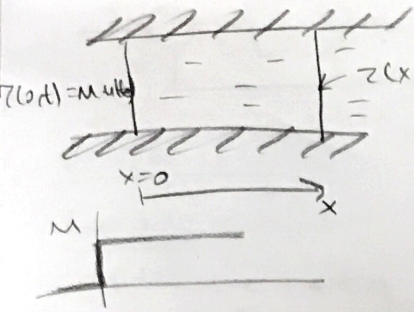
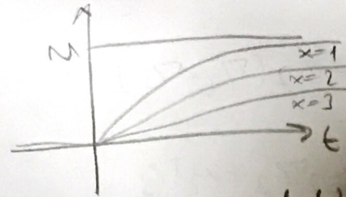
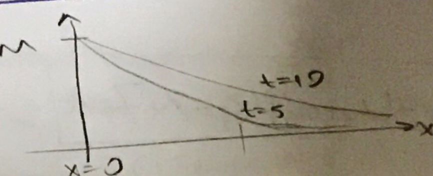
Ex: Unit step response at a fixed distance x inside the slab.





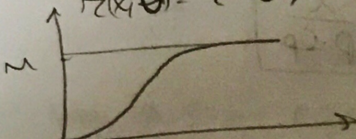
Example: Fixed time t, variable distance x.



Let $\theta=\left(\frac{a}{x^2}\right)t$ a scaling of the time variable

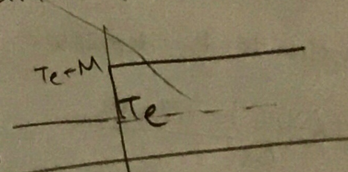
$$x^2=\frac{a}{\theta}\cdot t \qquad \tau(x,t)=M\cdot\operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{x^2}{at}}\right)=M\cdot\operatorname{erfc}\left(\frac{1}{2\sqrt{\theta}}\right)$$

$\tau(x,\theta)=\tau(\theta)$



We worked with "Delta variables" around the steady state:

$$T(0,t)=T_e+M\cdot u(t)$$
$$T(x,t)=T_e+M\operatorname{erfc}\left(\frac{x}{2\sqrt{at}}\right)$$

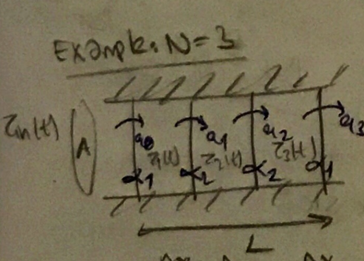


Lumped Parameter Approximation (of Distributed Parameter Models)

→ "Early Pass"

Low order PDE → Higher Order ODE (order will depend on the resolution of discretization)
→ System of interconnected lower order ODEs.

Example: N=3



$\tau_{out}(t)$ Assuming we again operate around the operating (steady state) point

$\tau_i(0)=0$

$\tau_i(t)$ is uniform inside each slide.

The energy balance eqn:

$$\underbrace{(A\,\Delta x\cdot\rho\cdot c_p)}_{\Delta C}\frac{d\tau_1}{dt}=\alpha_1 A(\tau_{in}-\tau_1)-\alpha_2 A(\tau_1-\tau_2)$$

$$\boxed{\Delta C\frac{d\tau_1}{dt}=\alpha_1\tau_{in}-(\alpha_1+\alpha_2)\tau_1+\alpha_2\tau_2}$$

For the second slice:

$$\Delta C \frac{dT_2}{dt} = \alpha_2 (T_1 - T_2) - \alpha_2 (T_2 - T_3)$$

$$\boxed{\Delta C \frac{dT_2}{dt} = \alpha_2 T_1 - 2\alpha_2 T_2 + \alpha_2 T_3}$$

Last Slice:

$$\boxed{\Delta C \frac{dT_3}{dt} = \alpha_2 T_2 - (\alpha_1 + \alpha_2) T_3 + \alpha_1 T_{out}}$$

If we take T_1, T_2, T_3 as the state variables

$$\begin{bmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \dot{T}_3 \end{bmatrix} = \frac{1}{\Delta C} \begin{bmatrix} -(\alpha_1+\alpha_2) & \alpha_2 & 0 \\ \alpha_2 & -2\alpha_2 & \alpha_2 \\ 0 & \alpha_2 & -(\alpha_1+\alpha_2) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} + \frac{1}{\Delta C} \overset{3\times 2}{\begin{bmatrix} \alpha_1 & 0 \\ 0 & 0 \\ 0 & \alpha_1 \end{bmatrix}} \begin{bmatrix} T_{in} \\ T_{out} \end{bmatrix}$$

Remember in the continuous (distributed) problem we need $\boxed{a = \frac{\lambda}{\rho \cdot C_p}}$ α_i

How to find the correspondence between λ and α_i's?

