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EE407 Process Control
HW 3

1. (a) -
   (b) The desired values can be calculated from the given figure as follows;

$$K_p = \frac{\Delta y}{\Delta u} = \frac{1}{1} = 1$$

$$\tau_p = \frac{1}{0.7}(t_{2/3} - t_{1/3}) = \frac{1}{0.7}(37.38 - 23.81) = 19.38 \ seconds$$

$$\theta_p = t_{1/3} - 0.4\tau_p = 23.81 - 0.4(19.38) = 16.06$$

   (c) -
   (d) The simulink model can be seen at *Figure 1.*

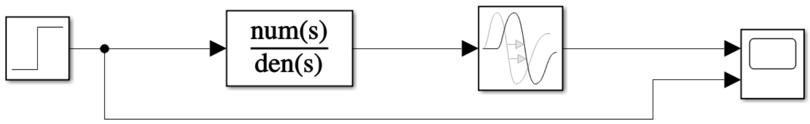


Figure 1: Simulink Model for the FOPDT Model

   (e) The Step Response of the FOPDT Model can be seen at *Figure 2.*

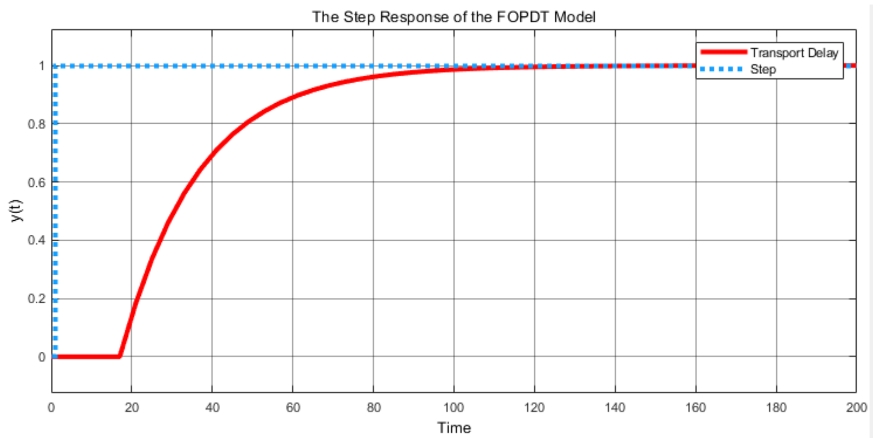


Figure 2: The Step Response of the FOPDT Model

The response in comparison with given figure can be seen at *Figure 3*.

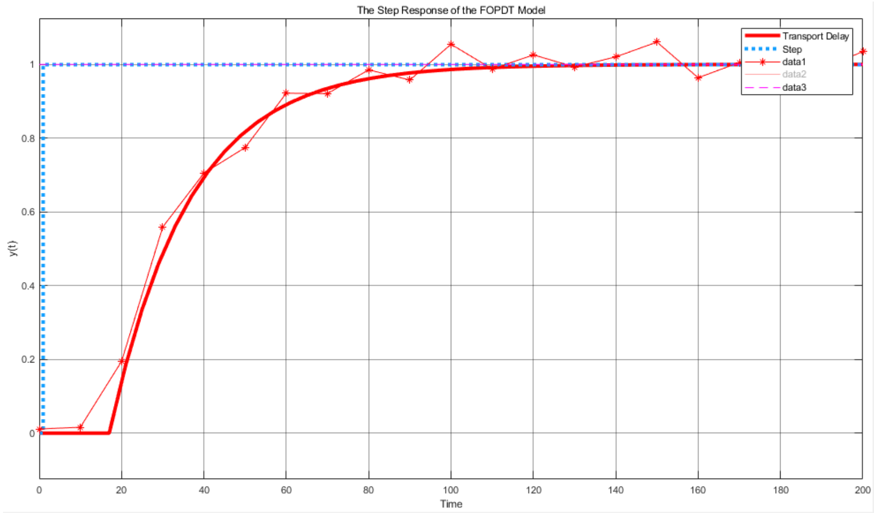


Figure 3: The Step Response of the FOPDT Model in comparison with Given Figure

(f) The existed version is pushed to remote repository with a commit message *"Question 1 part e is completed"*.

(g) K_c for servo control problem can be calculated with the formula based on Integral of Time Weighted Absolute Error (ITAE) as follows;

$$K_{c,moderate} = \frac{0.2}{K_p}\left(\frac{\tau_p}{\theta_p}\right)^{1.22} = \frac{0.2}{1}\left(\frac{19.38}{16.06}\right)^{1.22} = 0.25$$

$$K_{c,aggressive} = 2.5K_{c,moderate} = 0.628$$

(h) The updated simulink model with P-controller can be seen *Figure 4*. The step response for this system can be seen at *Figure 5*.

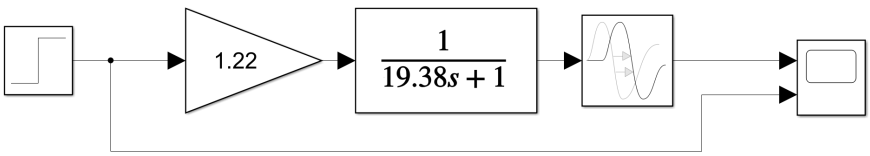


Figure 4: Simulink Model for the FOPDT Model with P-Controller

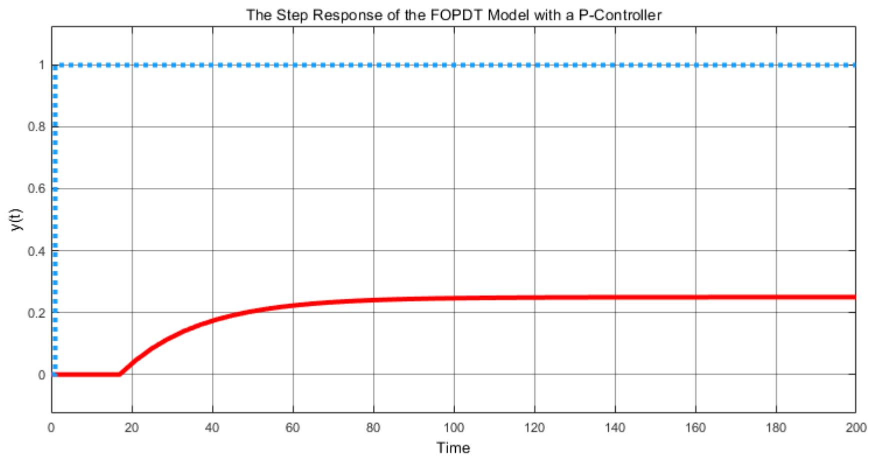
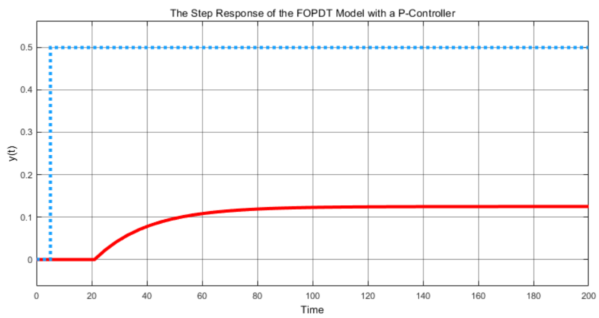


