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EE407 Process Control
HW 3

1. (a) $m = 2$, $n = 2$, $m + n + 1 = 5$

$$e^{-\theta s} = 1 - \theta s + \theta^2 \frac{s^2}{2!} - \theta^3 \frac{s^3}{3!} + \theta^4 \frac{s^4}{4!} \approx \frac{a_0 + a_1 s + a_2 s^2}{1 + b_1 s + b_2 s^2}$$

$$(1+b_1 s+b_2 s^2)(1-\theta s+\theta^2 \frac{s^2}{2!}-\theta^3 \frac{s^3}{3!}+\theta^4 \frac{s^4}{4!})+HOT(greater\ than\ s^2) \approx a_0+a_1 s+a_2 s^2$$

$$a_0 = 1\ ,\ a_1 = b_1 - \theta\ ,\ a_2 = b_2 - b_1\theta + \frac{\theta^2}{2!}$$

$$0 = -b_2\theta + b_1 \frac{\theta^2}{2!} - \frac{\theta^3}{3!}$$

$$0 = b_2 \frac{\theta^2}{2!} - b_1 \frac{\theta^3}{3!} - \frac{\theta^4}{4!}$$

By solving them:

$$\boxed{a_0 = 1}\ ,\ \boxed{a_1 = -2}\ ,\ \boxed{a_2 = 212}\ ,\ \boxed{b_1 = 2}\ ,\ \boxed{b_2 = 212}$$

(b) $m = 0$, $n = 1$, $m + n + 1 = 2$

$$e^{-\theta s} = \frac{1}{e^{\theta s}} = \frac{1}{1^\theta s} \approx \frac{a_0}{1 + b_1 s}$$

$$\boxed{a_0 = 1} \quad \boxed{b_1 = \theta}$$

$m = 0$, $n = 2$, $m + n + 1 = 3$

$$e^{-\theta s} = \frac{1}{e^{\theta s}} = \frac{1}{1^\theta s + \theta^2 \frac{s^2}{2!}} \approx \frac{a_0}{1 + b_1 s}$$

$$\boxed{a_0 = 1} \quad \boxed{b_1 = \theta}$$

(c) The magnitude response of .e-s is always 1,R[1/1] and R[2/2] magnitude response are also exactly 1 and fit the original system. When m=n, the all pass system do not change. The result is expected since smaller m and n values represent more approximation. R[1/1] and R[2/2] have more closer phase response to the original system than R[0/1] and R[0/2]. From the figure 1, we understood that for small frequency, the approximation is valid. However, it became invalid for high frequency.

