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Q1. (25 Points)
This question is composed of <u>two independent</u> parts.

Part I. Using the Taylor series approximation(s) to the exponential function, <u>derive</u> the First Order Plus Deadtime (FOPDT) approximation to the higher order linear process model given by

$$G(s) = \frac{K(-0.1s+1)}{(5s+1)(2s+1)(0.5s+1)}$$

$z = 10$

$\boxed{P_1 = -1/5}$ $P_2 = -1/2$ $P_3 = -2$ → dominant one

(Hint: Note that $e^{-\theta s} = 1/e^{\theta s}$.)

$e^{-\theta s} = 1 - \theta_s + \frac{\theta_s^2}{2} - \frac{\theta_s^3}{6} + \frac{\theta_s^4}{24} - - -$ // from Tayle series exp.

Using our knowladge that the higher order systems can be approximated to a lower order system with dead time, we can approximate

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$G(s)$ as

$G(s) = \frac{K e^{-\theta_d s}}{(5s+1)} = \frac{K}{(5s+1)} e^{-\theta_{d_2}} \cdot e^{-\theta_{d_3}} = \frac{K}{5s+1} \cdot e^{-(-10+2.5)s} = \cancel{\boxed{\frac{K e^{+7.5s}}{5s+1}}}$

* for $G_1(s) = \frac{K_1}{\underbrace{(\tau_s+1)}_{\text{dominant one}}(\tau_2 s+1) - - \tau_s n+1} \simeq \frac{K_1 e^{-\theta_d s}}{(\tau_1 s+1)}$ $\boxed{\theta_d = \sum_{n \neq 1} \tau_n}$

$G_2(s) = \frac{1}{K_2}(-0.1s+1) \simeq \frac{1}{K_2} e^{+10s}$

$\frac{1}{G_2(s)} = G_3(s) = \frac{K_2'}{(0.1s+1)} \simeq K_2' e^{-\theta_{d_2} s}$ $\boxed{\theta_{d_2} = -10}$

$\boxed{G_2(s) \simeq e^{+10s}}$ w/ $K_2 = 1$

$G_3(s) = \frac{K_3}{(5s+1)(2s+1)(0.5s+1)} \simeq \frac{K_3}{5s+1} e^{-\theta_{d_3} s}$ $\boxed{\theta_{d_3} = -\frac{1}{2} - 2 = +2.5}$

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Q1. Part II.

Consider each one of the following process models. By inspection, indicate with justification whether or not the model can be reasonably expressed by a First Order Plus Deadtime(FOPDT) model (YES/NO). For each acceptable case, also determine the best estimate time delay θ and the time constant τ.

(a) $\dfrac{K}{(10s+1)(10s+1)}$

(b) $\dfrac{K}{(10s+1)(8s+1)(s+1)}$ $-1/10$ $-1/8$ -1

(c) $\dfrac{K}{(10s+1)(s+1)^2}$

(d) $\dfrac{K}{(10s^2+11s+1)}$

(e) $\dfrac{K}{(100s^2+10s+1)}$

a $\rightarrow$ $\boxed{\text{NO}}$ / there is no one dominant pole / already in SOPDT $\rightarrow$ / $\theta = 0$

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b $\rightarrow$ can be approximated but the closeness of the two poles ($s_1 = -1/10$) & ($s_2 = -1/8$) effect the closeness of approximation to original system a lot

$\rightarrow$ $\boxed{\text{NO}}$

3/3

c $\rightarrow$ $\boxed{\text{Yes}}$ $\boxed{s_1 = -1/10}$ is a good dominant pole $\rightarrow$ $\boxed{\tau_c = 10}$

$\boxed{\theta_c = -1 - 1 = -2}$

3/3

d $\rightarrow$ $= \dfrac{K}{(10s+1)(s+1)}$ $\rightarrow$ $\boxed{\text{Yes}}$ $s_1 = -1/10$ is dominant pose

$\rightarrow$ $\boxed{\tau_d = 10}$ & $\boxed{\theta_c = -1}$

3/3

Complex conjugate

e $\rightarrow$ the roots of denominator is imaginary, i.e., $P_{1,2} = \frac{-1}{20} \mp \frac{\sqrt{3}}{20} J$

Also no dominant pole.

