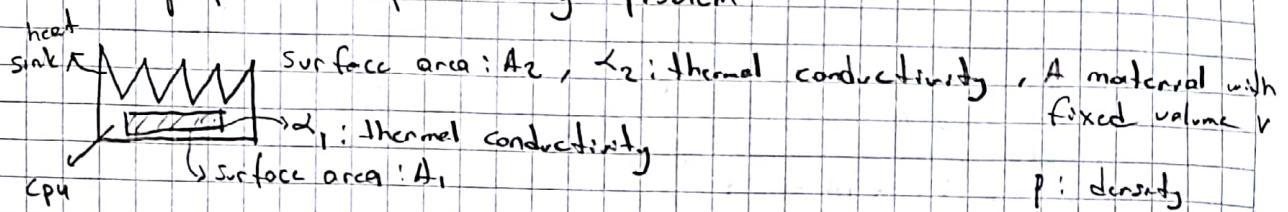


$$T - \bar{T} = T'(\bar{t})$$

* Distributed Parameter Systems and Lumped parameter Approximation

Ex Simplified CPU cooling problem



CPU temp: $T_{in}(t)$, heat sink temperature: $T(t)$

C_p : specific heat.

Ambient temp: $T_{amb}(t)$

dynamic variable of interest

The energy conservation law:

Heat accumulation = (in-flow of heat) - (out-flow of heat)

$$(V \cdot C_p \cdot p) \underbrace{\frac{d}{dt} T(t)}_{C: \text{ thermal cap.}} = A_1 \kappa_1 (T_{in}(t) - T(t)) - A_2 \kappa_2 (T(t) - T_{amb}(t))$$

C : thermal cap.

$$C \frac{d}{dt} T(t) + (\kappa_1 A_1 + \kappa_2 A_2) T(t) = A_1 \kappa_1 T_{in}(t) + A_2 \kappa_2 T_{amb}(t)$$

At steady-state we have $\frac{d}{dt} T(t) = 0$

$$(\kappa_1 A_1 + \kappa_2 A_2) T_{ss} = \kappa_1 A_1 T_{in,ss} + \kappa_2 A_2 T_{amb,ss}$$

$$T_{ss} = \frac{\kappa_1 A_1 T_{in,ss} + \kappa_2 A_2 T_{amb,ss}}{\kappa_1 A_1 + \kappa_2 A_2}$$

We define the deviation values:

$$Z(t) = T(t) - T_{ss}$$

$$Z_{in}(t) = T_{in}(t) - T_{in,ss}$$

$$Z_{amb}(t) = T_{amb}(t) - T_{amb,ss}$$

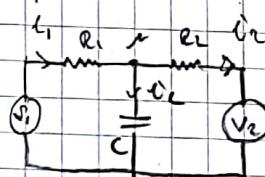
$$C \frac{dZ(t)}{dt} + (\zeta_1 A_1 + \zeta_2 A_2) Z(t) = \zeta_1 A_1 Z_{in}(t) + \zeta_2 A_2 Z_{out}(t)$$

↓
Zero initial cond → L.S. I

$$(s + \zeta_1 A_1 + \zeta_2 A_2) Z(s) = \zeta_1 A_1 Z_{in}(s) + \zeta_2 A_2 Z_{out}(s)$$

$$Z(s) = \frac{\zeta_1 A_1 Z_{in}(s)}{(s + \zeta_1 A_1 + \zeta_2 A_2)} + \frac{\zeta_2 A_2 Z_{out}(s)}{(s + \zeta_1 A_1 + \zeta_2 A_2)}$$

Tf w.r.t. the input . Tf w.r.t " disturbance"



kcl at N

$$i_1 - i_C - i_2 = 0$$

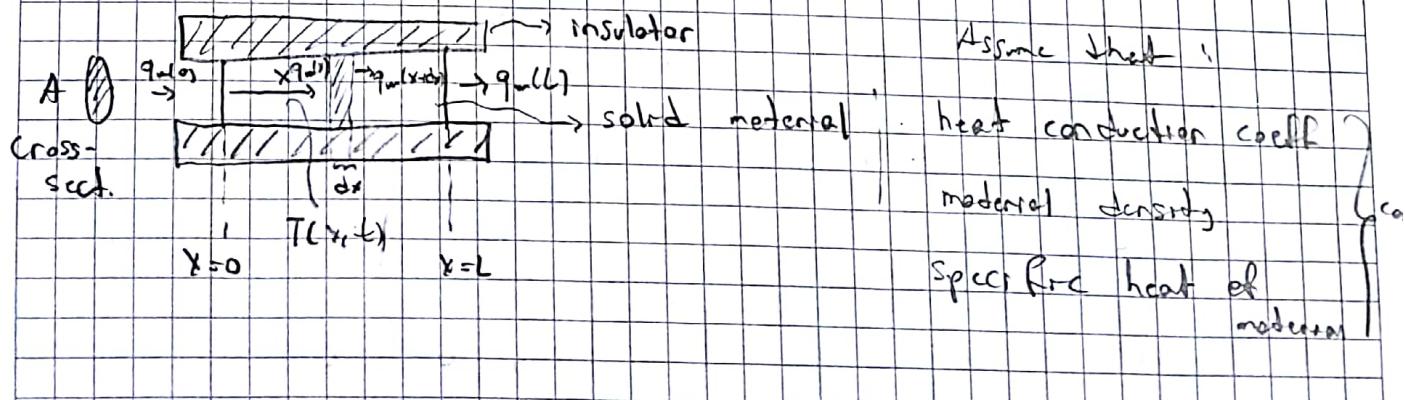
$$\frac{V_1 - V_C}{R_1} - C \frac{dV_C}{dt} - \frac{V_C - V_2}{R_2} = 0$$

$$C \frac{dV_C(t)}{dt} = \frac{1}{R_1} (V_1(t) - V_C(t)) - \frac{1}{R_2} (V_C(t) - V_2(t))$$

