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# EE407 Process Control
# HW 1

1. We will analyse the system shown in the *Figure 1*.

   (a) To write the SS model of the system, let us begin with writing fundamental equation describing the system.

$$F_{Net} = m\ddot{x} = F - b\dot{x} - kx$$

   Choosing $\underline{x} = [x \;\; \dot{x}]^T$ and $\underline{y} = [1 \;\; 0]$x, we can build our Space-State Model for the system as

$$\underline{\dot{x}} = Ax + Bu \;\; \& \;\; \underline{y} = Cx + Du$$

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u$$

   where u is the input force F.

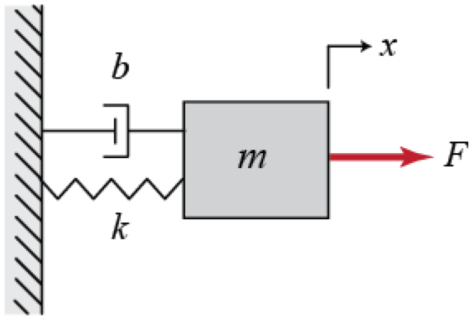



Figure 1: Mass Spring Damper System

   (b) Simulink Model for the Mass Spring Damper System can be seen at *Figure 2*

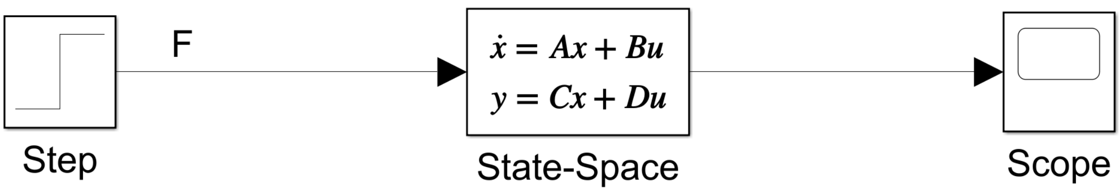
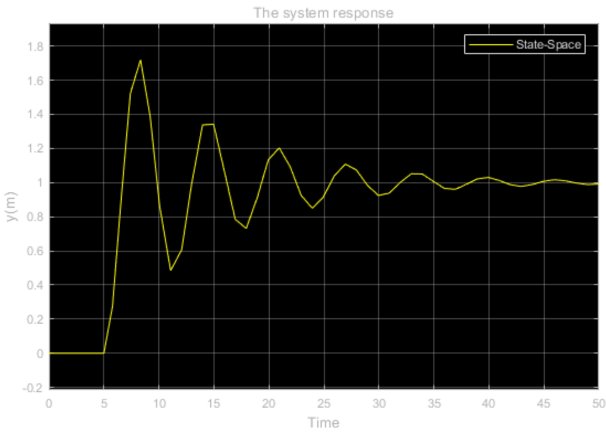


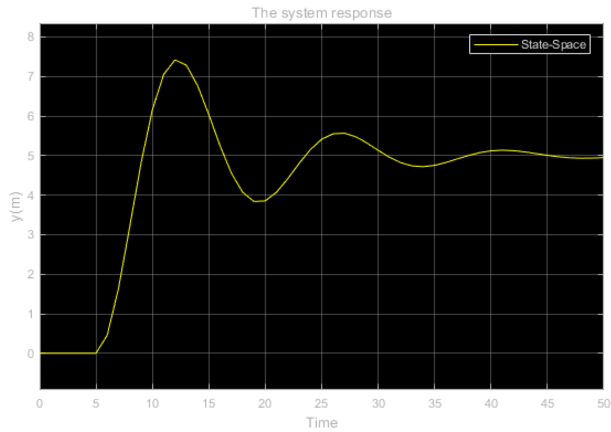

Figure 2: Simulink Model for the Mass Spring Damper System

(c) For the following subsections, the simulations are for the model when the applied force is a unit step function starting at t = 5 sec, i.e., $u(t-5)$.

i. The spring force $F_S = kx$ with the direction opposite to the input force $F$ is proportional to the "k" which is the spring force constant . Therefore, when the spring constant decreased, $F_{Net}$ is increased and the displacement $x$ is increased. The figures are consistent with these, *Figure 3b* has small $k$ value and it's final value is bigger than that of *Figure 3a*.