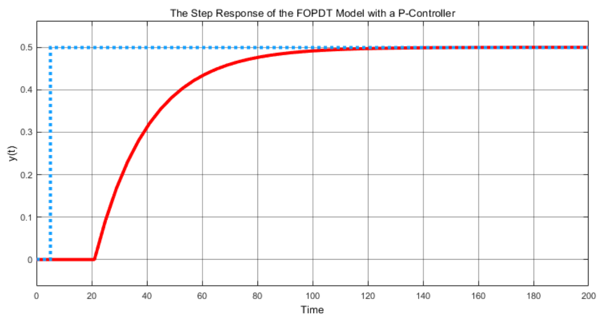
Figure 5: The Step Response of the FOPDT Model with P-Controller

The existed version is pushed to remote repository with a commit message *"Question 1 part h is completed(Simulink Update) together with Iclal"*.

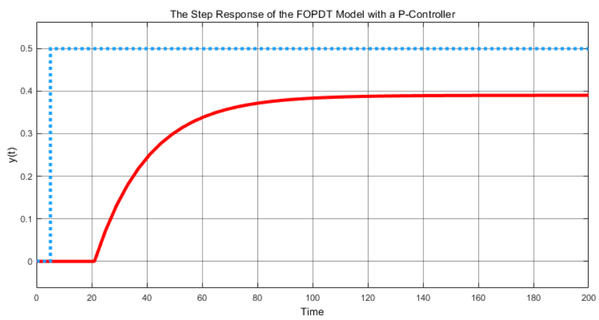
(i) The final value for K_c was decided to be 1. The different responses for given input signal with different gain values can be seen at *Figure 6*



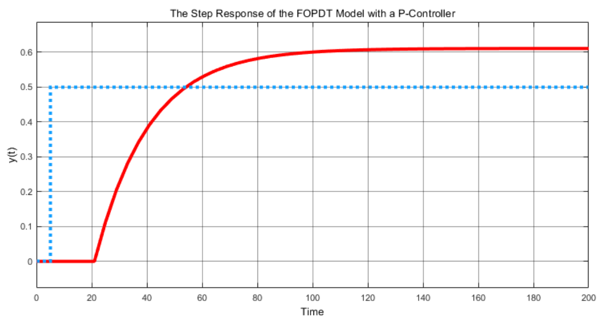
(a) The Step Response with K_c=0.25



(b) The Step Response with K_c=1.0



(c) The Step Response with K_c=0.78



(d) The Step Response with K_c=1.22

Figure 6: The Step Response of the FOPDT Model with Varying K_c

2. (a)

$$\dot{h} = \begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \\ \dot{h}_3 \\ \vdots \\ \dot{h}_n \end{bmatrix} = \begin{bmatrix} -\frac{1}{A_1R_1} & 0 & 0 & \dots & 0 \\ \frac{1}{A_2R_1} & -\frac{1}{A_2R_2} & 0 & \dots & 0 \\ 0 & \frac{1}{A_3R_2} & -\frac{1}{A_3R_3} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{A_2R_1} & \frac{1}{A_2R_1} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ \vdots \\ h_n \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} q_i$$

$$y = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} h$$

(b)

$$\begin{bmatrix} sH_1(s) \\ sH_2(s) \\ sH_3(s) \\ \vdots \\ sH_n(s) \end{bmatrix} = \begin{bmatrix} -\frac{1}{A_1R_1} & 0 & 0 & \dots & 0 \\ \frac{1}{A_2R_1} & -\frac{1}{A_2R_2} & 0 & \dots & 0 \\ 0 & \frac{1}{A_3R_2} & -\frac{1}{A_3R_3} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{A_2R_1} & \frac{1}{A_2R_1} \end{bmatrix} \begin{bmatrix} H_1(s) \\ H_2(s) \\ H_3(s) \\ \vdots \\ H_n(s) \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} Q_i(s)$$

$$\begin{bmatrix} s+\frac{1}{A_1R_1} & 0 & 0 & \dots & 0 \\ -\frac{1}{A_2R_1} & s+\frac{1}{A_2R_2} & 0 & \dots & 0 \\ 0 & -\frac{1}{A_3R_2} & s+\frac{1}{A_3R_3} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & -\frac{1}{A_2R_1} & s+\frac{1}{A_2R_1} \end{bmatrix} \begin{bmatrix} H_1(s) \\ H_2(s) \\ H_3(s) \\ \vdots \\ H_n(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{A_1} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} Q_i(s)$$

From the matrix ,let us start with $G_1(s) = \dfrac{H_1(s)}{Q_i(s)}$

$$(s + \frac{1}{A_1R_1})H_1(s) = \frac{1}{A_1}Q_i(s)$$

$$\boxed{G_1(s) = \frac{H_1(s)}{Q_i(s)} = \frac{R_1}{s(A_1R_1)+1}}$$

$$(-\frac{1}{A_2R_1})H_1(s) + (s + \frac{1}{A_2R_2})H_2(s) = 0$$

$$\boxed{G_2(s) = \frac{H_2(s)}{H_1(s)} = \frac{R_2/R_1}{s(A_2R_2)+1}}$$

$$(-\frac{1}{A_3R_2})H_2(s) + (s + \frac{1}{A_3R_3})H_3(s) = 0$$

$$\boxed{G_3(s) = \frac{H_3(s)}{H_2(s)} = \frac{R_3/R_2}{s(A_3R_3)+1}}$$

iteratively, it can be observed that,

$$\boxed{G_n(s) = \frac{H_n(s)}{H_{n-1}(s)} = \frac{R_n/R_{n-1}}{s(A_nR_n)+1}}$$

Thus, the general transfer function can be written as

$$G(s) = G_1(s)G_2(s)G_3(s)......G_n(s) = \prod_{k=1}^{n} G_k(s)$$

$$\boxed{G(s) = R_n \prod_{k=1}^{n} \frac{1}{s(A_kR_k)+1}}$$

(c) Let us assume a new transfer function $G'(s) = \dfrac{Q_o(s)}{Q_i(s)}$

$$G'(s) = \frac{Q_o(s)}{Q_i(s)}$$

Also remember that,

$$q_k = \frac{h_k}{R_k}$$

$$Q_k(s) = \frac{H_k(s)}{R_k}$$

$$G'_1(s) = \frac{Q_1(s)}{Q_i(s)} = \frac{H_1(s)/R_1}{Q_i(s)} = \frac{G_1(s)}{R_1} = \frac{1}{s(A_1R_1)+1}$$

$$G'_2(s) = \frac{Q_2(s)}{Q_1(s)} = \frac{H_2(s)/R_2}{H_1(s)/R_1} = \frac{G_2(s)}{R_2/R_1} = \frac{1}{s(A_2R_2)+1}$$

also iteratively,

$$\boxed{G'_n(s) = \frac{Q_n(s)}{Q_{n-1}(s)} = \frac{H_n(s)/R_n}{H_{n-1}(s)/R_{n-1}} = \frac{G_n(s)}{R_n/R_{n-1}} = \frac{1}{s(A_nR_n)+1}}$$