L

$$(s + \frac{1}{R_1} + \frac{1}{R_2}) \tilde{V}_C(s) = \frac{1}{R_1} \tilde{V}_1(s) + \frac{1}{R_2} \tilde{V}_2(s)$$

* Heat Conduction in a solid Body



- Heat balance equation for the element of length Δx

$$(A \cdot \Delta x \cdot \rho \cdot c_p) \frac{\partial T(x,t)}{\partial t} = \underbrace{A q_w(x)}_{\substack{\text{in-flow} \\ \text{accumulation}}} - \underbrace{A q_w(x+\Delta x)}_{\substack{\text{out-flow} \\ \text{C}}}$$

$$- A \frac{\partial q_w(x)}{\partial x} \Delta x$$

$$\rho c_p \frac{\partial T(x,t)}{\partial t} = - \frac{\partial q_w}{\partial x} \quad \begin{matrix} \text{due to} \\ \text{definition} \\ \text{of derivative} \end{matrix} \quad \frac{\partial q_w(x)}{\partial x} \xrightarrow{x=0} \frac{q_w(x+\Delta x) - q_w(x)}{\Delta x}$$

Using Fourier law of Thermal conduction

$$q_w \rightarrow - \frac{\partial T(x,t)}{\partial x} \quad \lambda: \text{coeff. of thermal cond.}$$

↳ space continuous version of heat conduction over an interface

$$\rho c_p \frac{\partial T(x,t)}{\partial t} = - \frac{\partial}{\partial x} \left[- \frac{\partial T(x,t)}{\partial x} \right]$$

$$\frac{\partial T(x,t)}{\partial t} = \frac{\lambda}{\rho c_p} \frac{\partial^2 T(x,t)}{\partial x^2}, \quad \alpha = \frac{\lambda}{\rho c_p} : \text{factor of heat conductivity}$$

partial diff eqn in x and t of heat propagation within solid body

$$+ \equiv \mathbb{F} \equiv \mathbb{E}$$

- The solution of the PDE requires boundary conditions

(space) at $x=0$ and $x=L$ and initial condition (time)

for $t=0$

$$\left. \begin{array}{l} T(0,t) = T^0(t) \\ T(L,t) = T^L(t) \end{array} \right\} \text{still function of } t$$

The initial condition:

$T(x, 0) = T_0(x)$ is still func. of the space variable

- The steady-state conditions inside the slab

Assume constant steady-state boundary temperature.

T^0 and T^L give us

steady-state implies that I have $\frac{\partial T(x, t)}{\partial t} = 0$

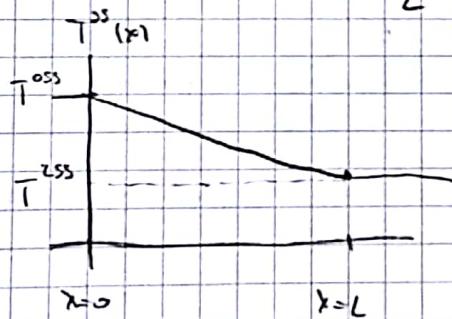
$$0 = \rho \left(\frac{\partial^2 T(x, t)}{\partial x^2} \right) \rightarrow 0 = \frac{\partial^2 T(x, t)}{\partial x^2}$$

$T^{ss}(x) = C_1 x + C_2$, use boundary conditions to determine C_1, C_2

$$x=0, T^{ss}(0) = T^{oss} = (C_1 x + C_2)|_{x=0} = C_2$$

$$x=L, T^{ss}(L) = T^L = (C_1 x + C_2)|_{x=L} = C_1 L + C_2 \rightarrow C_1 = \frac{T^L - T^{oss}}{L}$$

$$T^{ss}(x) = T^{oss} + \frac{T^L - T^{oss}}{L} x$$



- Note: Two approaches to design controllers

- a) "Later-pass" to use PDE model directly to design controllers and use numerical approximations later in the process (Begin our approach)
- b) "Early-pass", Discretize space variable at the beginning and obtaining a higher order ordinary diff eqn (lumped parameter finite element) \leftarrow approximation nested

- Solution of the PDE

Assume an infinite slab ($L \rightarrow \infty$)

Assume steady-state: $T^{ss} = T_c$

Define "Delta variable" $\bar{T}(x,t) = T(x,t) - T_c$

$$\frac{\partial \bar{T}(x,t)}{\partial t} = a \frac{\partial^2 \bar{T}(x,t)}{\partial x^2}$$

$$\left. \begin{aligned} \bar{T}(x,s) - \bar{T}(x,0) &= a \frac{\partial^2 \bar{T}(x,s)}{\partial x^2} \\ 0 \text{ due to} \\ \text{operating point} \end{aligned} \right\} \Rightarrow \text{PDE reduced to an ordinary diff eqn. in } x$$

Since we have $L \rightarrow \infty$ we assume that the end of the infinite slab is not effected by transient changes

$$\bar{T}(x,t) = 0 \text{ as } x \rightarrow \infty$$

$$\frac{\partial^2 \bar{T}(x,s)}{\partial x^2} = \left(\sum_{n=0}^{\infty} \right)^2 \bar{T}(x,s)$$

