EE 407

Homework 3

Due 23:55, 30.11.2018

1 Introduction

The objective of this homework is to get you acquainted with modeling of interacting and non-interacting processes as well as designing PID controllers using Ziegler-Nichols (continuous cycling) method and Internal Model Control (IMC, a model-based design method).

The fourth question contains short questions regarding the videos you were expected to watch. The links to these videos can also be found below.

- P-Only Controller Introduction
  `http://apmonitor.com/che436/index.php/Main/LectureNotes5`
- Introduction to Proportional Integral (PI) Control
  `http://apmonitor.com/che436/index.php/Main/LectureNotes7`
- PID Control: Derivative Action
  `http://apmonitor.com/che436/index.php/Main/LectureNotes8`

Before getting started, make sure that you go through the steps below.

- One of the partners should accept the invitation of this assignment via below link and *create a new team*. It will give you access to a repository, namely *ee-407-hw3-2018-YourGroupName*, in the @METU-EE407 organization on GitHub. Then, the other partner clicks the same below link and joins the corresponding team. After these steps, both partners will be able to work on the same repository.
  `https://classroom.github.com/g/muYudPi6`
- Don't forget to put your names and student IDs in the README file in your repository.

Please don't forget that we expect each member of a team to make at least one commit to the repository.

2 Questions

1. **Heat Exchanger System** In this question, a heat exchanger system will be investigated. A chemical reactor called "stirring tank" is depicted in Fig. 1. The top inlet delivers liquid to be mixed in the tank. The tank liquid must be maintained at a constant temperature by varying the amount of steam supplied to the heat exchanger (bottom pipe) via its control valve. Variations in the temperature of the inlet flow are the main source of disturbances in this process.

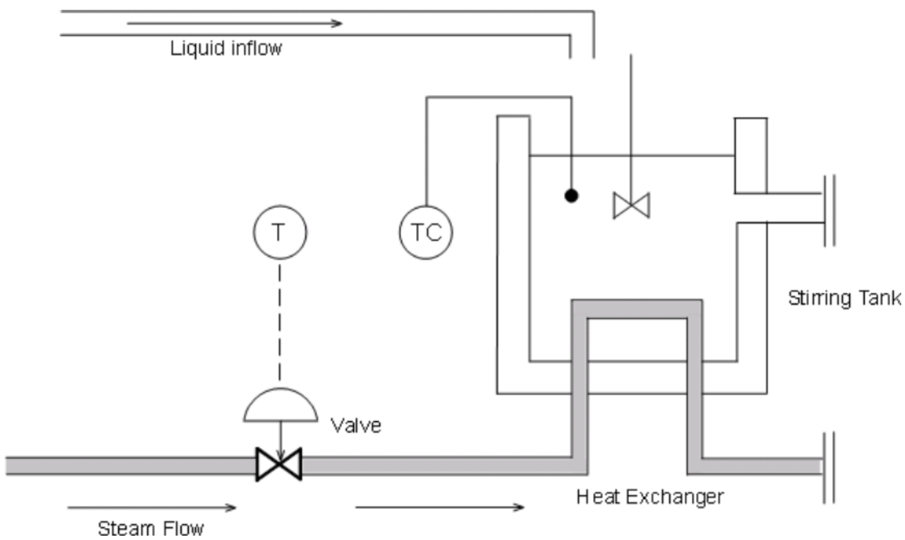

Figure 1: Schematic of the heat exchanger system *(Courtesy of Mathworks)*

   (a) Open the figure called *HeatExchanger_StepResponse.fig* that you will find in your repository. This figure reveals the normalized tank temperature, T, when a step disturbance is injected in valve voltage, V.

   (b) In this step you are required to derive a first-order-plus-deadtime (FOPDT) model of the heat exchanger characteristics from the given data embedded in the figure. Find process gain, K_p, process time constant, τ_p and process dead time, θ_p.

   (c) Create a new Simulink model and name it as *Question1*.

   (d) Design the model realizing FOPDT response with the parameters you found. (**Hint**: See *Transport Delay* block available in Simulink Library.)

   (e) Simulate the model for 200 seconds when the input signal is unit step, and plot the response of the FOPDT model along with the real data provided by the given figure. Insert your figure into your report with a relevant title, labeled axes, and reasonable axis limits.

   (f) Commit the model and push the local repo to the remote repository on GitHub.

(g) Now, calculate a proportional controller, K_c, for servo control problem with the formula based on Integral of Time Weighted Absolute Error (ITAE).

(h) Modify the Simulink model to simulate closed loop response of the FOPDT model under the control of the proportional controller. Commit current version of the model and push the local repo to the remote repository on GitHub.