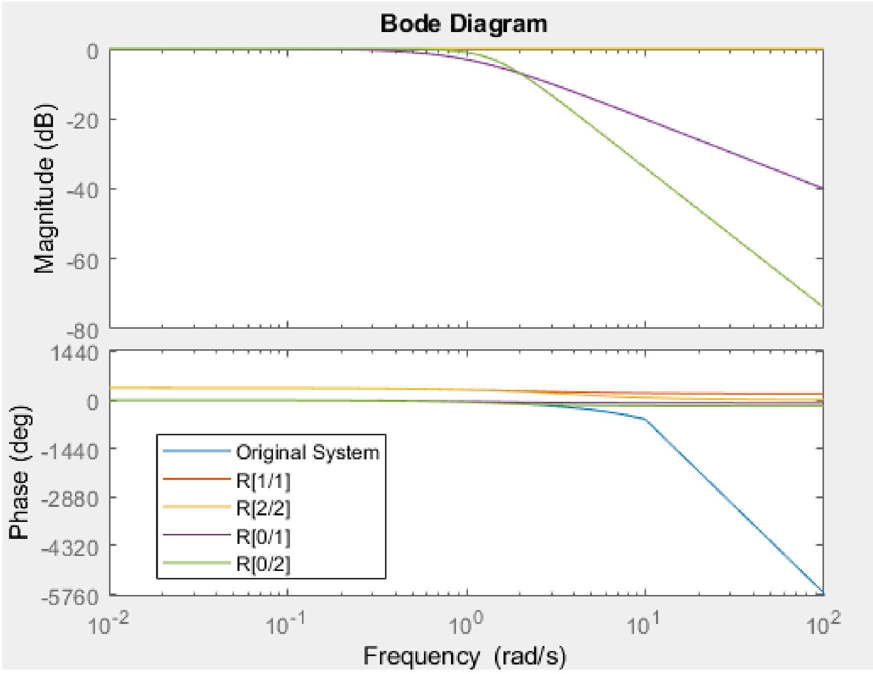


Figure 1: The magnitude and phase responses of e^{-s} and corresponding $R[0/1]$, $R[0/2]$, $R[1/1]$ and $R[2/2]$

(d) The justification hold from the previous step. The systems with non-minimum phase zeros which are R[1/1] and R[2/2] behave oscillatory behaviour around t = 0, then fit the corresponding data. The start point for non-minimum phase zeros do not fit with the original system.

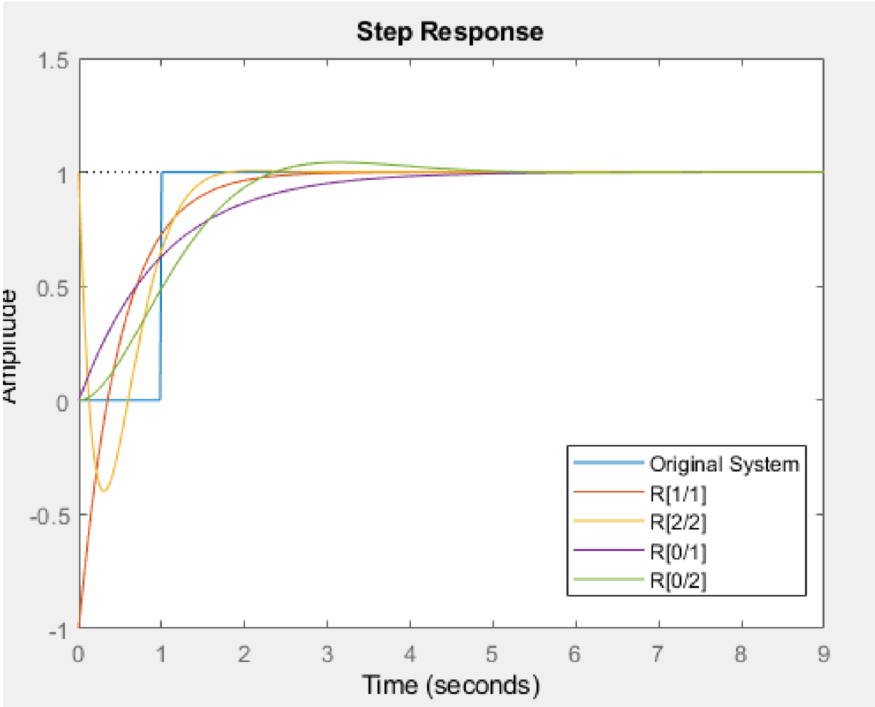


Figure 2: The step responses of e^{-s} and corresponding $R[0/1]$, $R[0/2]$, $R[1/1]$ and $R[2/2]$

2. Model-Based Design Methods

Q2

a) Derive PI controller parameters using IMC design method r=1

$$\tilde{G}_p(s) = K_p \frac{1}{\tau_p s+1}$$

$$\tilde{G}_p(s) = \tilde{G}_{p+} \tilde{G}_{p-} \qquad \tilde{G}_{p-} = K_p \frac{1}{\tau_p s+1}$$

$\tilde{G}_{p+}$: part with positive zeros; $\tilde{G}_{p-}$: part with negative zeros

$$G_c^*(s) = \frac{1}{\tilde{G}_{p-}} \left(\frac{1}{1+\tau_c s}\right)^{r=1} = \frac{\tau_p s+1}{K_p(1+\tau_c s)}$$

$$G_c(s) = \frac{G_c^*(s)}{1-G_c^*(s)\tilde{G}_p(s)} = \frac{\dfrac{\tau_p s+1}{K_p(1+\tau_c s)}}{1-\dfrac{1}{(1+\tau_c s)}} = \frac{\tau_p s+1}{K_p \tau_c s}$$