The homogeneous solution to this D.E. is

$$\bar{T}(x,s) = k_1 e^{-x\sqrt{\frac{s}{a}}} + k_2 e^{x\sqrt{\frac{s}{a}}}$$

We have $\bar{T}(x,0) = 0$ as $x \rightarrow \infty$

$$\lim_{x \rightarrow \infty} \bar{T}(x,0) = 0 \rightarrow k_2 = 0$$

Now use boundary condition at $x=0$

$$\bar{T}(0,t) = T^o(t) \xrightarrow{\text{Laplace}} \bar{T}(0,s) = T^o(s)$$

$$\bar{T}(x,s) = k_1 e^{-x\sqrt{\frac{s}{a}}} \Big|_{x=0} = T^o(s) - k_1$$

$$\bar{T}(x,s) = T^o(s) e^{-x\sqrt{\frac{s}{a}}}$$

十二月五日

Now let us consider a step temperature function at the boundary

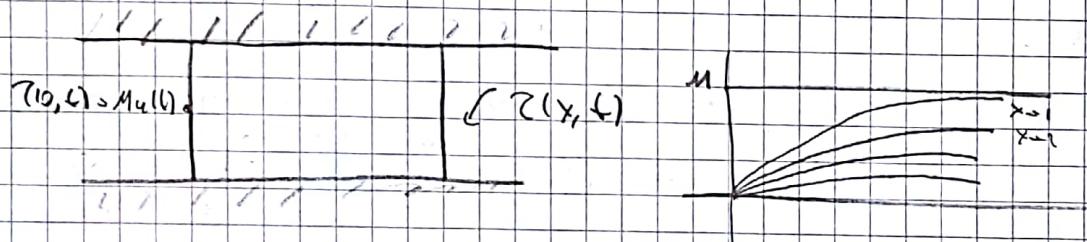
$$T(0, t) = M u(t) \xrightarrow{\mathcal{L}} T(0, s) = T(s) = \frac{M}{s}$$

$$\rightarrow T(x, s) = \frac{M}{s} e^{-x \sqrt{\frac{s}{\alpha}}} \quad \text{S. 3}$$

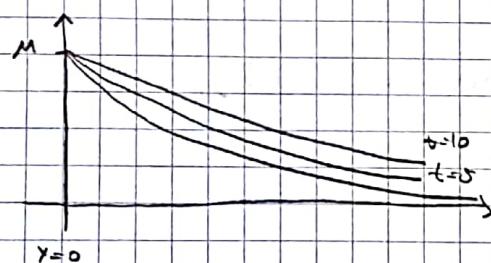
$$\operatorname{erfc}\left(\frac{k}{2\sqrt{\alpha t}}\right) \xrightarrow{\mathcal{L}} \frac{1}{s} e^{-k\sqrt{s}}, \operatorname{erfc}(t): \text{complementary error func.}$$

$$T(x, t) = M \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Ex Unit-step response at a fixed distance x inside slab

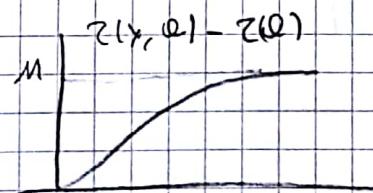


Ex Fixed time t , variable distance x .



- Let $\vartheta = (\frac{\alpha}{\pi c})^{\frac{1}{2}} t$ a scaling of the time variable

$$T(x, t) = M \operatorname{erfc}\left(\frac{1}{2} \sqrt{\frac{x^2}{\alpha t}}\right) \rightarrow M \operatorname{erfc}\left(\frac{1}{2} \sqrt{\frac{\alpha t}{\vartheta c t}}\right) = M \operatorname{erfc}\left(\frac{1}{2} \sqrt{\frac{1}{\vartheta}}\right)$$



- we worked with "Delta variable" around the steady-state

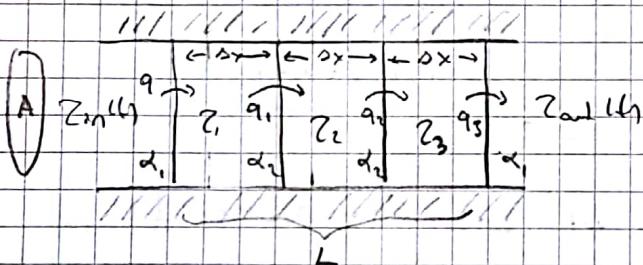
$$T(0, t) = T_c + \Delta u(t)$$

$$T(x, t) = T_c + M \operatorname{erfc} \left(\frac{x}{2\sqrt{\alpha t}} \right)$$

Lumped Parameter Approximation (of distributed Models)

Low order PDE \rightarrow Higher order ODE (order will depend on the resolution of discretization)
 \rightarrow System of interconnected lower order ODE

Ex $N=3$



Assuming we again operate around the operating point

$$z_i(0) = 0 \quad z_i(t) \text{ is uniform inside}$$

Energy Balance eqn:

$$\underbrace{\frac{dC}{dt}}_{\Delta C} \frac{dz_1}{dt} = \underbrace{\lambda_1 A}_{\Delta_1} (z_{in} - z_1) - \underbrace{\lambda_2 A}_{\Delta_2} (z_1 - z_2)$$

$$= \Delta_1 z_{in} - (\Delta_1 + \Delta_2) z_1 + \Delta_2 z_2$$

For second slice

$$\frac{dC}{dt} \frac{dz_2}{dt} = \Delta_2 (z_1 - z_2) - \Delta_2 (z_2 - z_3) = \Delta_2 z_1 - \Delta_2 z_2 + \Delta_2 z_3$$

Last slice

$$\frac{dC}{dt} \frac{dz_3}{dt} = \Delta_2 (z_2 - z_3) - \Delta_1 (z_3 - z_{out}) = \Delta_2 z_2 - (\Delta_1 + \Delta_2) z_3 + \Delta_1 z_{out}$$

if we take z_1, z_2, z_3 as the state variables

$$\begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \\ \dot{Z}_3 \end{bmatrix} = \frac{A}{\Delta t} \begin{bmatrix} -2, -\alpha_2 & \alpha_2 & 0 \\ \alpha_2 & -2\alpha_2 & \alpha_2 \\ 0 & \alpha_2 & -2, -\alpha_2 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} + \frac{A}{\Delta t} \begin{bmatrix} \alpha_1 & 0 \\ 0 & 0 \\ 0 & \alpha_1 \end{bmatrix} \begin{bmatrix} Z_{in} \\ Z_{out} \end{bmatrix}$$

Remember in the continuous problem we had $\frac{\partial^2 Z}{\partial x^2}$
(distributed)

How to find the correspondence between λ and ω ?