Using the finite difference approximation to the derivative:

$$\frac{\partial^2 T(x,t)}{\partial x^2}$$

$$\frac{\partial T(x,t)}{\partial x} \cong \frac{T(x+\Delta x, t) - T(x,t)}{\Delta x}$$

$$\frac{\partial^2 T(x,t)}{\partial x^2} \cong \frac{T(x+2\Delta x, t) - T(x+\Delta x, t) - [T(x+\Delta x, t) - T(x,t)]}{(\Delta x)^2}$$

$$\cong \frac{T(x+2\Delta x) - 2T(x+\Delta x, t) + T(x,t)}{(\Delta x)^2}$$

$$\cong \frac{T(x+\Delta x, t) - 2T(x,t) + T(x-\Delta x, t)}{(\Delta x)^2}$$

Define:

$T_i(t) = T(i \cdot \Delta x, t)$

and $T(x,t) = T_i(t)$ for $((i-1)\Delta x) < x < (i \cdot \Delta x)$

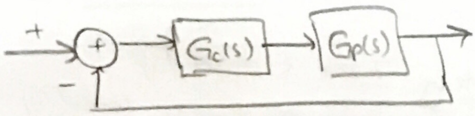
$\Delta C \triangleq A \cdot \rho \cdot C_p \cdot \Delta x$

$$\frac{\partial T_i(t)}{\partial t} = \frac{a}{(\Delta x)^2} \left[T_{i+1}(t) - 2T_i(t) + T_{i-1}(t) \right]$$

inside the slab

$$\frac{\lambda}{\rho \cdot C_p (\Delta x)^2} = \frac{\alpha_2}{\Delta C} = \frac{\alpha_2}{\rho \cdot C_p \cdot \Delta x} \qquad \frac{\lambda}{\Delta x} = \alpha_2$$

Multi-loop Systems (Chan, Ch. 10)



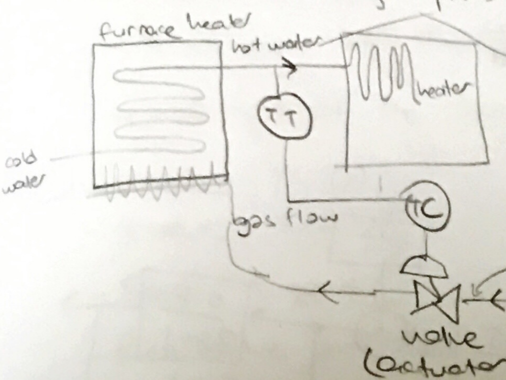
- Cascade Control
- Disturbance Feedforward
- Feedforward-Feedback

(Smith Predictor to compensate for ideal delay)

Process Notation

Cascade Control

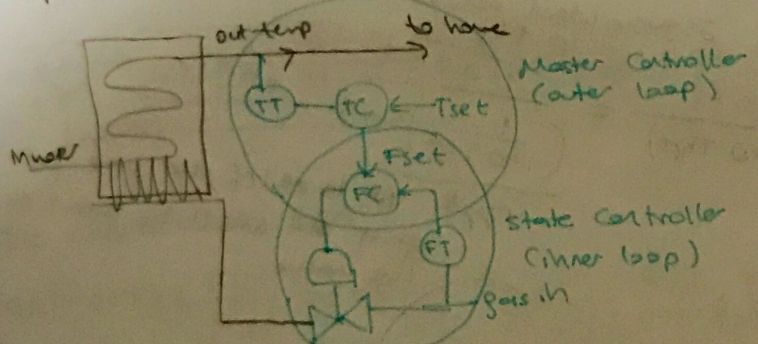
Consider the heating problem (Temperature control)