$\boxed{\rightarrow \text{NO}}$

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Q2. (25 points)
A process has a transfer function

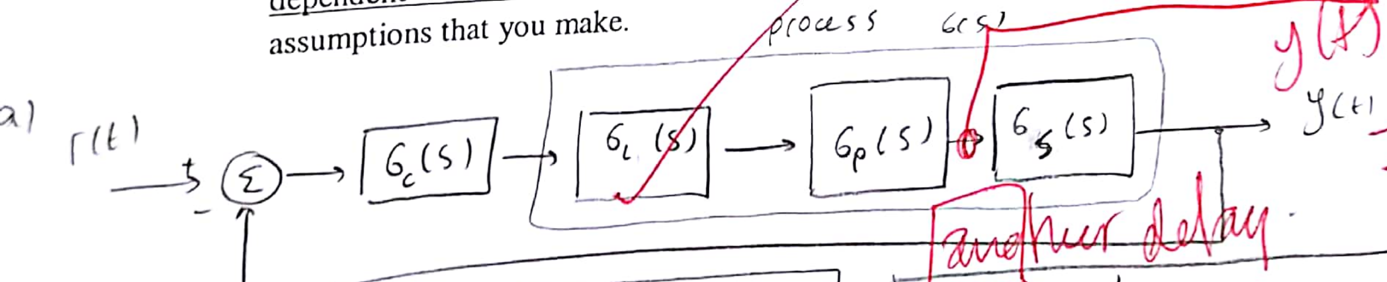
$$G(s) = \frac{1}{(10s+1)(5s+1)}$$

Where time constants are in seconds. The process is to be controlled from a control center through a communication link. The link introduces a 0.5 sec. latency (time delay) each way. The process output is measured through a sensor (mounted on the process side) with first order dynamics and with a gain of 2 and a time constant of 1 sec.

(a) Construct the block diagram of the closed-loop controlled system described.

(b) Using Direct Design method, derive a full Proportional-Integral-Derivative (PID) controller for the plant such that the closed loop system exhibits first order response with time constant $\tau = 4$ secs. and with a suitable amount of pure delay. Give the PID controller in dependent ideal PID form. Show all your steps and clearly indicate all assumptions that you make.

a)



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b) $G_L(s) = e^{-0.5s}$ & $G_s(s) = \frac{2}{s+1}$ & $G_p(s) = \frac{1}{(10s+1)(5s+1)}$

$G_c(s) = K_c + \frac{K_c}{\tau_I}\frac{1}{s} + K_c \tau_D s$

→ from this point G(s) of mine is diff than G(s) = Gp(s) given | A | Sorry for inconvenience

w/ given $\tau_c = 4$ secs / let us assume $u_{ss,0}' = 0$ for convenience

$G(s) = G_L(s)\, G_p(s)\, G_s(s) = \frac{2e^{-0.5s}}{(s+1)(10s+1)(5s+1)} \cong \boxed{\frac{2e^{-1.7s}}{10s+1}}$ FOPDT model for process

$s=-1$ $s=-0.1$ $s=-0.2$

$\frac{1}{(s+1)(10s+1)(5s+1)} \cong \frac{1}{10s+1} e^{-1.1s}$ [from Apprx of higher order poly to lower order polyns w/ dead time ~ from Taylor series exp.]

$\tau_p = 10$

$\theta_p = 1.7$

$\tau_c = 4$ given

$K_p = 2$

$K_c = \frac{1}{K_p}\frac{\tau_p}{\theta_p + \tau_c} = \frac{1}{2}\frac{10}{5.7} \cong 0.877$