Thus,

$$G'(s) = G'_1(s)G'_2(s)G'_3(s)......G'_n(s) = \prod_{k=1}^{n} G'_k(s)$$

$$\boxed{G'(s) = \prod_{k=1}^{n} \frac{1}{s(A_kR_k)+1}}$$

It can be seen that

$$G(s) = R_nG'(s)$$

(d) Assuming $n = 2$, The steps for drawing Root Locus for this system are as follows;

$$G(s) = \frac{R_2}{(sA_1R_1+1)(sA_2R_2+1)}$$

Poles are:

$$\boxed{s_1 = \frac{-1}{A_2R_2}}\ ,\ \boxed{s_2 = \frac{-1}{A_1R_1}}$$

The both poles are at left half plane.

- $m = \#ofpoles = 2$
- $n = \#ofpoles = 0$
- $Branches = |m - n| = 2$
- Asymptotes Angles are 90 and 270
- $\sigma_0 = \dfrac{-1}{2(A_1R_1 + A_2R_2)}$
- Break away will be same with σ_0

As can be seen from the Root-Locus at *Figure 7*, the system never cross jw axis. Thus, the Ziegler-Nichols method can not be used.

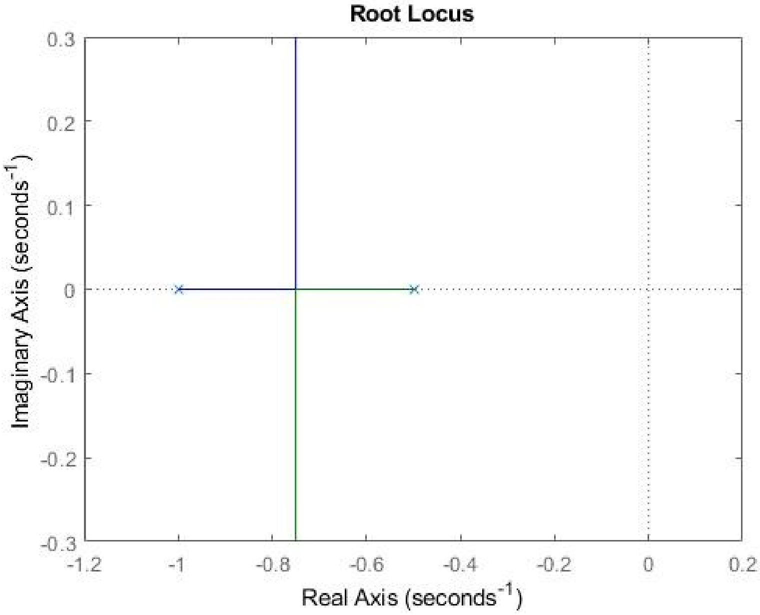


Figure 7: An approximate root locus plot for $n = 2$

As can be seen from the approximate bode plot at *Figure 8*, the phase response plot reaches the -180 degree phase as $f \to \infty$. Thus, it is not possible to use the Ziegler-Nichols method for this system.

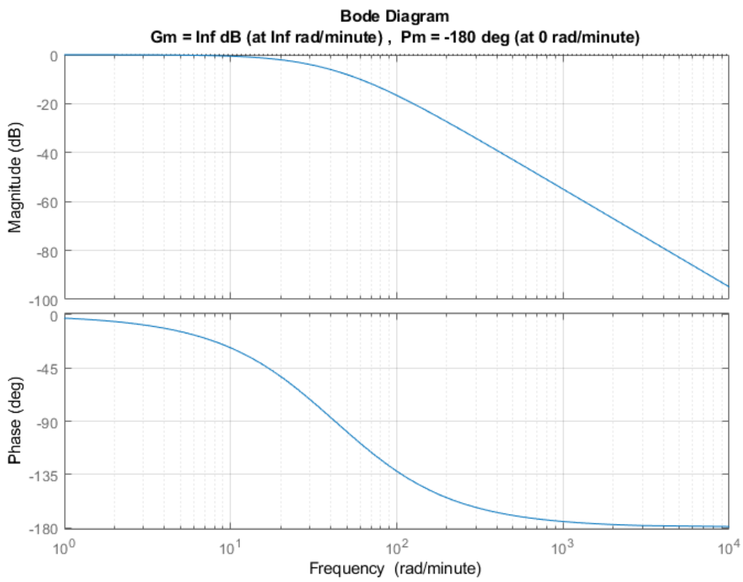


Figure 8: An approximate bode plot for $n = 2$

(e) With $R_iA_i = 1(min)$ and $R_i = 1(min/m^2)$ for $i = 1, 2, 3$.

$$G(s) = \frac{R_3}{(sA_1R_1+1)(sA_2R_2+1)(sA_3R_3+1)} = \frac{1}{(s+1)^3}$$

Poles are:

$$\boxed{s_1 = \frac{-1}{A_1R_1 = -1}}, \boxed{s_2 = \frac{-1}{A_2R_2 = -1}}, \boxed{s_3 = \frac{-1}{A_3R_3 = -1}}$$

The poles are at left half plane.

- $m = \#\ of\ poles = 3$
- $n = \#\ of\ zeros = 0$
- $Branches = |m - n| = 3$
- Asymptotes Angles=60, 180 and 300
- $\sigma_0 = \frac{-1}{3(A_1R_1 + A_2R_2 + A_3R_3)} = -\frac{1}{9}$

Then, using these informations ,the approximate Root-Locus drawing for $n = 3$ at *Figure 9* can be constructed. It can be observed from the same figure that the critical stability point is approximately $\boxed{K_c = 8}$ point. The system becomes unstable for the values of K_c greater than that.

