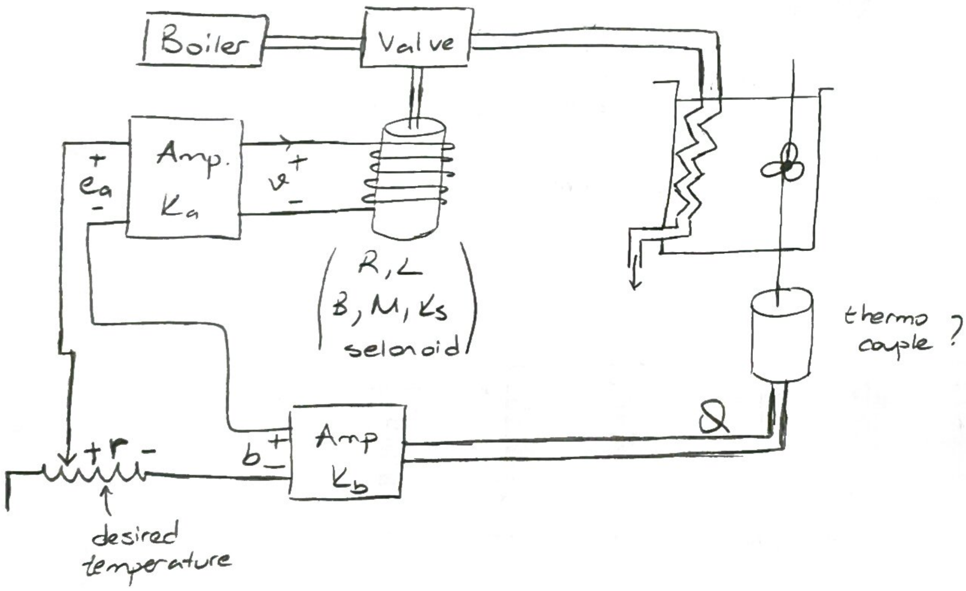
Process Control

2018 - 2019 Fall

03.10.2018 / Mitwoch

physical system | experiment
model | simulation
solution

System Diagram:



1. Voltage generated by the thermo caple is amplified:

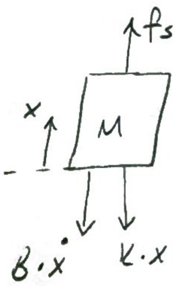
$$b = K_b \cdot \theta \longrightarrow \text{Terminal Equation}$$

2. Actuating signal "e" is the difference between the desired "r" and actual "b":

$$-r + e + b = 0 \longrightarrow \text{Topolepical Equation}$$

3. Voltage "v" energizes the selonoid and generates a force "fs" proportional to the current "i":

$$I(s) = \frac{1}{R + sL} V(s) \quad , \quad f_s = K_s \cdot I$$



x: position of the valve

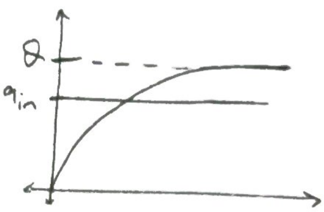
$$\mathcal{L}\left\{ \begin{array}{l} f_{net} = M \cdot \ddot{x} = f_s - B\dot{x} - Kx \\ F_s = (Ms^2 + Bs + K)\, X(s) \end{array} \right\} \quad X(s) = \frac{1}{Ms^2 + Bs + K} F(s)$$

" Force from the seleroid acts on the mass of the valve, damping B and restraining spring with constant K, moving the valve by an amount of x."

4. The valve in turn controls the flow q of hot steam into the tank in a proportional manner.

$$q = K_q \cdot x$$

5. Temperature in the tank

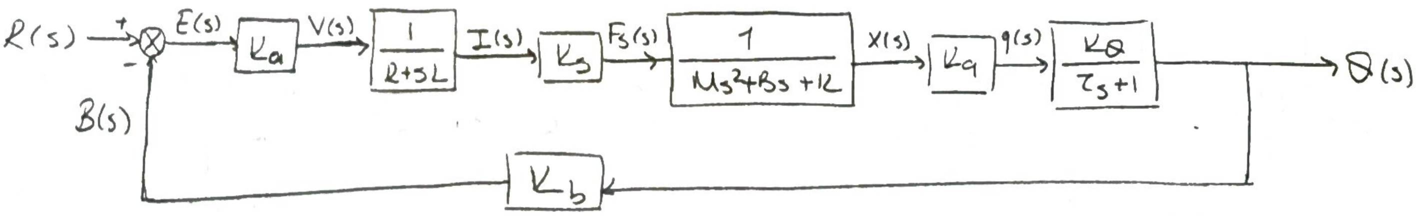


$$\theta(s) = K_\theta \cdot \frac{Q(s)}{\tau s + 1}$$

↑ time costant of the system

(Note: Behaves like an R, C circuit.)

Block Diagram:



Block Diagram Mathematical Model of the physical system

08.10.2018 / Montag

How do we build models?

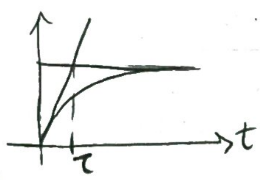
I. { Collective experience of experts in the field (lots of experiments) (inc. laws of nature worked out by generations of scientists.)

II. { The actual system being modeled and experiments on it.

III. Mathematical Model of System

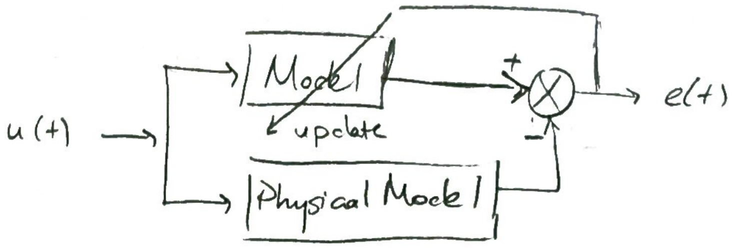
(I - II) Domain spesific!

(II - III) Knowledge or control engineer



$$G(s) = \frac{1}{\tau s + 1}$$

III → Physical Modelling → System Identification → Model Validation

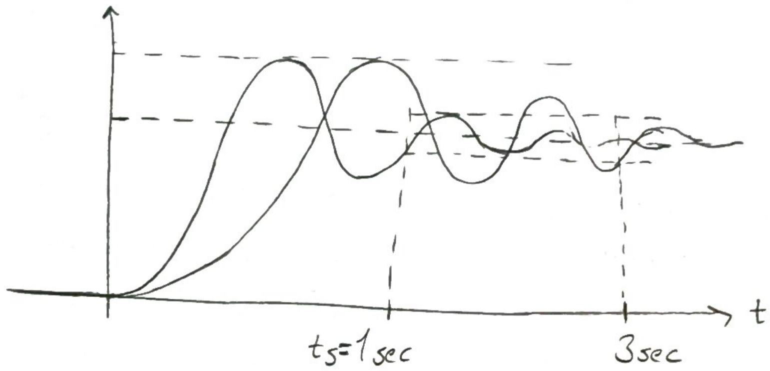


Example:
(System Identification)

From first principles the model of a physical system is found to be (with best guess parameter values:)

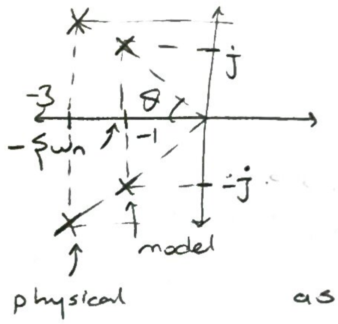
$$G_m(s) = \frac{1}{s^2+2s+2}$$

When experiments are performed, the system show the following response: Find (update) the system model.



Model Response: as^2+bs+c

poles =? $\Delta = b^2-4ac = 4-4.1.2 = -4 \rightarrow s_{1,2} = \frac{-2 \pm j2}{2} = -1 \pm j$



%5 settling time

$$t_s \simeq \frac{3}{\xi w_n} = 3\,secs$$

$\xi = \cos\theta \rightarrow \xi$ & M_p are directly related

Observe that physical system has the same max overshoot as the model

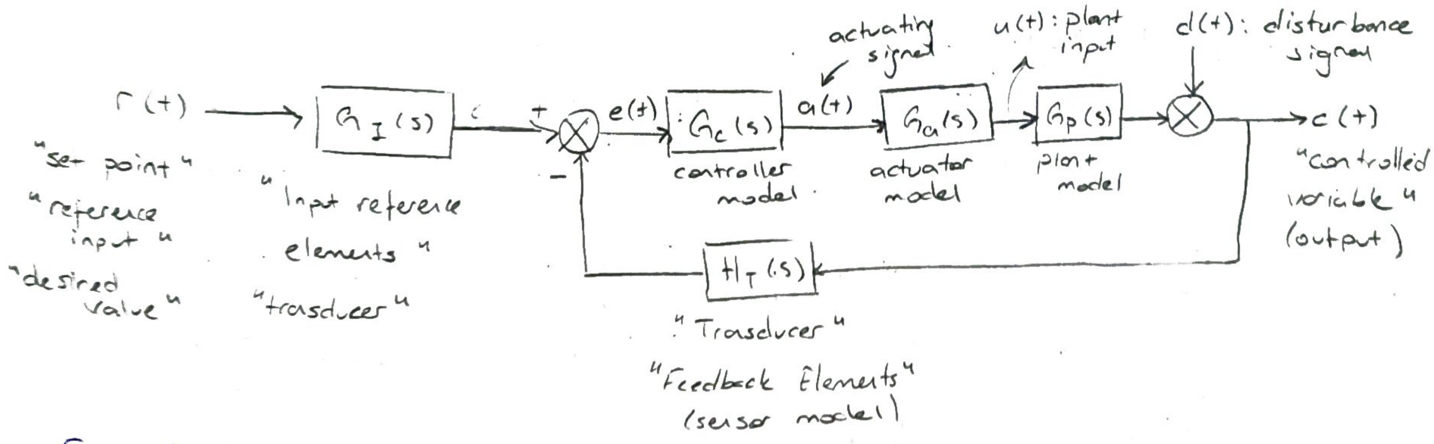
→ We need to be on the same angle for the pole but

$$t_s = 1\,sec = \frac{3}{\xi w_n} \rightarrow \boxed{\xi w_n = 3}$$

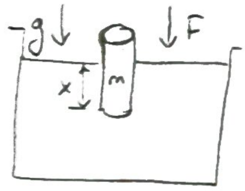
↳ Use the correct set of poles to construct the new denominator for the model.

$$D_{new}(s) = (s+3+3j)(s+3-3j) = s^2+6s+18$$

General Structure of a Process Control System



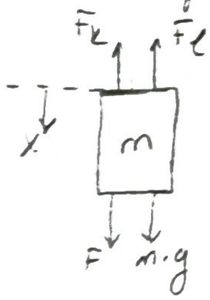
Example:



A floating object has a cylindrical body shape with radius R and mass m. There is viscous friction between the object and the liquid with coefficient K per unit area. A downward force F is applied to the object (input) and we are interested in how the object floats assuming we also have gravity.

Solution: Mostly a mechanical problem except how to determine the force exerted by the liquid to float the object F_ℓ.