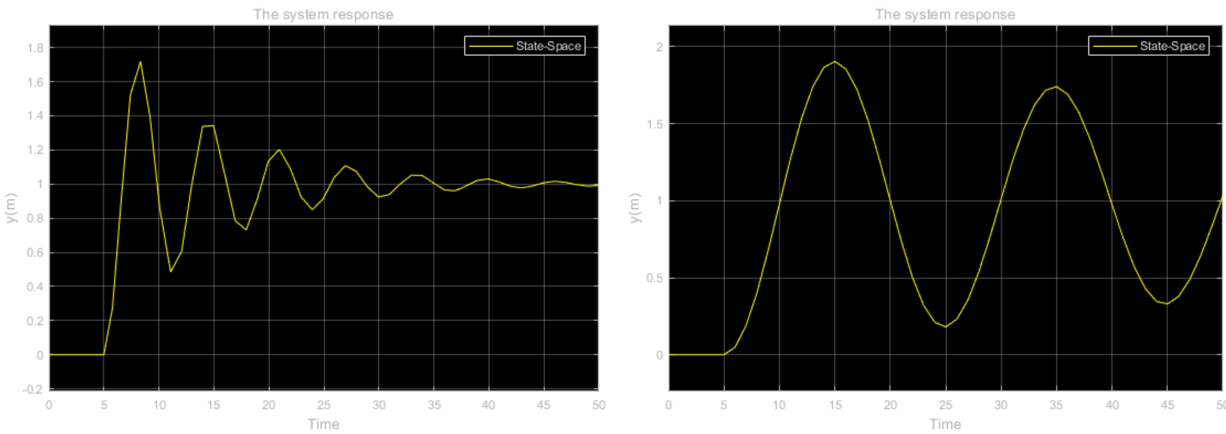

(a) The System Response for MSD as m = 1kg, b = 0.2Ns/m, k = 1N/m




(b) The System Response for MSD as m = 1kg, b = 0.2Ns/m, k = 0.2N/m

Figure 3: The System Response for MSD with varying spring force constant

ii. It is known that $F_{net} = ma = m\ddot{x}$ , then mass and acceleration that is related to position are oppositely proportional, so when mass increased, the rate of change in the velocity of the oscillation is decreased. The *Figure 4a* and *Figure 4b* are shows the expected effect. While both systems expected to reach same final steady state value, the system in *Figure 4a* oscillates quickly and reaches the steady state value in shorter time tant the system *Figure 4b*.

