Q1) Part 1

Taylor series expansion (s)

1

$$G(s) = \frac{k(-0,1s+1)}{(5s+1)(2s+1)(0,5s+1)}$$ FOPDT

Largest $\tau = 5$ is dominant

$e^{-\theta s} \cong 1-\theta s + H,\cancel{O,T}$ $\quad e^{\theta s} \cong 1+\theta s \quad e^{-\theta s} \cong \frac{1}{e^{\theta s}}$

$1-0,1s = e^{-0,1s}$

$\frac{1}{1+2s} \cong \frac{1}{e^{2s}} = e^{-2s}$

$\frac{1}{1+0,5s} \cong \frac{1}{e^{0,5s}} = e^{-0,5s}$

Total Delay $e^{-2,6s}$

$$G(s) = \frac{k e^{-2,6s}}{(5s+1)}$$

Bonus is how correct is approximation,

Part 2

Reasonably express FOPDT approximation is possible or not

a) $\frac{K}{(10s+1)(10s+1)}$ NO, no dominant pole

b) $\frac{K}{(16s+1)(8s+1)(s+1)}$ No dominant single pole

c) $\frac{K}{(10s+1)(s+1)^2}$ Yes, $\tau = 10$ $\quad \theta = 2$

d) $\frac{K}{(10s^2+11s+1)} \rightarrow$ $s_1 = -0,1 \rightarrow \tau_1 = 10$

$s_2 = -1 \qquad \tau_2 = 1$

e) $\frac{K}{(100s^2+10s+1)}$ $\Delta = 300$ no ming

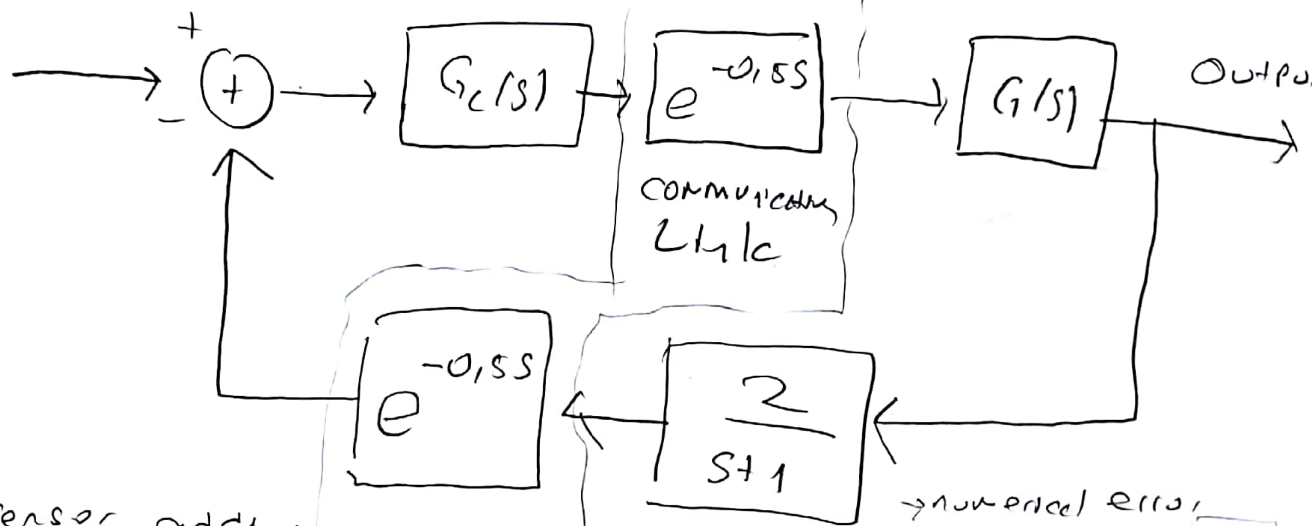
Complex conjugate poles $\rightarrow$ oscillatory response

No FOPDT Model possible

Q2)

$$G(s)=\frac{-1}{(10s+1)(5s+1)}$$

a) G(s) given process is to be controlled through communication link first order dynamics



→ numerical error, take sensor $\frac{1}{s+1}$

b) sensor adds unnecessary sensor gain
Assumptions neglect sensor time constant
Using direct design method,

$$T(s)=e^{-s}\cdot e^{-s}=\frac{e^{-2s}}{4s+1}$$ overall desired transfer function

