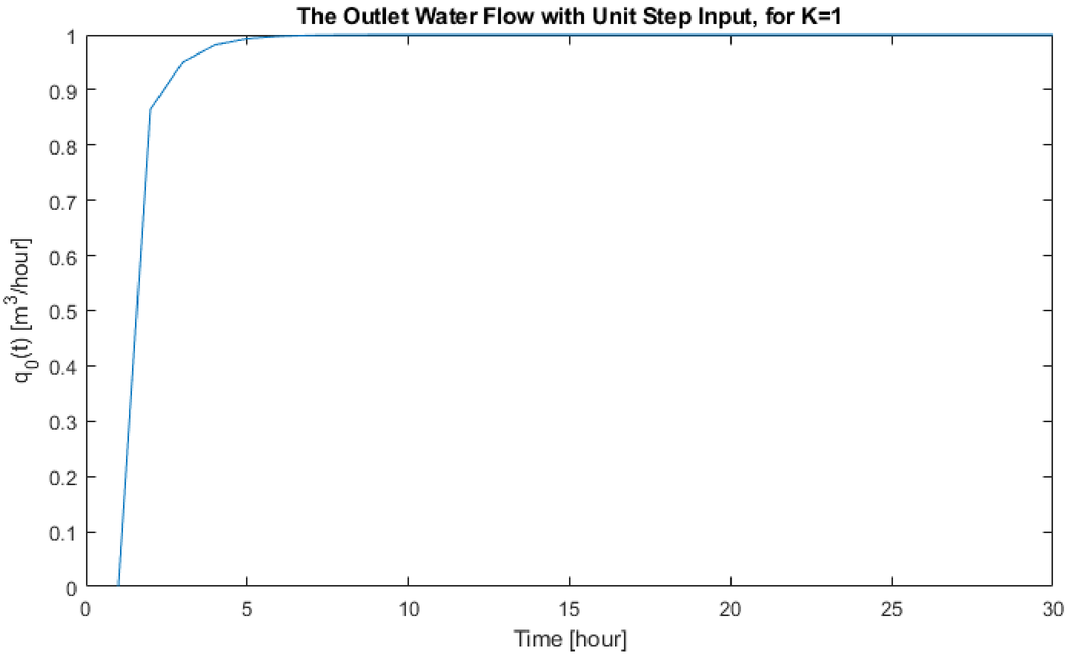

Figure 1: Output of the Reservoir System with Unit Step Input

For particular solution:

$$\frac{dw_p(t)}{dt} + Kw_p(t) = BKw_{max}$$

$$w_p(t) = Bw_{max}u(t)$$

It is known that

$$w(0) = w_{min} = w_{initial}$$

$$A = w_{min} - Bw_{max}$$

The maximum value will be at time=120

$$w_{max} = (w_{min} - Bw_{max})e^{-120K} + Bw_{max}$$

$$w_{min} = \frac{(1 + e^{-120K} - B)w_{max}}{e^{-120K}}$$

$$\boxed{w_\gamma = w_{max} - w_{min} = w_{max}(1 - e^{120K} + Be^{120K} - B)}$$

2. (a) Resulted Simulink model can be observed at *Figure 2*.