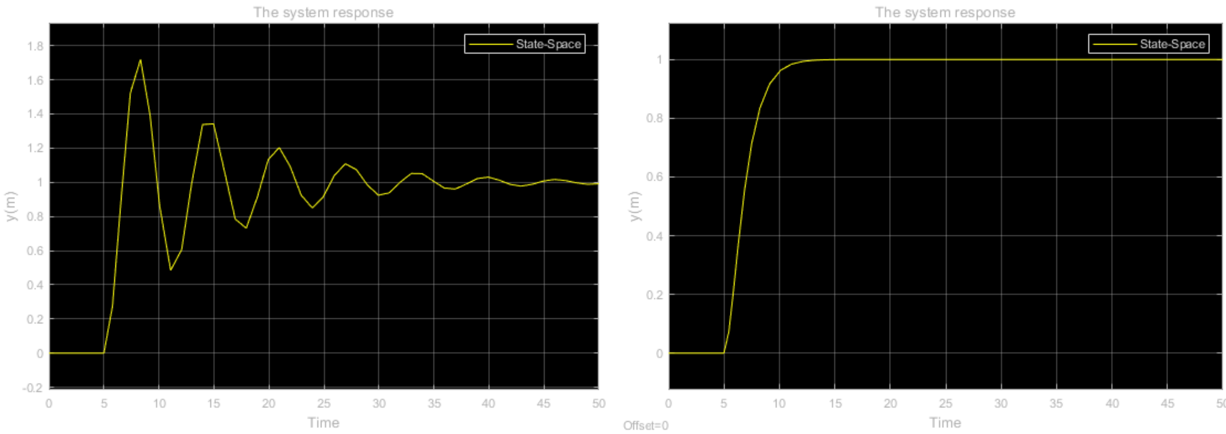



(a) The System Response for MSD as m = 1kg, b = 0.2Ns/m, k = 1N/m

(b) The System Response for MSD as m = 10kg, b = 0.2Ns/m, k = 1N/m

Figure 4: The System Response for MSD with varying mass

iii. The viscous damping force is proportional to the velocity of the mass, $v = \dot{x}$ with the direction of opposite to the F. In this case, firstly this opposite direction is not so much because of the velocity is small and after some point this velocity value increases and effect the system with more opposite force. Therefore, *Figure 5a* and *Figure 5b* are expected.




(a) The System Response for MSD as m = 1kg, b = 0.2Ns/m, k = 1N/m

(b) The System Response for MSD as m = 1kg, b = 2Ns/m, k = 1N/m

Figure 5: The System Response for MSD with varying viscous damping constant

(d) In this part, the system response of the MSD is observed when an input force are applied for certain time and stopped. We observe that the input force forces system to stabilize its position at non-zero steady-state value. When the input force is removed from system at $t = 20$ *sec*, the steady-state value that the system tries to reach moves back again to zero. The simulink model constructed can be seen at *Figure 6* and the the system response for this system with input change at $t = 20$ *sec* can be seen at *Figure 7*.