$$G_c(s) = \frac{\tau_p}{K_p \tau_c}\left(1+\frac{1}{\tau_p}\cdot\frac{1}{s}\right) \qquad \boxed{K_c = \frac{\tau_p}{K_p \tau_c}} \qquad \frac{1}{\tau_I} = \frac{1}{\tau_p} \qquad \boxed{\tau_I = \tau_p}$$

b) Derive PID controller parameters using IMC design method r=1

$$\tilde{G}_p(s) = K_p \frac{1}{(\tau_1 s+1)(\tau_2 s+1)}$$

$$\tilde{G}_{p-}(s) = \tilde{G}_p(s)$$

$$G_c^*(s) = \frac{1}{\tilde{G}_{p-}(s)} \cdot \left(\frac{1}{1+\tau_c s}\right)^{r=1} = \frac{(\tau_1 s+1)(\tau_2 s+1)}{K_p(1+\tau_c s)}$$

$$G_c(s) = \frac{G_c^*(s)}{1-G_c^*(s)\tilde{G}_p(s)} = \frac{\dfrac{(\tau_1 s+1)(\tau_2 s+1)}{K_p(1+\tau_c s)}}{1-\dfrac{1}{(1+\tau_c s)}} = \frac{(\tau_1 s+1)(\tau_2 s+1)}{K_p \tau_c s}$$

$$G_c(s) = \frac{\tau_1+\tau_2}{K_p \tau_c}\left(1+\frac{1}{\tau_1+\tau_2}\cdot\frac{1}{s}+\frac{\tau_1\tau_2}{\tau_1+\tau_2}s\right)$$

$$\boxed{K_c = \frac{\tau_1+\tau_2}{K_p \tau_c}} \qquad \boxed{\tau_I = \tau_1+\tau_2} \qquad \boxed{\tau_d = \frac{\tau_1\tau_2}{\tau_1+\tau_2}}$$

(c) Derive the PID controller parameter using IMC, r = 1

$$\tilde{G}_p(s) = K_p \frac{(-\beta s+1)}{(\tau_1 s+1)(\tau_2 s+1)}, \quad \beta > 0$$

$$\tilde{G}_p(s) = \tilde{G}_{p-}(s)\,\tilde{G}_{p+}(s) \qquad \tilde{G}_{p-} = K_p \frac{1}{(\tau_1 s+1)(\tau_2 s+1)} \;\&\; \tilde{G}_{p+} = (-\beta s+1)$$

$$G_c^*(s) = \frac{1}{\tilde{G}_{p-}(s)}\left(\frac{1}{(\tau_c s+1)}\right)^{r=1} = \frac{(\tau_1 s+1)(\tau_2 s+1)}{K_p(\tau_c s+1)}$$

$$G_c(s) = \frac{G_c^*(s)}{1-G_c^*(s)\tilde{G}_p(s)} = \frac{\frac{(\tau_1 s+1)(\tau_2 s+1)}{K_p(\tau_c s+1)}}{1-\frac{(-\beta s+1)}{\tau_c s+1}} = \frac{(\tau_1 s+1)(\tau_2 s+1)}{K_p(\tau_c+\beta)s}$$

$$G_c(s) = \frac{\tau_1+\tau_2}{K_p(\tau_c+\beta)}\left(1+\frac{1}{(\tau_1+\tau_2)}\cdot\frac{1}{s}+\frac{\tau_1\tau_2}{\tau_1+\tau_2}\cdot s\right)$$

$$\boxed{K_c = \frac{\tau_1+\tau_2}{K_p(\tau_c+\beta)}} \qquad \boxed{\tau_I = \tau_1+\tau_2} \qquad \boxed{\tau_d = \frac{\tau_1\tau_2}{\tau_1+\tau_2}}$$

$$\tilde{G}_p(s) = \frac{1}{(10s+1)(5s+1)}$$

$\tau_c = 5$ min Find PID controller parameter using Direct Synthesis Method.