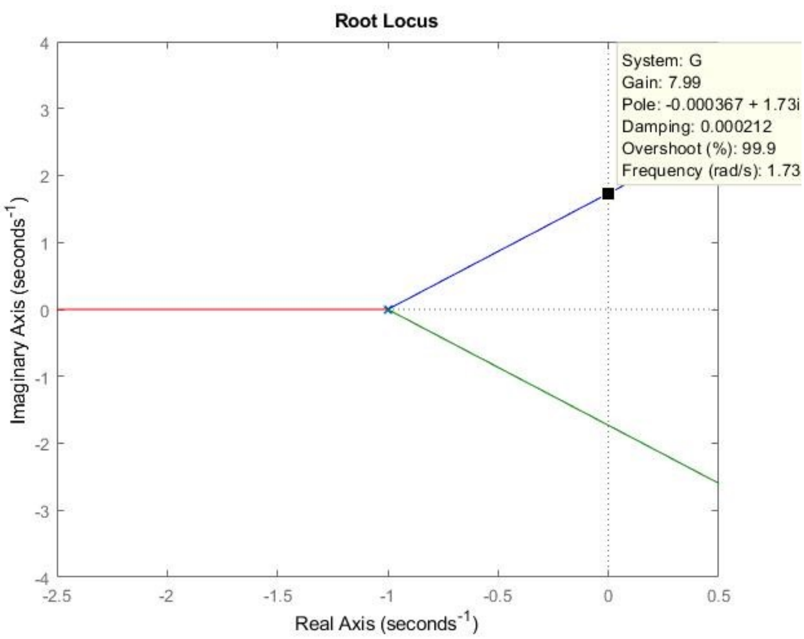


Figure 9: An approximate root locus plot for n=3

The basic block diagram for the system with $G(s) = \dfrac{1}{(s+1)^3}$ and proportional gain K_c can be seen at *Figure 10.*

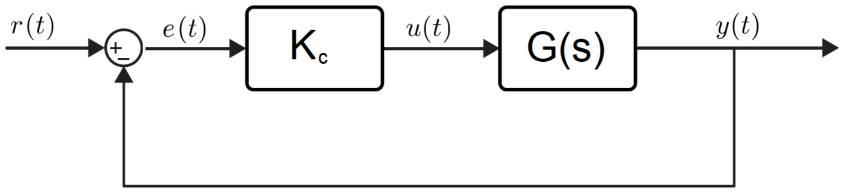


Figure 10: Block Diagram for the System

The closed loop transfer function then becomes;

$$H(s) = \frac{K_c G(s)}{1 + K_c G(s)}$$

$$H(s) = \frac{K_c \dfrac{1}{(s+1)^3}}{1 + K_c \dfrac{1}{(s+1)^3}}$$

From this transfer function, characteristic equation can be found to be;

$$D(s) = 1 + K_c \frac{1}{(s+1)^3} = 0$$

equivalently,

$$\boxed{D(s) = s^3 + 3s^2 + 3s + 1 + K_c = 0}$$

The Routh Hurwitz Table can be constructed for this equation to find stability conditions for K_c values.

$$\begin{array}{c|ccccc}
s^3 & 1 & 3 & 0 & 0 & \cdots \\
s^2 & 3 & K_c + 1 & 0 & \cdots & \\
\hline
s^1 & \dfrac{K_c - 8}{3} & 0 & \cdots & & \\
s^0 & K_c + 1 & \cdots & & &
\end{array}$$

For stability,

- $\dfrac{K_c - 8}{3} > 0$
$$K_c > 8$$
- $K_c + 1 > 0$
$$K_c > -1$$

It can be understood from the Routh Hurwitz Table that for $\boxed{K_c = 8}$ the system is critically stable. The system becomes unstable for the values of K_c greater than that. The result is also consisted with the findings of root locus.

Let us also examine the bode plot for the system with $n = 3$ at *Figure 11.* The frequency response is equal to 180 degree as the magnitude response of the system is 18.1 dB

$$Magnitude\ Response = -20log|H(jw_{cu})| = 18.1\ dB$$

Or equivalently, $10^{\frac{18.1}{20}}$ $\boxed{\approx 8.02}$ (m^2/min).
Also from the graph $w_{cu} \approx 1.73\ (rad/min)$
The results are also agree with the previous findings.

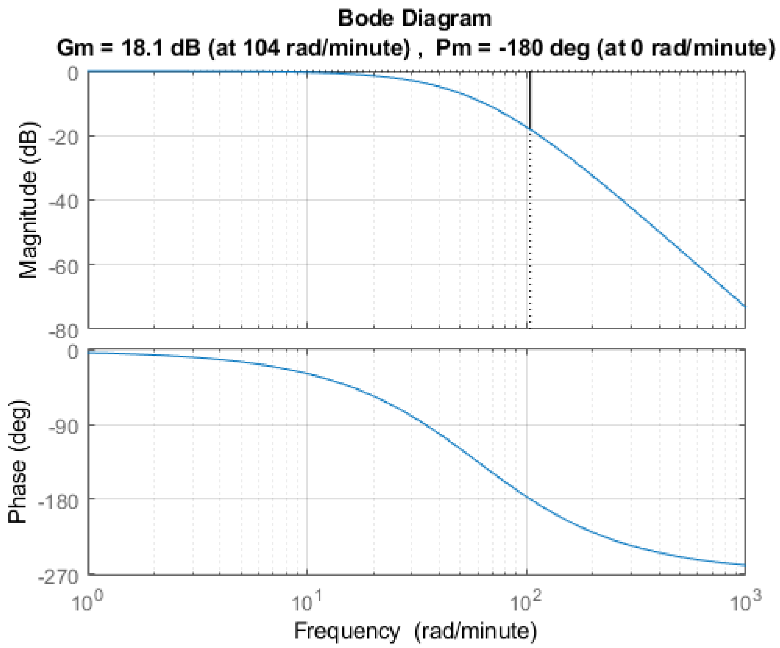


Figure 11: An approximate bode plot for n=3

(f) Let us examine the never ending oscillations of the step response of the system with newly found $K_c = 8$ to find the period P_u. The response can be seen at *Figure 12*. The period can be calculated from the figure as

$$P_u = 5.74 - 2.12 = 3.62 \; minutes$$

Since the design is actually for minutes, let us convert it to seconds,

$$P_u = 60(3.62) = 217.8 \; seconds$$

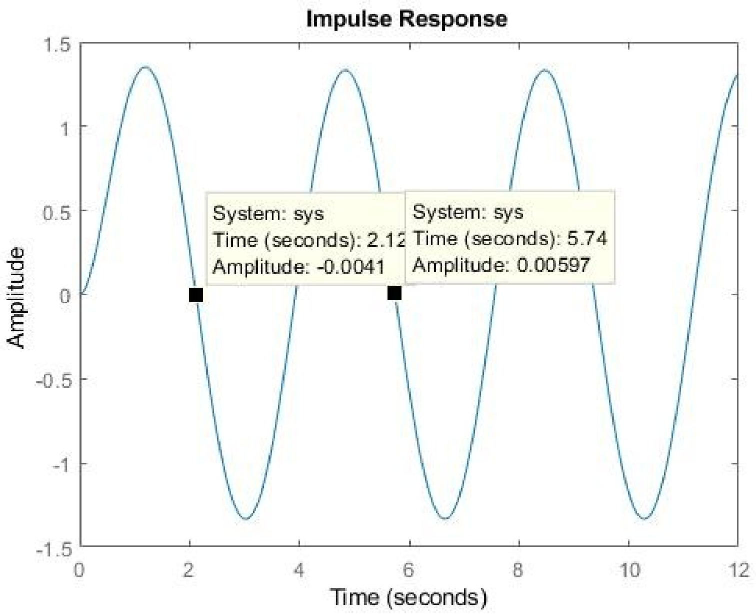


Figure 12: The impulse response of the system for n=3

According to Ziegler-Nichols method for a suitable PID controller.

