EE 407
Homework 1 - Solutions

Introduction to Simulink

1 Answers to the Questions

1. **Mass Spring Damper System**

   The Mass Spring Damper (MSD) system is illustrated in Fig. 1, [1].

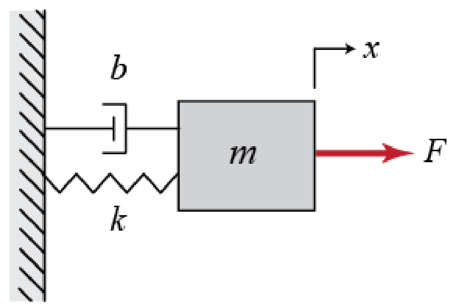

Figure 1

   (a) The state-space model of the system is as given in (1)-(2).

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F(t) \tag{1}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \tag{2}$$

   Note that the state vector is determined to comprise of position and velocity, i.e., $\mathbf{x} = \begin{bmatrix} x & \dot{x} \end{bmatrix}^T$, $F(t)$ denotes the force exerted on the mass, and the output is position.

   (b) See the attached Simulink model named as $EE407\_HW1\_Q1$.

   (c) Run the model to observe requested output signals. Once you run the simulation, it will load the output signals to the *Workspace* in case you want to further investigate or process.

      i. When spring constant is decreased, the settling point of the position increases as expected. Besides, frequency of the oscillations in position decreases.

      ii. When mass is increased, it takes much more time for the oscillations to fade out. On the other hand, the settling point (which depends on spring constant) of position does not change.

iii. By increasing damping coefficient, the system is rendered to exhibit overdamped characteristics.

(d) Simulate the given model to acquire the output signal.

(e) **Prior Information:**

You simulate a dynamic system by computing its states at successive time steps over a specified time span. This computation uses information provided by a model of the system. The time steps are time intervals when the computation happens. The size of this time interval is called the step size. The process of computing the states of a model in this manner is known as solving the model... A solver applies a numerical method to solve the set of ordinary differential equations that represent the model. Through this computation, it determines the time of the next simulation step....

Type of the step size used in the computation is one of the major criteria used to classify solvers.

Variable-step solvers vary the step size during the simulation. They reduce the step size to increase accuracy when the states of a model change rapidly and during zero-crossing events. They increase the step size to avoid taking unnecessary steps when the states of a model change slowly. Computing the step size adds to the computational overhead at each step. However, it can reduce the total number of steps, and hence the simulation time required to maintain a specified level of accuracy for models with piecewise continuous or rapidly changing states.

Observation: Time differences between successive data points varies throughout the simulation if you created a new Simulink model from scratch and did not modify its *Solver Settings*. This is because every new model comes with *Variable-Step Solver* as default.

(f) Fixed-step solvers solve the model at fixed step sizes from the beginning to the end of the simulation. When you specify the Solver to be *Fixed Step* with sampling time of 0.01 sec, your model guarantees to produce a solution for the differential equations at each 0.01 sec. By doing so, a smoother output signal is obtained.

You can specify the step size or let the solver choose the step size. Generally, decreasing the step size increases the accuracy of the results and increases the time required to simulate the system.

Note that the sections typed as italic in (e) and (f) are copied from
`https://www.mathworks.com/help/simulink/ug/choosing-a-solver.html`
Please also see below link for further details.
`https://www.mathworks.com/help/simulink/ug/types-of-solvers.html`

2. **Propeller Levitated Arm Simulation**

The system is illustrated in Fig. 2.

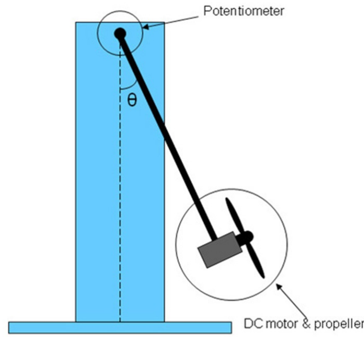

Figure 2

(a) See the attached Simulink model named as $EE407\_HW1\_Q2$.

(b) In Fig. 3, X, Y and Z stand for angular position (θ), angular velocity ($\dot{\theta}$) and angular acceleration ($\ddot{\theta}$) respectively.

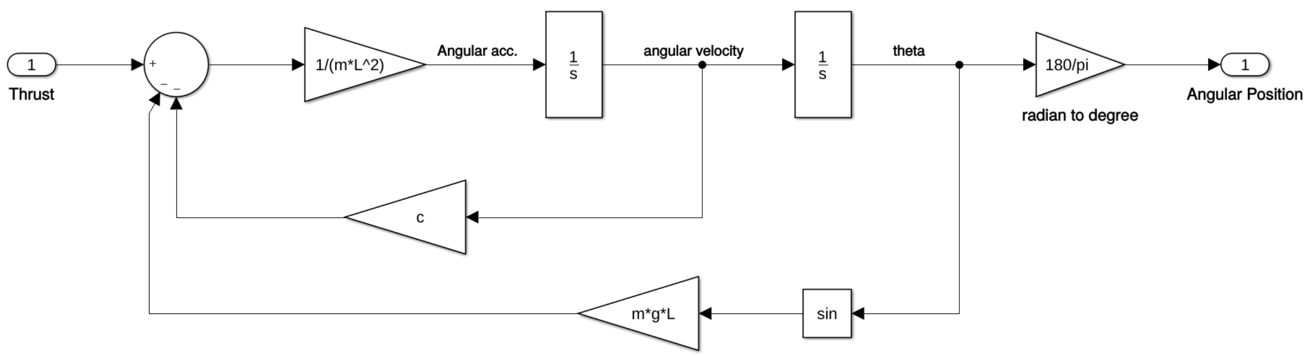

Figure 3

(c) The model is based on the *genuine* nonlinear differential equation of the system; it does not refer to an approximation of the characteristics. Therefore, it is valid for all values of the state vector.

(d) Simulate the model to observe resulting output signals for input thrusts of 14 Nm and 15 Nm when $m = 1\ kg,\ L = 2\ m,\ g = 9.81\ m/s^2,\ c = 0.5\ kgm^2/s$. For 14 Nm, angle of the arm settles down to 45 deg after some oscillations; however, angular position is detected to increase in an unbounded fashion when the input thrust is set to 15 Nm. This observation leads to the following interpretation: in the latter case, the input thrust is large enough to make the arm exceed 180 deg. Therefore, the arm happens to rotate regularly.

References

[1] "Lecture notes in ee302," 2017.

[2] "Simulink - How to Create a Subsystem," https://www.mathworks.com/help/simulink/ug/creating-subsystems.html#f4-7371, accessed: 2017-10-09.