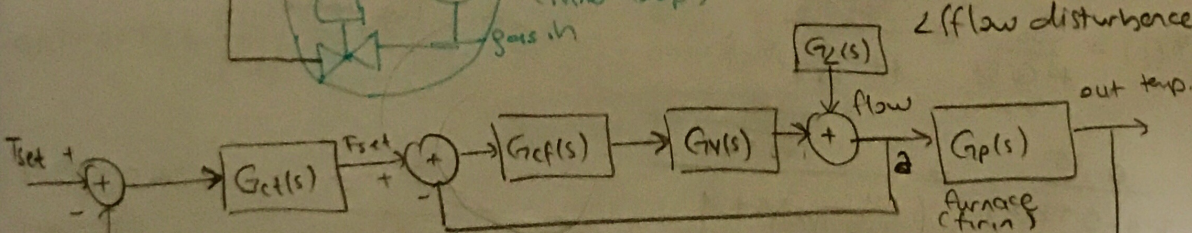


TT: Temperature Transmitter (sensor)

TC: Temperature Controller

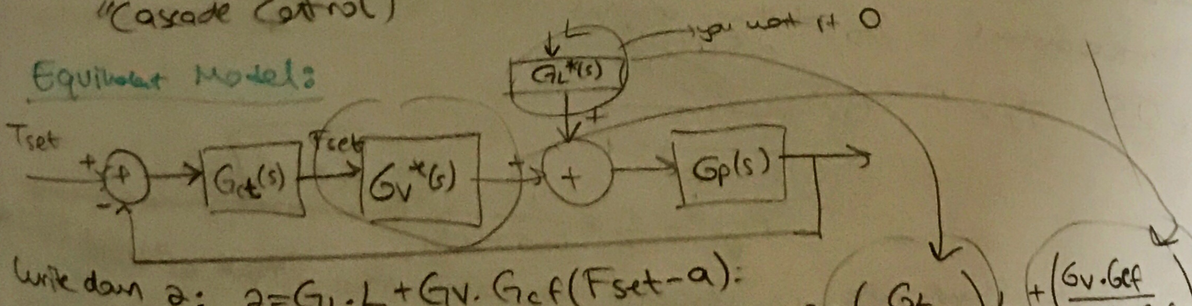
Idea: Add a flow measurement & a flow controller and combine with the temperature controller.





"Cascade Control"

Equivalent Model:



Write down a: $a = G_L \cdot L + G_v \cdot G_{cf}(F_{set} - a)$:

$$a(1 + G_{cf} G_v) = G_L \cdot L + G_v G_{cf} F_{set} \qquad a = \underbrace{\left(\frac{G_L}{1 + G_v G_{cf}}\right)}_{G_L^*(s)} L + \underbrace{\left(\frac{G_v \cdot G_{cf}}{1 + G_v G_{cf}}\right)}_{G_v^*(s)} F_{set}$$

Claims:

1) Cascade control can make the system more stable(?)

2) Cascade control can make the valve appear to be much faster(?)

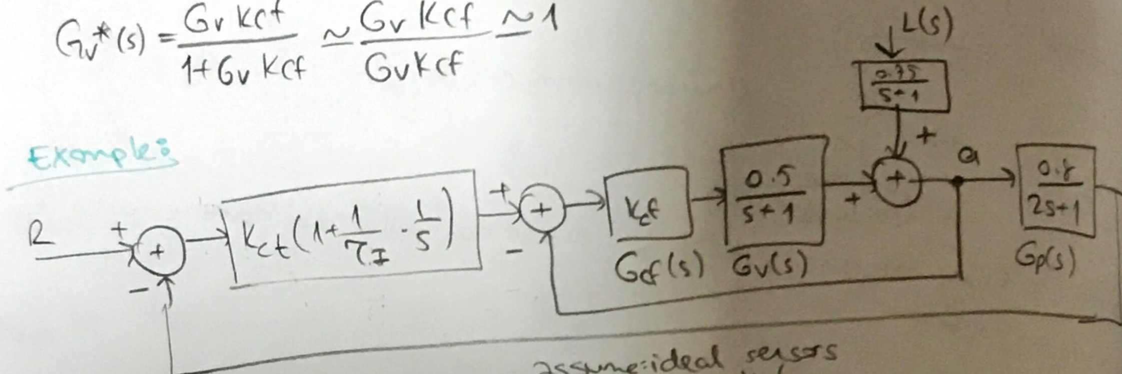
Assume $G_{cf}=K_{cf}$ a proportional controller