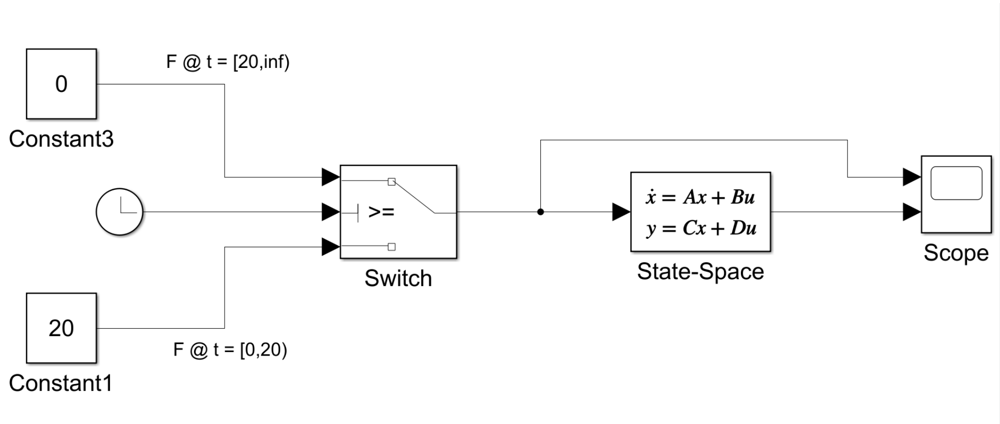



Figure 6: Simulink Model for the MSD with Varying Input Force

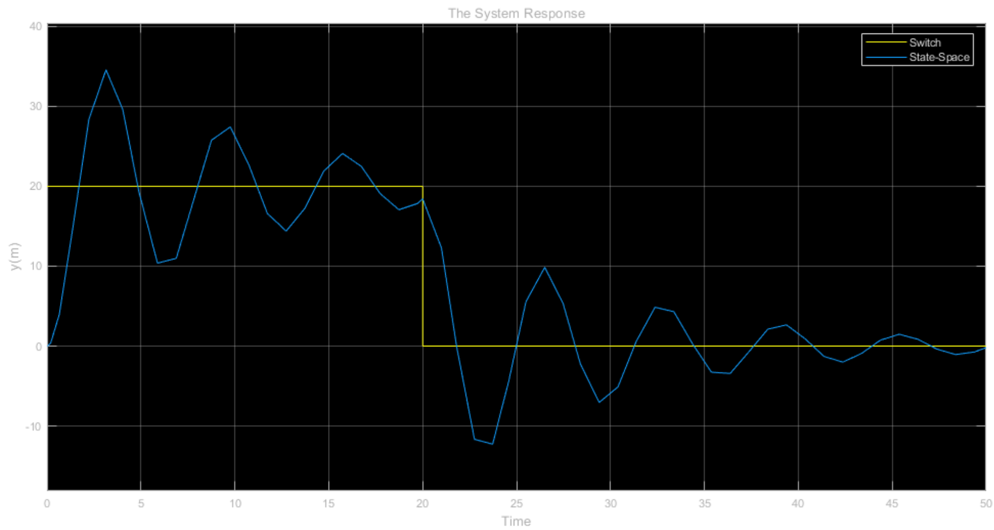



Figure 7: The System Response for MSD as the Input Changes at t=20 s

(e) Time differences between successive data points seems a varying behaviour in simulation results in step d. This behaviour results with a simulation result that is not as smooth as expected.

(f) As the fixed step is used, the system response can be seen at *Figure 8*. The difference in simulation results with using fixed step option and with using varying step opt,on can be clearly seen at *Figure 9*. The simulation result looks smoother when the fixed step option is used.

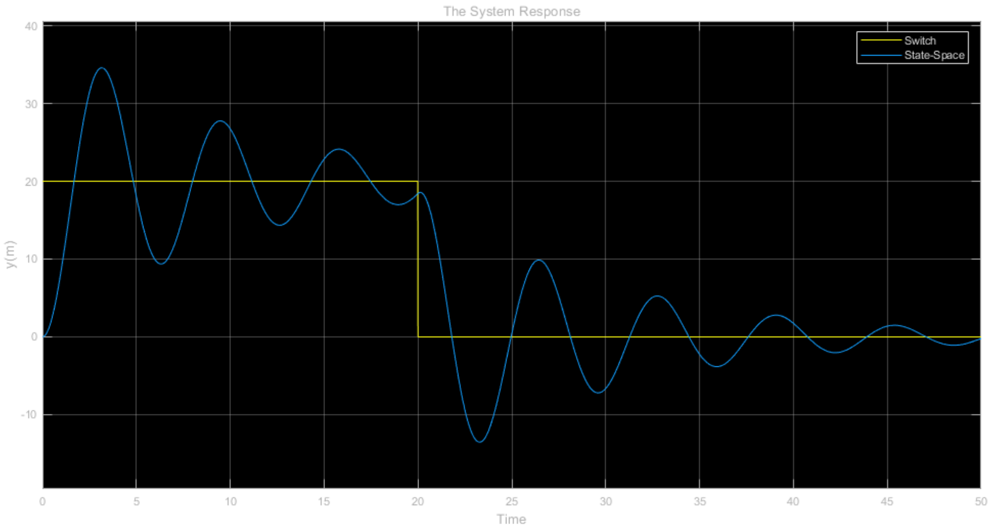
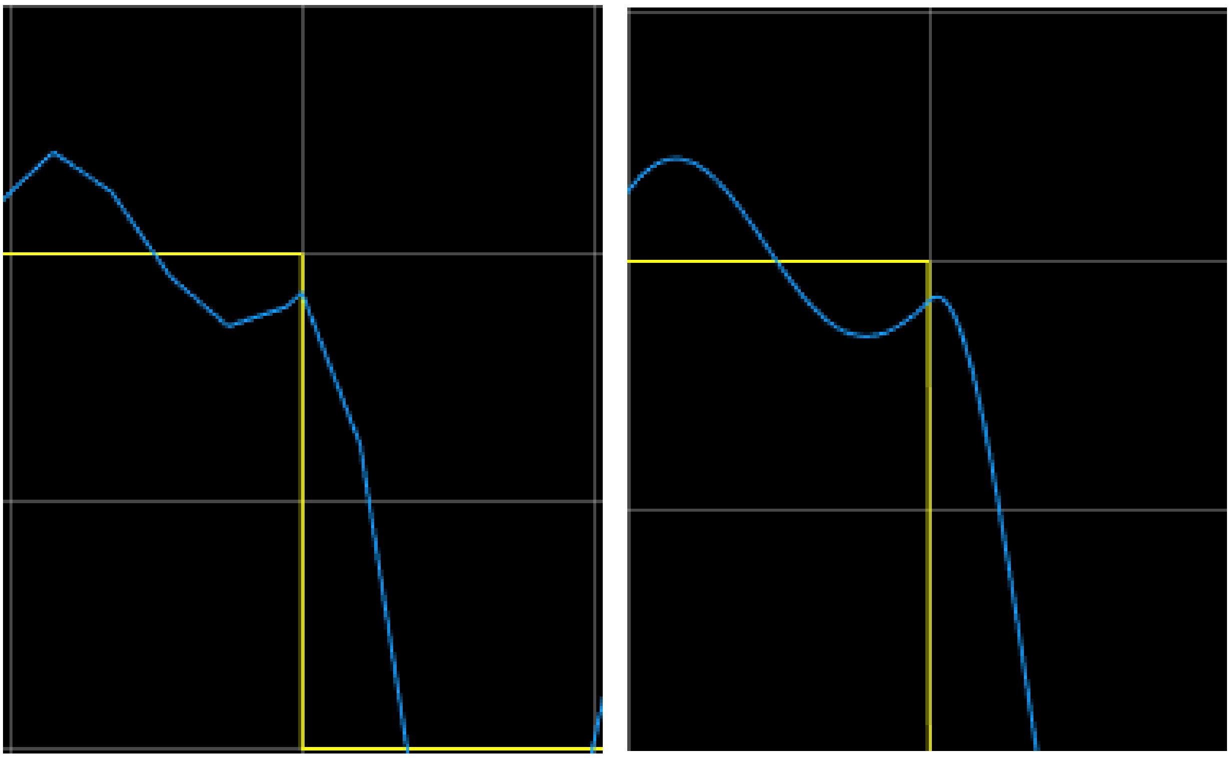