$\tau_I = \tau_p + 0.5\theta_p = 10 + \frac{1.7}{2} = 10.85$ secs

$\tau_{..}$ $\tau_p \theta_p$ $10.(1.7)$

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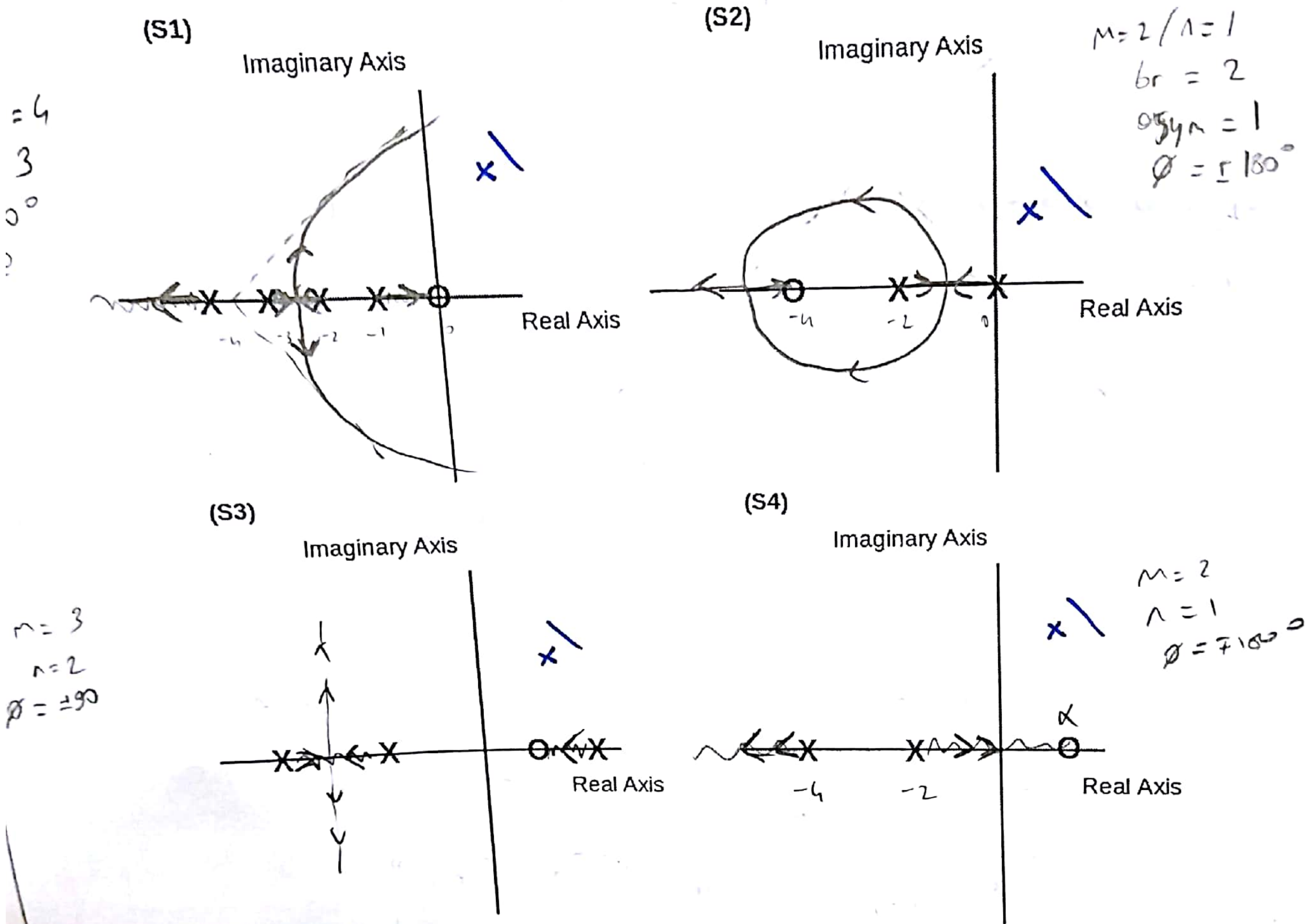
Q3. (25 Points)
Consider the four independent open-loop process models with pole-zero plots given in the following figure.



Part I.

(a) For a proportional controller, ($K_p > 0$), sketch the approximate root-locus plots for each case directly on the figures above. ✓

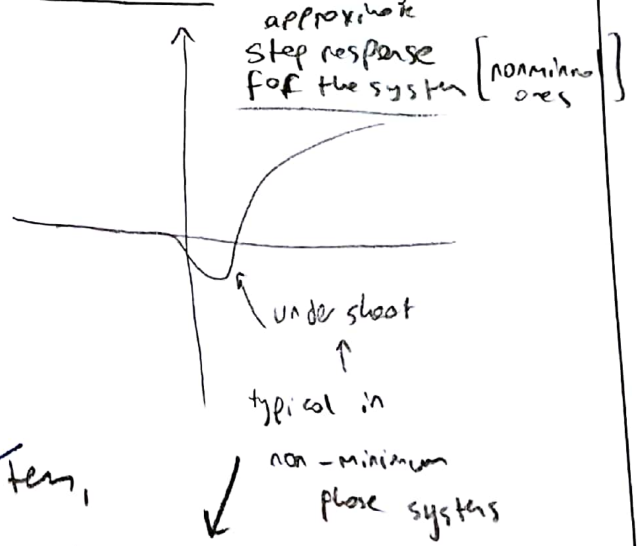
(b) State the system(s) which are minimum-phase / non-minimum phase. Justify your answer.