(i) For a desired temperature signal of $0.5u(t-5)$ (when initial temperature is 0), run simulations to fine tune the controller. Insert a figure into your report exhibiting the response of the close loop system for the fine-tuned K_c. What is the final value of K_c? Did you end up in a point that is far away from the initial guess of K_c? Comment on the tuning procedure?

2. **Non-interacting Tanks** Consider the non-interacting tanks system depicted in Fig. 2. Let $A_k(m^2)$ denote the cross-sectional area of tank k, $q_k(m^3/min)$ and $q_{k-1}(m^3/min)$ its infow and outflow, respectively, and $R_k(min/m^2)$ its outlow valve resistance. Assuming that the outflow is linearly dependent to tank water height, $q_k = h_k/R_k$. Assume also that the actuating valve and the level transmitter are ideal; that is, level transmitter can give instantaneous measurements and actuator can supply the desired water flow without any limits or delay.

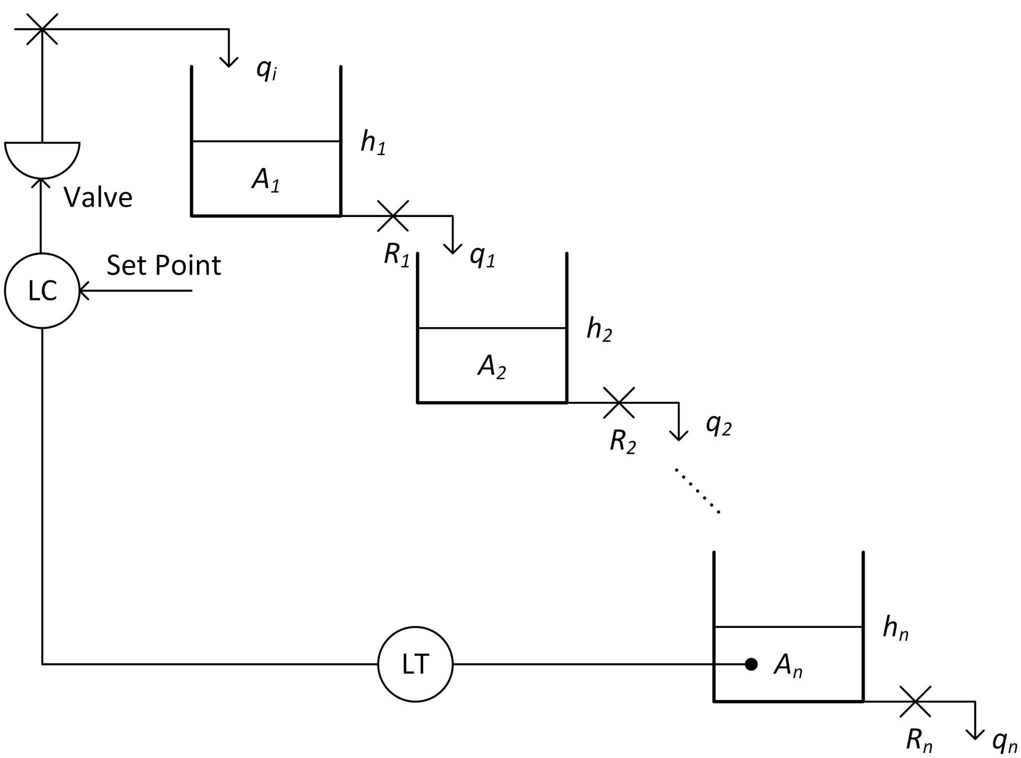

Figure 2: Schematic of the non-interacting tanks

(a) Write down the differential equations satisfied by water heights of all tanks and obtain the state space representation of this system in the following form $\dot{\mathbf{h}} = \mathbf{A}\mathbf{h} + \mathbf{B}q_i$, where $\mathbf{h} = [h_1, h_2, \ldots, h_n]^T$.

(b) Find the transfer function, $G(s) = H_n(s)/Q_i(s)$, of the system. If you observe the stucture of state space representation (or the governing differential equations), each state is dependent only on itself and its previous state. Thus, starting from $G_1(s) = H_1(s)/Q_i(s)$ and proceeding sequentially may ease up the calculations.

(c) If you were to modify the previous transfer function as $G'(s) = Q_n(s)/Q_i(s)$, what would be its DC gain? Does its DC gain make sense on physical grounds and why (not)?

(d) In Ziegler-Nichols method, the idea is to control the system under proportional-only control and increase its gain slowly until sustaining oscillations are observed at the output. The system has to be open loop stable (as it is the case in our self regulating process) and one has to be able to push the system to critical stability. Now, suppose that $n = 2$ and note that all of the constants are positive. Show that continuous cycling method cannot be applied to this system (with transfer function $G(s)$) using:

- an approximate root locus plot and
- an approximate bode plot. (**Hint:** At critical stability, a system has zero phase margin.)