Let K_{cf} be very large!

$$G_L^*(s) = \frac{G_L}{1+G_vK_{cf}} \simeq 0$$ will be reduced to very small.

$$G_v^*(s) = \frac{G_vK_{cf}}{1+G_vK_{cf}} \simeq \frac{G_vK_{cf}}{G_vK_{cf}} \simeq 1$$

Example:



assume: ideal sensors

$$G_v^*(s) = \frac{\frac{0.5}{s+1}\cdot K_{cf}}{1+\frac{0.5}{s+1}K_{cf}} = \frac{0.5K_{cf}}{s+(1+0.5K_{cf})} = \frac{\frac{0.5K_{cf}}{1+0.5K_{cf}}}{\underbrace{\left(\frac{1}{1+0.5K_{cf}}\right)}_{\tau_v^*}s+1}$$

for large K_{cf}, we have unit gain and small the constant for the valve

$$G_L^*(s) = \frac{G_L}{1+G_vK_{cf}} = \frac{\frac{0.75}{s+1}}{1+\frac{0.5}{s+1}\cdot K_{cf}}$$

$$= \frac{0.75}{s+(1+0.5K_{cf})} = \frac{\frac{0.75}{1+0.5K_{cf}}}{\left(\frac{1}{1+0.5K_{cf}}\right)s+1}$$

Q: Determine the slave-controller gain K_{cf} such that the "virtual valve" G_v^* time constant is 1/10 of the original valve time constant

$G_v = \frac{0.5}{s+1}$ $\quad \tau_v = 1$ $\quad G_v^* = \frac{\frac{0.5K_{cf}}{1+0.5K_{cf}}}{\frac{1}{1+0.5K_{cf}}s+1}$

$$\tau_v^* = \frac{1}{10} = \frac{1}{1+0.5K_{cf}}$$ $\boxed{K_{cf}=18}$

Steady state flow rate in response to unit step input as "desired flow

$$A(s)=\frac{\frac{0.5}{s+1}K_{cf}}{1+K_{cf}\frac{0.5}{s+1}}\cdot\frac{1}{s}=\frac{0.5K_{cf}}{s+1+0.5K_{cf}}\cdot\frac{1}{s}$$

$$\lim_{t\to\infty} a(t)=\lim_{s\to 0} s\cdot\frac{1}{s}\cdot\frac{0.5K_{cf}}{s+1+0.5K_{cf}}=\frac{(0.5)18}{10}=0.9 \qquad 10\%$$

steady-state error

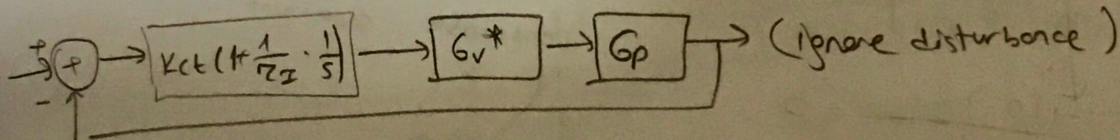
The inner controller is Type $\emptyset$ and has 10% steady state error on the desired flow rate. But the outer controller makes the system Type 1 (Integral Term) and will eliminate this error at the output

The design of the master PI controller?

Requirements:

- Stable System
- Fast Enough
- Reasonable overshoot

Approach: Make attempts for τ_I a) $\tau_I=0.05$ b) $\tau_I=0.5$ c) $\tau_I=5$ } → Use fast Root-Locus sketches to make a decision



(ignore disturbance)