$\tau_1 = 10 \quad \tau_2 = 5$

$$\boxed{K_c = 3} \qquad \boxed{\tau_I = 15\text{ min}} \qquad \boxed{\tau_d = \frac{10}{3} = 3.33\text{ min}}$$

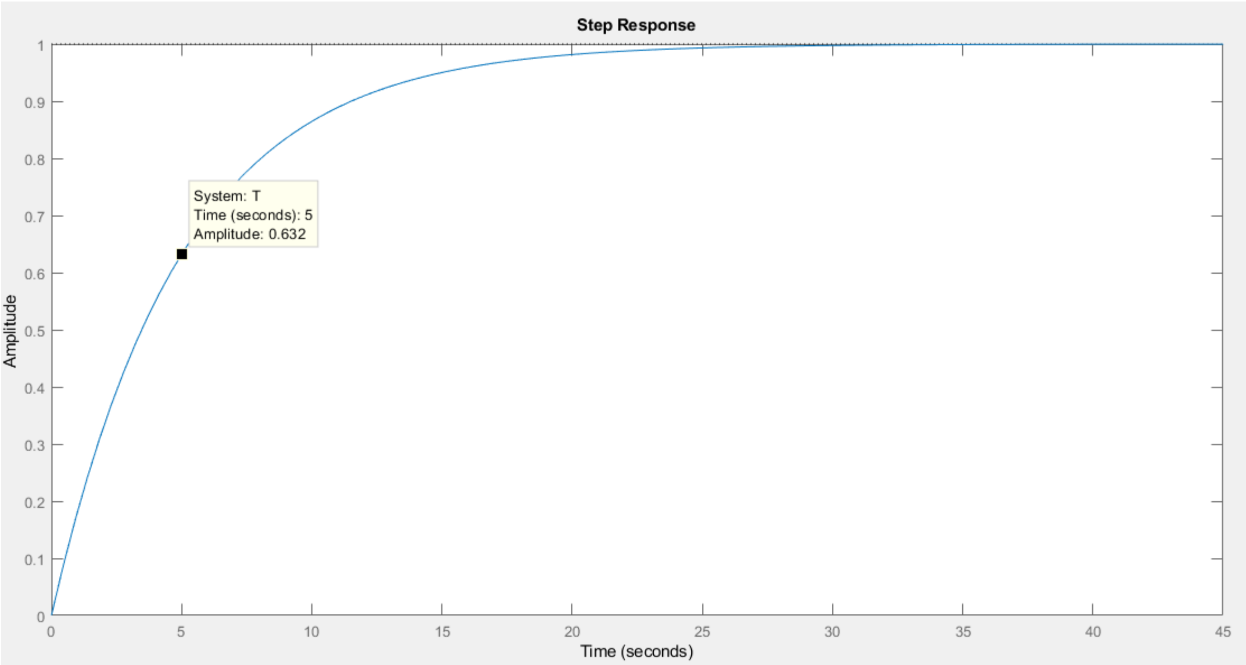


Figure 3: The step response of the system in Part c The time is in minute , not in second. I write the coefficient according to minute.

(e) $\tilde{G}_p(s) = \frac{1+\alpha}{(10s+1)(5s+1)}$

$1 + G_c \tilde{G}_p(s) = 0 \longrightarrow 1 + \frac{1+\alpha}{(10s+1)(5s+1)} \cdot \frac{3+45s+150s^2}{15s} = 0$

$15s(10s+1)(5s+1) + 3 + 45s + 150s^2 + \alpha(3+45s+150s^2) = 0$

$5s(50s^2+15s+1) + 50s^2+15s+1 + \alpha(50s^2+15s+1) = 0$

$\underbrace{(50s^2+15s+1)}_{(10s+1)(5s+1)}(5s+1+\alpha) = 0$

$s = -\frac{1}{10}$ $s = -\frac{1}{5}$ $\boxed{s = \frac{-1-\alpha}{5}} \longrightarrow$ check this for stability

$\frac{-1-\alpha}{5} < 0$ $\boxed{\alpha > -1}$

f) What is the limiting value of τ_c if $|\alpha|<0.2$

$\tilde{G}_p(s) = \frac{1+\alpha}{(10s+1)(5s+1)}$ $\qquad G_c(s) = \frac{15}{\tau_c}\left(1+\frac{1}{15s}+\frac{10}{3}s\right)$