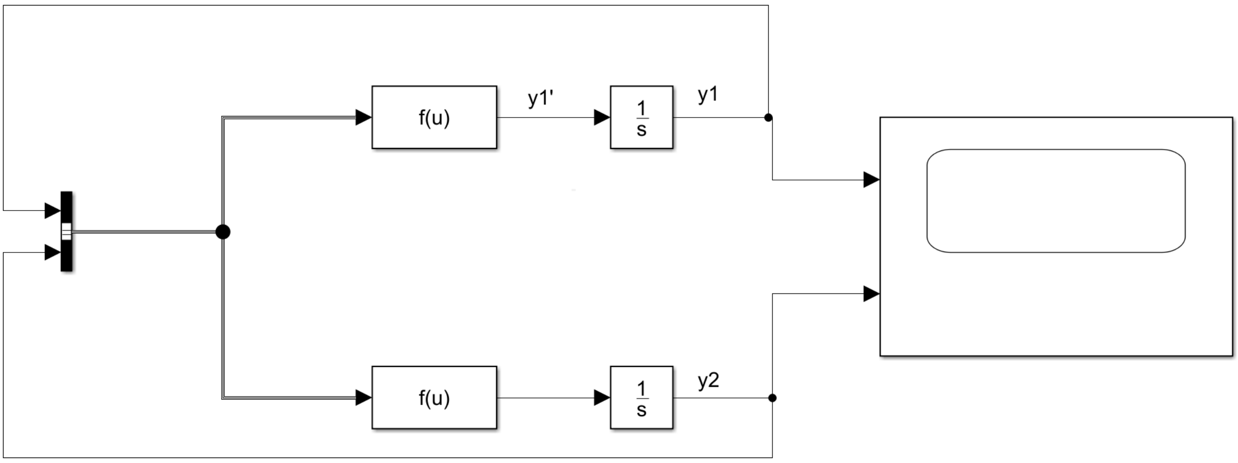

Figure 2: Simulink Model for the Ecological Prey & Predator Model

(b) Result for the simulation with given coefficients, can be seen at *Figure 3*.

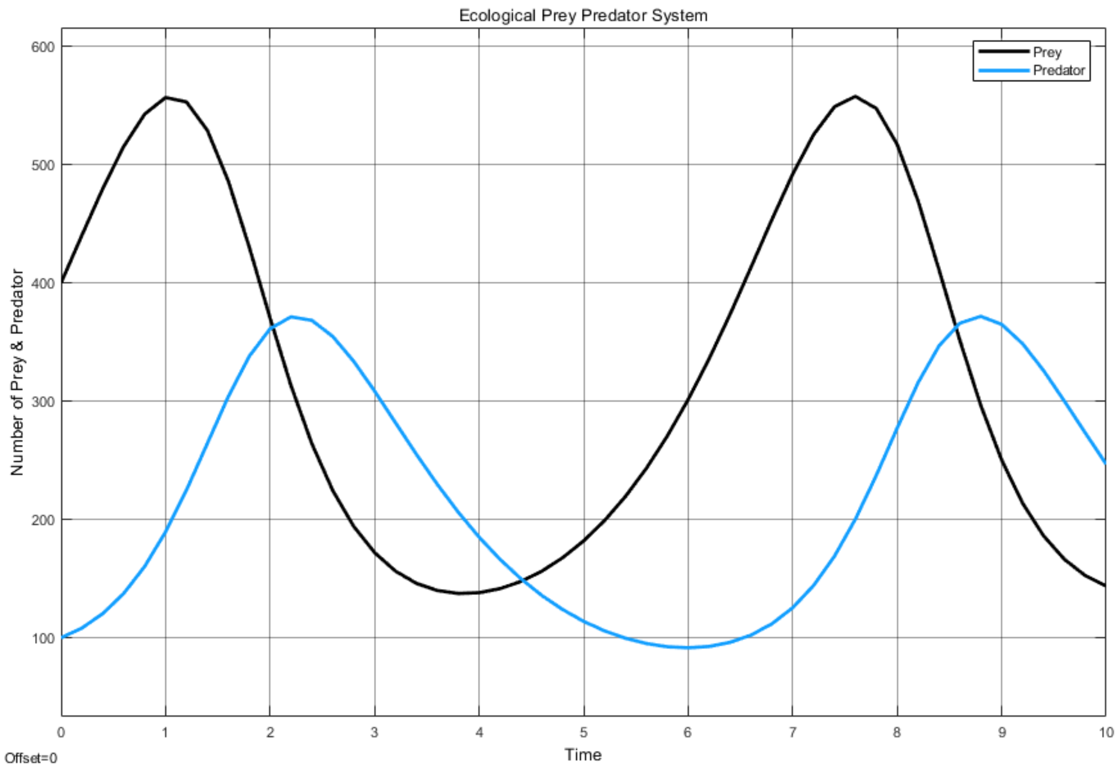

Figure 3: Prey & Predator Model as $\eta_1 = 300$,$\eta_2 = 100$,$y_{1,initial} = 400$,$y_{2,initial} = 100$

(c) The pattern of population reminded us the "cyclic population behaviour" and this non-linear system provides us sustainable oscillations. When the prey increase, firstly the predator also increase as can be seen from *Figure 3*, after some point the increase in the predators results in the decrease in the prey. Then, this decrease in the preys also results in decrease in predator population. And, it goes in this circulation behaviour.