Free Body Diagram:



F_k: viscous friction

F_ℓ: lifting force → Archimede's Principle

$$F_k = [K \cdot (2\pi R x)] \cdot \dot{x}$$

frictional constant, surface area

$$F_\ell = V \times \text{density} \times g$$

if water

$$= \pi \cdot R^2 \cdot x \cdot 1 \cdot g$$

System Model is from Newton's 2^{nd} Law:

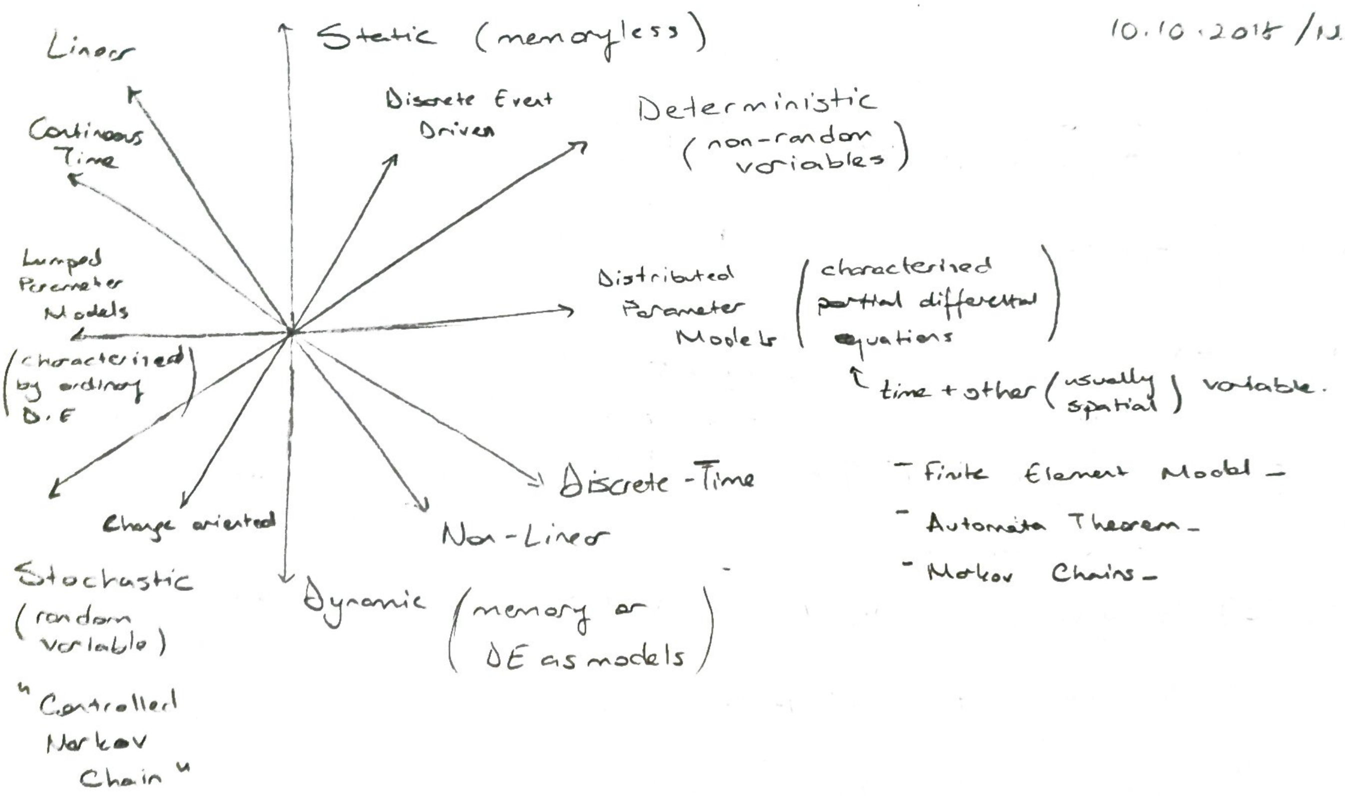
$$F_{net} = m \cdot \ddot{x} = F + mg - F_k - F_\ell$$

$$= F + mg - K \cdot 2\pi R x \cdot \dot{x} - \pi R^2 \cdot x \cdot g$$

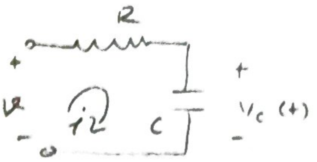
$$\rightarrow \boxed{m \cdot \frac{d^2x(t)}{dt} + 2\pi R K x(t) \cdot \frac{dx(t)}{dt} + \pi R^2 g x(t) = mg + F(t)}$$

Types of System Models

08.10.2018 / Monday
10.10.2018 / Wednesday



Example:



$$-v(t) + \underbrace{V_R(t)} + V_C(t) = 0$$

$$R\, i_R(t) = R \cdot i_c(t) = R \cdot C \cdot \frac{dV_c(t)}{dt}$$

$$\rightarrow -v(t) + RC \cdot \frac{dV_c(t)}{dt} + V_c(t) = 0$$

$$\rightarrow \boxed{RC \cdot \frac{dV_c(t)}{dt} + V_c(t) = V(t)}$$ Differential Equations

Assume zero initial conditions:

$$\mathcal{L}\{\cdot\} \Rightarrow RCsV_c(s) + V_c(s) = V(s) \longrightarrow \frac{V_c(s)}{V(s)} = \boxed{T(s) = \frac{1}{RCs+1}}$$ Transfer Function Representation

Q: Can I find a discrete-time approximation to the model

A: Newton's Difference Quotient

Approximate:

$$\frac{d}{dt}v(t) \cong \frac{v(t+T) - v(t)}{T} \cong \frac{v(t) - v(t-T)}{T} = \frac{v[k] - v[k-1]}{T}$$

This approximation becomes more accurate with smaller T.

Former is non-causal versus the latter which is causal approximation.

$$RC \cdot \frac{dV_c(t)}{dt} + V_c(t) = V(t)$$

$$\downarrow \quad RC \cdot \left[\frac{V_c[k] - V_c[k-1]}{T} \right] + V_c[k] = V[k]$$

$$\rightarrow RC \cdot \left[V_c[k] - V_c[k-1] + T V_c[k] \right] = T \cdot V[k]$$

$$\rightarrow (RC + T)\, V_c[k] = RC \cdot V_c[k-1] + T V[k]$$

$$\rightarrow \boxed{V_c[k] = \frac{RC}{RC+T} V_c[k-1] + \frac{T}{RC+T} \cdot V[k]}$$ "Difference Equation"

If $V_c[0]$ is given, also $V[k]$, for $\forall\, k \geq 1$ is given, you can calculate any V_c value for the future.

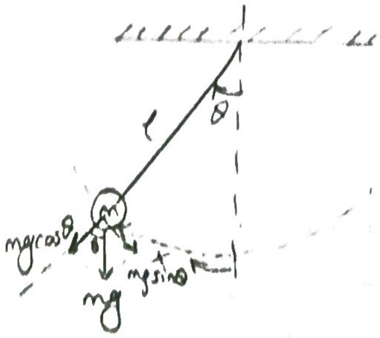
$\downarrow$ Z.T. assuming $V_c[0] = 0$

$$V_c(z) = \frac{RC}{RC+T} \cdot z^{-1} \cdot V_c(z) + \frac{T}{RC+T} \cdot V(z)$$

$$\frac{V_c(z)}{V(z)} = \boxed{T(z) = \frac{\frac{T}{RC+T}}{1 - \frac{RC}{RC+T} \cdot z^{-1}}}$$

Q: How the system behaves in terms of stability?

Q: How does the behaviour of the cont. time system relates to its DT appr.



$x = l \cdot \theta$

$\dot{x} = l \cdot \dot{\theta}$

$\ddot{x} = l \cdot \ddot{\theta}$

$F_{net} = m \cdot \ddot{x} = -mg\sin\theta = m \cdot l \cdot \ddot{\theta}$

$ml \cdot \ddot{\theta} + mg\sin\theta = 0$

$\boxed{l \cdot \ddot{\theta} + g\sin\theta = 0}$ "Non-Linear Differential Equation"

Q: Can we find a linear approximation?

A: If we choose an operating point.

Around $\theta \approx 0 \rightarrow \sin\theta \simeq \theta \longrightarrow \boxed{l \cdot \ddot{\theta} + g \cdot \theta = 0}$ "Linearized Equation"

15.10.2018 / Monday

Example: (Ecological Model)

V. Volterra "Variations and Fluctuations of the Number of Individuals in Animal Species Living Together" 1928

→ Population Dynamics

→ Two animal species with population $N_1(t)$ and $N_2(t)$

→ Birth rate of species, constants λ_1 and λ_2

↳ Means; inflow of members $\lambda_1 \cdot N_1(t)$ and $\lambda_2 N_2(t)$

→ Mortality rates are more complex. μ_1 and μ_2 and they depend on "the availability of foods" and "the risk of being eaten" in general.

$$\mu_1 = \mu_1(N_1(t), N_2(t))$$
$$\mu_2 = \mu_2(N_1(t), N_2(t))$$

↳ Outflow of members $\quad out_1 = \mu_1(N_1(t), N_2(t)) \cdot N_1(t)$
$\quad out_2 = \mu_2(N_1(t), N_2(t)) \cdot N_2(t)$

→ Conservation Law:

Inflow − outflow = Accumulation. $\rightarrow \frac{d}{dt} N_1(t)$

↳ Using the "Conservation Law"

$$\left.\begin{aligned} \frac{d}{dt} N_1(t) &= \lambda_1 \cdot N_1(t) - \mu_1(N_1(t), N_2(t)) \cdot N_1(t) \\ \frac{d}{dt} N_2(t) &= \lambda_2 \cdot N_2(t) - \mu_2(N_1(t), N_2(t)) \cdot N_2(t) \end{aligned}\right\} \text{Turned out as a "Space State Model"}$$

Case 1: Species compete for the same food.

A simple model → Total # of both species → Availability of Food → Mortality Rates

$$\mu_1(N_1, N_2) = \gamma_1 + \delta_1 \cdot (N_1 + N_2) \quad \text{where } \gamma_i > 0 \text{ \& } \delta_i > 0$$

Now we have

$$\left.\begin{aligned} \frac{d}{dt} N_1(t) &= (\lambda_1 - \gamma_1) N_1(t) - \delta_1 (N_1(t) + N_2(t)) N_1(t) \\ \frac{d}{dt} N_2(t) &= (\lambda_2 - \gamma_2) \cdot N_2(t) - \delta_2 (N_1(t) + N_2(t)) \cdot N_2(t) \end{aligned}\right\} \text{Non-Linear State Space Model}$$

It can be shown that $\frac{\lambda_1 - \gamma_1}{\delta_1} > \frac{\lambda_2 - \gamma_2}{\delta_2}$ then the 2nd species (N_2) will die-out and the 1st species (N_1) will approach $\left(\frac{\lambda_1 - \gamma_1}{\delta_1}\right)$.

Equilibrium Point: A condition where the rate of change of the state variable is arrived to zero.

If $N_2 = 0$ as $t \to \infty$
$N_1 = \dfrac{\lambda_1 - \gamma_1}{\delta_1}$ as $t \to \infty$ $\Big\}$ $\begin{bmatrix} N_1 \\ N_2 \end{bmatrix}$ equilibrium point.

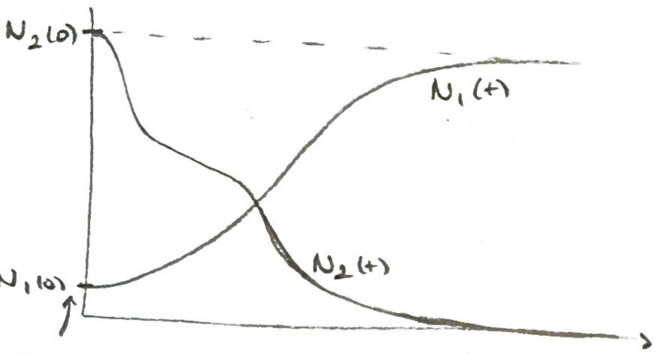
System gets stuck in equilibrium point. There is no way out.

Put into differential equations:

$$\frac{d}{dt} N_1(t) = (\lambda_1 - \gamma_1) \cdot \left(\frac{\lambda_1 - \gamma_1}{\delta_1}\right) - \delta_1 \cdot \left(\frac{\lambda_1 - \gamma_1}{\delta_1}\right)^2 = 0$$

$$\frac{d}{dt} N_2(t) = 0 - 0 = 0$$

If we simulate this: $\lambda_1 = 3$, $\lambda_2 = 2$, $\gamma_1 = \gamma_2 = \delta_1 = \delta_2 = 1$



Note that very small initial pop. for N_1

Case II: Predator & Prey Case

Assume that one (N_1) species feeds on the second

* The supply of food for species 1 is proportional to $N_2(t)$ and their mortality rate diminishes with more food (N_2 increase)

$$\mu_1(N_1, N_2) = \gamma_1 - \alpha_1 N_2(t) \quad , \quad \alpha_1 > 0 \ \& \ \gamma_1 > 0$$

* But the mortality rate of species (2) increases when $N_1(t)$ increases.

$$\mu_2(N_1, N_2) = \gamma_1 + \alpha_2 N_1(t) \quad , \quad \alpha_2 > 0 \ \& \ \gamma_2 > 0$$

$$\frac{d}{dt} N_1(t) = \lambda_1 \cdot N_1(t) - (\gamma_1 - \alpha_1 N_2(t)) \cdot N_1(t)$$

$$\frac{d}{dt} N_2(t) = \lambda_2 \cdot N_2(t) - (\gamma_2 + \alpha_2 N_1(t)) \cdot N_2(t)$$

↳ rewrite

$$\frac{d}{dt} N_1(t) = (\lambda_1 - \gamma_1) N_1(t) + \alpha_1 N_1(t) \cdot N_2(t)$$

$$\frac{d}{dt} N_2(t) = (\lambda_1 - \gamma_2) N_2(t) - \alpha_2 \cdot N_1(t) \cdot N_2(t)$$

We ~~ca~~ further assume the followings:

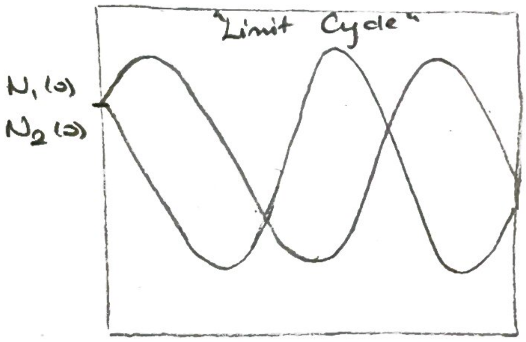
* If there is no prey ($N_2 = 0$); predators will die out:

$$\left.\begin{array}{l} \frac{d}{dt} N_1(t) < 0 \\ N_1(t) > 0 \end{array}\right\} \lambda_1 - \gamma_1 < 0 \rightarrow \lambda_1 < \gamma_1$$

* If there is no predator ($N_1 = 0$); preys will multipy:

$$\left.\begin{array}{l} \frac{d}{dt} N_2(t) > 0 \\ N_2(t) > 0 \end{array}\right\} \lambda_2 - \gamma_2 > 0 \longrightarrow \lambda_2 > \gamma_2$$

Simulation for a typical case satisfying the conditions:

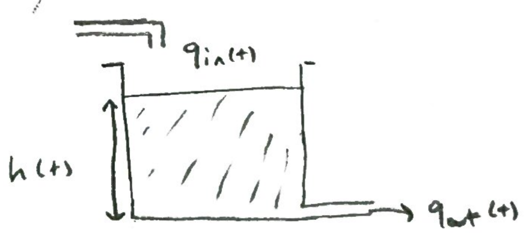


$\lambda_1 = 1$, $\gamma_1 = 2$, $\lambda_2 = 2$, $\gamma_2 = 1$

$\alpha_1 = \alpha_2 = 1$

Linearization	Operating Points	Nominal
Equilibrium point	Delta Variable	

Example: "Simple Flow System" - Tank with free out-flow



in-flow rate: $q_{in}(t)$ (m^3/sec)

out-flow rate: $q_{out}(t)$ (m^3/sec)

tank cross-section A (m^2)

out-flow crossection a (m^2)

? in-flow crossection a (m^2) ?