Combine all delay into single delay e^{-2s}
wire out to wire in plant model = open loop transfer function what controller sees,

$$G_c(s)=\frac{1}{G(s)}\cdot\frac{T(s)}{1-T(s)}$$

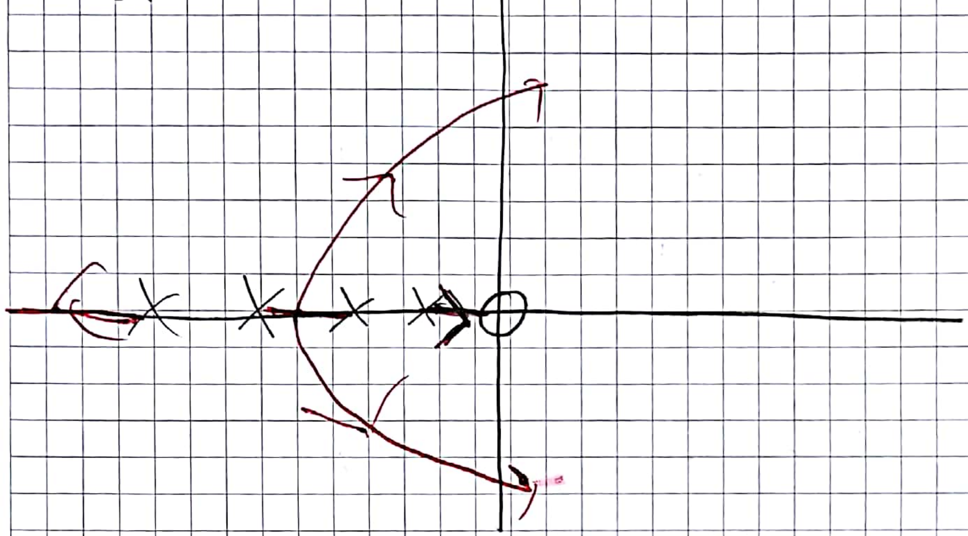
$$G_c(s)=\frac{(10s+1)(5s+1)}{e^{-s}}\cdot\frac{\frac{e^{-s}}{4s+1}}{\frac{4s+1-e^{-s}}{4s+1}}$$

$$G_c(s)=\frac{(10s+1)(5s+1)}{4s+1-(1-s)}=\frac{50s^2+15s+1}{5s}=10s+3+\frac{1}{5s}$$

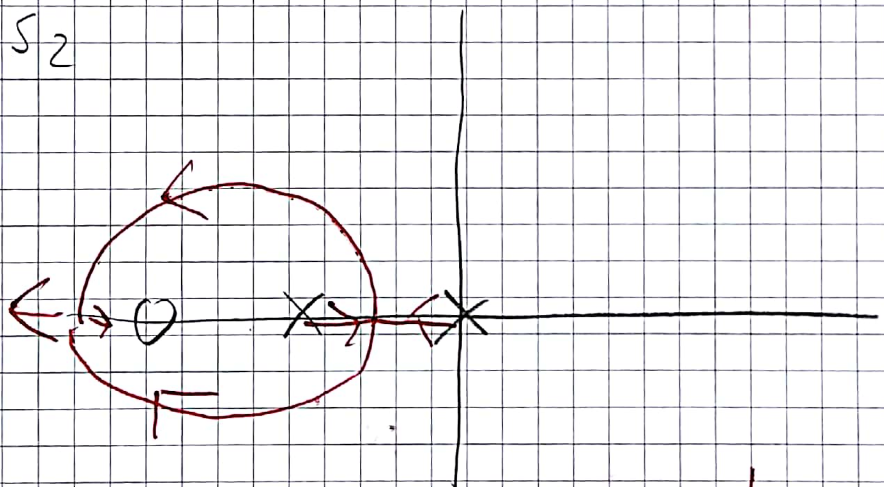
$$= 3\left[1 + \frac{1}{15}\cdot\frac{1}{s} + \frac{10}{3}s\right]$$