Figure 8: The System Response for MSD with Desired Parameters in Q1f



(a) Simulation with Variable Step (b) Simulation with Fixed Step

Figure 9: Simulation with Variable and Fixed Step

2. We will analyse the system shown in the *Figure 10*.

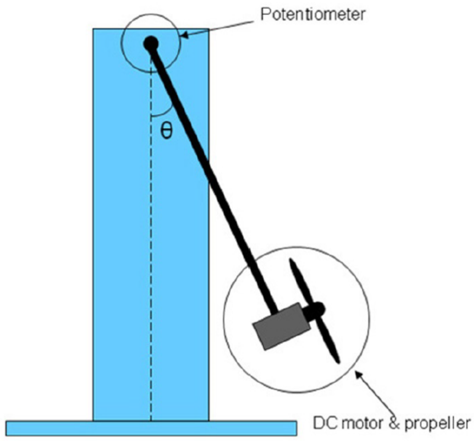



Figure 10: Propeller Levitated Arm System

The system can be modelled mathematically as

$$mL^2\ddot{\theta} = -c\dot{\theta} - mgLsin(\theta) + u$$

where $\theta$ is angular position of the arm, $m$ is the total mass of the propeller and DC motor, $L$ is length of the arm, $c$ is viscous damping coefficient, $g$ is gravitational acceleration and $u$ is thrust produced by the propeller.

(a) Simulink Model for the Propeller Levitated Arm can be seen at *Figure 11* and its more compact version can be seen at *Figure 12*.