Liquid level $h(t)$ (m)

1) Bernollie's Law; determines the relationship between $h(t)$ and out-flow speed $V_{out}(t)$ (m/s)

$$V_0(t) = \sqrt{2gh(t)} \rightarrow \sqrt{\frac{m}{s^2} \cdot m} = (m/s)$$

→ Determine what happens to the $h(t)$ as a function of $q_{in}(t)$?

2) Volumetric out flow rate:

$$q_o(t) = a \cdot (m^2) \cdot V_o\left(m/s\right) = a \cdot V_o(t) \left(m^3/s\right)$$

3) Volume of liquid (accumulation) in the tank:

$$T(t) = A \cdot h(t) \quad (m^2 \cdot m) = (m^3)$$

4) Mass-balance Equation (Conservation Law)

Assumption: Liquid density is constant

inflow − outflow = accumulation

$$\frac{d}{dt} T(t) = q_{in}(t) - q_{out}(t)$$

$$\rightarrow \boxed{A \cdot \frac{d}{dt} h(t) = q_{in}(t) - a \cdot V_o(t) = q_{in}(t) - a \cdot \sqrt{2g \cdot h(t)}}$$

Non-Linear System.

Simulate this system:

$$q_{in}(t) = 1, \quad t > 0 \quad \& \quad A = 1 \quad \& \quad a \cdot \sqrt{2g} = 1$$

i) $h(0) = 0\ m$

ii) $h(0) = 2\ m$

Simplest way is to use Newton's Formula: (Let $\Delta t = 0.2$)

$$\frac{h[k] - h[k-1]}{\Delta t} = 1 - \sqrt{h[k-1]}$$

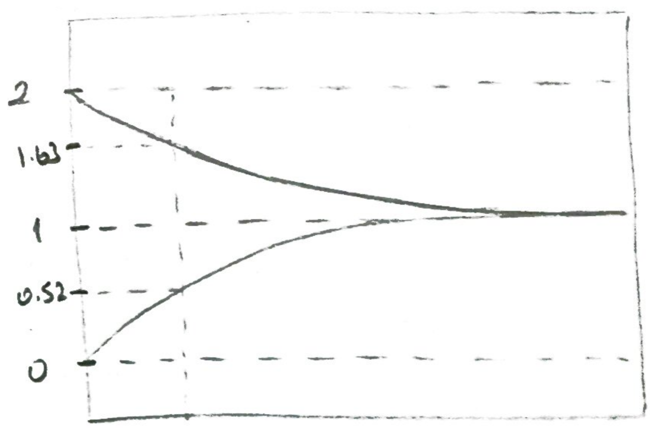
$$\rightarrow h[k] = h[k-1] + 0.2 - 0.2 \cdot \sqrt{h[k-1]}$$ → Difference Equation Approximation

	$h[0] = 0\ m$ $h[k]$	$h[0] = 2\ m$ $h[k]$
k=1	0.2	1.92
k=2	0.31	1.84
k=3	0.39	1.76
k=4	0.46	1.69
k=5	0.52	1.63

$$h[1] = h[0] + 0.2 - 0.2 \cdot \sqrt{h[0]} = 0.2$$

$$h[2] = h[1] + 0.2 - 0.2 \cdot \sqrt{h[1]} = 0.31$$

⋮



17.~~00~~.2018 /Mitwoch

We obtained:

$$A \cdot \frac{d}{dt} h(t) = q_{in}(t) - a \cdot \sqrt{2g\,h(t)}$$

We will assume a "nominal" in flow rate $\rightarrow q_{in}(t) = q_{in,op} \rightarrow$ operating value

Q: Is there a steady-state $h(t)$?

Yes. "self stabilizing" systems

↳ System has a equilibrium point.

Definition: "Equilibrium Point" $\rightarrow \boxed{\frac{d}{dt} h(t) = 0}$

$$\hookrightarrow q_{in}(t) - a\sqrt{2g} \cdot \sqrt{h(t)} = 0$$

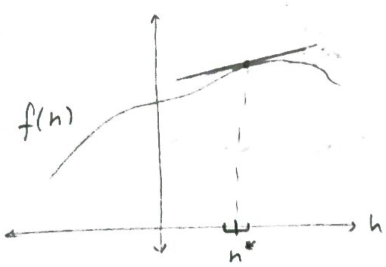
$$h(t) = \frac{(q_{in}(t))^2}{2a^2 g}$$

Operating point: $q_{in}(t) = q_{in,op} \longrightarrow \boxed{h_{op} = \frac{(q_{in,op})^2}{2a^2 g}} \quad (*)$

$\left.\begin{matrix} q_{in}(t) = q_{in,op} \\ h(t) = h_{op} \end{matrix}\right\}$ Chosen Equilibrium Point $\Big\}$ Operating Point

Definition: An operating point is a chosen equilibrium point around which we will derive a linear approximation to a non-linear system.

How: To use the Taylor Series Expansion



$$f(h) = f(h^*) + \left.\frac{df(h)}{dh}\right|_{h=h^*} \cdot (h - h^*) + H.O.T.$$

$$f(h, q_{in}) \cong \underbrace{f(h_{op}, q_{in,op})}_{\text{is 0 because of } (*)} + \left.\frac{df(h,q_{in})}{dh}\right|_{h=h_{op}} \cdot (h(t) - h_{op}) + \left.\frac{df(h,q_{in})}{dq_{in}}\right|_{q_{in}=q_{in,op}} \cdot (q_{in} - q_{in,op})$$

$$f(h, q_{in}) = \frac{dh(t)}{dt} \cong \underbrace{\left.\frac{a}{2A} \cdot \frac{1 \cdot 2g}{(2g\,h(t))^{1/2}}\right|_{h(t)=h_{op}}}_{\text{Constant } K_1} \cdot \underbrace{(h(t) - h_{op})}_{\Delta h(t)} + \underbrace{\frac{1}{A}}_{K_2} \underbrace{(q_{in}(t) - q_{in,op})}_{\Delta q_{in}(t)}$$

"Delta Variable"

$\tilde{h}(t) \triangleq h(t) - h_{op}$

$\tilde{q}_{in}(t) \triangleq q_{in}(t) - o$

$$\frac{d}{dt}\tilde{h}(t) = K_1 \cdot \tilde{h}(t) + K_2\, \tilde{q}_{in}(t)$$ Linear Ordinary Differential Equation.

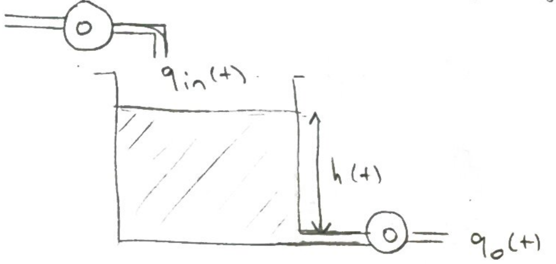
$\tilde{h}(0) = 0$ if you start at the operating point.

$$s\tilde{H}(s) - \underbrace{\tilde{h}(0)}_{0} = K_1 \cdot \tilde{H}(s) + K_2 \cdot \tilde{Q}_{in}(s)$$

$$\rightarrow (s - K_1)\tilde{H}(s) = K_2\, \tilde{Q}_{in}(s) \longrightarrow G(s) = TF(s) = \frac{K_2}{s - K_1} = \frac{\tilde{H}(s)}{\tilde{Q}_{in}(s)}$$

Provided that we start and remain in the close proximity of the operating point.

Example: Tank with pumps. (Integrating process.) 22.10.2018 / Monday



We have:

$$A \cdot \frac{dh(t)}{dt} = q_{in}(t) - q_o(t)$$

→ $q_{in}(t)$ and $q_o(t)$ are dictated by the pumps.

Note:

$q_{in}(t) = q_o(t) \Rightarrow h(t) \rightarrow \;\Rightarrow$ no change

$q_i(t) > q_o(t) \Rightarrow h(t) \nearrow \;\Rightarrow$ increasing indefinetely

$q_i(t) < q_o(t) \Rightarrow h(t) \swarrow \;\Rightarrow$ decreasing indefinetely.

↳ $x(t) \triangleq q_i(t) - q_o(t)$

We have $A\,\dfrac{dh(t)}{dt} = x(t) \xrightarrow{\mathcal{L}} sA \cdot H(s) - \underbrace{h(0)}_{\text{problem}} = X(s)$

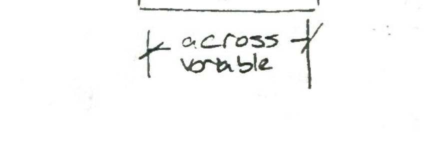
single input / single output

→ Define a operating point where $h_{op} = h(0)$ and define the "Delta Variable" as $\tilde{h}(t) = h(t) - h_{op} \longrightarrow \tilde{h}(0) = 0$

$$\frac{d\tilde{h}(t)}{dt} = \frac{dh(t)}{dt}$$

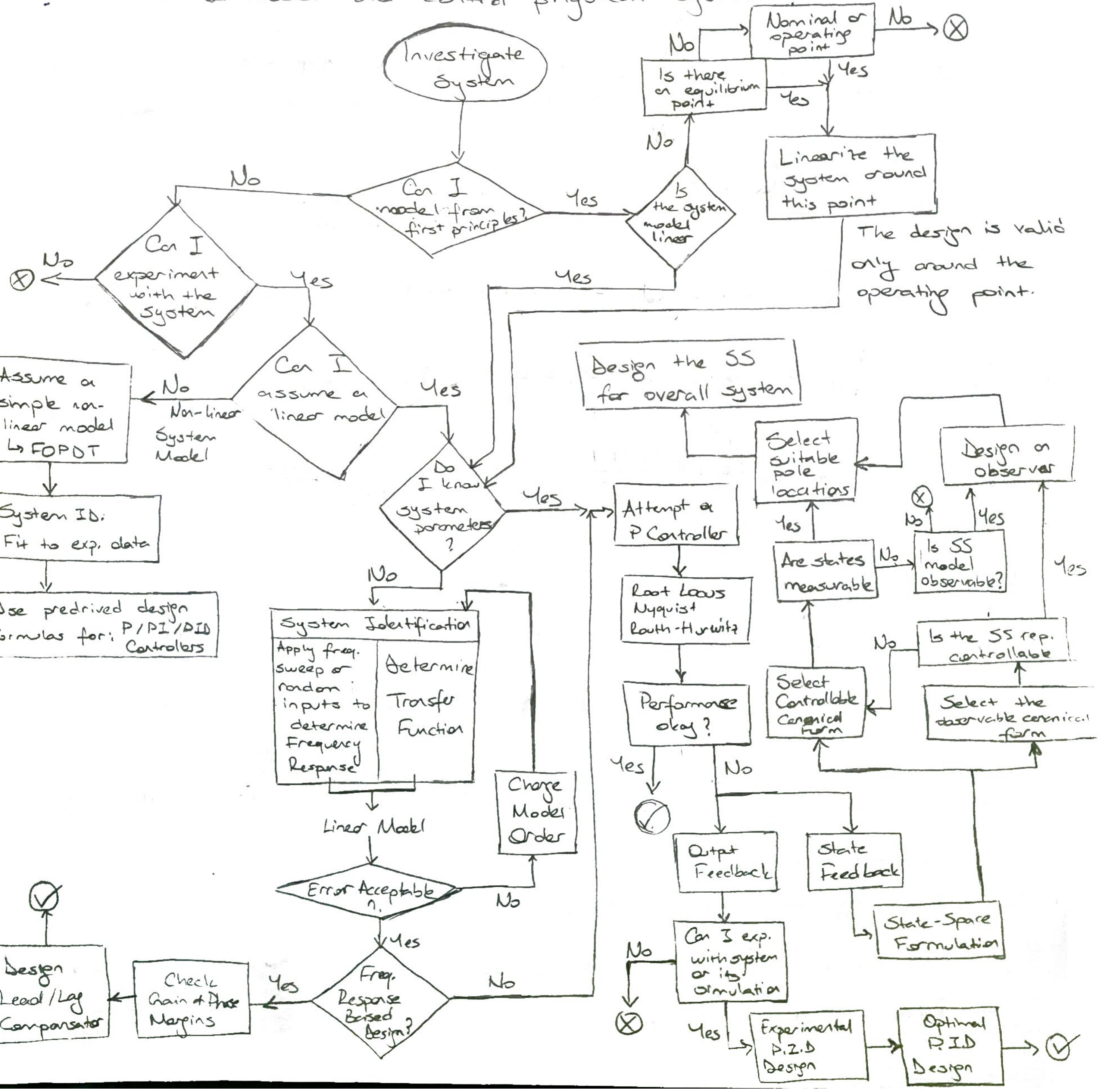
$$\longrightarrow sA \cdot \tilde{H}(s) = X(s) \longrightarrow G(s) = \frac{\tilde{H}(s)}{X(s)} = \frac{1}{As}$$ integrator