An basic open market economical model can also be similar example. In this model, as the demand for the product increases the price of this product also increases for a point where the customers will not be willing to pay more money. Then, the decrease in the customers force suppliers to reduce their prices. To give a recent example, the excess increase in the Iphone prices in recent years resulted in the decrease in the sales of Iphones in very first time since its launch in 2008.[1]

(d) The desired model committed with a *"The resulting simulink model for Q2b"* message.

(e) The equilibrium point can be found by equating the change in the variable to the zero, that is to find the point where the value of the desired variable is not changing afterwards. For our interest, we would like to find the values where the change in the population of preys and predators are zero.

$$\frac{d}{dt}y_1(t) = (1 - \frac{y_2}{\eta_2})y_1 = 0$$

To satisfy the equation without equating $y_1 = 0$, the only possibility is to use $y_2 = \eta_2 = 200$,

$$\frac{d}{dt}y_2(t) = -(1 - \frac{y_1}{\eta_1})y_2 = 0$$

To satisfy the equation without equating $y_2 = 0$, the only possibility is to use $y_1 = \eta_1 = 300$,

When the initial value of populations are equated to the equilibrium point values found above, the populations did not change for any t as expected as in *Figure 4*. When the initial values equated to a value such that the value is very close the equilibrium points found above, the period of the oscillation does not change in comparison to the original system at *Figure 3* and the magnitude of oscillations decreases as expected as in *Figure 5*.

[1] https://www.engadget.com/2018/11/01/apple-will-stop-reporting-how-many-iphones-ipads-and-macs-it-se/

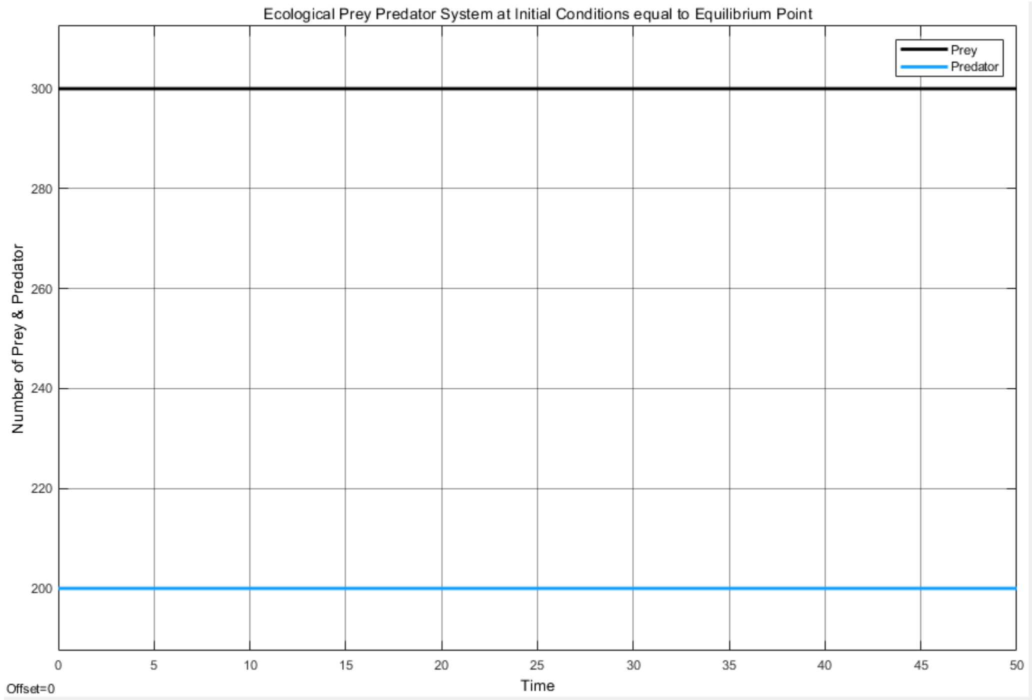

Figure 4: Prey & Predator Model as $\eta_1 = 300$,$\eta_2 = 100$,$y_{1,initial} = 300$,$y_{2,initial} = 200$