(S1) ⟶ Minimum-phase ✓

(S2) ⟶ Minimum-phose ✓

(S3) ⟶ non-minimum phose ✓

(S4) ⟶ Minimum-phose ✗



* By ~~guessing the step response of the system, only (S3) might show unfamilior response like above which includes undershoot. Other systems are more familier to us which we analysed their responses in EE302.~~

Not a scientific reasoning

(c) Consider each system. Indicate whether or not one can use the Zygler-Nichols (ZN) Continuous Oscillations method for designing a PID controller for the system? Justify your answer in each case. ×1

the method can be used for (S1) & (~~S4~~) since the system can become oscillatory by increasing gain. [Root locus plot crosses $j\omega$-axis]

On the contrary, for the same reasons, the method cannot be used on (S2) & (S3) +0.5 +0.5

• One might try to use (S2) w/ $K=0 \rightarrow G_c(s)=0$ [unusable] !

(d) Assume the above systems are given as black boxes such that a bump test (step response) method is to be used to identify FOPDT models. Indicate the systems for which this method can/cannot be used. Justify your answers.

• the test can be applied for (S1) & (~~S4~~) for small K_c values - for higher gain, these systems becomes unstable, thus, the step response would no longer valuable for design purposes. ×1 It is an open-loop test!

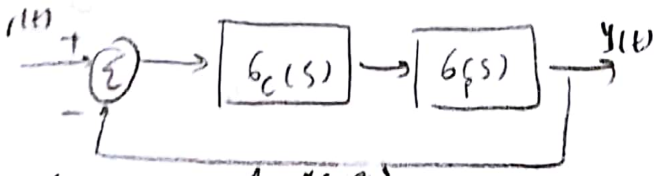
• the test can be applied on (~~S2~~) for all K_c values. [no stability issue.]

• the test can not be applied on (S3), since it is unstable for all K_c values. ×1

Part II. Now, you will modify the Zygler-Nichols tuning method to design a Dependent Ideal Proportional-Derivative (PD) controller for (S4). The following extra information is given:

- The open-loop plant has two poles at (-2, -4) and a zero at (2).
- The steady-state process gain of the open-loop plant is given as (+1).
- The transfer function of the Dependent Ideal PD controller is given as $G_c(s) = K_p\left(1 + \frac{K_d}{K_p}s\right)$ and the ratio of $\frac{K_d}{K_p}$ is constant at (-1/4).

(a) Obtain the transfer function of open-loop plant, (S4).



$$G_p(s) = \frac{1\cdot(s-2)}{(s+2)(s+4)}$$

OLTF $= G_c(s)\cdot G(s)$ ×1

$$\boxed{G_{OL}(s) = K_p\left(1-\frac{1}{4}s\right)\left(\frac{(s-2)}{(s+2)(s+4)}\right)}$$

$$\boxed{G_{OL}(s) = K_p\frac{(4-s)(s-2)}{4(s+2)(s+4)}}$$ ×1

(b) Draw the root-locus for varying $K_p > 0$. (while the open-loop plant is being controlled by the given PD controller). Find the ultimate gain, K_{pu}.

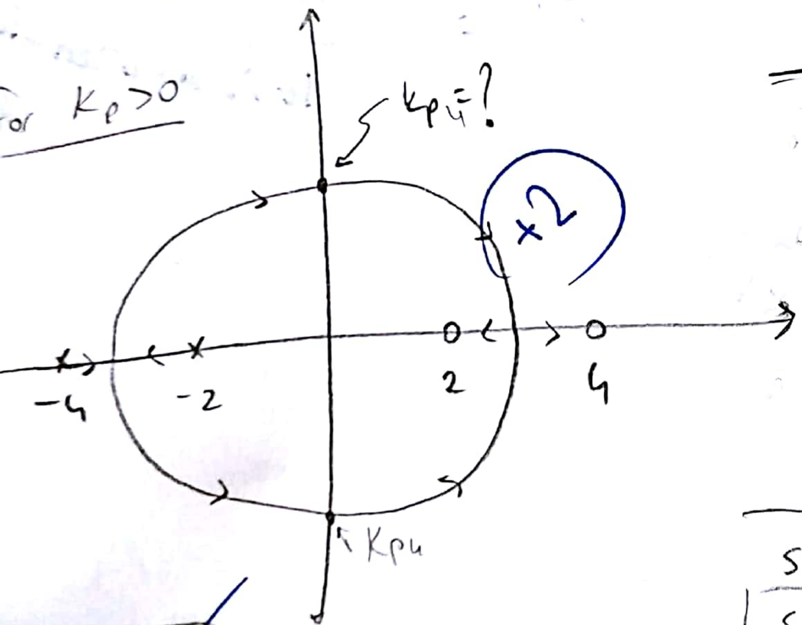
$$G_{OL}(s) = K_p \left[\frac{-1}{4} \frac{(s-2)(s-4)}{(s+2)(s+4)} \right]$$