Note: "Generalization of System Models"

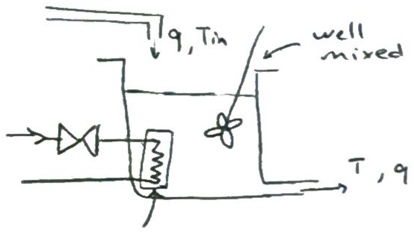


System	Thru Variable	Across Variable
Electrical	Current i (A)	Voltage Difference V_{12} (V)
Translational Mechanical	Force F (N)	Velocity Difference V_{21} $\left(\frac{m}{s}\right)$
Rotational Mechanical	Torque T (N.m)	Angular Velocity Difference ω_{21} $\left(\frac{rad}{sec}\right)$
Fluid	Volumetric Rate of Flow q $\left(\frac{m^3}{s}\right)$	Pressure Difference P_{21} $\left(\frac{N}{m^2}\right)$
Thermal	Heat Flow Rate q (J/sec)	Temperature Difference T_{21} (°C)

? How do I model and control physical systems?



Example: The Heat Transfer Process (Fikor)



T_h: Temperature of heater
F: Area of contact with the liquid.

T_{in}: temperature of inflowing liquid (°C)
q: Volumetric in/out flow rate (m^3/sec)
T: temperature of well-mixed liquid in tank (°C)
V: volume of the liquid (m^3)
ρ: density of the liquid (kg/m^3)
c_p: spesific heat (J/kelvin)

Aim: Regulating the temperature inside the tank.

→ Heat accumulation = inflow of heat energy − outflow of heat.

r: Heat flow rate

$$C \cdot \frac{dT(t)}{dt} = r_{net}$$

T(t): temperature of the liquid (Accross variable)
r_{net}: net flow of heat energy (Thru variable)

$$C = (\underbrace{V}_{\text{volume}} \cdot \underbrace{\rho}_{\text{density}} \cdot \underbrace{c_p}_{\text{spesific heat}})$$

$$(V \cdot \rho \cdot c_p) \cdot \frac{dT(t)}{dt} = r_{in} - r_{out} = \left[q \cdot \rho \cdot c_p \cdot T_{in} + \underbrace{\alpha}_{\text{heat transfer coeffecent}} \cdot F \cdot (T_h - T)\right] - \left[q \cdot \rho \cdot c_p \cdot T\right]$$

$$(V \cdot \rho \cdot c_p) \cdot \frac{dT(t)}{dt} = q \cdot \rho \cdot c_p \cdot T_{in}(t) - \underbrace{(q \rho \cdot c_p + \alpha F)}\, T(t) + \alpha F T_h(t)$$

$$\underbrace{\frac{V \rho \cdot c_p}{q \cdot \rho \cdot c_p + \alpha F}}_{T_1} \frac{dT(t)}{dt} + T(t) = \underbrace{\frac{q \cdot \rho \cdot c_p}{q \cdot \rho \cdot c_p + \alpha F}}_{z_1} T_{in}(t) + \underbrace{\frac{\alpha F}{q \cdot \rho \cdot c_p + \alpha F}}_{z_2} T_h(t)$$

$$\Rightarrow T_1 \cdot \frac{dT(t)}{dt} + T(t) = z_1 \cdot T_{in}(t) + z_2 \cdot T_h(t)$$

Usually $T(0) \neq 0$

We need to define a steady-state (operating point) by letting $\frac{dT(t)}{dt} = 0$.

$$T_{op} = z_1 T_{in,op} + z_2 T_{h,op}$$

Given $T_{in,op}$ and T_{op}; find the corresponding $T_{h,op}$ from this Eq. to determine the operating point.

At the operating point define "Delta Variables"

$$\tilde{T}(t) = T(t) - T_{op} \; ; \; \tilde{T}_{in}(t) = T_{in}(t) - T_{in,op} \; ; \; \tilde{T}_h(t) = T_h(t) - T_{h,op}$$

$$\rightarrow \tilde{T}(0) = 0 \quad \text{if} \quad T(0) = T_{op}$$

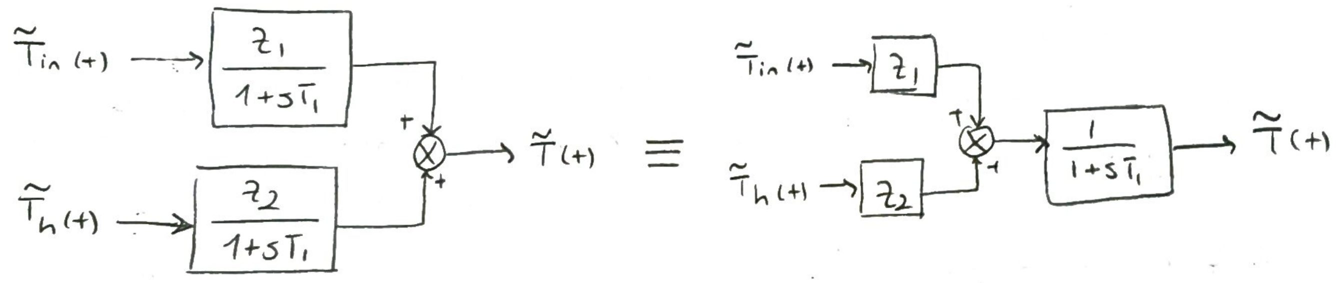
Apply superposition:

Let $\tilde{T}_{in}(t) = 0 \longrightarrow T_1 \cdot \frac{d\tilde{T}(t)}{dt} + \tilde{T}(t) = Z_2 \cdot \tilde{T}_h(t)$

$$\boxed{H_1(s) = \frac{\tilde{T}(s)}{\tilde{T}_h(s)} = \frac{Z_2}{1+sT_1}}$$

Let $\tilde{T}_h(t) = 0 \longrightarrow T_1 \cdot \frac{d\tilde{T}(t)}{dt} + \tilde{T}(t) = Z_1 \cdot \tilde{T}_{in}(t)$

$$\boxed{H_2(s) = \frac{\tilde{T}(s)}{\tilde{T}_{in}(s)} = \frac{Z_1}{1+sT_1}}$$



31.10.2018 / Mitwoch

"Modes" of Control
ON-OFF → PID Family

Example: Simplified Economic Model
↳ Simplified Kaynesian Model: Multiplier Accelerator Model (P. Samuelson)

Fundamental Variables:

$y[k]$: GNP, Gross National Product : Total monetary production of a country

$c[k]$: Total consumption (at time k)
not-cumulative

$i[k]$: Investments at time k

$g[k]$: Government expenses at time k

inflow - outflow = accumulation ⟷ income - expenses = savings

$$y[k] - (c[k] + g[k]) = i[k]$$

Due to Samuelsson: M.A. Model:

1) The consumption in the current year is proportional to the GNP of the previous year.

$$c[k] = a \cdot y[k-1] \quad , \quad a > 0$$

2) Investments are proportional to increase in consumption from the previous year.

$$i[k] = b\left[c[k] - c[k-1]\right] \quad , \quad b > 0$$

Note: Examples of input variables:

Government can influence $c[k]$ via taxes

$i[k]$ via interest rates

directly change $g[k]$ (rest of question)

$$y[k] = c[k] + i[k] + g[k]$$

$$= a\cdot y[k-1] + b\left[a\cdot y[k-1] - a y[k-2]\right] + \overset{\text{input}}{g[k]}$$

$\rightarrow$ $\boxed{y[k] - (ab+a)\,y[k-1] + (ab)\,y[k-2] = g[k]}$ $\leftarrow$ linear difference equation

Transform to state-space:

Define the states as:

$$\left.\begin{aligned} x_1[k] &= c[k] = a\cdot y[k-1] \\ x_2[k] &= y[k] \end{aligned}\right\} \qquad \begin{aligned} \underline{x}[k] &= A\cdot \underline{x}[k-1] + B\underline{u}[k] \\ \underline{y}[k] &= C\,\underline{x}[k] \end{aligned}$$

$$x_1[k] = a\cdot y[k-1] = a\cdot x_2[k-1]$$

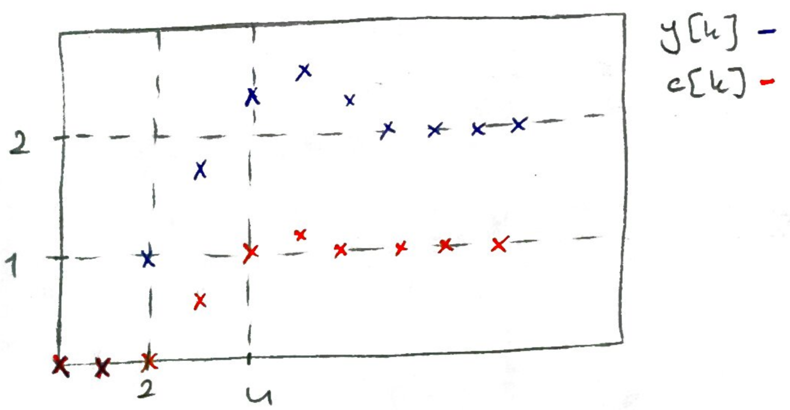
$$x_2[k] = y[k] = [a+ba]\cdot y[k-1] - b\cdot x_1[k-1] + g[k]$$

$$= [a+ba]\cdot x_2[k-1] - b\cdot x_1[k-1] + g[k]$$

$$\begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} = \begin{bmatrix} 0 & a \\ -b & a(1+b) \end{bmatrix} \cdot \begin{bmatrix} x_1[k-1] \\ x_2[k-1] \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} g[k]$$

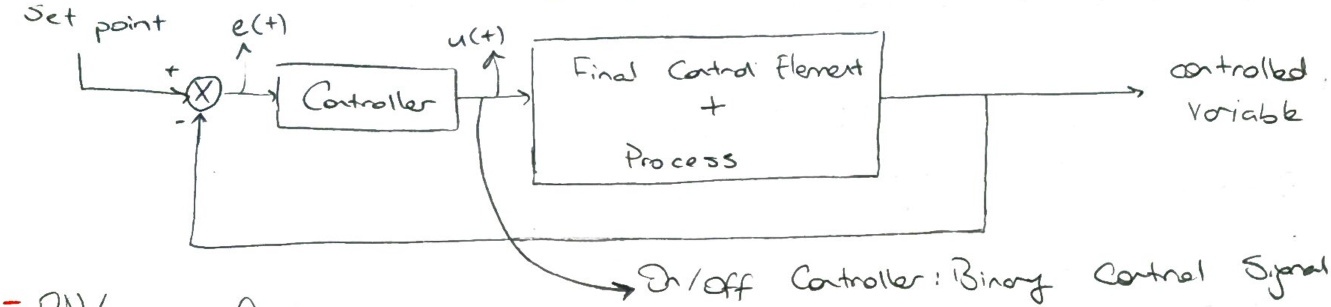
$$y[k] = \begin{bmatrix} 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix}$$

