EE407 Process Control
Experiment 5

1 Mathematical Modelling

1. A first order transfer function can approximate the temperature of the chamber at the n^{th} sensor for the purpose of designing a temperature controller.

$$T_n(s) = \frac{K_n}{\tau_n s + 1} V_h(s)$$

where K_n is the steady-state gain and τ_n is the time constant of the n^{th} sensor.

2 Bump Test Identification

2. -

3 Time Response Characteristics of 2^{nd} Order Systems

3. The output of the given system at *Figure 1* can be written in Laplace domain as

$$Y(s) = \frac{C(s)P(s)}{1 + C(s)P(s)} R(s)$$

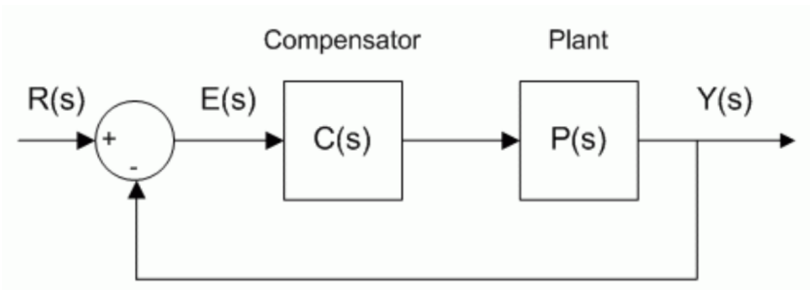

Figure 1: Block diagram of a negative unit feedback close loop system

4. Error transfer function for *Figure 1* can found as follows;

$$E(s) = R(s)_Y(s) = R(s)\left[1 - \frac{C(s)P(s)}{1 + C(s)P(s)}\right]$$

$$\boxed{E(s) = \frac{1}{1 + C(s)P(s)} R(s)}$$

5. With $C(s) = K_p$, $P(s) = \dfrac{K}{\tau s + 1}$, by using the final-value theorem steady state error can be found as follows;

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{sR(s)}{1 + K_p \dfrac{K}{\tau s + 1}}$$

With $R(s) = \dfrac{R_0}{s}$

$$e_{ss} = \frac{R_0}{1 + K_p \ \lim_{s \to 0} \left(\dfrac{K}{\tau s + 1} \right)}$$

$$\boxed{e_{ss} = \frac{R_0}{1 + KK_p}}$$

6. Investigating the steady state error and open loop transfer function, it can be concluded that the system is a Type 0 system. The table at *Table 1*[1] can be examined for more understanding.

Table 1: Steady-State Error Table

Input	Type 0: $e(\infty)$	Type 1: $e(\infty)$	Type 2: $e(\infty)$
Step, $u(t)$	$\frac{1}{1+K_p}$	0	0
Ramp, $tu(t)$	∞	$\frac{1}{K_v}$	0
Parabola, $t^2u(t)$	∞	∞	$\frac{1}{K_a}$

7. The closed-loop transfer function can be casted into the standard form given by

$$G(s) = \frac{Y(s)}{R(s)} = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}$$

where w_n is the natural frequency and ξ is the damping ratio.

[1] Memorial University of Newfoundland / Control Systems I / Unit 6: Steady-State Error

8.

$$\boxed{\%OS = \frac{y_{peak} - yss}{y_{ss}} = 100\ e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)}}$$

$$\boxed{t_{peak} = \frac{\pi}{w_n\sqrt{1-\xi^2}} = \frac{\pi}{w_d}}$$

4 Performance Requirements for the Heat Flow Experiment

9. In this experiment, time-domain performance requirements are speci ed as follows.

$$e_{ss} = 0\ ,\ t_p = 15\ secs\ ,\ PO(\%) = 15.0$$

5 Temperature Control

5.1 On-Off Control

10. -

11. -

6 Proportional-Integral Control

12. -

13.