- $$\boxed{K_c = \frac{K_{cu}}{1.74} = 4.7}$$
- $$\boxed{\tau_I = \frac{P_u}{2} = 108.9 \; seconds = 1.815 \; minutes}$$
- $$\boxed{\tau_D = \frac{P_u}{8} = 27.23 \; seconds = 0.454 \; minutes}$$

$$\boxed{G_{PID}(s) = K_c\left(1 + \frac{1}{\tau_I}\frac{1}{s} + \tau_D s\right)}$$

(g) The step response of the updated system with PID Controller designed in *step e* can be seen at *Figure 13*.

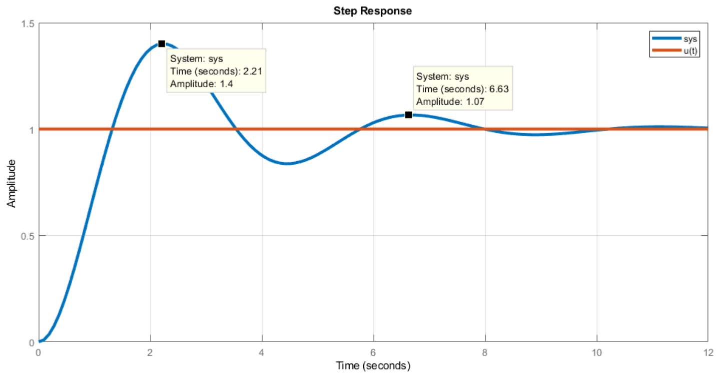


Figure 13: The step response of the system with PID Controller

The decay ratio, then, can be calculated from the same figure as the ratio of the first two peaks ;

$$Decay\ ratio = \frac{1.07 - 1}{1.4 - 1} = \frac{0.07}{0.4} = 0.175$$

(h) The step responsen of the systems at *part e* and *part g* can be seen at *Figure* **??**.

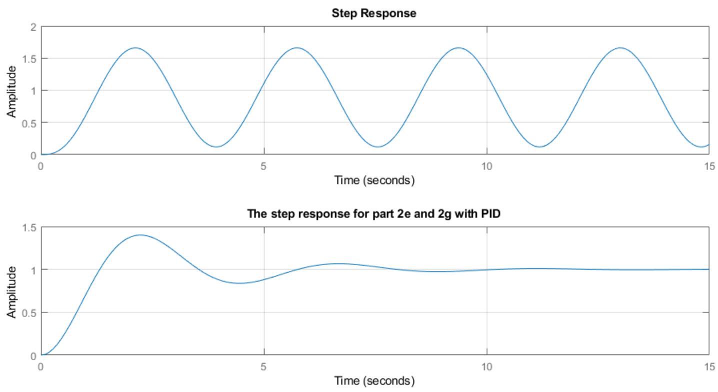


Figure 14: The step responses of the system for part 2e and for part 2g with PID Controller

3. Let us analyse the system at *Figure 15.*

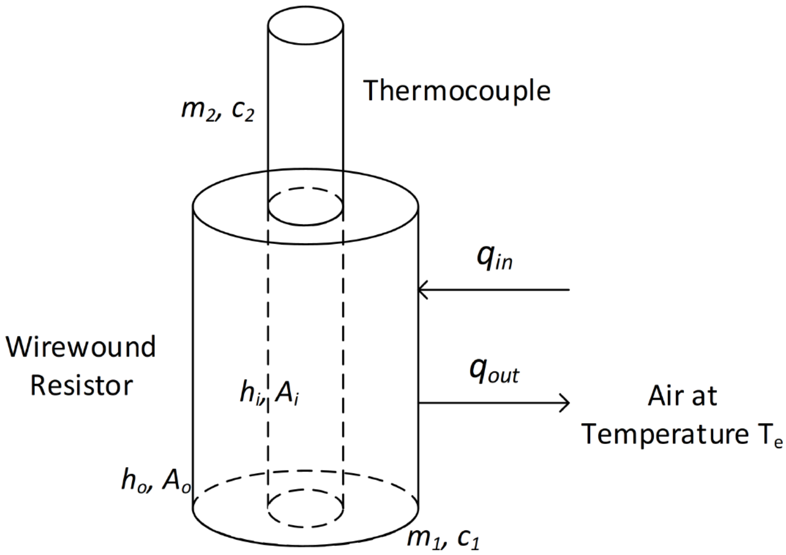


Figure 15: Illustration of the interacting masses being heated

(a) Differential equation that governs the system are as follows;

- $$Q_1 = m_1 c_1 \frac{\partial T_1}{\partial t} = h_i A_i (T_2 - T_1) + h_0 A_0 (T_e - T_1) + q_{in}$$
- $$Q_2 = m_2 c_2 \frac{\partial T_2}{\partial t} = h_i A_i (T_1 - T_2)$$

(b) It is important to notice that, the equations above resembles the differential equations of a circuit with two capacitor. Let us assume some parameters for convenience;

- $Q_1(t) = i_1(t)$ namely, current of first capacitor(C_1)
- $T_1(t) = v_1(t)$ namely, voltage of first capacitor (C_1)
- $Q_2(t) = i_2(t)$ namely, current of second capacitor (C_2)
- $T_2(t) = v_2(t)$ namely, voltage of second capacitor (C_2)
- q_{in} can be left as an independent current source
- $T_e(t)$ can be left as an independent voltage source

Notice that,

$$i_1(t) = C_1 \frac{dV_1(t)}{dt} = h_i A_i (v_2(t) - v_1(t)) + h_0 A_0 (T_e(t) - v_1(t)) + q_{in}(t)$$

$$i_2(t) = C_2 \frac{dv_2(t)}{dt} = h_i A_i (v_1(t) - v_2(t))$$

with

$$\boxed{C_1 = m_1 c_1}$$

$$\boxed{C_2 = m_2 c_2}$$

From these equations, the below equations can be derived;

$$\frac{i_2(t)}{h_i A_i} = V_1(t) - V_2(t)$$

$$\frac{i_1(t) - q_{in}(t) - i_2(t)}{h_0 A_0} = \frac{i_3(t)}{h_0 A_0} = T_e(t) - V_1(t)$$

Let us further assume some resistor values as;

$$\boxed{R_1 = \frac{1}{h_o A_o}}$$

$$\boxed{R_2 = \frac{1}{h_i A_i}}$$

Thus,

$$i_2(t) R_2 = V_1(t) - V_2(t)$$

$$i_3(t) R_1 = T_e(t) - V_1(t)$$

From these equations, the circuit at *Figure 16* can be drawn.