$\rightarrow$ Simulation with step response ($a=0.5$ and $b=0.5$): Unit increase in government expenses



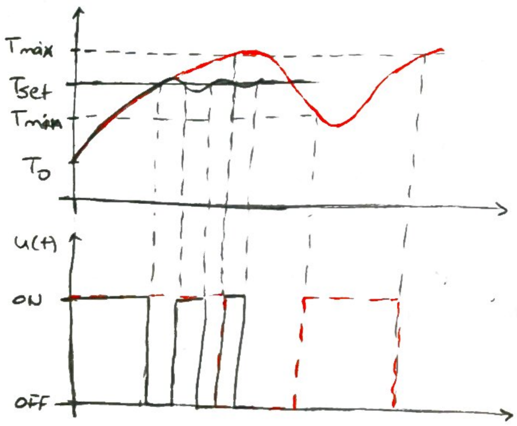
05.11.2018 / Monteeg

Modes of Control (design of Controllers)



- ON/OFF Controller

- PID Controller (Continuous Value Control)



→ Intermediate (Continuous) Value Control:

Quite general linear case: PID Controller

$$u(t) = \underbrace{u_{bias}}_{\text{(steady state)}} + \underbrace{K_c \cdot e(t)}_{\text{Proportional Mode}} + \underbrace{\frac{K_c}{\tau_I} \int_0^t e(t) \cdot dt}_{\text{Integral Mode}} + \underbrace{K_c \cdot \tau_D \cdot \frac{de(t)}{dt}}_{\text{Derivative Value}} \longrightarrow \text{"Dependent Ideal PID Form"}$$

"Dependent Interactive PID Form"

$$u(t) = u_{bias} + K_c \left(1 + \frac{\tau_D}{\tau_I}\right) e(t) + \frac{K_c}{\tau_I} \cdot \int_0^t e(t) \cdot dt + K_d \cdot \frac{de(t)}{dt}$$

"Independent PID Form"

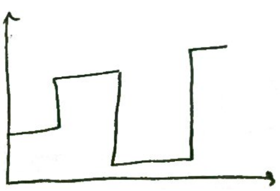
$$u(t) = u_{bias} + K_c \cdot e(t) + K_I \cdot \int_0^t e(t) \cdot dt + K_d \cdot \frac{de(t)}{dt}$$

Note: "Proportional Band" $\longrightarrow K_c = \frac{100}{PB}$

However: $e(t) = r(t) - y(t)$

If $r(t)$ is piecewise constant, $\frac{d}{dt} e(t) = \underbrace{\frac{d}{dt} r(t)}_{} - \frac{d}{dt} y(t)$

↳ 0, except for transitions in $r(t)$, where we have an impulse function.

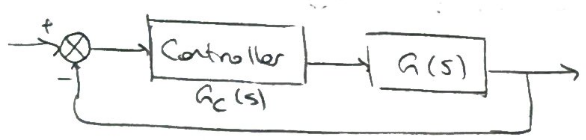


$\longrightarrow \frac{d}{dt} e(t) \approx -\frac{d}{dt} y(t)$

Behavior and Effects of Control Actions

- Use root-locus analysis to consider these different effects.

Example: Consider $G(s) = \dfrac{1}{(s+1)(s+2)}$



Case 1: Proportional Controller $\rightarrow G_c(s) = K_c$

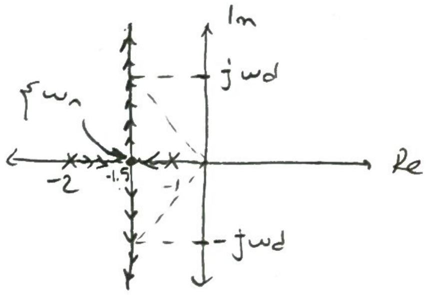
$$G_c(s) \cdot G(s) = \frac{K_c}{(s+1)(s+2)}$$

Sketch the root-locus : # of branches $= 2$

# of asymptotes $= 2 = |n-m| = |2-0|$

Angle of asymptotes $= \dfrac{\pm 180}{|n-m|} \cdot (2\ell+1) \nearrow +90° \searrow -90°$

Centroid : $\gamma = \dfrac{\sum p - \sum z}{\# p - \# z} = \dfrac{-3-0}{2-0} = -1.5$



Aim:

1) A settling time as small as possible

2) A steady-state error as small as possible

$t_s = \dfrac{3}{\xi \cdot w_n} \longrightarrow$ Settling time can be reduced until

$\xi w_n = 1.5 \rightarrow t_s = 2$ sec.

" Increasing K_c further will have no effect on the settling time "

Steady-state error ?

$e_{ss} = \dfrac{1}{1 + K_c/2}$ " Type 0 "

$K_c \uparrow \; e_{ss} \downarrow$

$\hookrightarrow$ Also increases the overshoot (related with w_d) and the oscillation frequency.

Case II: " Proportional Derivative Controller "

$$G_c(s) = K_c + K_c \cdot \tau_D \cdot s = K_c \tau_D \left(s + \frac{1}{\tau_D}\right)$$

Assume a small τ_D

$$= K_d \cdot (s + \alpha)$$

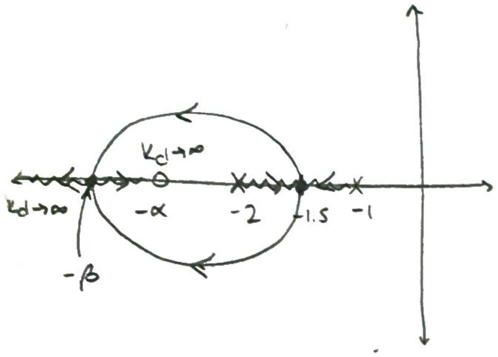
$G_c(s)$ have a zero at $s = -\dfrac{1}{\tau_D}$ $\longrightarrow$ at the left half plane

$\longrightarrow$ hopefully far away.

$$G_c(s) \cdot G(s) = \frac{K_d \cdot (s+a)}{(s+1)(s+2)}$$

Sketch the root locus: # of asymptotes $= |n-m| = |2-1| = 1$

$$\text{Angle} = \frac{\pm 180^\circ (2l+1)}{1} = \pm 180^\circ$$



?. Settling time

? Steady-State Error

t_s is minimum $\rightarrow \xi w_n = \beta \rightarrow t_s \approx \frac{3}{\beta}$

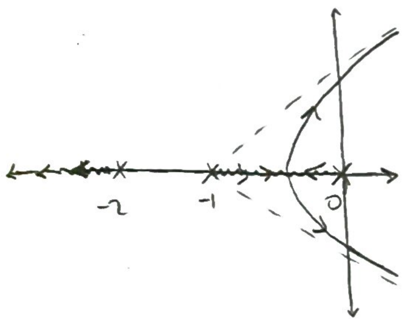
$e_{ss} = \frac{1}{1 + \frac{K_d}{2}} \longrightarrow$ first increases then decreases with increasing K_d.

" $-\beta$ seems like a good point to stop. "

Case 3: "Integral Controller"

$$G_c(s) = \frac{K_c}{\tau_I} \cdot \frac{1}{s} = K_I \cdot \frac{1}{s(s+1)(s+2)}$$

$e_{ss} = 0$!! $\rightarrow$ "Type 1"
(to a unit step input) $\leftarrow$ perfect



# of asymptotes $= 3$

$$\text{angle} = \frac{\pm 180^\circ}{3} \cdot (2l+1) \rightarrow \pm 60, \ 180^\circ$$

$$\text{centroid} = \sigma_0 = \frac{-1-2-0}{3-0} = -1$$

$t_s \gtrless 3$ secs / Stability bad, cannot increase gain much

Case IV: "Proportional Integral Controller"

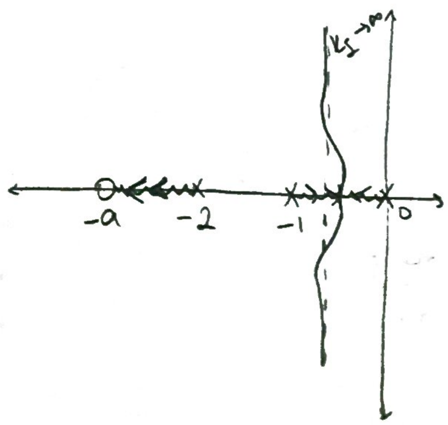
$$G_c(s) = K_c + \frac{K_c}{\tau_I} \cdot \frac{1}{s} = \frac{K_c}{\tau_I}\left(\frac{\tau_I s + 1}{s}\right) = K_I \cdot \left(\frac{s+a}{s}\right)$$

$K_I = \frac{K_c}{\tau_I}$

$a = \frac{1}{\tau_I}$

$$G_c(s) \cdot G_c(s) = \frac{K_I \cdot (s+a)}{s(s+1)(s+2)}$$

$\longrightarrow$ "Type 1" $\longrightarrow e_{ss} = 0$ (to a unit step input)



# of asymptotes $= 2$

angles $= \pm 90^\circ$

$$\text{centroid} = \sigma_0 = \frac{-3+a}{3-1} = -1.5 + \frac{a}{2}$$

$\nearrow$ can be adjusted with choice of α

$\rightarrow e_{ss} = 0$ is preserved and $t_s \cong \frac{3}{-\sigma_0}$ (minimum value)

and also, we avoided any instability problem.

07.11.2018 / Mitwoch

Case V: "Proportional - Integral - Derivative (PID)"

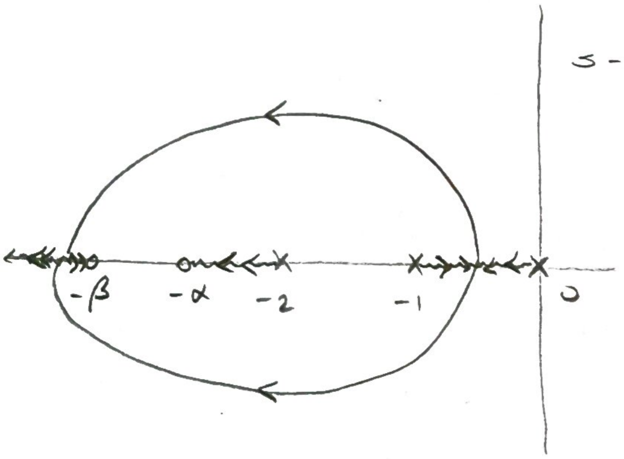
$$G_c(s) = K_c + \frac{K_c}{\tau_I}\cdot\frac{1}{s} + K_c\cdot\tau_D\cdot s = \frac{K_c\cdot s + \frac{K_c}{\tau_I} + K_c\cdot\tau_D\cdot s^2}{s}$$

$$= \frac{K_c\cdot\tau_D s^2 + K_c s + \frac{K_c}{\tau_I}}{s} = \underbrace{K_c\,\tau_D}_{K_D}\cdot\left(\frac{s^2 + \frac{1}{\tau_D}s + \frac{1}{\tau_I\tau_D}}{s}\right)$$

$$= \frac{K_c\cdot(s+\alpha)(s+\beta)}{s} \rightarrow \text{Two zeros \& one pole (at zero)}$$

Now a much richer set of behaviours possible with two zeros. (can be real or complex conjugate)

Consider simple case (zeros are real)



$$G_c(s)\cdot G(s) = \frac{K_D(s+\alpha)(s+\beta)}{s(s+1)(s+2)}$$

# of asymptotes : $|1-2| = 1$

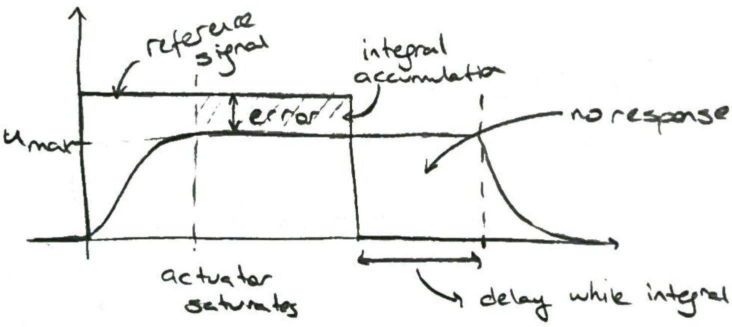
angle $= \pm 180°$

Notes about Integral Term:

→ Very popular when combined with P-term (PI), gives us $e_{ss} = 0$!

→ Problem: "Integral Windup"

Case of actuator saturation, $u(t) = u_{max}$



↳ Solution: "Anti-Windup" / "Reset-Windup" → 1. Stop the integral when the actuator saturation is detected.