$1+\frac{\cancel{15}}{\tau_c}\left(\frac{\cancel{1+15s+50s^2}}{\cancel{15}s}\right)\cdot\frac{1+\alpha}{\cancel{(10s+1)(5s+1)}}=0$

$|\alpha|<0.2$
$-1-\alpha$ is always ~~negative~~

$\frac{-1-\alpha}{\tau_c}<0$

$\tau_c s+1+\alpha=0 \qquad s=-\frac{1-\alpha}{\tau_c}$

$\boxed{\tau_c>0}$

$\tau_c=0$ is limiting value.

3. Distributed Parameter Systems

value.

3. $\frac{\partial T}{\partial x}+\frac{\partial T}{\partial t}=0$ $\qquad T(x,0)=T_e$ for $0<x\leq 1$ (initial condition)
$T(0,t)=V(t)+T_e$ for $t>0$ (boundary condition)

$\eta(x,t)=T(x,t)-T_e$. Rewrite Diff eqn & boundary condition. Find $T(x,t)$

$\frac{\partial\eta}{\partial x}=\frac{\partial T}{\partial x} \qquad \frac{\partial\eta}{\partial t}=\frac{\partial\eta}{\partial t} \Rightarrow \frac{\partial\eta}{\partial x}+\frac{\partial\eta}{\partial t}=0$

$\eta(x,0)=0$ for $0\leq x\leq 1 \qquad \eta(0,t)=V(t)$ for $t>0$

$\mathcal{L}\{\eta(x,t)\}=T^*(x,s) \Rightarrow sT^*(x,s)-\cancel{\eta(x,0)}^{0}=\mathcal{L}\left\{\frac{\partial\eta(x,t)}{\partial t}\right\}$

Take $T^*(x,s)=\lambda.e^{kx}$ and try to find solution

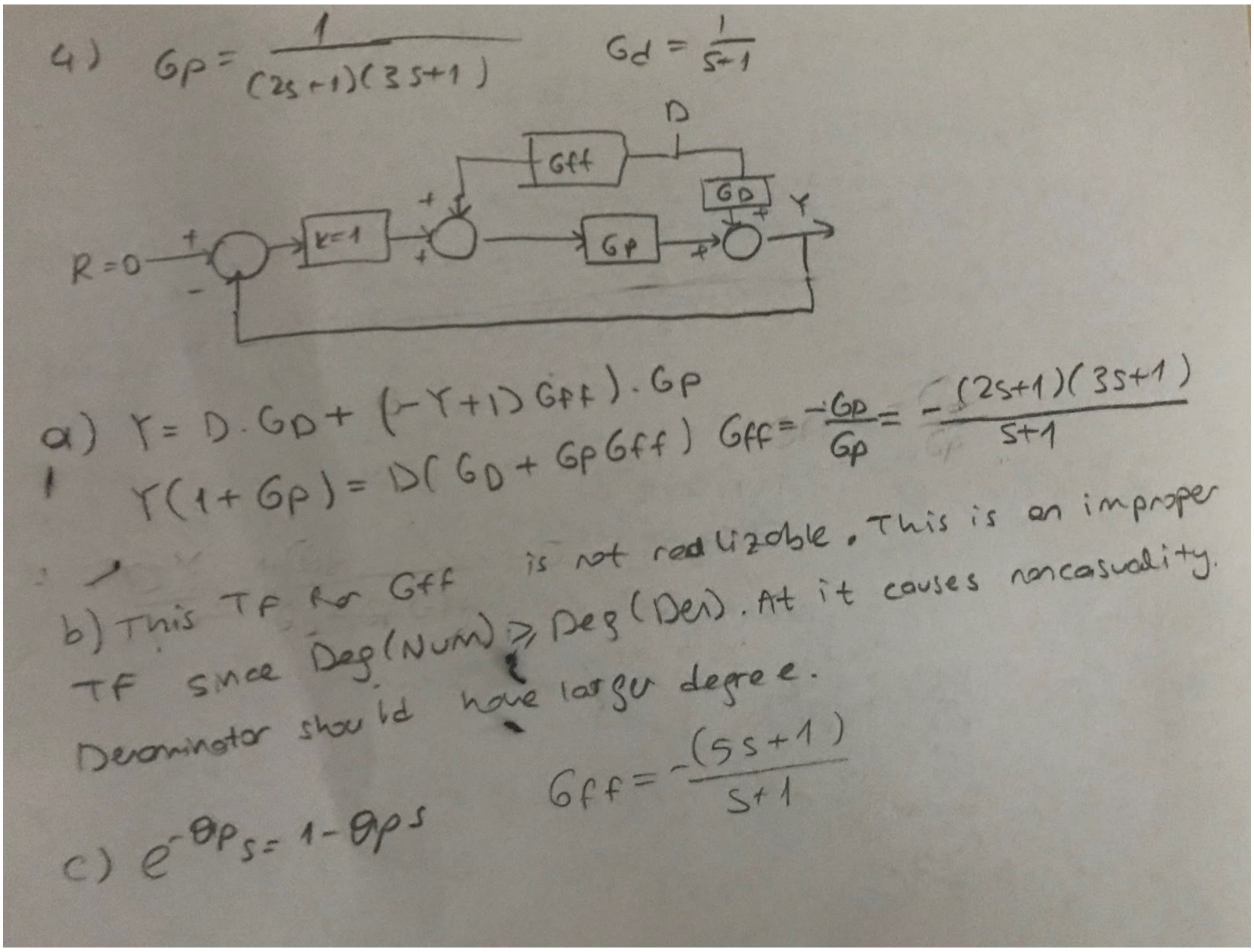
$x.\lambda e^{kx}+s.e^{kx}=0 \qquad k=-s$