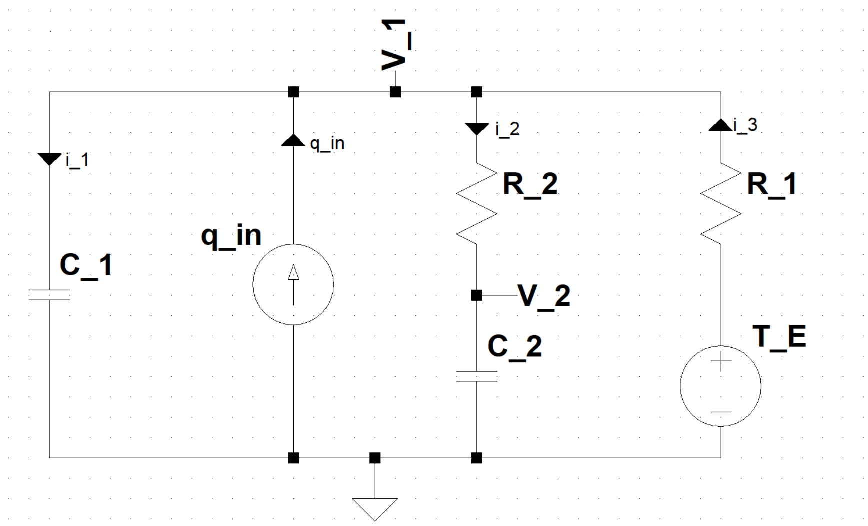


Figure 16: Electrical Equivalent Circuit of Heat Exchanger System

(c) Knowing that you can superpose the effect of independent sources in an electrical circuit, the active voltage source corresponding to the environmental temperature ($T_e(t)$) can be killed. The new circuit can be seen at *Figure 17.*

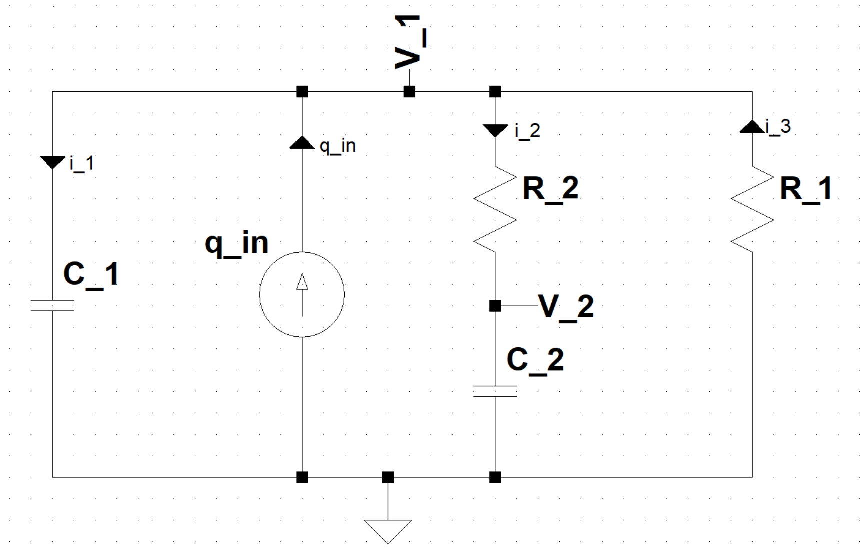


Figure 17: Electrical Equivalent Circuit of Heat Exchanger System without Environmental Temperature Effect

Let us analyse the new circuit in the Laplace domain

$$I_1(s) = sC_1V_1(s)$$

$$I_2(s) = sC_2V_2(s)$$

$$I_3(s) = \frac{V_1(s)}{R_1}$$

$$\frac{V_1(s) - V_2(s)}{R_2} = I_2(s)$$

$$V_2(s) = V_1(s) - R_2I_2(s)$$

$$I_2(s) = sC_2\left[V_1(s) - R_2I_2(s)\right]$$

$$I_2(s) = \frac{sC_2}{1 + sC_2R_2}V_1(s)$$

$$Q_{in}(s) = I_1(s) + I_2(s) + I_3(s)$$

$$Q_{in}(s) = \left[sC_1 + \frac{sC_2}{1 + sC_2R_2} + \frac{1}{R_1}\right]V_1(s)$$

$$Q_{in}(s) = \frac{sC_1(1+sC_2R_2)R_1 + sC_2R_1 + sC_2R_2 + 1}{(sC_2R_2+1)R_1} V_1(s)$$

$$G(s) = \frac{V_1(s)}{Q_{in}(s)} = \frac{(sC_2R_2+1)R_1}{sC_1(1+sC_2R_2)R_1 + sC_2R_1 + sC_2R_2 + 1}$$

$$\boxed{G(s) = \frac{R_1[sC_2R_2+1]}{s^2[C_1R_1C_2R_2] + s[C_1R_1 + C_2R_1 + C_2R_2] + 1}}$$

(d) Let us assume the transfer function $G(s)$ is in the following form and approximate it as FOPDT model from its step response.

$$G(s) = \frac{K_p(\tau_1 s + 1)}{\tau^2 s^2 + 2\xi\tau s + 1} = \frac{0.5(10s+1)}{2000s^2 + 410s + 1}$$

The FOPDT approximation model is as follows, the parameters can be found from the step response at *Figure 18.*

$$G_p(s) = \frac{K_p e^{-\theta_p s}}{\tau_p s + 1}$$

From *Figure 18*, $t_{1/3}$ and $t_{2/3}$ can be found as

$$t_{1/3} = 160 \ secs$$
$$t_{2/3} = 439 \ secs$$

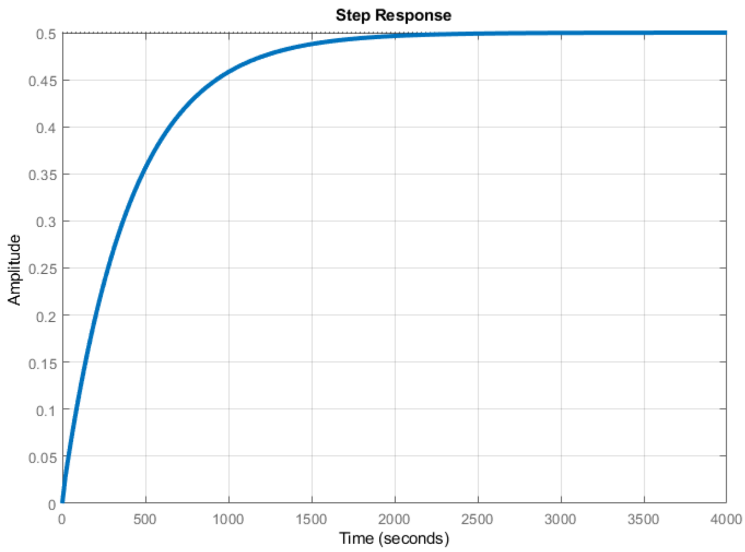


Figure 18: Step Response of the System

$$\boxed{K_p = 0.5}\ ,\ from\ the\ equation$$

$$\boxed{\tau_p = \frac{1}{0.7}(t_{2/3} - t_{1/3}) = \frac{279}{0.7} = 398,5714\ secs}$$