2. Stop the integral term when the controller is "off."

Notes about the derivative term and PID Controllers:

Problem 1: "Derivative kick"

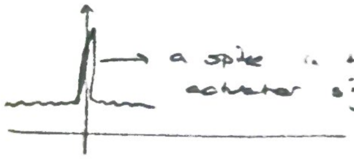
→ e changes instantaneously

→ Use $y(t)$ instead of $e(t)$ in the derivative:

$$\frac{de(t)}{dt} \rightarrow \frac{-dy(t)}{dt}$$

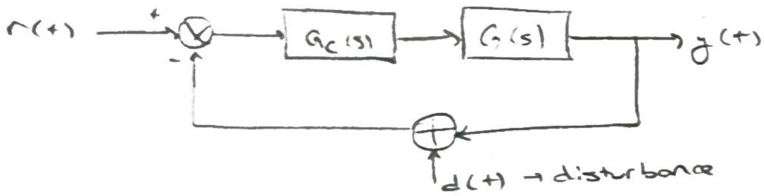
$$e(t) = r(t) - y(t)$$

$$\frac{de(t)}{dt} = \cancel{\frac{dr(t)}{dt}}^{\,0} - \frac{dy(t)}{dt}$$



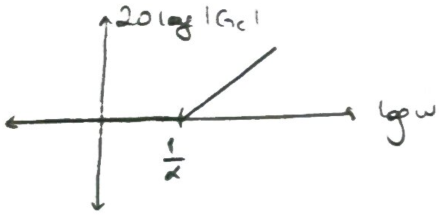
→ a spike in the actuator signal is noted.

Problem 2:



PD Controller:

$G_c(s) = K_d \cdot (s + \alpha)$



→ High frequency components in the disturbance (e.g. sensor noise) is exponentially amplified.

Solutions:

- Use a low pass filter before the PD Controller.

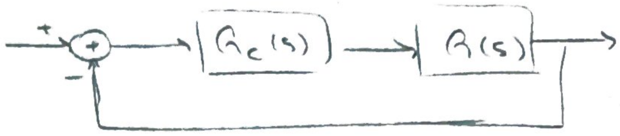
- Do not use derivative operation. Use (if available) corresponding variable instead. (Example: if a velocity sensor is available, use $\dot{\theta}$ instead of $\frac{d\theta}{dt}$)

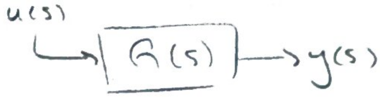
PD Controller ⟶ PV Controller

12.11.2018 / Montag

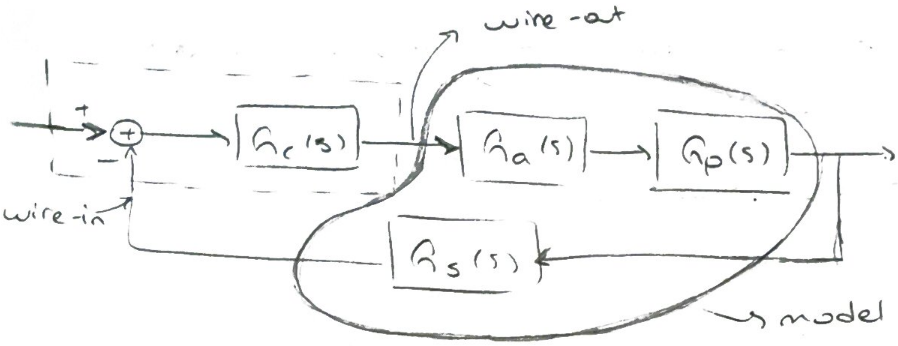
Design of PID Controllers for General Case:

- Open Loop Testing (Process Reaction Curve Based Design)
  - → Linear / Linearized Model (all previous methods)
  - → $\boxed{\text{FOPDT}}$ Simplest non-linear model
    - → Cohen-Coon Tuning Formula
    - → ITAE (P) (III)
    - IMC (PI + PID)
- Closed Loop Testing
  - → Manual (Zygler-Niokols Continuous Cycling Method) (II)
  - → Iterative Optimization (I)





- Openloop testing → may be unstable without a controller
↓
Closed Loop Testing



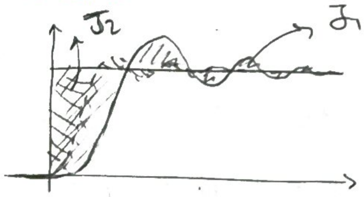
Wire-out to wire-in model.

1. Design by Optimization:

1. Define an Objective Function (measures the performance of the controller)

scalar = $J(\theta_1, \theta_2, \ldots \theta_N)$ ← parameters of the controller.

Example:



J_1 "Integral of Time Error"

& A combination of rise-time, settling time, overshoot etc.

↳ "Optimal Control Course"

"Optimization"

↳ { Minimize $J(K_c, \tau_I, \tau_D)$ with respect to K_c, τ_I & τ_D.

Subject to other constraints (such as actuator limits etc.)

Requires: a) An accurate system model and closed form time solution of the error $(e(t))$

$e(t, K_c, \tau_I, \tau_D)$

OR

$$\frac{\partial J}{\partial K_c} = 0 \quad , \quad \frac{\partial J}{\partial \tau_I} = 0 \quad , \quad \frac{\partial J}{\partial \tau_D} = 0 \longrightarrow \text{Neccessity Conditions.}$$

b) Ability to continuously experiment with the system. (safely)

c) Ability to simulate the system with a computer model.

→ Model-in-the-loop (Simulation)

→ Hardware-in-the-loop

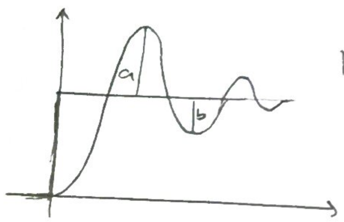
$\boxed{\text{Simulink - PID Tuner}}$

2. Zyggler - Nichols Continuous Cycling Method:

1. Shut off integral and derivative control.
2. Turn-on Control (Automatic Mode rather than Manual Mode)
3. Adjust the K_c such that you obtain non-decaying oscillations.
4. Obtain gain K, $\longrightarrow$ Ultimate Gain K_u

   Obtain period $\longrightarrow$ Ultimate Period P_u

The design parameters that gives us $0.25\left(\frac{1}{4}\right)$ decay ratio (DR)



$$DR = \frac{b}{a} = \frac{1}{4}$$

	K_c	τ_I	τ_D
P	$\frac{K_u}{2}$		
PI	$K_u/2.2$	$P_u/1.2$	
PID	$K_u/1.7$	$P_u/2$	$P_u/8$

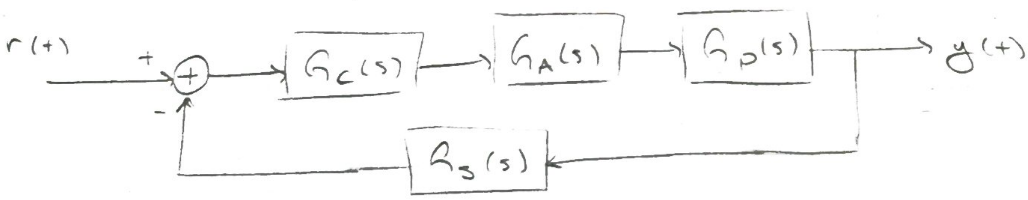
Example: PID with Zyggler - Nikhols Method (but on a given model) ⇝ Try at Simulink

Actuator Model: $G_a(s) = \dfrac{5}{2s+1}$ → time constant (unit?) ← Should be same with used P_u

$\boxed{\tau_A = 2\,\text{mins}}$

Sensor Model: $G_s(s) = \dfrac{0.4}{5s+1}$

Process Model: $G_p(s) = \dfrac{2}{s+1}$



Task: Determine the ultimate gain and the ultimate period.

↳ Sustained oscillations when two closed-loop poles are on the $j\omega$-axis.

↳ We have the full model with $G_c(s) = K_c$

Open-loop transfer function $= K_c \cdot \dfrac{5}{2s+1} \cdot \dfrac{0.4}{5s+1} \cdot \dfrac{2}{s+1}$

$$q(s) = 1 + K_c \frac{4}{(2s+1)(5s+1)(s+1)}$$

$$q(s) = (2s+1)(5s+1)(s+1) + 4K_c$$

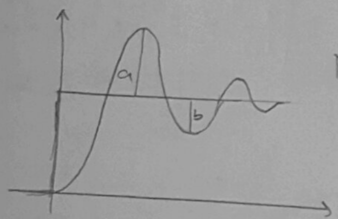
$$= 10s^3 + 17s^2 + 8s + 1 + 4K_c$$

2. Zyggler - Nichols Continuous Cycling Method:

1. Shut off Integral and derivative control.
2. Turn-on Control (Automatic Mode rather than Manual Mode)
3. Adjust the K_c such that you obtain non-decaying oscillations.
4. Obtain gain K, $\longrightarrow$ Ultimate Gain K_u

Obtain period $\longrightarrow$ Ultimate Period P_u

The design parameters that gives us $0.25\left(\frac{1}{4}\right)$ decay ratio (DR)



$$DR = \frac{b}{a} = \frac{1}{4}$$

	K_c	τ_I	τ_D
P	$\frac{K_u}{2}$		
PI	$K_u/2.2$	$P_u/1.2$	
PID	$K_u/1.7$	$P_u/2$	$P_u/8$

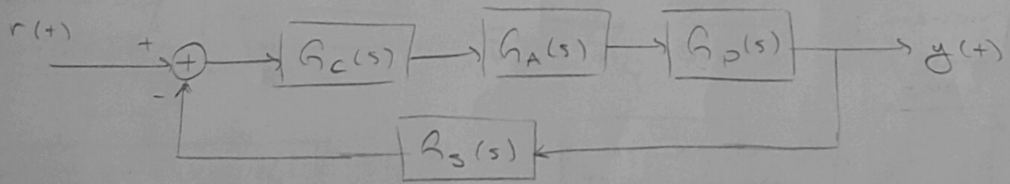
Example: PID with Zyggler - Nikhols Method (but on a given model) ⤳ Try at Simulink.

Actuator Model: $G_a(s) = \dfrac{5}{2s+1}$ $\longrightarrow$ time constant (unit?) $\longleftarrow$ Should be same with used P_u

$\boxed{\tau_A = 2\text{mins}}$

Sensor Model: $G_s(s) = \dfrac{0.4}{5s+1}$

Process Model: $G_p(s) = \dfrac{2}{s+1}$



Task: Determine the ultimate gain and the ultimate period.

↳ Sustained oscillations when two closed-loop poles are on the $j\omega$-axis.

↳ We have the full model with $G_c(s) = K_c$

Open-loop transfer function $= K_c \cdot \dfrac{5}{2s+1} \cdot \dfrac{0.4}{5s+1} \cdot \dfrac{2}{s+1}$

$$q(s) = 1 + K_c \frac{4}{(2s+1)(5s+1)(s+1)}$$

$$q(s) = (2s+1)(5s+1)(s+1) + 4K_c$$

$$= 10s^3 + 17s^2 + 8s + 1 + 4K_c$$

Let $s = j\omega$

$$q(j\omega) = 10(j\omega)^3 + 17(j\omega)^2 + 8(j\omega) + 1 + uKc$$

$$= -10j\omega^3 - 17\omega^2 + 8j\omega + 1 + uK_c$$

Complex Equation $\longrightarrow$ Two Equations:

Real Part: $-17\omega^2 + 1 + uK_c = 0$

Imaginary Part: $-10\omega^3 + 8\omega = 0 = \omega(-10\omega^2 + 8) \to \omega = 0$

$\to 10\omega^2 = 8$

$\omega = \pm\sqrt{\frac{8}{10}} \approx \pm 0.894 = \omega$ rad/min

Ultimate Period:

$$\boxed{T_u = \frac{2\pi}{\omega_u} = \frac{2\pi}{0.894} = 7.03 \text{ mins}}$$

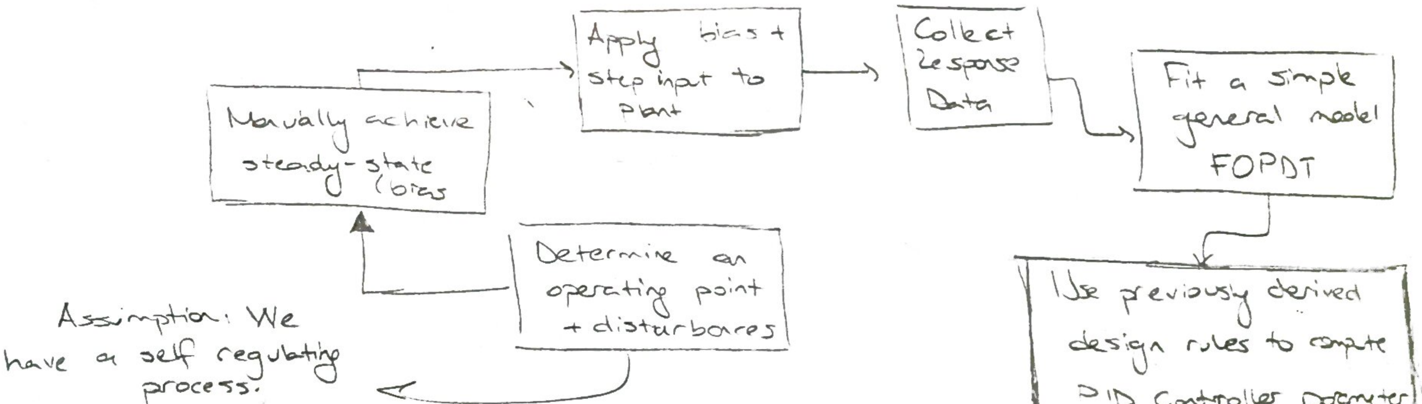
From real part equation: $-17 \cdot (0.894)^2 + 1 + 4K_u = 0$

$$\boxed{K_u = 3.15}$$

For a full PID Controller (from the table)