The characteristic Eqn

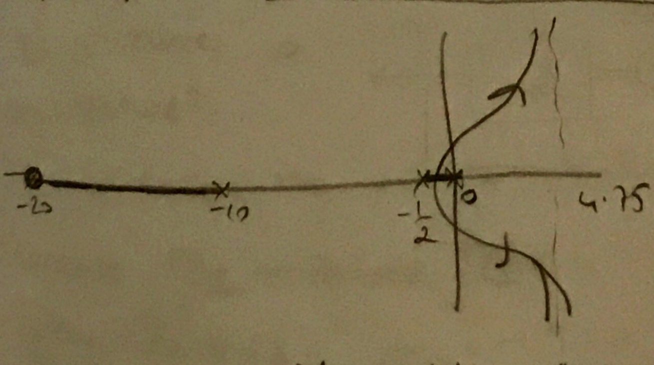
$$q(s)=1+K_{ct}\frac{Z(s)}{P(s)}=1+\underbrace{K_{ct}\left(\frac{\tau_I s+1}{\tau_I s}\right)\left(\frac{0.9}{0.1s+1}\right)\left(\frac{0.8}{2s+1}\right)}_{\text{open-loop TF}}$$

Note: $K=0.72K_{ct}$

(a) $\tau_I=0.05$

$\boxed{\text{open-loop zeros: } s=-1/\tau_I}$ → zero at $\boxed{s=-20}$

open-loop poles: $\boxed{s=0 \quad s=-10 \quad s=-1/2}$



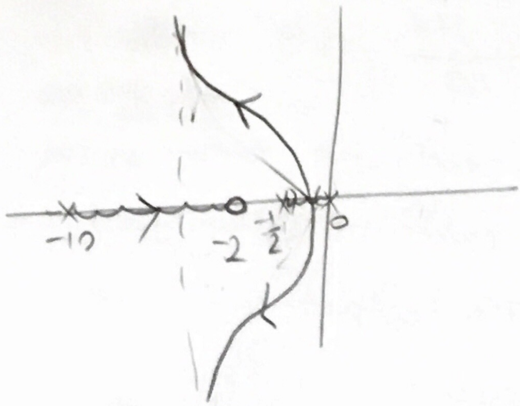
3-branches
2 asymptotes

$$\sigma_0=\frac{\sum p-\sum z}{\#p-\#z}=\frac{-10.5+20}{2}\simeq 4.75$$

→ large ~~settling~~ settling time → system goes unstable

(b) $\tau_I = 0.5$ zero at $\boxed{s = -1/\tau_I = -2}$ poles are the sa

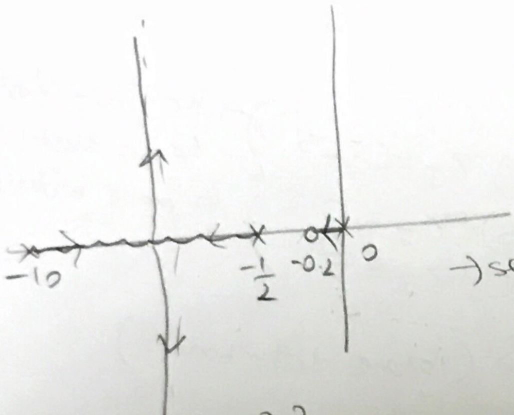
$$\sigma_0 = \frac{\Sigma p - \Sigma z}{\#p - \#z} = \frac{-10.5+2}{2} = -4.25$$



→ more overshoot

(c) $\tau_I = 5$ zero at $s = -1/\tau_I = -0.2$ poles are the

$$\sigma_0 = \frac{\Sigma p - \Sigma z}{\#p - \#z} = \frac{-10.5+0.2}{2} = -5.15$$



→ settling the problem

small settling time

reasonable overshoot.

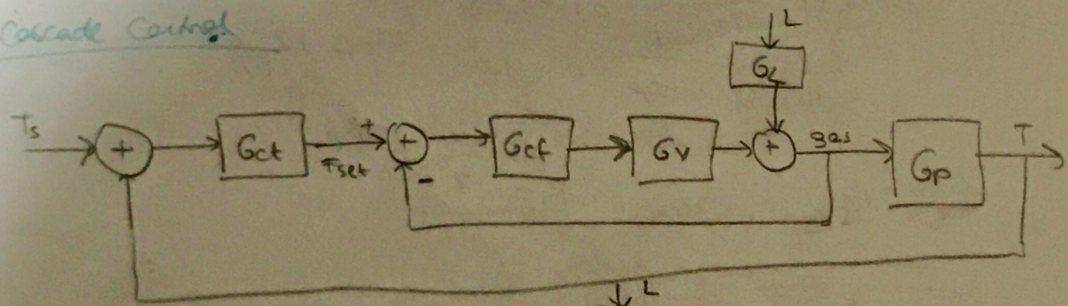
$$1 + K_c \frac{(5s+1)\,0.72}{5s(0.1s+1)(2s+1)} = 0$$