$$\boxed{\theta_p \approx t_{1/3} - 0.4\tau p = 0.5714\ secs}$$

(e) IMC Based PID Controller for FOPDT plant;

$$K_c = \frac{\tau_p + \frac{\theta_p}{2}}{K_p\left(\tau_c + \frac{\theta_p}{2}\right)} = \frac{398.8571}{0.5(199.5714)} \approx 3.997\ for\ \tau_c = \frac{\tau_p}{2}$$

$$K_c = \frac{\tau_p + \frac{\theta_p}{2}}{K_p\left(\tau_c + \frac{\theta_p}{2}\right)} = \frac{398.8571}{0.5(50.1071)} \approx 15.92\ for\ \tau_c = \frac{\tau_p}{8}$$

$$\tau_I = \tau_p + \frac{\theta_p}{2} = 398.8571\ secs$$

$$\tau_D = \frac{\tau_p\theta_p}{2(\tau_p + \frac{\theta_p}{2})} = \frac{227.7436}{2(398.8571)} \approx 0.2854\ secs$$

$$\boxed{G_{PID}(s) = K_c\left(1 + \frac{1}{\tau_I}\frac{1}{s} + \tau_D s\right)}$$

The simulink model can be seen at *Figure 19* and the step response of the bare system and system with PID controller can be seen at *Figure 20*.

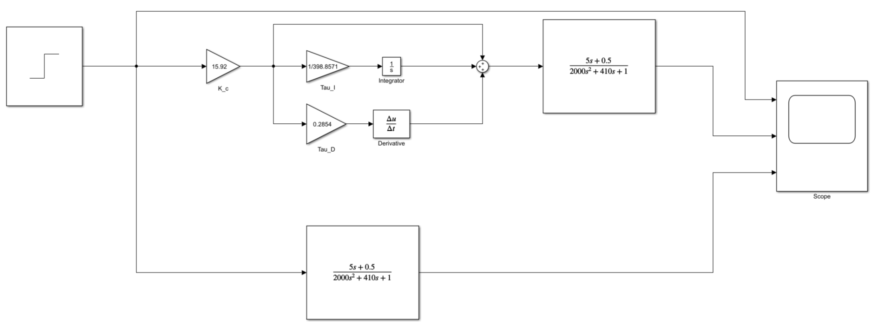


Figure 19: Simulink Model for the system and system with PID controller

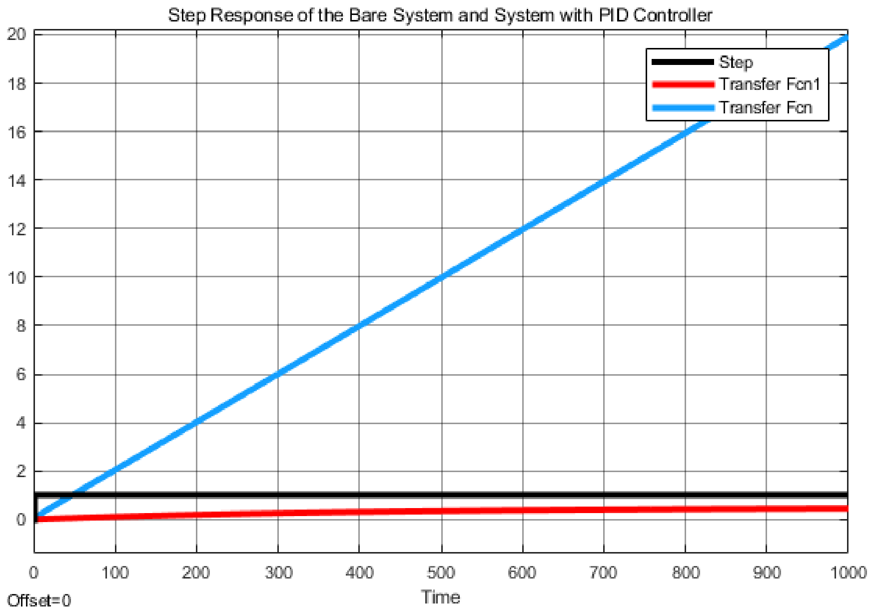


Figure 20: Step Response of the Bare System and System with PID Controller

4. 4

(a)

$$y(t) = (1 - e^{-(t-t_0-\theta_p)/T_p})\ u^{(t-t_0-\theta_p)}\ \Delta y + y_0$$

Let us derive the formulae for T_p and θ_p.

$$u^{(t-t_0-\theta_p)} = 1 \quad for \quad t > t_0 + \theta_p$$

$$\frac{(y(t) - y_0)}{\Delta y} = (1 - e^{-(t-t_0-\theta_p)/T_p})$$

$$t = -T_p\ ln\left(1 - \frac{y(t) - y_0}{\Delta y}\right) + t_0 + \theta_p$$

Derive for $t_{1/3}$:

$$y(1/3) = y_0 + \frac{1}{3}\Delta y$$

$$ln\left(\frac{2}{3}\right) \approx -0.4$$

$$t_{1/3} = 0.4T_p + t_0 + \theta_p$$

Derive for $t_{2/3}$:

$$y(2/3) = y_0 + 2/3\delta y$$

$$ln\left(\frac{1}{3}\right) \approx -1.1$$

$$t_{2/3} = 1.1T_p + \tau_0 + \theta_p$$

Now, we can derive them as:

$$t_{2/3} - t_{1/3} = 0.7T_p => T_p = 1/0.7(t_{2/3} - t_{1/3})$$

$$\theta_p = t_{1/3} - 0.4T_p - t_0$$

(b) i. The on-off controller mechanism is shutting down the operation when the desired point reached and turning otherwise. This acts like a digital system. If the operation is not critical in the sense of overshooting, the dead band is very good idea. Dead band solve the problem of turning on and off frequently. It could be problem for some components to change its state very quickly and the dead band solve this problem. An example of on-off controller is light with sensor, that turn on and off automatically when a person or an object appears.

ii. A self-regulating process may observe non-zero e_{ss} when the a P-only control is chosen according to set point. This set point should be equal to controller output bias.

iii. For the problem of driving a car, the optimal (in the sense of ITAE) P-control parameters for

A. The intention is to change the speed of the car regularly where the set point tracking (servo control) is objective. In this case, set point is changing regularly. This might be good correlation:

$$\boxed{K_c = \frac{0.202}{K_p}\left(\frac{\theta_p}{\tau_p}\right)^{-1.219}}$$

B. The slope of the road changes as the intention is to drive the car at a constant speed where disturbance rejection (regulatory control) is the objective. In this case, it is expected to constant set point and driving way is according to this set point. This might be good correlation:

$$\boxed{K_c = \frac{0.490}{K_p}\left(\frac{\theta_p}{\tau_p}\right)^{-1.084}}$$

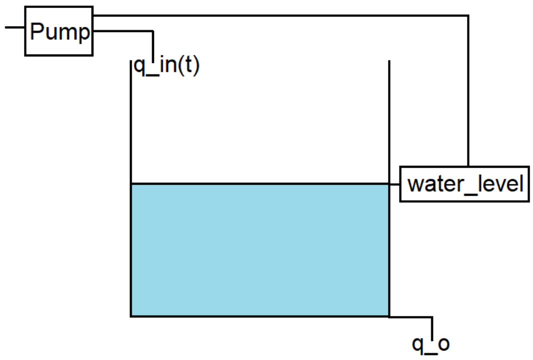
iv. **ITAE**, **IAE** and **ISE** controller parameters are found for a certain type of a system by the minimization of