Apply boundary condition to find λ

$T^*(0,s)=\mathcal{L}\{V\}=V^*=\lambda \qquad \Rightarrow T(x,t)=V(t-x)+T_e$

$\eta(x,t)\mathcal{L}^{-1}\{V.e^{-sx}\}=\underbrace{V(t).\delta(t-x)}_{V(t-x)}$

4. Feedforward Control



d) The simulink diagram for the system can be seen at *Figure 4*. The results agrees with the expectations, as the feedforward controller added to the system, the steady state error due to step disturbance was compensated. The results can be seen at *Figure 5*.

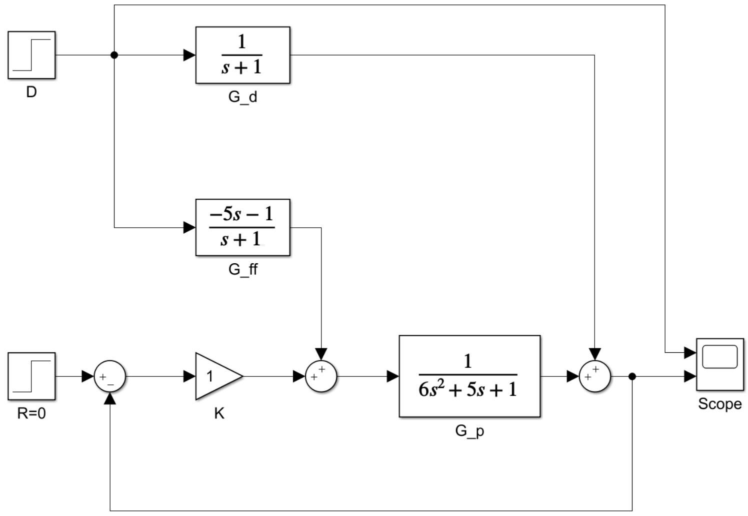
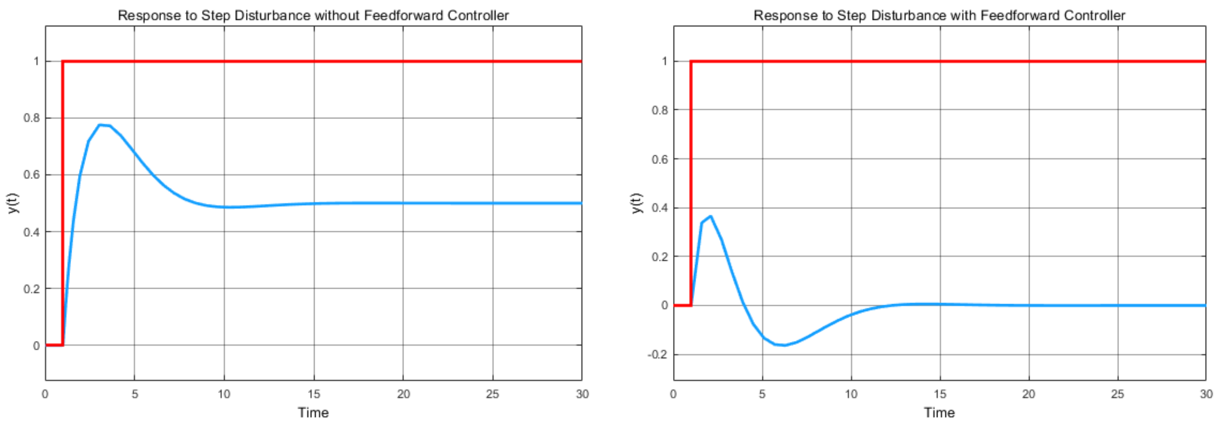