Using the finite difference approximation to the derivative

$$\begin{aligned} \frac{\partial^2 Z(x,t)}{\partial x^2} &\approx \frac{Z(x+2\Delta x, t) - 2Z(x+\Delta x, t) + Z(x, t)}{(\Delta x)^2} \\ &= \frac{Z(x+2\Delta x, t) - 2Z(x+\Delta x, t) + Z(x, t)}{(\Delta x)^2} \end{aligned}$$

Define: $Z_i(t) = Z(i\Delta x, t)$ and $Z(x, t) = Z_i(t)$ for
 $(i-1)\Delta x < x < i\Delta x$

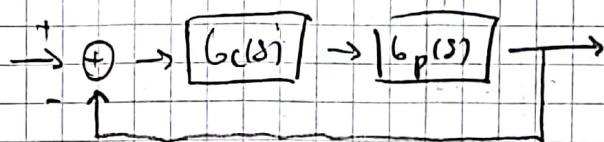
$$\frac{\partial^2 Z(x,t)}{\partial x^2} \approx \frac{Z(x+\Delta x, t) - 2Z(x, t) + Z(x-\Delta x, t)}{(\Delta x)^2}$$

$$\frac{\partial^2 Z_i(t)}{\partial t^2} = \frac{a}{(\Delta x)^2} [Z_{i+1}(t) - 2Z_i(t) + Z_{i-1}(t)]$$

$$\frac{\lambda}{fcp(\Delta x)^2} = \frac{\omega}{fcp\Delta x} \rightarrow \frac{\lambda}{\Delta x} = \omega_2$$