$m = 2$ | # of branches $= 2$

$n = 2$

# of asymptotes $= 0$

For $K_p > 0$



$\times 2$

$q(s) = 1 + G_{OL}(s)$

$4(s^2+6s+8) - k_p(s^2-6s+8)$

$q(s) = 4(s+2)(s+4) - K_p(s-2)(s-4)$

let us use Routh-Hurwitz for finding stability edge

$+1$ for the effort

s^2	$4-k_p$	$32-8k_p$
s^1	$24+6k_p$	0
s^0	$32-8k_p$	0

$\to k_p = -4$

$\to \boxed{k_p = 4}$

let us equate the row to zero

let us equate to zero

$k_p \leq 0$

$\boxed{K_{pu} = 4}$ is the ultimate point.

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(c) Considering the K_{pu} you have found in the previous step, and the fact that we have modified the ZN procedure, what are the final values of the PD controller gains K_p and K_d?

take as

$K_p = K_{pu} / 1.7$ [as in PID case]

$\boxed{K_p \cong 2.\cancel{3}53 = \frac{40}{17}}$

$\boxed{K_d = -\frac{1}{4} K_p \cong -0,588 = \frac{-10}{17}}$

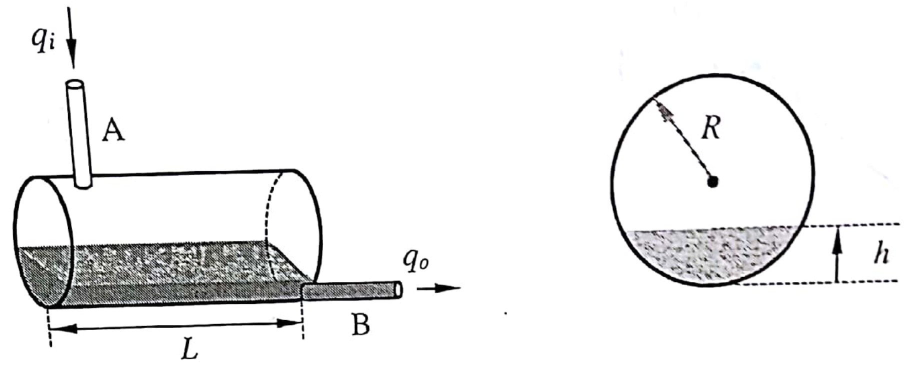
x0.5

(d) Briefly explain the differences between a conventional ZN tuning procedure and the one you utilized in this question. Comment on possible reasons why we preferred this special scheme for the open-loop plant, (S4).

~~In classical one, we have to find the period of the oscillations as the K_{pu} is used as controller to find derivative parameter. Since we are not able to see the step response experimentally, we would have to compute step response by hand and it would take very long time. Also, even if we can see the response experimentally or by computation, it wouldn't stabilize by itself since it is unstable for all K_{pu} values?~~

Q4. (25 Points)

Consider the liquid level system depicted in the Figure below where the diagonal and cross-section views are given. A horizontal cylindrical tank with fixed length (L) and Radius (R) is supplied with liquid from the inlet (A) with in-flow rate $q_i(t)$. Liquid flows out of the tank with outflow rate $q_o(t)$. The liquid level in the tank at any time is given by $h(t)$.



a) Derive the differential equation model for the height of liquid, $h(t)$, in the tank at any time with the inlet and outlet volumetric flow rates, $q_i(t)$ and $q_o(t)$ as model inputs (Outflow does not necessarily depend on liquid height).