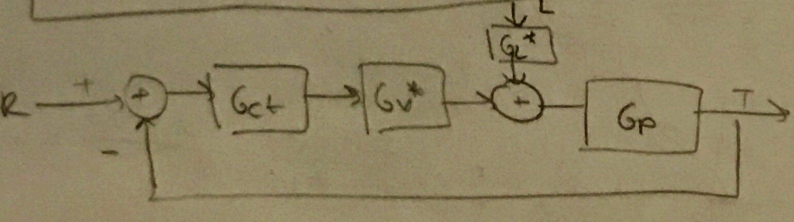
$0.2s^2 +$

$$K_c = \frac{-5s(0.1s+1)(2s+1)}{(5s+1)\,0.72}$$

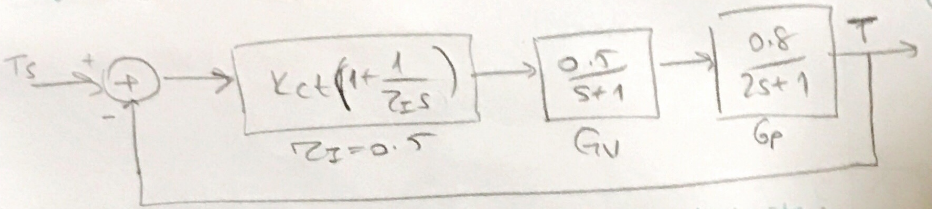
12/12/2018

Cascade Control





Q: What would be the behaviour without the slave loop. (what would be the new range of K_{ct}?)

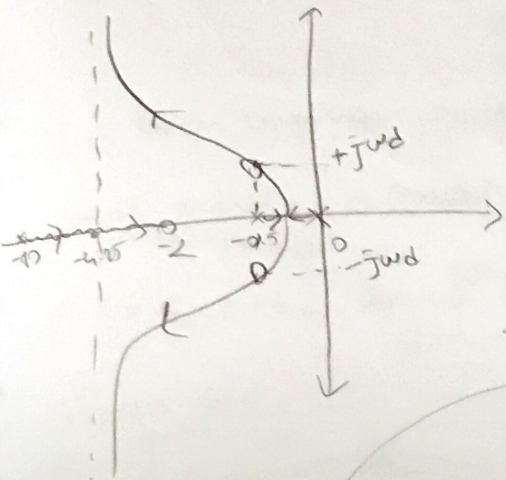


Use Routh Hurtwitz:

Closed loop characteristic Eqn:

$$q(s)=1+G_v G_c G_p = 1+K_{ct}\left(\frac{\tau_I s+1}{\tau_I s}\right)\left(\frac{0.5}{s+1}\right)\left(\frac{0.8}{2s+1}\right)$$

$$q(s)=s^3+1.5s^2+(0.5+0.2k_{ct})s+0.4k_{ct}$$



s^3	1	$0.5+0.2K_{ct}$
s^2	1.5	$0.4K_{ct}$
s^1	a	
s^0	$0.4K_{ct}$	

$0.4K_{ct} \rightarrow 0.4K_{ct}>0$

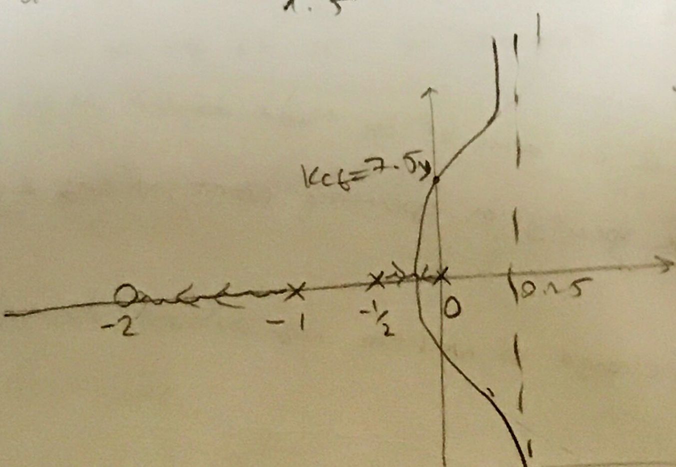
$\boxed{K_{ct}>0}$

$$a=\frac{1.5(0.5+0.2K_{ct})-0.4K_{ct}}{1.5}$$

$a>0 \rightarrow 0.75+0.3K_{ct}-0.4K_{ct}>0$

$-0.1K_{ct}>-0.75$

$\boxed{\rightarrow K_{ct}<7.5}$



Root Locus.