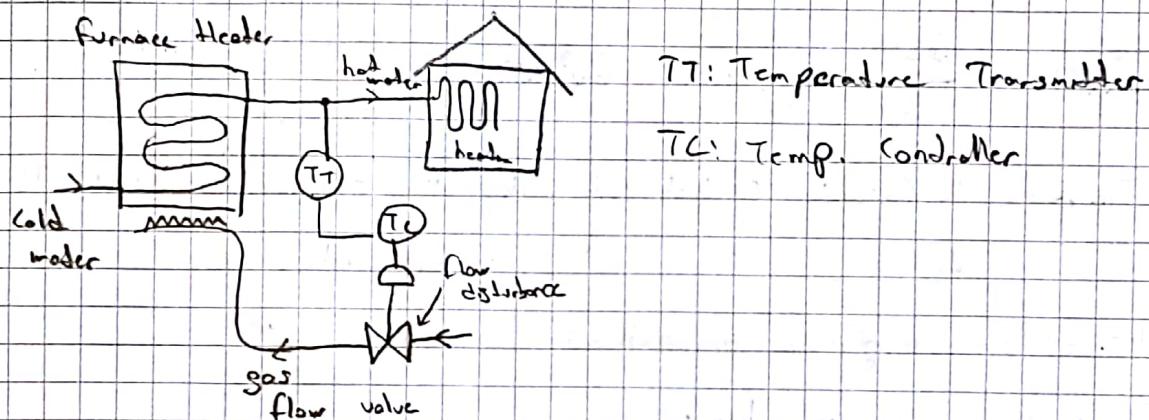
$$T = \text{Set} + \Delta$$

Multi-Loop Systems

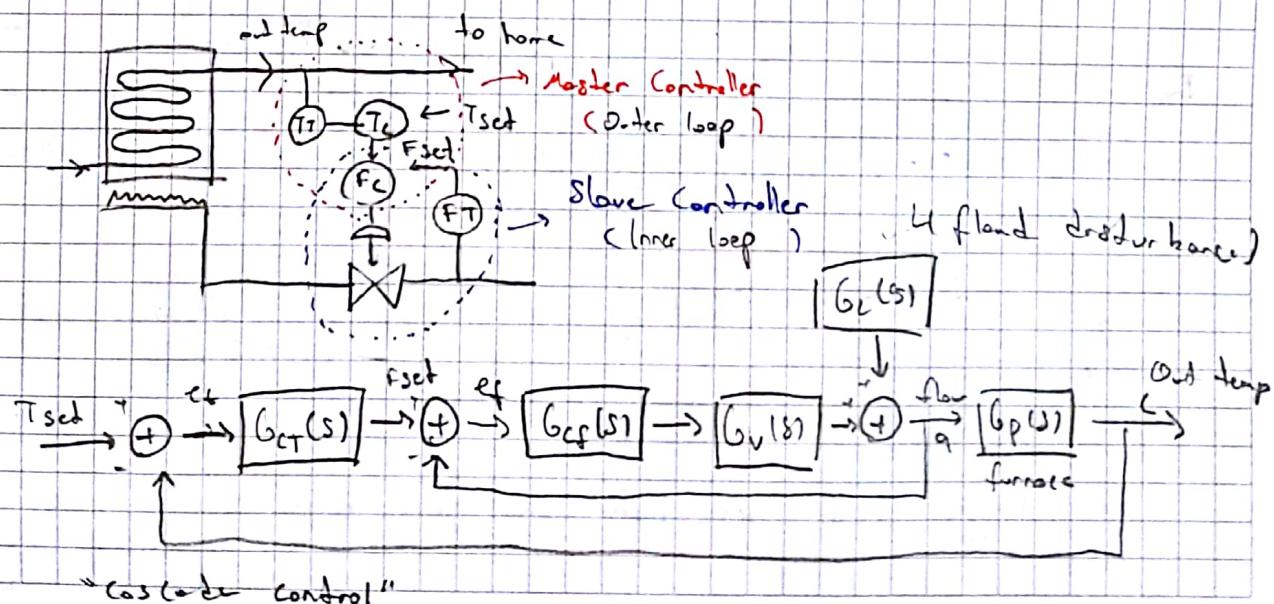


* Cascade Control

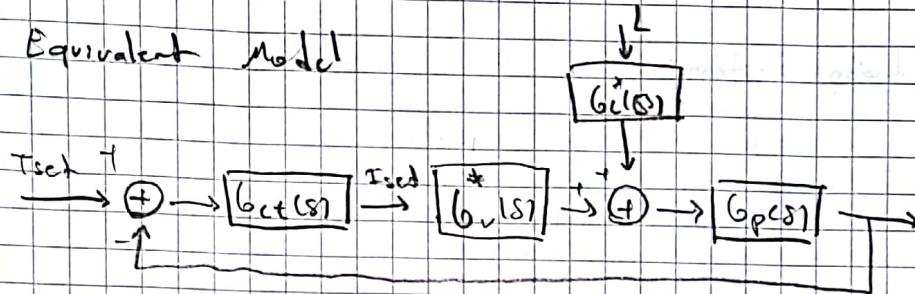
Consider the heating problem Temp. Control



Idea is that add a flow measurement and a flow controller and combine with TC



- Equivalent Model



$$q = G_L \cdot L + G_v \cdot G_{cf} (F_{set} - a)$$

$$(1 + G_v G_{cf}) q = G_c \cdot L + G_v G_{cf} F_{set} \rightarrow q = \frac{G_L}{1 + G_v G_{cf}} L + \frac{G_v G_{cf}}{1 + G_v G_{cf}} k_{set}$$

$$G_c(s) = \frac{G_L(s)}{1 + G_v(s) G_{cf}(s)}$$

$$G_v(s) = \frac{G_v(s) G_{cf}(s)}{1 + G_v(s) G_{cf}(s)}$$

- Claims: 1- Cascade Control can make the system more stable (?)

2- " " " " " the value appear to be much faster (?)

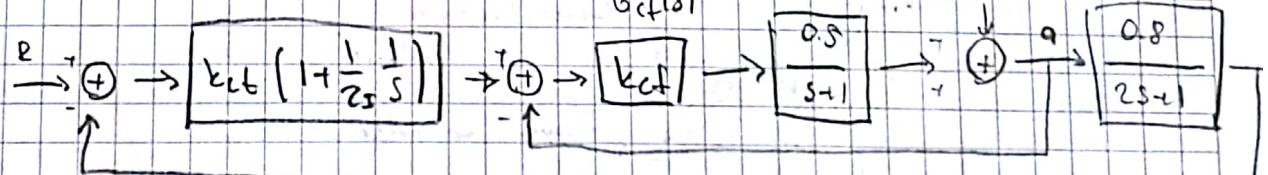
Assume: $G_{cf} = k_{cf}$ a proportional controller

Let k_{cf} be very large!

$$G_c(s) = \frac{G_L}{1 + G_v k_{cf}} \approx 0 \text{ will reduce to very small}$$

$$G_v(s) = \frac{G_v k_{cf}}{1 + G_v k_{cf}} \approx \frac{G_v k_{cf}}{G_v k_{cf}} = 1$$

Ex:



$$G_v = \frac{\frac{0.5}{s+1} k_{cf}}{1 + \frac{0.5}{s+1} k_{cf}} = \frac{0.5 k_{cf}}{(1 + 0.5 k_{cf}) s + 1}$$

Assume ideal sensor

for large k_{cf} , we have unit gain and small time constant for value

$$G_C^+ = \frac{6L}{1+6kcf} = \frac{\frac{0.75}{s+1}}{1 + \frac{0.5}{s+1} kcf} = \frac{0.75 / 1 + 0.5 kcf}{\left(\frac{1}{1 + 0.5 kcf} \right) s + 1}$$

- Q: Determine slave controller gain such that the virtual "valve" time constant is $1/10$ of the original time constant.

$$G_U L \rightarrow \frac{0.5}{s+1} \rightarrow T_U = 1$$

$$G_U^+ (s) = \frac{0.5 kcf / 1 + 0.5 kcf}{\left(\frac{1}{1 + 0.5 kcf} \right) s + 1} \rightarrow T_U^+ = \frac{1}{1 + 0.5 kcf} = \frac{1}{10}, kcf = 18$$

- Steady-state flow rate in response to unit step input as "desired flow rate"

$$A(s) = \frac{0.5 kcf}{s + 1 + 0.5 kcf} \frac{1}{s}, \text{ from } A(s) \rightarrow \text{from } S.A.C(s) = s \frac{1}{s} \frac{0.5 kcf}{s + 1 + 0.5 kcf} = 0.5$$

To b
corner

- The inner loop \rightarrow Type 0 and has no effect on the desired flow rate. But the outer controller makes the system Type I and will eliminate this error at the output.

- The design of master (outer) loop controller requires: stable system, fast enough, reasonable overshoot.

- Approach 1: Make attempts for T_S

$$\left. \begin{array}{l} T_S = 0.05 \\ T_S = 0.5 \\ T_S = 5 \end{array} \right\} \begin{array}{l} \text{use fast root locus} \\ \text{Sketches to make decision} \end{array}$$

Assume $L \approx 0$

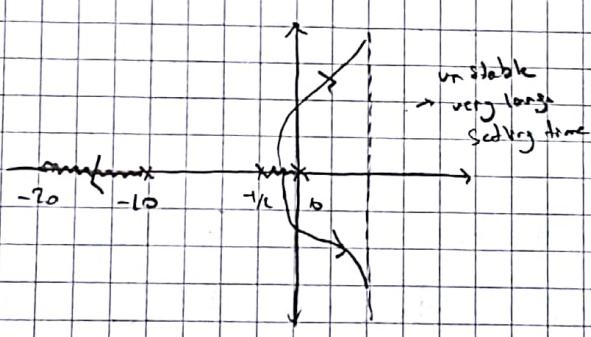
$$\rightarrow \frac{1}{s} \oplus \rightarrow \boxed{k_{ct} \left(1 + \frac{1}{2s} \frac{1}{s} \right)} \rightarrow \boxed{6u} \rightarrow \boxed{6p}$$

- Characteristic Eqn.