$K_c = 3 \qquad \tau_I = 15 \qquad \tau_D = \frac{10}{3}$

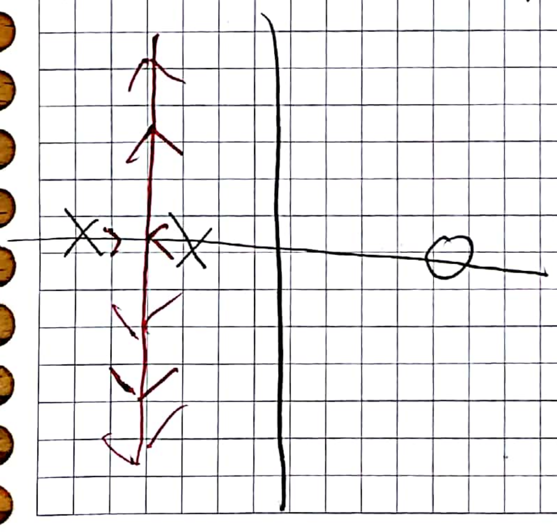
S_1



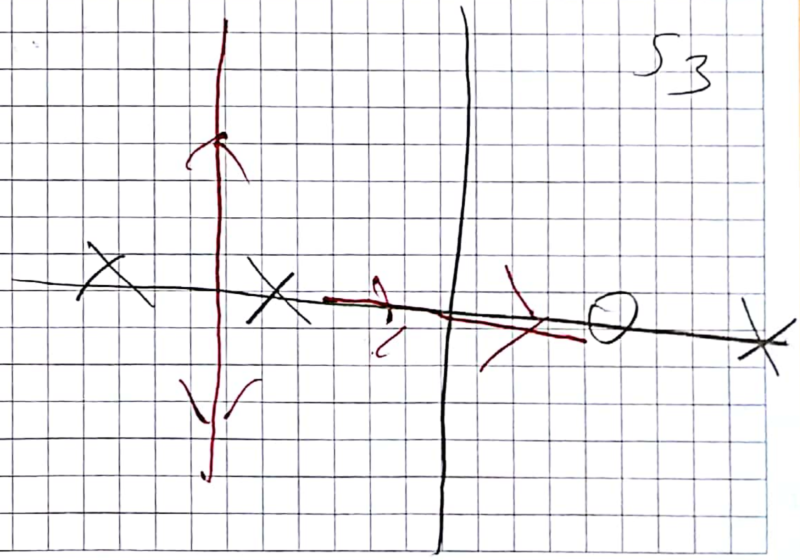
S_2



S_4



S_3



4

5) It should not have any unstable zeros

pole on right half plane unstable case

Zero in RHP then non minimum phase system

zero on the origin does not effect the design or make non minimum phase system

~~4pts 4pts~~ C) Ziegler Nichols

S1 a) It is feasible to apply

S3 system it is not stable

S2 not cross jw axis

S4 NO oscillations no Ziegler nichols

d) Bumptest is used to identify FOPDT models

a integrating model can not be analyzed

S3 step response test

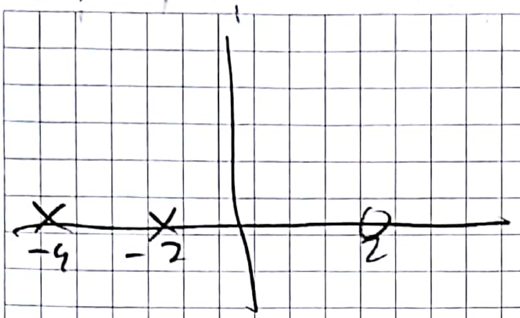
unstable step response can not be applied

a pole that is closer

S4 we can apply FOPDT model

Part 2 : V will design, we cannot apply directly Ziegler nichols methods, (

Open loop plant pole -2 and -4 and zero at



a) Obtain the TF of at all open-loop plant

steady state process gain $+1$

a) Obtain the TF of open loop plant

$$G_c(s) = K_p\left(1 + \frac{K_d}{K_p} s\right) \quad \text{where} \quad \frac{K_d}{K_p} = -\frac{1}{4}$$

$$\lim_{t\to\infty} \lim_{t\to\infty} g(t) = \lim_{s\to 0} Y(s) = \lim_{s\to 0} G(s)$$

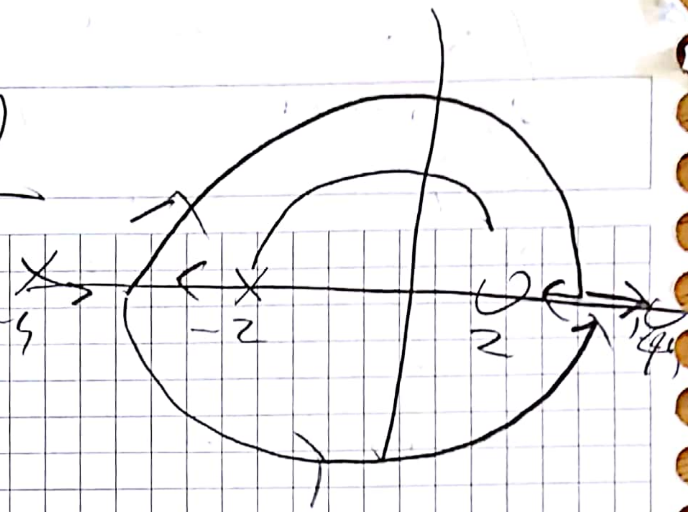
$$\lim_{s\to 0} \frac{K(s-2)}{(s+2)(s+4)} = 1 \quad K = \frac{8}{-2} = -4$$

$$G_p(s) = \frac{(-4)(s-2)}{(s+2)(s+4)}$$

$$G_{ol}(s) = G_c(s)G(s) = K_p\left(1 - \frac{1}{4}s\right)\frac{(-4)(s-2)}{(s+2)(s+4)}$$