(e) Now, let $n = 3$. This time you have a system which can be made unstable/critically stable. Also let $\tau_i = R_i A_i = 1(min)$ and $R_i = 1(min/m^2)$ for $i = 1, 2, 3$. Find the value of gain K_c (critical/ultimate gain), using both of the methods of step 2d, to make the system critically stable. Also find the angular frequencies of oscillation, $\omega_{cu}(rad/sec)$. Simulate the system to verify and report your result. Note that slight numerical inaccuracy may make the system unstable; you may have to decrease the gain slightly to have a bounded response. In practical systems, this is of no concern as the actuator output is limited unlike your simulation, and you do not even need a model of the system to find these parameters.

(f) From your simulation result, determine the so-called critical/ultimate period P_u (period of oscillations under critical gain). Using K_{cu} and P_u, find the controller parameters for a suitable PID controller.

(g) Simulate the system and find the decay ratio (decay ratio is explained in the video Introduction to Proportional Integral Control).

(h) Commit your simulation model. In this model, you are required to show both your simulation results (of steps 2e and 2g)

3. **Heating of Interacting Masses**

Consider the heating process of the masses shown in Fig. 3 (which is the setup of experiment #4). Let m_1 and c_1 denote the mass and heat capacity of the wirewound resistor (outer mass), and m_2 and c_2 be those of the thermocouple (inner mass). h_i and A_i are the heat transfer coefficient and the area of inner contact, and h_o and A_o are those of the outer one. Assume that each body is at a uniform temperature and heat is transferred only by convection. q_{in} is the power supplied to the resistor (by the means of Joule heating, ohmic heating or resistive heating) and $q_{out} = h_o A_o (T_1 - T_e)$ is the power lost to the environment. A similar equation of heat transfer governs the heat transfer between the masses, which is left for you to find at the first step.

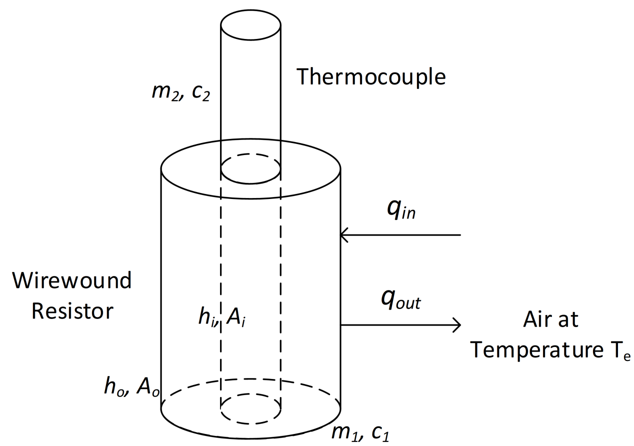

Figure 3: Illustration of the interacting masses being heated

(a) Find the differential equations governing the temperatures of each mass.

(b) Find the electrical equivalent circuit representing this system. Find the values of circuit elements $R_1, \ldots, C_1, \ldots, V_1, \ldots$ in terms of the given quantities.

(c) Knowing that you can superpose the effect of independent sources in an electrical circuit, kill the active circuit element corresponding to the environmental temperature (set its value to 0). Find the transfer function $G(s) = T_1(s)/Q(s)$ in terms of the circuit variables you introduced.

(d) The transfer function you found in the previous step should be of the following form. Suppose that the system parameters are as follows. Approximate the system as a FOPDT system.

$$G(s) = \frac{K_p(\tau_1 s + 1)}{(\tau^2 s^2 + 2\xi\tau s + 1)} = \frac{0.5(10s + 1)}{2000s^2 + 410s + 1} (^\circ\mathrm{C/W})$$

(e) Find the Internal Model Control PID parameters using this approximation. As the desired closed loop time constants, use the following two $\tau_c = T_p/2$ and $\tau_c = T_p/8$. Simulate the actual plant with these controllers and comment on your results.

(f) Commit your simulation model of step 3e. Both controlled outputs are expected to be observed in this model.

4. **Miscellaneous Questions**

(a) Consider the step response of a bump test shown in Fig. 4. One way that you can obtain the FOPDT parameters would be to draw a tangent line at the inflection point (that is, the point where the step response has the highest scope), and intersect it with the y_0 line to find θ_p. Then, find the time of $0.632\Delta y = (1 - e^{-1})\Delta y$, and, from the results of the two, find T_p. Interested readers are encouraged to have a look at **this video** giving the explanations of these steps and **here** to see how to achieve this programmatically. However, this would either require you to draw the tangent line rudimentarily or be prone to errors due to spikes caused by noise. Instead, finding a mathematical expression for the time constant and the dead time may be more beneficial. To achieve this, let $\Delta u = u_1 - u_0$ and $\Delta y = y_1 - y_0$. Also let $y_{1/3} = y_0 + \Delta y/3$ and $y_{2/3} = y_0 + 2\Delta y/3$. You are also given that the response of $y(t)$ is of the following form, where t_0 is the input step time and $u'(.)$ is the unit step function. Rearrange this equation to find an expression for t where $t > t_0 + \theta_p$, and then find $t_{1/3}$ & $t_{2/3}$ (corresponding to $y_{1/3}$ & $y_{2/3}$, respectively). Then, rearrange these results to find T_p and θ_p. Using these results and shifting the plots as $t_0 = 0$, show that the formulae given for them in your lectures are valid.