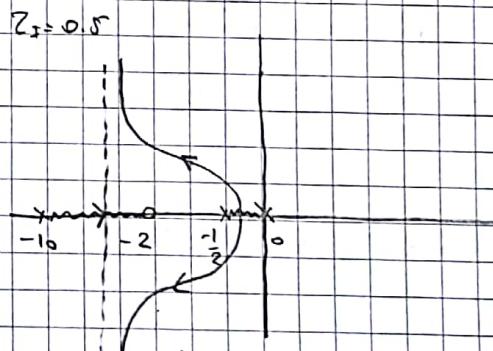
$$q(s) = 1 + k_{ct} \frac{2(s)}{p(s)} = 1 + k_{ct} \left(\frac{2s+1}{2s} \right) \left(\frac{0.9}{0.1s+1} \right) \left(\frac{0.8}{2s+1} \right)$$

$$\rightarrow Z_s = 0.05, \text{ open loop zeros} \rightarrow s = -1/Z_s = -20$$

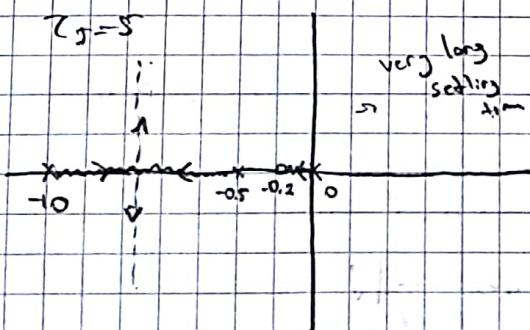
$$\text{open loop poles} \rightarrow s = 0, -10, -1/2$$



$$2 \text{ asymptotes} \quad j_0 = \frac{\zeta_p - \zeta_z}{\zeta_p - \zeta_z} \approx 1.25$$



$$j_0 = \frac{-\frac{1}{2} - 10 + 12}{2} = -4.25$$

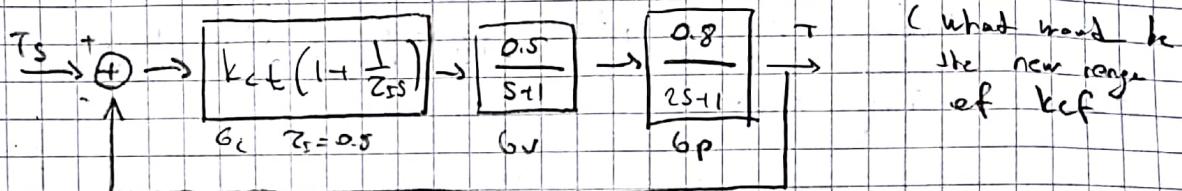


$$j_0 = \frac{-10.5 + 0.2}{2} = -5.15$$

$$j_0 = 0.15$$

+ = / + = \

- Q: What would be the behaviour without above loop.



Use Routh-Hurwitz

Closed loop characteristic eqn:

$$q(s) = 1 + G_c G_v G_p = 1 + kcf \left(\frac{2s^2 + 1}{2s} \right) \left(\frac{0.5}{s+1} \right) \left(\frac{0.8}{2s+1} \right)$$

$$q(s) = s^3 + 1.5s^2 + (0.5 + 0.2kcf)s + 0.6kcf$$

$$s^3 \quad | \quad 1 \quad 0.5 + 0.2kcf$$

$$a_1 = \frac{(1.5)(0.5 + 0.2kcf) - 0.6kcf}{1.5}$$

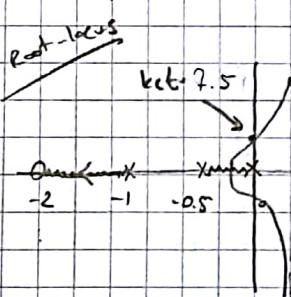
$$s^2 \quad | \quad 1.5 \quad 0.6kcf$$

$$a_{22} \rightarrow 0.75 + 0.3kcf - 0.6kcf > 0$$

$$s \quad | \quad 9 \quad 0$$

$$7.5 > kcf$$

$$s^0 \quad | \quad 0.6kcf \rightarrow 0.6kcf + 1 > 0, kcf > 0$$



* Note in root locus

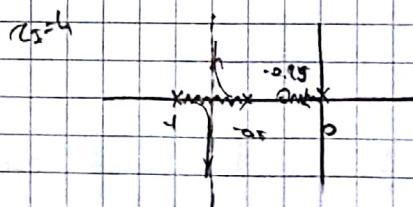
$$k = \frac{(0.5)(0.8)}{2} kcf$$

$$J_0 = \frac{-1 - 0.5 + 2}{2} = 0.25$$

Slow system with very limited stable range
for kcf

- Is there a better J_0 for the current classical feed back control?

Find J_0 so date centroid to the left



$$J_0 = -0.625$$

more stable (?)

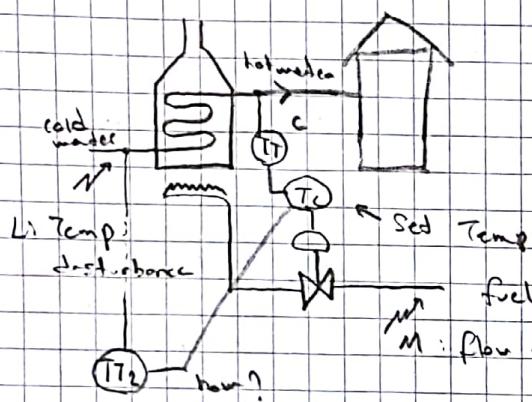
much slower.

$$t = f + t_f$$

Disturbance Feedforward Control

Idea: Take a very proactive approach to eliminating the disturbance.