zero at $s=-\frac{1}{\tau_I}=-2$

$$\sigma_0=\frac{-\frac{1}{2}-1+2}{2}=0.25$$

Slow system with very limited stable range for the gain K_{ct}!!

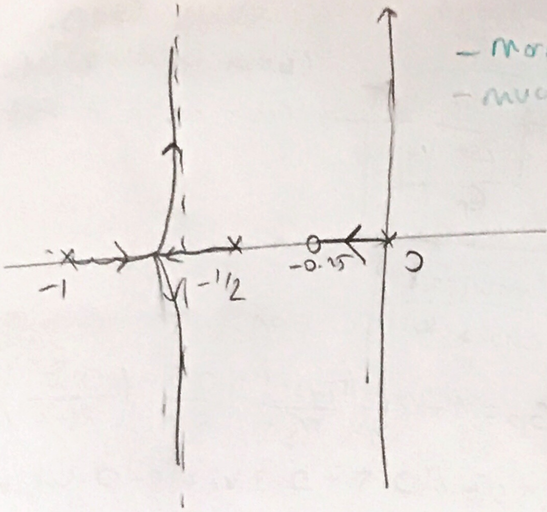
Note: in Root Locus. $k=\frac{(0.5)(0.8)}{2\tau_I}\cdot K_{ct}$

Q: Is there a better τ_I for the current classical feedback architecture?

Idea: to take the centroid to the left improving stability.

Increase $\tau_I \longrightarrow$ take $\tau_I=4 \rightarrow$ zero at $s=-0.25$

New centroid: $\sigma_0=\frac{-1.5+0.25}{2}=-0.625$

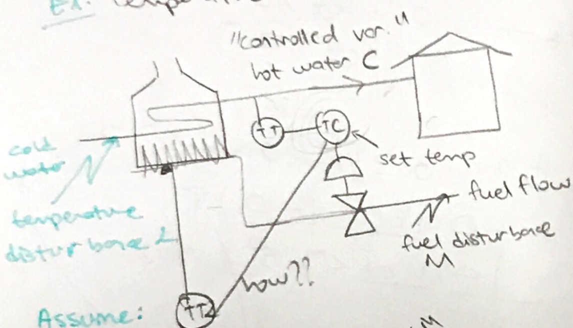


- More stable(?)
- much slower!!

Disturbance Feedforward Control (Chen, Ch10)

Idea: Take a very practive approach to eliminating the disturbance

EX: Temperature control



Assume:

- We have a "Process Model"

→ "we can write the process variable C in terms of the "input variables" M and L

→ If the plant is nonlinear or we specify an operating point, M and L are delta variable.

Idea: Introduce a pre-computed change to M (the manipulated variable) to compensate the disturbance L.

We have:

$C = G_L \cdot L + G_p \cdot M$ we want $C = R$

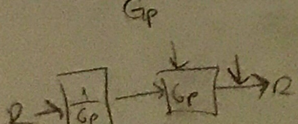
If we operate around the operating point

$C = R = G_L \cdot L + G_p \cdot M$ I will try to express M in terms of L

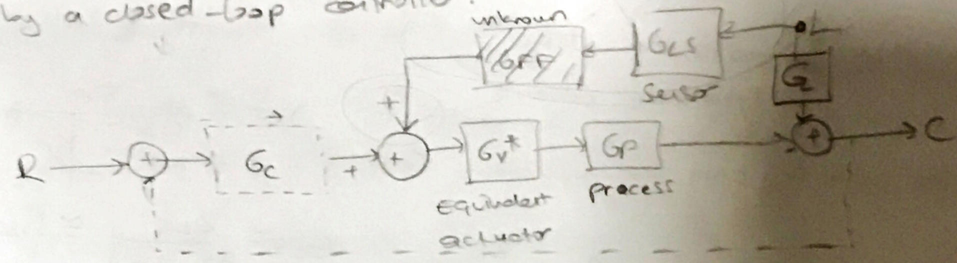
$M = \frac{1}{G_p}(R - G_L \cdot L)$ $\quad$ $M = \frac{1}{G_p} \cdot R - \frac{G_L}{G_p} \cdot L$