Figure 4: Simulink Model for the System



(a) Response to Step Disturbance without Feedforward Controller

(b) Response to Step Disturbance with Feedforward Controller

Figure 5: Response to Step Disturbance

5. -

(a) The open loop and closed loop transfer functions for the inner loop are

$$G_{OL,inner} = G_{c2}G_v = \frac{20}{s+1}$$

$$G_{CL,inner} = \frac{G_{c1}G_v}{1 + G_{m2}G_{c2}G_v} = \frac{\frac{20}{s+1}}{1+\frac{4}{s+1}} = \frac{20}{s+5} = \frac{4}{0.2s+1}$$

as can be seen from the transfer functions also, the time constant of closed loop is five time faster than that of open loop's.

(b)

(c) with given constants closed loop transfer function of the system becomes

$$G_{CL} = \frac{G_{c2}(\frac{4}{0.2s+1})G_p}{1 + G_{c1}\frac{4}{0.2s+1}G_pG_{m1}}$$

$$G_{CL} = \frac{G_{c2}(\frac{20}{s+5})(\frac{4}{(4s+1)(2s+1)})}{1 + G_{c1}(\frac{20}{s+5})(\frac{4}{(4s+1)(2s+1)})0.05}$$

Then characteristic equation becomes *(for D=0);*

$$q(s) = 1 + G_{c1}\left(\frac{4}{(s+5)(4s+1)(2s+1)}\right) = 0$$

$$(s+5)(4s+1)(2s+1) + 4G_{c1} = 0$$

$$8s^3 + 46s^2 + 31^s + 5 + 4G_{c1} = 0$$

From Routh-Hurwitz stbility crietarian, critical gain G_{c1} can be found as

$$G_{c1,max} = 43.31$$

$$\boxed{K_c \approx \frac{G_{c1,max}}{2} = 21.66}$$

(d) with given numbers, the CLTF becomes

$$G_{CL} = \frac{4(\frac{5}{s+1})\frac{4}{(4s+1)(2s+1)}}{1 + \frac{5}{s+1}\frac{4}{(4s+1)(2s+1)}0.05}$$

Then characteristic equation becomes *(for D=0);*

$$q(s) = 1 + G_{c1}\frac{1}{(s+1)(4s+1)(2s+1)}$$

$$(s+1)(4s+1)(2s+1)+G_{c1}=0$$

$$8s^3+14s^2+7^s+1+G_{c1}=0$$

Again from Routh-Hurwitz

$$G_{c1,max}=11.25$$

$$\boxed{K_c \approx \frac{G_{c1,max}}{2}=5.625}$$

(e)