- Ex: Temp. Cont.



Assume

→ we have a "Process Model"

→ "we can write the proper variable c in terms of the "input variable" M, L

→ if the plant is non-linear
 M : flow disturbance or we specify an operating point;
 M, L and delta variable

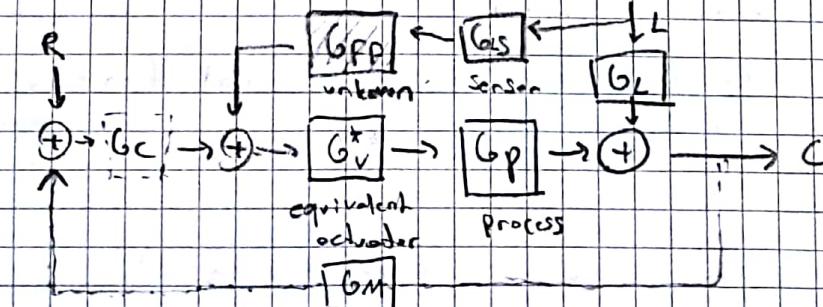
- Idea: Introduce a pre-computed change to M (the manipulated variable) to compensate the disturbance L

We have:

$$c = G_L L + G_p M \quad (\text{if we operate around the op} \mid \text{we want cst.})$$

$$c = R = G_L L + G_p M \rightarrow M = \frac{1}{G_p} (R - G_L L) = \underbrace{\frac{1}{G_p} R}_{\text{reference}} - \underbrace{\frac{G_L}{G_p} L}_{\text{disturbance}}$$

- Note: We will assume that the reference tracking with later be handled by a closed-loop controller



G_{FF} : "Disturbance feedforward controller"

ideal scenario is $C=0$ (at ss the delta output is at zero and stay at 0)

$$C = G_C L + G_{LS} G_{FF} G_V G_P \cdot L = (G_C + G_{LS} G_{FF} G_V G_P) L = 0$$

$$G_{FF} = -\frac{G_C}{G_{LS} G_V G_P} \rightarrow \text{unfortunately this } f \text{ for } G_{FF} \text{ is not realizable}$$

- Ex: $G_V = \frac{1}{0.1s+1}$; $G_P = \frac{0.8}{2s+1}$; $G_L = \frac{1}{2s+1}$ usually have the same time constant as the plant

$$G_{LS} = \frac{1}{0.1s+1} \quad (\text{Dy turbine sensor})$$

$$G_{FF} = -\frac{\frac{1}{2s+1}}{\frac{1}{0.1s+1} \cdot \frac{1}{0.1s+1} \cdot \frac{0.8}{2s+1}} = -\frac{(0.1s+1)^2}{0.8}$$

improper TF
 $\text{Deg}(num) > \text{Deg}(den)$
 $\rightarrow \text{not causal}$

- Ex: $G_P(s) = \frac{0.8}{2s+1} e^{-\zeta s}$ FOPDT Model

$$G_{FF} = -\frac{1}{0.8} (0.1s+1)^2 e^{-\zeta s}$$

a time advance, you need time into future to implement G_{FF}

$$G_P = \frac{k_P}{2s+1}, \quad G_V = \frac{k_V}{2s+1}, \quad G_L = \frac{k_L}{2s+1}, \quad G_{LS} = \frac{k_{LS}}{2s+1}$$

$$G_{FF} = \frac{\frac{k_L}{2s+1}}{k_{LS} k_V k_P \cdot \frac{1}{(2s+1)(2s+1)(2s+1)}} = \frac{(2s+1)(2s+1)(2s+1)}{k_{LS} k_V k_P (2s+1)}$$

$G_{LS} \approx k_{LS}$ for a "very fast" sensor (small ζ)

1. If we assume a fast disturbance sensor

2. If we use a fast actuator (which also can be actuated by cascade controller)

$$G_V \approx k_V$$

With these assumptions ; the FF controller

$$G_{FF} \approx -\frac{k_c}{k_L k_v k_p} \frac{(Z_p s + 1)}{(Z_L s + 1)} \Rightarrow \text{dynamic compensator.}$$

- In general G_{FF} transfer function is a lead-lag compensator.

$$G_{FF} = k_{FF} \frac{Z_{lead}}{Z_{lag} + 1}$$

- Note we can also have $G_{FF} = k_{FF}$ "steady-state compensator" if G_L and G_p has some time constant and sensor and actuator time constant Z_L , Z_v much smaller than Z_p
- Other practical note) we can tune G_{FF} , if we consider $Z_{lead} \approx Z_L + Z_v^+$, $Z_{lag} \approx 0.1 Z_{lead}$

Feedforward - Feedback Architecture

- Note: Disturbance feedback architecture is almost always used with the output feedback, because
- We do not a very precise model G_p of plant.
- Measurement errors (output disturbance)
- Error in feedforward components
- v_r measured load variables.