$$G(s) = \frac{T(s)}{T_d(s)}$$

$$\left[K_p(T_d(s)b_{sp} - T(s)) + K_I\left(\frac{T_d(s)_T(s)}{s}\right)\right]\frac{K}{\tau s+1} = T(s)$$

$$T_d(s)\left[K_pb_{sp} + \frac{K_I}{s}\right]\frac{K}{\tau s+1} = T(s)\left[1 + \frac{K}{\tau s+1}\left(K_p + \frac{K_I}{s}\right)\right]$$

$$G(s) = \frac{\left[K_pb_{sp} + \frac{K_I}{s}\right]\frac{K}{\tau s+1}}{1 + \frac{K}{\tau s+1}\left(K_p + \frac{K_I}{s}\right)} = \frac{K\left[K_pb_{sp} + \frac{K_I}{s}\right]}{(\tau s+1) + K\left(K_p + \frac{K_I}{s}\right)}$$

$$G(s) = \frac{KK_pb_{sp}s + KK_I}{\tau s^2 + (1+KK_p)s + KK_I}$$

$$\boxed{G(s) = \frac{\frac{KK_p b_{sp}}{\tau}s + \frac{KK_I}{\tau}}{s^2 + \frac{1+KK_p}{\tau}s + \frac{KK_I}{\tau}}}$$

14.

$$\boxed{G(s) \approx \frac{\frac{KK_I}{\tau}}{s^2 + \frac{1+KK_p}{\tau}s + \frac{KK_I}{\tau}}}$$

$$G(s) = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}$$

with

$$\boxed{w_n = \sqrt{\frac{KK_I}{\tau}}} \quad , \quad \boxed{\xi = \frac{1+KK_p}{2\tau w_n}}$$

Thus,

$$\boxed{K_I = \frac{w_n^2 \tau}{K}} \quad , \quad \boxed{K_p = \frac{2\xi w_n \tau - 1}{K}}$$

15.

$$\boxed{\xi = \frac{-\ ln(\frac{\%OS}{100})}{\sqrt{\pi^2 + ln^2(\frac{\%OS}{100})}}}$$

$$\boxed{w_n = \frac{\pi}{t_p\sqrt{1-\xi^2}}}$$

16. To meet the requirements at step 9

$$e_{ss} = 0\ ,\ t_p = 15\ secs\ ,\ PO(\%) = 15.0$$

$$\boxed{\xi = \frac{-\ ln(0.15)}{\sqrt{\pi^2 + ln^2(0.15)}} = 0.589}$$

$$\boxed{w_n = \frac{\pi}{t_p\sqrt{1-(0.589)^2}} = 0.355}$$

17. With $K = 0.8$ and $\tau = 50$

$$\boxed{K_I = \frac{w_n^2\tau}{K} = \frac{(0.355)^2 50}{0.8} = 7.8765}$$

$$\boxed{K_p = \frac{2\xi w_n\tau - 1}{K} = \frac{2(0.589)(0.355)50 - 1}{0.8} = 24.8868}$$

18.

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G_{ol}(s)} = \frac{1}{1 + G_{PI}G(s)}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1 + \left(G_p + \dfrac{G_I}{s}\right)\left(\dfrac{K}{\tau s + 1}\right)}$$

with given K and τ

$$\frac{E(s)}{R(s)} = \frac{50s^2 + s}{50s^2 + [1 + 0.8K_p]s + 0.8K_I}$$

19.

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} sR(s)\frac{50s^2 + s}{50s^2 + [1 + 0.8K_p]s + 0.8K_I}$$

Using L'Hospital Rule, the limit can be evaluated as follows

$$e_{ss} = \lim_{s\to 0} \frac{50s^2 + s}{50s^2 + [1 + 0.8K_p]s + 0.8K_I} = \lim_{s\to 0} \frac{100s + 1}{100s + 1 + 0.8K_p}$$

$$\boxed{e_{ss} = \lim_{s\to 0} \frac{100}{100} = 1}$$

6.1 Anti-Windup

20. -

21. -

22. -