$\frac{1}{G_p}$ → reference tracking Controller

$\frac{G_L}{G_p}$ → Disturbance Feedforward Controller



Note: We will assume that the reference tracking will later be handled by a closed-loop controller.



G_{FF}: "Disturbance Feedforward Controller"

Ideal scenario is $C=0$ (at steady state the delta

$$C = G_L \cdot L + G_{LS} \cdot G_{FF}\, G_V^* \cdot G_P \cdot L = \underbrace{\left(G_L + G_{LS}\, G_{FF}\, G_V^* G_P\right)}_{=0} \cdot \cancel{L} = 0 \qquad L \neq 0$$

Express G_{FF}:

$$\boxed{G_{FF} = -\frac{G_L}{G_{LS} G_V^* G_P}}$$

Unfortunately; this TF for G_{FF} is not realizable!!

(Let us see it on an example)

Example:

$$G_V^* = \frac{1}{0.1s+1}\ ; \quad G_P = \frac{0.8}{2s+1}\ ; \quad G_L = \frac{1}{2s+1}$$ ← usually have the same time constant as the plant.

Disturbance sensor

$$G_{LS} = \frac{1}{2s+1}$$

$$G_{FF} = -\frac{\frac{1}{\cancel{2s+1}}}{\frac{1}{0.1s+1} \cdot \frac{1}{0.1s+1} \cdot \frac{0.8}{\cancel{2s+1}}} = -\frac{1}{0.8}(0.1s+1)^2$$ → Improper TF! Deg(Num) > Deg(Den)

problem? → Not causal

Two zero, very high gain at high freq (40 dB dec) amplifying all the high freq components. Improper TF, it is not causal

Proper TF, ~~we~~ denominator degree ~~has~~ should bigger

Another example: $G_p(s) = \frac{0.8}{2s+1} \cdot e^{-\theta s}$ FOPDT Model.

$G_{FF} = -\frac{1}{0.8}(0.1s+1)^2 e^{+\theta s}$ ← a positive delay → a time advance.

You need θ time into the future to implement GFF.

$G_p = \frac{k_p}{\tau_p s+1}$; $G_v^* = \frac{k_v}{\tau_v s+1}$; $G_L = \frac{K_L}{\tau_L s+1}$ $G_{LS} = \frac{k_{LS}}{\tau_{LS} s+1} \cong$

Hence:

$$G_{FF} = -\frac{G_L}{G_{LS}\, G_v^* G_p} = \frac{\frac{K_L}{\tau_L s+1}}{K_{LS} \cdot K_v \cdot K_p \frac{1}{(\tau_{LS} s+1)(\tau_v s+1)(\tau_p s+1)}}$$

$$= \frac{\overbrace{(\tau_{LS} s+1)}^{\cong 1}\overbrace{(\tau_v s+1)}^{\cong 1}(\tau_p s+1)}{K_{LS} K_v \cdot K_p (\tau_L s+1)}$$

1.) If we assume a fast disturbance sensor AND
2.) " " a fast actuator (which can also be achieved by cascade control)

$G_v^* \cong K_v$ with these assumptions; the FF controller

$$\boxed{G_{FF} \cong -\frac{K_L}{K_{LS} K_v^* K_p} \frac{(\tau_p s+1)}{(\tau_L s+1)}}$$ Dynamic Compensator

In general, the GFF transfer function (FF Controller) is a Lead-Lag "compensator"

$$\boxed{G_{FF} = K_{FF} \cdot \frac{\tau_{lead}\, s+1}{\tau_{lag}\, s+1}}$$

Note: We can also have

$\boxed{G_{FF} = K_{FF}}$ "steady-state Compensator"

when? If G_L and G_p has the same denominator (time constants) and the sensor and actuator time constants τ_{LS} and τ_v much smaller than τ_p.