- $\int_0^\infty t|e(t)|dt$
- $\int_0^\infty |e(t)|dt$
- $\int_0^\infty e^2(t)dt$

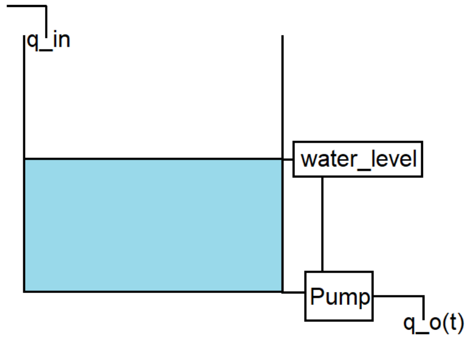
respectively.

- **ITAE** penalizes initial errors less than those at a later time since the terms is proportional with time.
- **ISE** penalizes large errors more since square term increases more with large error.
- **IAE** penalizes all errors equally.

v. The configuration of the pump in order to have a reverse acting happens when the actuator is a pump, which supplies water to the tank and direct acting controllers while taking feedback from the system. Basic set-up for reverse acting control can be seen at *Figure 21a*. While the configuration of the pump in order to have a direct acting happens when the actuator is a pump, which draining water from it while taking feedback from the system. Basic set-up for direct acting control can be seen at *Figure 21b*. If the pump is in reverse acting configuration and the controller is in direct acting mode, the system goes either to drain up and fill up. Because it increase and increase when an error happens or decrease and decrease like a positive feedback



(a) Reverse Acting Control

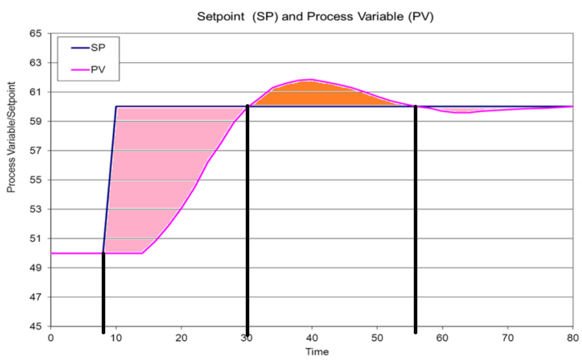


(b) Direct Acting Control

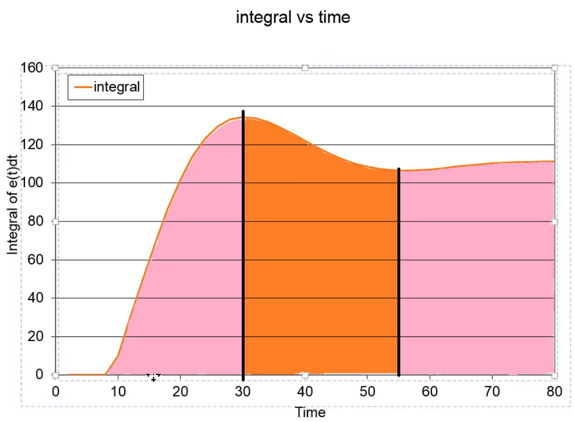
Figure 21: Reverse & Direct Acting Control

vi. The solution is *Anti Wind-up* or *Reset Wind-up*
   - Stop the integral when the actuator saturation is detected
   - Stop the integral term when the controller is **off**

vii. An example graph for SP (Setpoint) and PV (Process-Variable) from provided video lectures can be seen from *Figure 22a* while its corresponding error's integral graph can be seen at *Figure 22b*. As can be interpreted from the graphs, as the $SP > PV$ the integral of error increases [notice the purple sections] on the contrary while the $SP < PV$, the integral of error decreases [notice the orange section].



(a) SP (Setpoint) and PV (Process-Variable)



(b) Integral of Error $e((t)$

Figure 22: SP-PV and Integral of Error Graphs

viii. The solution is *Anti Wind-up* or *Reset Wind-up*
   - Stop the integral when the actuator saturation is detected
   - Stop the integral term when the controller is **off**

ix. The correlation between decay ratio and settling time is when decay ratio increases, settling time also increase. Since higher decay ration means that the system takes more time to settle. The correlation between rise time and peak time is also same. When the rise time increases, reaching the peak value is also increase.

x. Remember that

$$e(t) = SP - PV$$

If the set point is changed stepwise, an impulsive term due to differentiation would occur at the output of the controller. Due to this impulsive term, a pheromone called "derivative kick" would occur, that is large amount of initial CO value would exist for a single sampling time. The kick should be eliminated for the sake of mechanical problem that the system faces due to this phenomena. The effect can be eliminated by applying differentiation only on PV.

xi.

Appendices

A Source Code for Matlab Part of Question 2

```
%% Q2 Part d%

s=tf('s');
G= 1/((2*s+1)*(s+1));
rlocus(G);
margin(G);
[Gm,Pm,Wcg,Wcp]=margin(G);

%% Q2 Part e%
clear all;
close all;
s=tf('s');
G= 1/((s+1)*(s+1)*(s+1));
rlocus(G);
figure;
margin(G);
[Gm,Pm,Wcg,Wcp]=margin(G);

%% Q2 Part f%
clear all;
close all;
s=tf('s');
G= 1/((s+1)*(s+1)*(s+1));
K=8; %It is founded at part e
sys=K*G/(1+K*G);
impulse(sys);
xlim([0,12]);

%% Q2 Part g%
clear all;
close all;
s=tf('s');
u=tf(1);
G= 1/((s+1)*(s+1)*(s+1));
Kc=4.7; Ti= 1.815; Td=0.454; %It is founded at part f
PID=Kc*(1+(1/Ti)*(1/s)+Td*s);
sys=PID*G/(1+PID*G);
```

```
step(sys);
hold on;
step(u);
xlim([0,12]);

%% Q2 Part h%
clear all;
close all;
s=tf('s');
G= 1/((s+1)*(s+1)*(s+1));
K=8; %It is founded at part e
sys=K*G/(1+K*G);
figure;
subplot(2,1,1);
step(sys);
xlim([0,15]);
s=tf('s');
G= 1/((s+1)*(s+1)*(s+1));
Kc=4.7; Ti= 1.815; Td=0.454; %It is founded at part f
PID=Kc*(1+(1/Ti)*(1/s)+Td*s);
sys=PID*G/(1+PID*G);
subplot(2,1,2);
step(sys);
xlim([0,15]);
title('The step response for part 2e and 2g with PID');
```