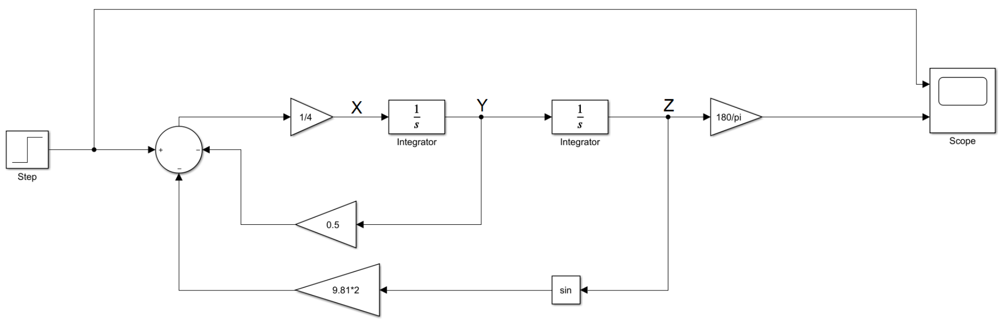



Figure 11: Simulink Model for the Propeller Levitated Arm

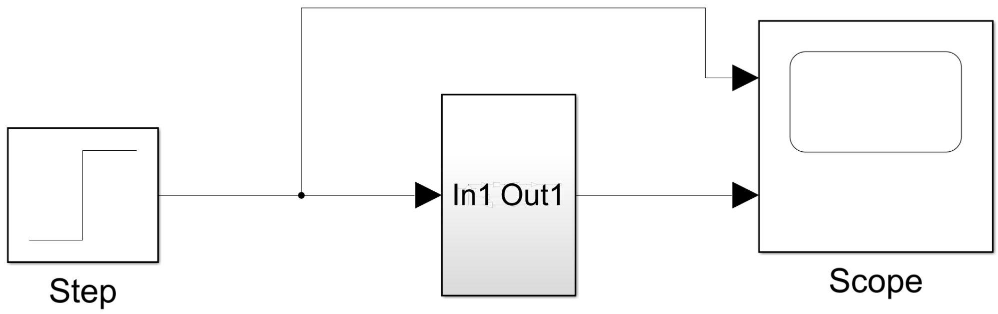


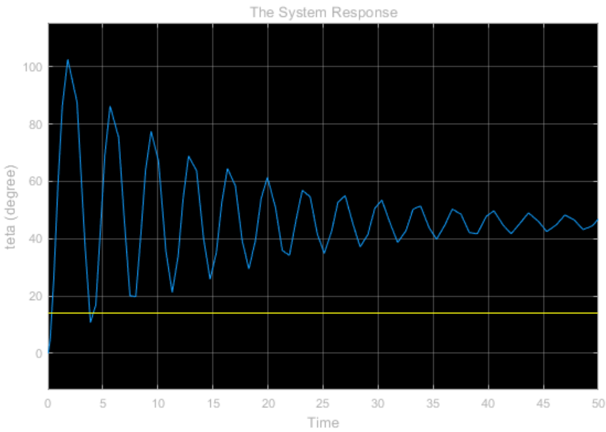

Figure 12: Simulink Model for the Propeller Levitated Arm using Subsystems

(b) X, Y and Z represent the followings:
   - X represents $\ddot{\theta}$
   - Y represents $\dot{\theta}$ and
   - Z represents $\theta$

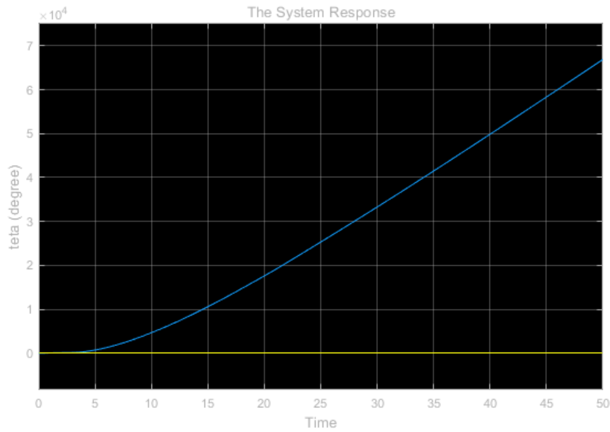
(c) This model is valid for all values of the state vector. Number of the state variable and independent row number are equal so the system is consistent with the real dynamics of the system.

(d) Simulation results for the system as $F = 14Nm$ and as $15Nm$ can be seen at *Figures 13a and 13b* respectively.

   The slight difference in the thrust results in extremely large change in the system because the system is unstable. When the input force reaches some point, it will not stabilize.




(a) The System Response for PLA for m = 1, L =2, g = 9.81, c = 0:5 and input force=14




(b) The System Response for PLA for m = 1, L =2, g = 9.81, c = 0:5 and input force=15

Figure 13: The System Response for PLA with varying input