$$\frac{1}{4}K_p \frac{(4-s)(-4)(s-2)}{(s+2)(s+4}$$

$$\frac{K_p(s-4)(s-2)}{(s+2)(s+4)}$$



Two points
where we have
sustain oscillations

$$g(s)\Big|_{j\omega} = 0 \qquad 1 + K_p \frac{s^2-6s+8}{s^2+6s+8}$$

$$s^2+6s+8+K_p(s^2-6s+8) = 0$$

$$-\omega^2+6j\omega+8+(-K_p\omega^2)-6j\omega+8K_p$$

$$8K_p \left((1+K_p)(-\omega^2)+\right) = 0$$

$$8+8K_p = -\omega^2-\omega^2 K_p$$

$K_p = 1$ ultimate gain

Considering

$$K_p = \frac{K_{pu}}{2} = \frac{1}{2} \qquad \frac{K_d}{K_p} = -\frac{1}{4}$$

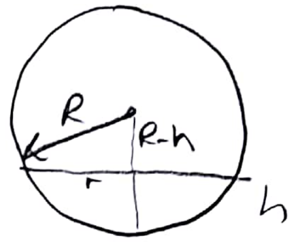
$$K_d = -\frac{K_p}{4} = -\frac{1}{8}$$

2 pts (d) open loop plant does not exhibit oscillation when we apply PD controller we can apply Ziegler Nichols step gain parameter to tune

derive differential equations
outflow dec, not necessarily

Apply conservation law Accumulation = inflow − outflow

need to relate $\frac{dV}{dt}$ with $\frac{dh}{dt}$



$$\frac{dV}{dt} = q_i(t) - q_o(t)$$

$$r^2 = R^2 - (R-h)^2 \qquad r = \sqrt{2Rh - h^2}$$

$$dV = 2rL\,dh = 2L\sqrt{2Rh-h^2}\,dh$$

$$\frac{dh}{dt} = \frac{1}{2L\sqrt{2Rh-h^2}}\left(q_i(t) - q_o(t)\right)$$

$$\frac{dh}{dt} = \frac{1}{2L\sqrt{2Rh-h^2}}\left(q_i(t) - q_o(t)\right)$$

b) Inflow

$$q_o(t) = h(t)\,|\sin(t)|$$

i) determine

$$\alpha(t)$$

ii) Time varying changes with time

$$x' = A(t)\,x(t) + B(t)\,u(t)$$

↳ Time varying

b) Static / dynamic → Dynamic system

c) S, S pts

$q_0 = q_o(t) = \frac{h(t)}{K}$ $K > 0$ model parameter

i) $\bar{h}$; stedy state

Self regulating we can find

equilibrium point $\frac{dh}{dt} = 0$ $q_i(t) = q_o(t)$

We can find a equilibrium point

ii) $\bar{q}_i = \frac{\bar{h}}{K}$ with setting $\frac{dh}{dt} = 0$

iii) Find a linear aproximation to system around the operating

$$f(h, q_i) = \frac{1}{2L\sqrt{2Rh - h^2}} \left(q_i(t) - \frac{h(t)}{K} \right)$$

$$f(h, q_i) = f(\bar{h}, \bar{q}_i) + \frac{\partial f(h, q_i)}{\partial h}\Big|(h - \bar{h}) + \frac{\partial f(h, q_i)}{\partial q_i}\Big|_{(\bar{h}, \bar{q}_i)} (q_i - \bar{q}_i)$$

$$f(h, a_i) = \frac{1}{2L\sqrt{2Rh - h^2}} \left(a_i(t) - \frac{h(t)}{K} \right)$$

at current operating point

$$\frac{\partial f}{\partial h} = \frac{1}{2L} \left[-\frac{1}{2} (2Rh - h^2)^{-3/2} (2R - 2h)(a_i - t) - \frac{1}{K} \left((2Rh - h^2)^{1/2} \right)^{?} \right.$$

$$(2Rh - h^2)^{-1/2}$$

$$\frac{\partial f}{\partial i} = \frac{1}{2L} \left(\frac{1}{\sqrt{2R - h^2}} \left(-\frac{1}{1/K} \right) \right]$$

$$\frac{\partial h}{\partial t} = f(\tilde{h}, \tilde{a_i}) \simeq \frac{\partial f}{\partial h} \tilde{h} + \frac{\partial f}{\partial i} \tilde{a_i}$$

$$\frac{d\bar{h}}{dt} = \frac{1}{2L\sqrt{2R\bar{h} - \bar{h}^2}} \left[\tilde{a_i} - \frac{1}{K} \tilde{h} \right]$$