Conservation low :

the accumulation = (in-flow) − (out-flow) 1

$$\boxed{A = \pi r^2 \frac{\alpha}{360} - (r-h)x}$$

$$x = \sqrt{r^2 - (r-h)^2}$$

$$\boxed{\alpha = \cos^{-1}(r-h/r)}$$

Volume : $T(t) = A \cdot h$

$$\frac{dT(t)}{dt} = q_{in}(t) - q_o(t)$$

$$\frac{d}{dt}\left[\left(\frac{\pi r^2}{360}\cos^{-1}\left[\frac{r-h}{r}\right] - [r-h]\sqrt{r^2-(r-h)^2}\right)h\right] = q_{in}(t)$$

where r = constant
h = a function of time

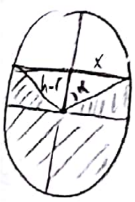
1.5

$$\frac{d}{dt}\left(\frac{\pi r^2}{360}\cos^{-1}\left(\frac{r-h}{r}\right)h\right) - \frac{d}{dt}\left(r\sqrt{r^2-(r-h)^2}\,h\right) + \frac{d}{dt}\left(\sqrt{r^2-(r-h)^2}\,h^2\right) = q_{in}(t) - q_o(t)$$

for $h(t) < r$

$$\frac{d}{dt}\left(\frac{\pi r^2}{360}h\right) - \frac{d}{dt}\left(\frac{\pi r^2}{360}\cos^{-1}\left(\frac{r-h}{r}\right)h\right) - \frac{d}{dt}\left(r\sqrt{r^2-(r-h)^2}\cdot h\right) + \frac{d}{dt}\left(\sqrt{r^2-(r-h)^2}\,h^2\right) = q_{in}(t) - q_o(t)$$

for $h(t) > r$



$$A = \frac{\pi r^2(360 - 2\alpha)}{360} + (h-r)x$$

$$\frac{d}{dt}\left[\left(\frac{\pi r^2\left(360 - \cos^{-1}\left[\frac{r-h}{r}\right]\right)}{360} + (r-h)x\right)h\right]$$

b) Assume now that the input of the system is inflow, $q_i(t)$ while the outflow is part of the system model and is given by $q_o(t) = h(t)|sin(t)|$. The process variable (output) of the system is chosen as the liquid height, $h(t)$. Answer the following questions by justifying your answers:

Is the system

(i) Stochastic or deterministic? Why?

Stochastic, since at random time t_1, the system might show different behaivour since $h(t_1) > r$ with probobility p & $h(t_1) < r$ with probobility $(1-p)$

(ii) Time-invariant or time-varying? Why?

Similarly, @ t_2, $h(t_2) < r$ while $h(t_2 + \Delta t) > r$ which results in different outputs.

So ?

(iii) Static or dynamic? Why?

Dynamic, because it has a memory

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c) Now, suppose that the outflow in the model is given as $q_o(t) = h(t)/K$ where $K > 0$ is a model constant. The system is at steady state when the liquid height is $\bar{h}$.

(i) Is this system self-regulating or integrating?

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$\boxed{\text{* Self regulating}}$ // Assuming constant q_{in} for the sake of clarification; as the h decreases, the $q_o(t)$ decreases, thus in return h increases. Similarly as the h increases, the $q_o(t)$ increases, thus in return h decreases.

(ii) What is the amount of inflow, $\bar{q}_i$, so that the system maintains the steady state?

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$$\boxed{\bar{q}_{in} = \bar{q}_o = \frac{\bar{h}}{K}}$$

no change in h.

Extra:

$\bar{h} = r$ // will be found @ part (iii)

$$\boxed{\bar{q}_{in} = r/K}$$

(iii) Find a linear approximation to the system in the form of a transfer function around the operating point $(\bar{h}, \bar{q}_i)$.

$\boxed{\tilde{h}(t) = h(t) - \bar{h}}$ $\quad \frac{d\tilde{h}(t)}{dt} = \frac{dh(t)}{dt}$ & $\tilde{h}(0) = 0$

0.5 $\boxed{\tilde{q}_i(t) = q_i(t) - \bar{q}_i}$ $\quad \frac{d\tilde{q}_i(t)}{dt} = \frac{dq_i(t)}{dt}$ & $\tilde{q}_i(t) = 0$

ssuming $\boxed{\bar{h} = r}$

$\boxed{\bar{A} = \frac{\pi r^2}{2}}$ $\quad \bar{A}\frac{d\tilde{h}(t)}{dt} = \tilde{q}_i(t) - \frac{\tilde{h}(t)}{k}$

$$\rightarrow \boxed{\frac{d\tilde{h}(t)}{dt} + \frac{1}{\bar{A}K}\tilde{h}(t) = \frac{1}{\bar{A}}\tilde{q}_i(t)}$$ First order ordinary differential equation

Linear apprx for the system ∇

~ only applicable around operating point $(\bar{h})$