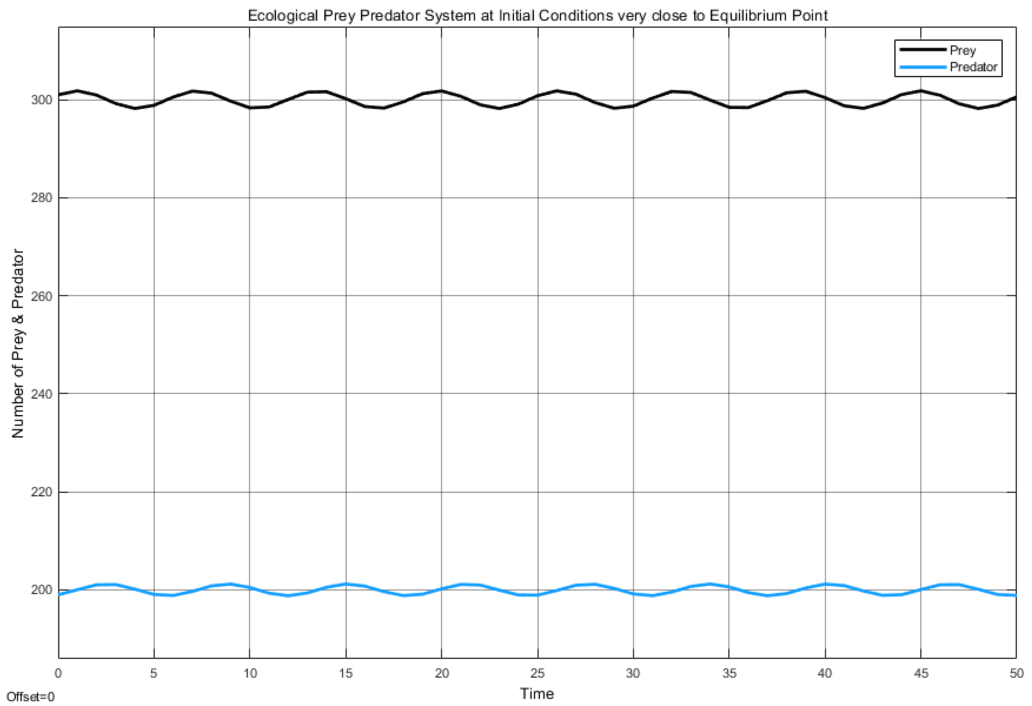

Figure 5: Prey & Predator Model as $\eta_1 = 300$,$\eta_2 = 100$,$y_{1,initial} = 303$,$y_{2,initial} = 197$

(f) Resulted Simulink model with changed variables can be seen at *Figure 6*.