$K_c = \frac{K_u}{1.7} \simeq 1.85$	$\tau_D = \frac{P_u}{8} \simeq 0.879$ mins
$\tau_I = \frac{P_u}{2} \simeq 3.52$ mins	

3. Process Reactive Curve Based Design (Open-Loop Testing)



→ First Order with Dead Time (FOPDT) Model (Note: Dead time = pure delay)

↳ This is an overdamped plant model !!!

$$u(t) \longrightarrow \boxed{G(s)} \longrightarrow y(t)$$

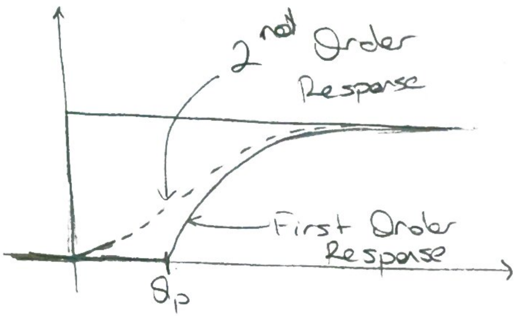
$$\tau_p \cdot \frac{dy(t)}{dt} + y(t) = K_p \cdot u(t - \theta_p)$$

$\mathcal{L}\{\cdot\}$

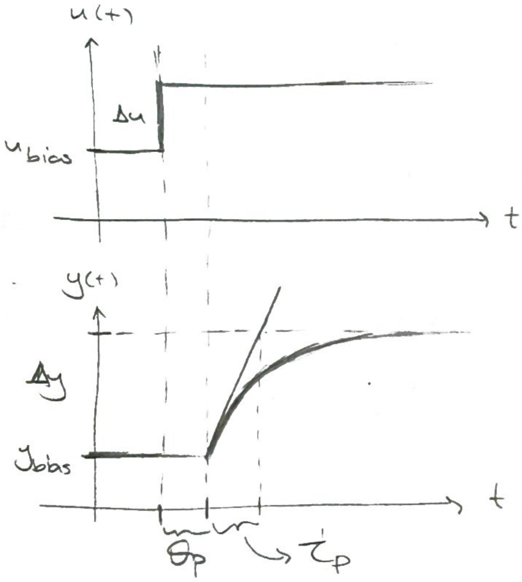
$$(\tau_p s + 1)\, Y(s) = K_p \cdot e^{-\theta_p s} \cdot U(s)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K_p \cdot e^{-\theta_p s}}{\tau_p \cdot s + 1}$$

FOPDT



Note: FOPDT can also approximate higher order (linear / nonlinear) processes.

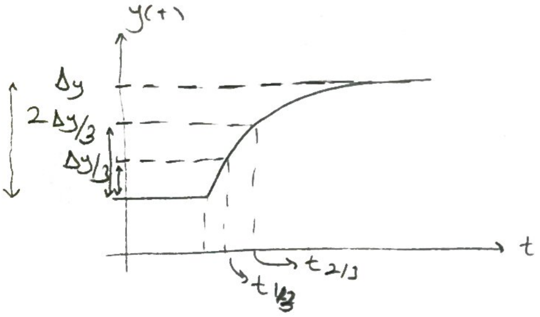


$$\boxed{K_p = \frac{\Delta y}{\Delta u}}$$

$\tau_p = ?$ → How to find it?

13.11.2018 / Mittwoch

What if the y(t) is noisy and cannot distinguish variable easily.

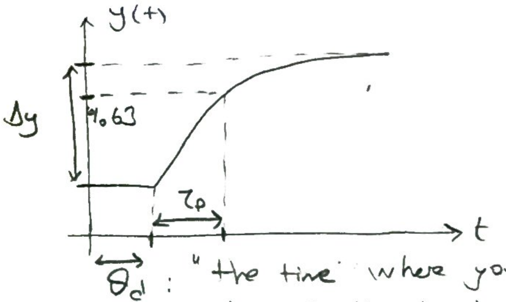


$$K_p = \frac{\Delta y}{\Delta u}$$

$$\tau_p \cong \frac{1}{0.7}\left(t_{2/3} - t_{1/3}\right)$$

$$\theta_d \cong t_{1/3} - 0.4\,\tau_p$$

Alternative:



θ_d : "the time where you observe the beginning of the response"

τ_p : time it takes to cover 63% of the response Δy

Open Loop Reaction Curve Design Procedure (Summary)

1) Establish an operating point with u_{ss}, y_{ss} ("Design Level of Operation")

2) "Bump the process" (apply a step around the operating point collect the response data from wire-out to wire-in.

3) Approximate the process $G_p(s)$ with FOPDT model.

4) Choose a desired closed-loop time constant τ_c.
(Assume near first-order closed-loop response)

5) Use "design rules" parametrized in τ_c to design P, PI, PID controller.

1. ITAE (Integral of Time Weighted Absolute Error) for P-Controller:

Moderate Response $\Rightarrow$ $$K_c = \frac{0.2}{K_p} \cdot \left(\frac{\tau_p}{\theta_d}\right)^{1.22}$$

Aggressive Response $\Rightarrow$ $K_c' = 2.5\,K_c$

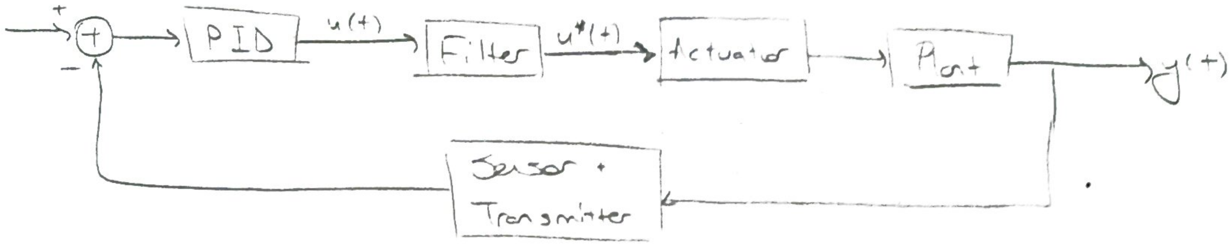
2. Internal Model Control (IMC) Rules for PI, PID and PID + CO Filter (controller output filter)

Given: $K_p, \tau_p, \theta_p, \tau_c$ (desired speed)

↳ Dependent Ideal PI, PID

↳ + CO filter time constant.

	Gain K_c	Reset Time τ_I	Deriv. Time τ_D	Filter Time Constant τ_f
PI	$\frac{1}{K_p} \cdot \frac{\tau_p}{(\theta_d + \tau_c)}$	τ_p	–	–
DI-PID	$\frac{1}{K_p} \cdot \left(\frac{\tau_p + 0.5\theta_d}{\tau_c + 0.5\theta_d}\right)$	$\tau_p + 0.5\theta_p$	$\frac{\tau_p \cdot \theta_d}{2\tau_p + \theta_d}$	–
DI-PID + CO filter	$\frac{1}{K_p} \cdot \left(\frac{\tau_p + 0.5\theta_d}{\tau_c + \theta_d}\right)$	$\tau_p + 0.5\theta_d$	$\frac{\tau_p\,\theta_d}{2\tau_p + \theta_d}$	$\frac{\tau_c(\tau_p + 0.5\theta_d)}{\tau_p(\tau_c + \theta_p)} \cdot \tau_d$ $\tau_f = \alpha \cdot \tau_d$



Filter:

$$\tau_f \cdot \frac{du^*(t)}{dt} + u^*(t) = u(t)$$

$\mathcal{L}$

$$(\tau_f \cdot s + 1)\, U^*(s) = U(s) \longrightarrow T_f(s) = \frac{1}{\tau_f \cdot s + 1}$$

How to select τ_c?

τ_c is the time constant of the "controlled" overall-closed loop system.

- Aggressive: $\tau_c = \max(0.1\tau_p ; 0.8\,\theta_d)$
- Moderate: $\tau_c = \max(\tau_p ; 8\theta_d)$
- Conservative: $\tau_c = \max(10\tau_p ; 80 \cdot \theta_d)$

x10

x10

Cohen-Coon Design Rules:

Similar design formulas parametrized on τ_c

Note: What about a discrete time implementation and sampling period T?

$\boxed{\tau < 0.1\,\tau_p}$

* Direct Synthesis of Controllers → IMC Controller design. *

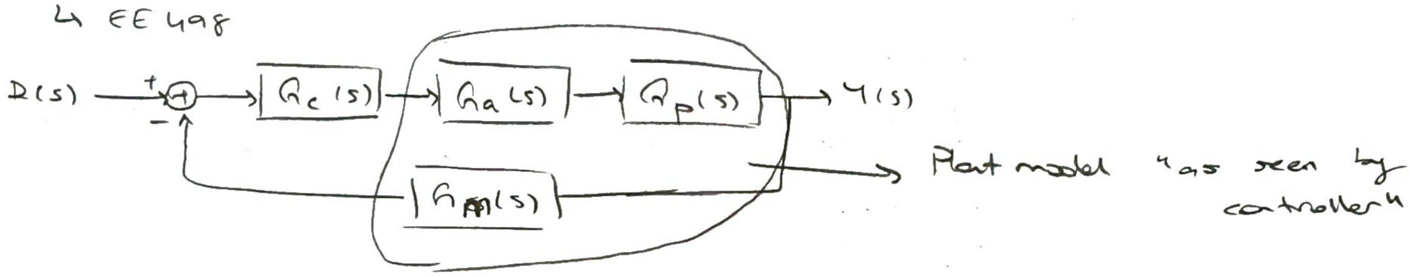
- Direct Synthesis → PI

IMC Controller Design + Padé' Approximation
(Intermediate Model Control) (for pure time delay)

→ PID Formulas

Note: Direct Synthesis method is more general and can result in controllers other than PI, PID.

↳ We will limit ourselves
↳ EE 498



Simplify: $G_a(s) = G_m(s) = 1$

Closed-loop transfer function?

$$T(s) = \frac{G_c(s) \cdot G_p(s)}{1 + G_c(s) \cdot G_p(s)}$$

Idea: Given $T(s)$ can I extract $G_c(s)$ as a function of other terms.

$$T(s) + T(s) \cdot G_c(s) \cdot G_p(s) = G_c(s) \cdot G_p(s)$$

$$(1 - T(s))\, G_c(s) \cdot G_p(s) = T(s) \rightarrow \boxed{G_c(s) = \frac{1}{G_p(s)} \cdot \frac{T(s)}{1 - T(s)}} \quad (*)$$

★ Poles of the controller are related with zeros of plant & zeros of controller are related with poles of transfer function. ★

★ Ideally we would want $T(s) = 1$. But (*) tells us this is not possible.

The closed loop transfer function $T(s)$ cannot be arbitrary because the CL characteristics equation should have a solution for some s,

$$q(s) = 1 + G_c(s) \cdot G_p(s) = 0$$

$$\Rightarrow 1 + \frac{T(s)}{1 - T(s)} = 0$$

$$\Rightarrow \frac{1}{1 - T(s)} = 0$$

→ for $T(s) = K$; no solution!!

→ $T(s) \neq 1$ or $T(s) \neq K$

Let $T(s) = \dfrac{1}{1 + \tau_c s}$, τ_c: desired overall time constant

$$T(s) = \frac{1}{1+\tau_c s}$$ τ_c: design parameter → desired overall time constant.

→ What about $G_c(s)$?

$$G_c(s) = \frac{1}{G_p(s)} \cdot \frac{T(s)}{1-T(s)} = \frac{1}{G_p(s)} \cdot \frac{1}{\tau_c \cdot s}$$ → Controller → Depends on inverse of plant and $\frac{1}{\tau_c s}$ inorder to give a first order response

→ Overall closed-loop transfer function → (char. equation)

$$q(s) = 1 + G_c(s) G_p(s) = 1 + \frac{1}{\cancel{G_p(s)}} \cdot \frac{1}{\tau_c s} \cdot \cancel{G_p(s)}$$

$$\rightarrow \tau_c \cdot s + 1 = 0 \rightarrow \boxed{s = \frac{-1}{\tau_c}}$$

Note 1: Too small τ_c would result too large gain for the controller, leading to actuator satration or worse.