$$E_1G_{c1}-Y_2G_{m2}=E_2$$

$$E_2G_{c2}G_v+D=Y_2$$

Assuming $R=0$

$$E_1=-G_{m1}G_pY_2$$

$$-G_{c1}G_{m1}G_pY_2-Y_2G_{m2}=E_2$$

$$Y_2=\frac{-1}{G_{c1}G_{m1}G_p+G_{m2}}E_2$$

$$D=-E_2G_{c2}G_v+\frac{-1}{G_{c1}G_{m1}G_p+G_{m2}}E_2$$

$$E_2(s)=\frac{-\,D(s)}{E_2G_{c2}G_v+\dfrac{1}{G_{c1}G_{m1}G_p+G_{m2}}}$$

with $D(s)=\frac{1}{s}$

$$e_{2,ss}=\lim_{s\to 0}sE_2(s)(\frac{-}{E_2G_{c2}G_v+\dfrac{1}{G_{c1}G_{m1}G_p+G_{m2}}})$$

with controller ($G_{c2}=4\ G_{m2}=0.2$)

$$e_{2,ss} \approx -0.0022$$

without controller ($G_{c2} = 1$ $G_{m2} = 0$)

$$e_{2,ss} \approx -0.0302$$

The simulink diagram for the system can be seen at *Figure 6*. The results agrees with the expectations, as the inner controller added to the system, the steady state error due to step disturbance was almost compensated. The results can be seen at *Figure 7*.

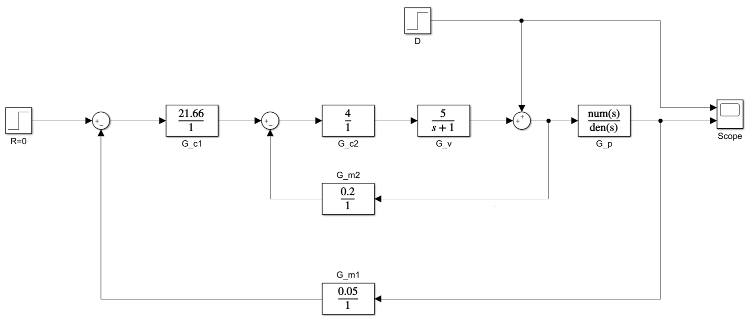
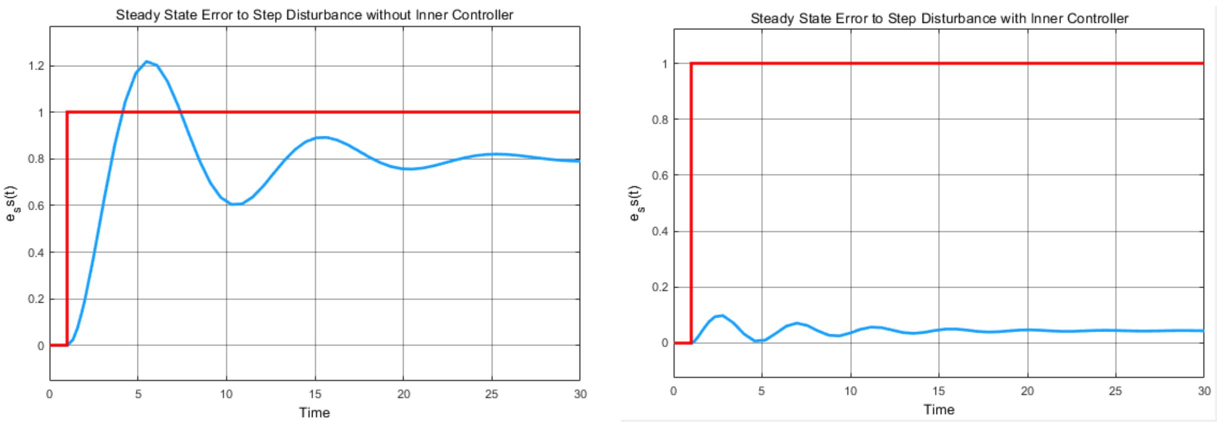


Figure 6: Simulink Model for the System



(a) Response to Step Disturbance without Inner Controller

(b) Response to Step Disturbance with Inner Controller

Figure 7: Response to Step Disturbance

(f) System become more stable and less oscillatory with the addition of Cascade Controller. The error rejectşon ratio increased significantly.