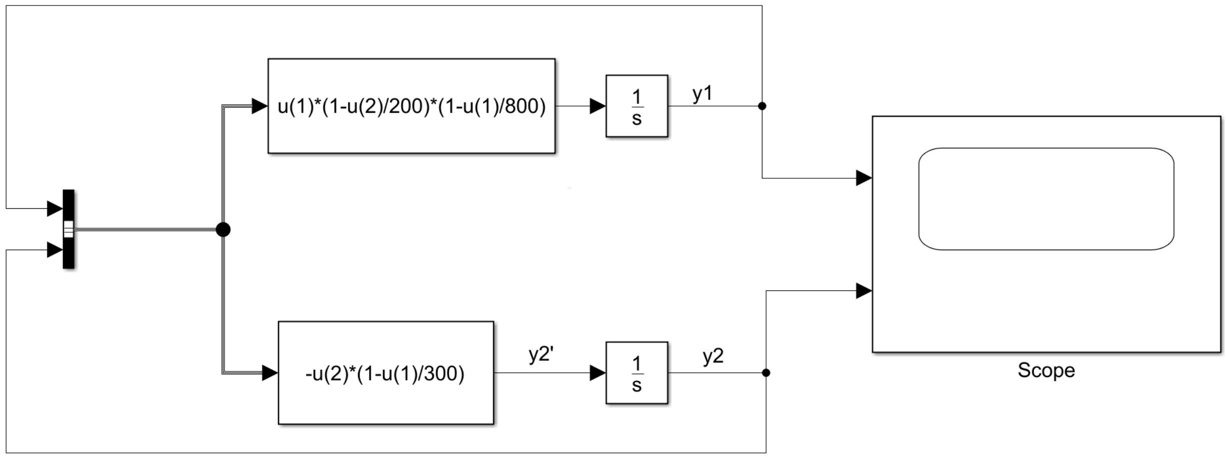

Figure 6: Simulink Model for the Ecological Prey & Predator Model with Introduced Growth Limiting Term

(g) Result for the updated system simulation with given coefficients, can be seen at *Figure 7*.

The shape of the solution curves again shows cylindrical behaviour similar to initial model and it is still periodic but the period of oscillations is increased and the magnitudes of oscillations of prey and predators became more similar.

(h) The desired model committed with a *"Simulink Model for Q2g, matlab part is completed together"* message.

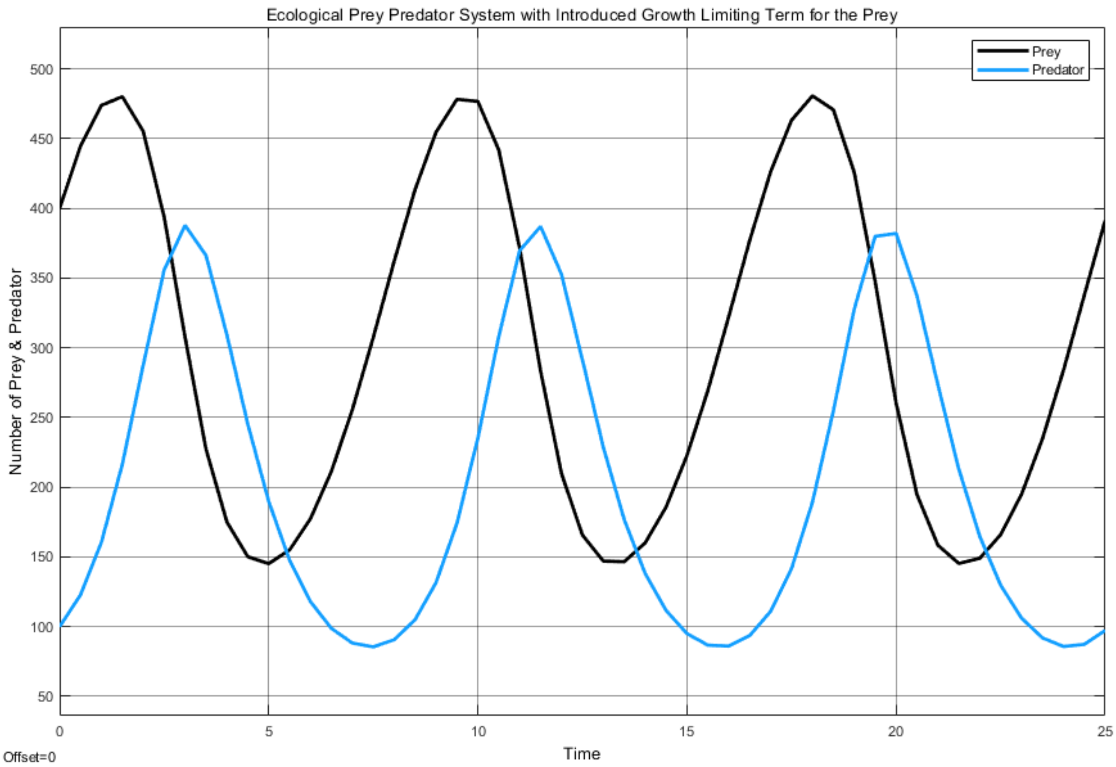

Figure 7: Prey & Predator Model as $\eta_1 = 300$,$\eta_2 = 100$,$y_{1,initial} = 400$,$y_{2,initial} = 100$