Note 2: Unstable zeros of plant become unstable poles of the controller.

Example: Given a plant with the transfer function $G_p(s) = \frac{K_p}{1+\tau_p s}$. Use direct synthesis to design a controller such that the closed loop system behaves with the constant τ_c.

$$G_c(s) = \frac{1+\tau_p s}{K_p} \cdot \frac{1}{\tau_c s} = \frac{1}{K_p \cdot \tau_c} + \frac{\tau_p}{K_p \tau_c} = \frac{\tau_p}{K_p \tau_c}\left(1 + \frac{1}{\tau_p} \cdot \frac{1}{s}\right)$$ → Dependent Ideal PI Controller

$$K_c\left(1 + \frac{1}{\tau_I} \cdot \frac{1}{s}\right)$$

★ $$\boxed{K_c = \frac{\tau_p}{K_p \tau_c} \text{ \& } \tau_p = \tau_I}$$

Example: Given 2^{nd} order plant: $$G_p(s) = \frac{K_p}{(1+\tau_1 s)(1+\tau_2 s)}$$

We want the CL system to exhibit ~~first order~~ first order response with desired time constant τ_c.

$$G_c(s) = \frac{1}{G_p(s)} \cdot \frac{1}{\tau_c s} = \frac{1+(\tau_1+\tau_2)s + \tau_1 \tau_2 s^2}{K_p \cdot \tau_c \cdot s}$$

$$= \frac{1}{K_p \tau_c} \cdot \frac{1}{s} + \frac{\tau_1+\tau_2}{K_p \tau_c} + \frac{\tau_1 \tau_2}{K_p \tau_c} \cdot s = \underbrace{\frac{\tau_1+\tau_2}{K_p \tau_c}}_{K_c}\left(1 + \underbrace{\frac{1}{\tau_1+\tau_2}}_{\frac{1}{\tau_I}} \cdot \frac{1}{s} + \underbrace{\frac{\tau_1 \tau_2}{\tau_1+\tau_2}}_{\tau_D} \cdot s\right)$$

→ Dependent Ideal PID with:

$$\boxed{K_c = \frac{\tau_1+\tau_2}{K_p \cdot \tau_c} \text{ \& } \tau_I = \tau_1 + \tau_2 \text{ \& } \tau_D = \frac{\tau_1 \tau_2}{\tau_1+\tau_2}}$$

Example: Obtain linear controller for the FOPDT plant model given by 17.11.2018 / Montag (3)

$$G_p(s) = \frac{K_p \cdot e^{-\theta_p s}}{1+\tau_p s}$$

→ Since the delay cannot be eliminated, let us assume $T(s) = \dfrac{e^{-\theta_p s}}{1+\tau_c s}$ where τ_c is the new desired time constant.

$$G_c(s) = \frac{1+\tau_p s}{K_p \cdot \cancel{e^{-\theta_p s}}} \cdot \frac{\cancel{e^{-\theta_p s}}}{1+\tau_c s - e^{-\theta_p s}} = \frac{1+\tau_p s}{K_p\left(1+\tau_c s - e^{-\theta_p s}\right)}$$

→ we wanted a linear controller so use Taylor series.

→ Assume a first order Taylor series approximation:

$$\boxed{e^{-\theta_p \cdot s} \simeq 1-\theta_p s}$$

→ This not the best approximation. Later we will cover a better approximation.

$$G_c(s) = \frac{1+\tau_p s}{K_p(\tau_c s+\theta_p s)} = \frac{1}{K_p \cdot (\tau_c+\theta_p)} \cdot \frac{1}{s} + \frac{\tau_p}{K_p(\tau_c+\theta_p)}$$

$$\rightarrow \frac{\tau_p}{K_p(\tau_c+\theta_p)}\left(1+\frac{1}{\tau_p}\cdot\frac{1}{s}\right) \xrightarrow[\text{PI Controller}]{\text{Dependent Ideal}} \boxed{K_c = \frac{\tau_p}{K_p(\tau_c+\theta_p)} \ \& \ \tau_I = \tau_p}$$

→ Same as in the design formulas table. Check!

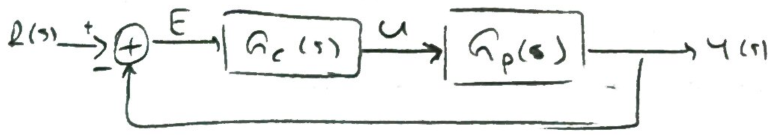
→ This is the PI design rules given before. In order to derive the PID design rules we will need:

1) Better app. of pure time delay
2) Better way of designing a controller.

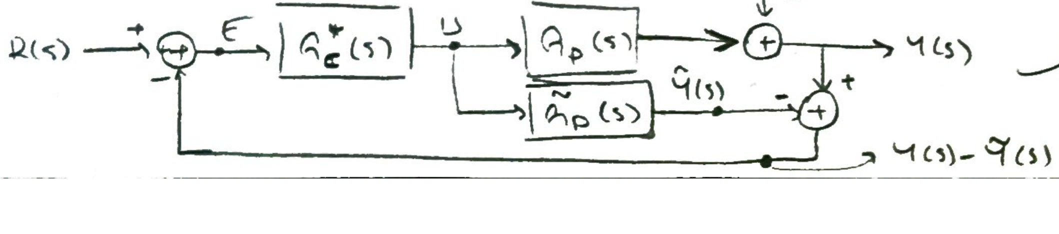
Internal Model Control (IMC):

IMC ist the generalization of Direct Synthesis, where we assume that we have an app. plant model instead of the actual model.

→ We assume a new closed loop architecture where the difference of the not having the perfect model made explicit.



→ with ideal perfect plant model.



→ Assumed architecture of the overall system (Internal Model Control Architecture)

How can we (first) derive $G_c^*(s)$?

Find the overall TF first?

$$E = R-(Y-\tilde{Y}) \quad \& \quad U = G_c^*(s)\cdot(R-(Y-\tilde{Y}))$$

$$\tilde{Y} = \tilde{G}_p(s)\cdot U \longrightarrow U = G_c^*(s)\cdot\left[R-Y+\tilde{G}_p(s)\cdot U\right]$$

$$U = G_c^*\cdot R - G_c^*\cdot Y + G_c^*\cdot\tilde{G}_p\cdot U \longrightarrow U\cdot(1-G_c^*\tilde{G}_p) = G_c^*(R-Y)$$

$$\boxed{U = \frac{G_c^*}{1-G_c^*\tilde{G}_p}\cdot(R-Y)}$$

Gives us we have $G_c(s) = \dfrac{G_c^*(s)}{1-G_c^*(s)\cdot\tilde{G}_p(s)}$

In the original control architecture: $\boxed{U = G_c\cdot E = G_c\cdot(R-Y)}$

Note: First find $G_c^*(s)$; the convert it to $G_c(s)$ to be used in the classical feedback architecture.

21.11.2018 / Mitwoch

$$\boxed{G_c(s) = \frac{G_c^*(s)}{1-G_c^*(s)\cdot\tilde{G}_p(s)}}$$

$$Y = D + G_p(s)\cdot U$$

$$U = \frac{G_c^*(s)}{1-G_c^*(s)\cdot\tilde{G}_p(s)}(R-Y)$$

$$Y = D + \frac{G_p\cdot G_c^*}{1-G_c^*\tilde{G}_p}(R-Y)$$

$$\left[1+\frac{G_pG_c^*}{1-G_c^*\tilde{G}_p}\right]Y = D + \frac{G_pG_c^*}{1-G_c^*\tilde{G}_p}R$$

$$Y = \left[\frac{1-G_c^*\tilde{G}_p}{1+G_c^*(G_p-\tilde{G}_p)}\right]\cdot D + \left[\frac{G_pG_c^*}{1+G_c^*(G_p-\tilde{G}_p)}\right]\cdot R$$

How we can extract $G_c^*(s)$?

Remember: Poles of G_c^* will be related with the zeros of $\tilde{G}_p$. To prevent an unstable controller for the plant having unstable zeros:

$$\tilde{G}_p = \underset{\substack{\text{parts with}\\ \text{positive zeros}}}{\tilde{G}_{p+}} + \tilde{G}_{p-} \leftarrow \text{part with negative zeros.}$$

We will derive a controller only based on the stable zeros of the plant. ($\tilde{G}_{p-}$)

We will define $G_c^*(s)$ in a similar way to direct design:

$$G_c^*(s) \triangleq \frac{1}{\tilde{G}_{p-}(s)}\cdot\left(\frac{1}{1+\tau_c s}\right)^r, \quad \underbrace{r = 1, 2, \ldots}$$

r can be increased if the $G_c^*(s)$ with rel can not be realized

You can define $G_c^*(s)$ like this only with the $G_p = \tilde{G}_p$ assumption.

Further Analysis of Time Delay 21.11.2018 / Mittwoch (5)

- A better model?
- What happens if the control loop has the delay?
- How can we use the "better model" to transform (approximate) higher order linear models to lower order models with time delay.

Padé Approximation (to the time-delay)

- First Order Plus Dead Time (FOPDT) : $G(s) = \dfrac{K \cdot e^{-\theta_d \cdot s}}{\tau s + 1}$

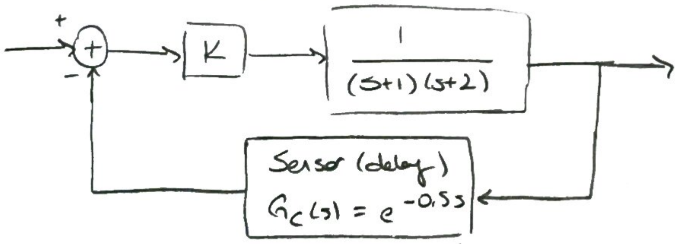
- Second Order Plus Dead Time (SOPDT) : $G_c(s) = \dfrac{K e^{-\theta_d s}}{\tau^2 s^2 + 2\xi\tau s + 1}$

1st Order Pade Approximation

$$\boxed{e^{-\theta_d \cdot s} \approx \dfrac{1 - \frac{\theta_d}{2} s}{1 + \frac{\theta_d}{2} s}}$$

Note: A stable pole and an unstable zero is introduced in the open loop path.

Example: In a closed loop system the output is measured by a sensor that transmitts the measurement to the controller through a communication link which introduces a delay of 0.5 sec. Analyse the effect of this delay on the stability of the closed-loop P-Controller.



★ Pade Approximation

$$e^{-0.5} = \frac{1 - 0.25s}{1 + 0.25s}$$

$$G_c(s) \cdot G(s) = \frac{K \cdot (1 - 0.25s)}{(s+1)(s+2)(1+0.25s)}$$

z: +4

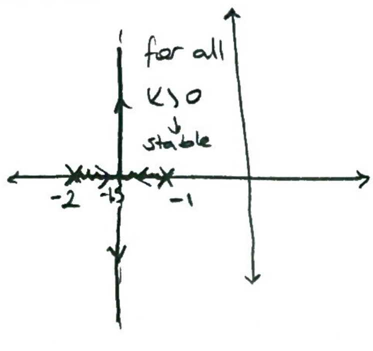
p: -1, -2, -4

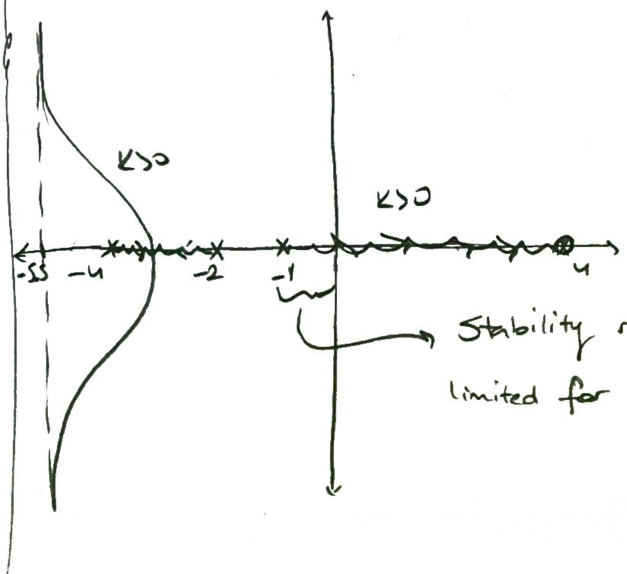
$$\sigma_0 = \frac{-1-2-4-4}{2} = -5.5$$

★ If there were no delay:

$$G_c(s) \cdot G(s) = \frac{K \cdot 1}{(s+1)(s+2)}$$

Root-Locus:





Stability range is much more limited for K>0.