$$y(t) = (1 - e^{-\frac{t - t_0 - \theta_p}{T_p}})u'(t - t_0 - \theta_p)\Delta y + y_0$$

(b) Answer the following questions after you have watched the videos on ODTUClass regarding P, PI and PID controllers.

i. Why is it a good idea to have a dead band in an on-off controller? Find an additional example of a practical on-off controller apart from those stated in the video.

ii. Which one of the following systems may observe non-zero e_{ss} with a properly chosen P-only control for different set points, and why? A self-regulating process or an integrating one.

iii. Consider the problem of driving a car. What are the optimal (in the sense of ITAE) P-control parameters for the cases of (a) you intend to change the speed of the car regularly and (b) slope of the road changes when you intend to drive the car at a constant speed?

iv. ITAE, IAE and ISE controller parameters are found for a certain type of a system by the minimization of $\int_0^\infty t|e(t)|dt$, $\int_0^\infty |e(t)|dt$ and $\int_0^\infty e^2(t)dt$, respectively. State which parameter tuning method (a) penalizes all errors equally, (b) penalizes large errors more and (c) penalizes initial errors less than those at a later time.

v. Consider the water level control of a tank where your actuator is a pump either supplying water to the tank or draining water from it. Show the configurations of

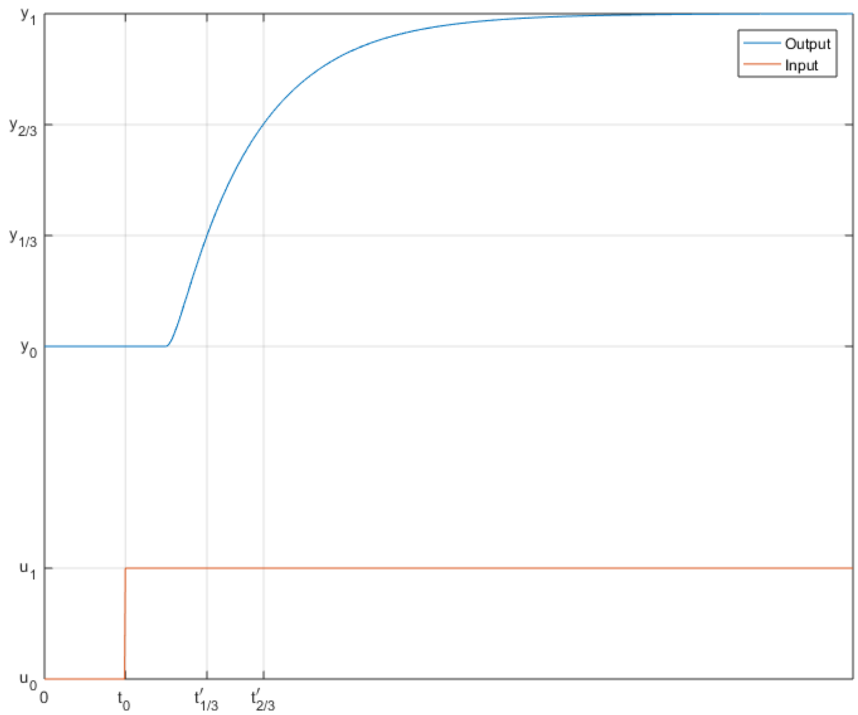

Figure 4: A step response obtain as a result of bump test

the pump in order to have a reverse acting and direct acting controllers. Describe what would happen if the pump is in reverse acting configuration and the controller is in direct acting mode.

vi. Suppose that $SP(t) > PV(t_{ss})$ where $PV(t_{ss})$ is the process variable when the controller output is $CO = CO_{bias}$. Draw approximately the integral of error and discuss how this achieves moving the bias term.

vii. What is reset or integral windup? Suggest a method to eliminate this.

viii. How do decay ratio and settling time generally correlate? How about rise time and peak time?

ix. What is the derivative kick that occurs when the set point changes stepwise? Why should you and how can you eliminate it?

x. When is the ideal derivative action detrimental to controller performance? What should you do if you want to use derivative action in such cases?