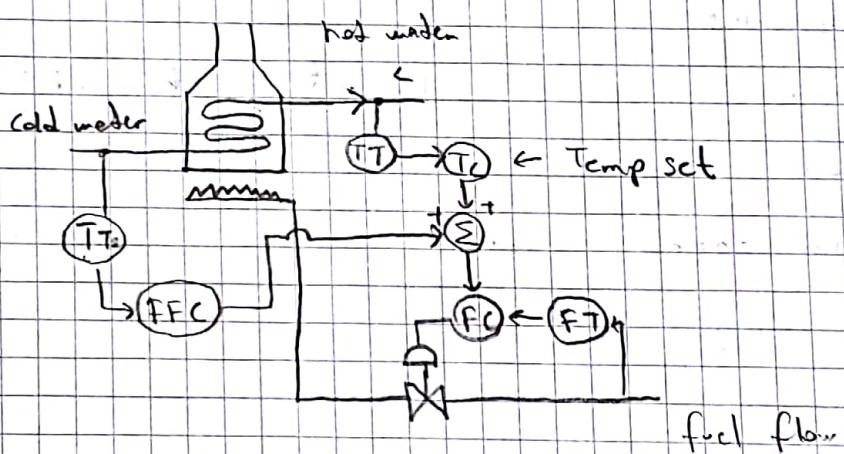
$$C = [G_L + G_{Ls} G_{FF} G_v^+ G_p] L + G_p G_v^+ G_c E, \quad E \rightarrow R - C G_M$$

$$C = \left[\frac{G_L + G_{Ls} G_{FF} G_v^+ G_p}{1 + G_m G_c G_v^+ G_p} \right] L + \left[\frac{G_c G_v^+ G_p}{1 + G_m G_c G_v^+ G_p} \right] R$$

- Note that the denominators of both TFs are the same and they do not have Gpp term.

→ Design of GFF does not effect the stability properties of the overall system. (Leaving us free to design it to achieve best disturbance rejection)

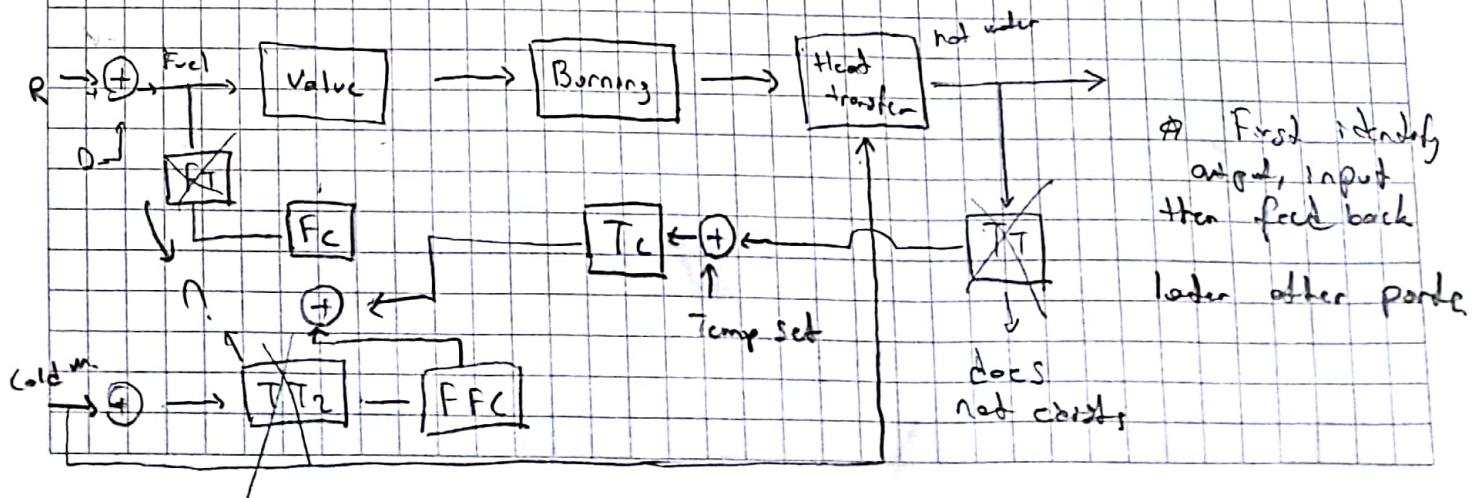
- All parts in some process diagram

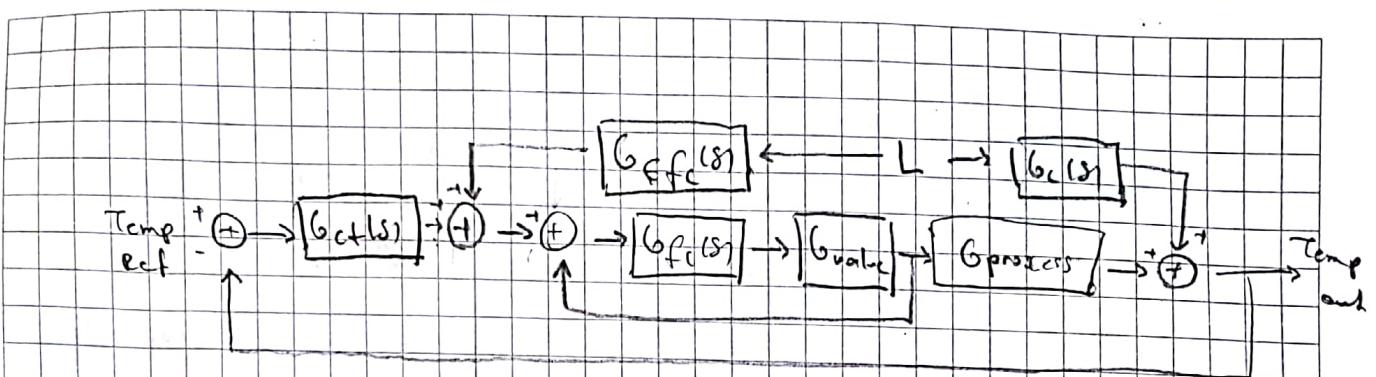


* Sketch the block diagram corresponding to the process diagram

Exercise

Wrong, blocks are S domain value \rightarrow b, (s) should be





* Engine ECU Design

- Task: control fuel flow to maintain efficient burn
(max power, min pollution)

Manipulated variables:

1. Fuel flow

2. Air flow

$$F = \frac{F_f}{F_A}$$

RC: "Ratio controller"

Process Variable: Engine Torque

