FREQUENCY DOMAIN REPRESENTATION OF LTI SYSTEMS

- EULER'S FORMULA
- EIGENFUNCTIONS OF LTI SYSTEMS
    - complex exponentials, $e^{j\omega n}$, sinusoids, $\sin(\omega n + \theta)$
- FREQUENCY RESPONSE
- FREQUENCY RESPONSE IS PERIODIC WITH 2π
- MAGNITUDE AND PHASE
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- FOR A REAL LTI SYSTEM, FREQUENCY RESPONSE IS ALSO CONJUGATE SYMMETRIC WRT π
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- FREQUENCY RESPONSE OF PURE DELAY
- LINEAR PHASE SYSTEMS
- “SUDDENLY” APPLIED COMPLEX EXPONENTIAL INPUTS

In the context of LTI systems, “Frequency Domain” refers to the representation of signals and analysis of LTI systems in terms of sinusoidal signals.

EULER’S FORMULA

Sinusoidal expressions are related to complex exponential expressions (Euler’s formula).

$$e^{j\omega n} = \cos(\omega n) + j\sin(\omega n)$$

$$\cos(\omega n) = \frac{1}{2}\left(e^{j\omega n} + e^{-j\omega n}\right)$$

$$\sin(\omega n) = \frac{1}{2}\left(e^{j\omega n} - e^{-j\omega n}\right)$$

Note:

Euler’s identity: $e^{j\pi} + 1 = 0$

Read the story of Leonhard Euler. You will find many interesting things about fundamental mathematical concepts, their rise/foundation and their relationships.

Born 1707, Basel, Switzerland
1727-1741, Imperial Russian Academy of Sciences, St. Petersburg, Russia
1741-1766, Berlin Academy, Berlin, Prussia
1766-1783 (death), St. Petersburg Academy, St. Petersburg, Russia

Leonhard Euler

SWISS MATHEMATICIAN

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Leonhard Euler, (born April 15, 1707, Basel, Switzerland—died September 18, 1783, St. Petersburg, Russia), Swiss mathematician and physicist, one of the founders of pure mathematics. He not only made decisive and formative contributions to the subjects of geometry, calculus, mechanics, and number theory but also developed methods for solving problems in observational astronomy and demonstrated useful applications of mathematics in technology and public affairs.

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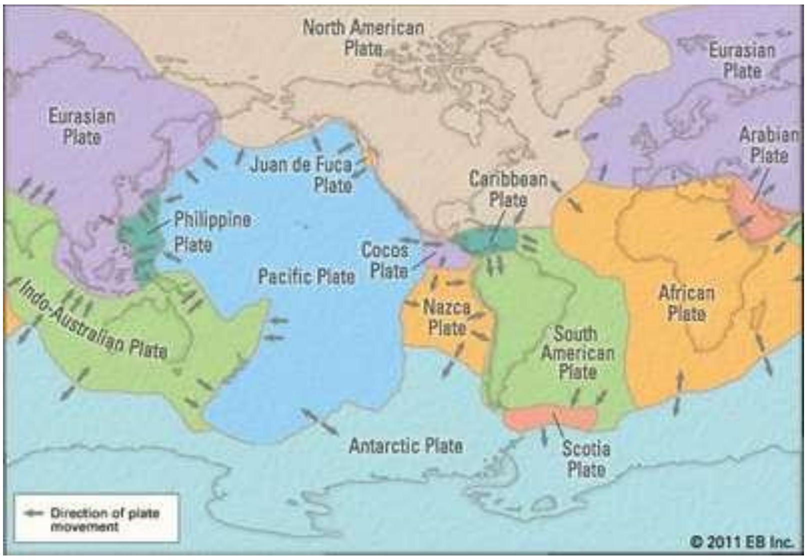


plate tectonics: Euler’s contributions

In the 18th century, Swiss mathematician Leonhard Euler showed that the movement of a rigid body across the surface of a sphere can be described as a rotation (or turning) around an axis that goes through the centre of the sphere, known as…

Euler’s mathematical ability earned him the esteem of Johann Bernoulli, one of the first mathematicians in Europe at that time, and of his sons Daniel and Nicolas. In 1727 he moved to St. Petersburg, where he became an associate of the St. Petersburg Academy of Sciences and in 1733 succeeded Daniel Bernoulli to the chair of mathematics. By means of his numerous books and memoirs that he submitted to the academy, Euler carried integral calculus to a higher degree of perfection, developed the theory of trigonometric and logarithmic functions, reduced analyticaloperations to a greater simplicity, and threw new light on nearly all parts of pure mathematics. Overtaxing himself, Euler in 1735 lost the sight of one eye. Then, invited by Frederick the Great in 1741, he became a member of the Berlin Academy, where for 25 years he produced a steady stream of publications, many of which he contributed to the St. Petersburg Academy, which granted him a pension.

In 1748, in his *Introductio in analysin infinitorum*, he developed the concept of function in mathematical analysis, through which variables are related to each other and in which he advanced the use of infinitesimals and infinite quantities. He did for modern analytic geometry and trigonometry what the *Elements* of Euclid had done for ancient geometry, and the resulting tendency to render mathematics and physics in arithmetical terms has continued ever since. He is known for familiar results in elementary geometry—for example, the Euler line through the orthocentre (the intersection of the altitudes in a triangle), the circumcentre (the centre of the circumscribed circle of a triangle), and the barycentre (the "centre of gravity," or centroid) of a triangle. He was responsible for treating trigonometric functions—i.e., the relationship of an angle to two sides of a triangle—as numerical ratios rather than as lengths of geometric lines and for relating them, through the so-called Euler identity ($e^{i\theta} = \cos\theta + i\sin\theta$), with complex numbers (e.g., 3 + 2Square root of$\sqrt{\boxed{-1}}$). He discovered the imaginary logarithms of negative numbers and showed that each complex numberhas an infinite number of logarithms.

Euler's textbooks in calculus, *Institutiones calculi differentialis* in 1755 and *Institutiones calculi integralis* in 1768–70, have served as prototypes to the present because they contain formulas of differentiation and numerous methods of indefinite integration, many of which he invented himself, for determining the work done by a force and for solving geometric problems, and he made advances in the theory of linear differential equations, which are useful in solving problems in physics. Thus, he enriched mathematics with substantial new concepts and techniques. He introduced many current notations, such as Σ for the sum; the symbol *e* for the base of natural logarithms; *a*, *b* and *c* for the sides of a triangle and A, B, and C for the opposite angles; the letter *f* and parentheses for a function; and *i* for Square root of$\sqrt{\boxed{-1}}$. He also popularized the use of the symbol π (devised by British mathematician William Jones) for the ratio of circumference to diameter in a circle.

After Frederick the Great became less cordial toward him, Euler in 1766 accepted the invitation of Catherine II to return to Russia. Soon after his arrival at St. Petersburg, a cataract formed in his remaining good eye, and he spent the last years of his life in total blindness. Despite this tragedy, his productivity continued undiminished, sustained by an uncommon memory and a remarkable facility in mental computations. His interests were broad, and his *Lettres à une princesse d'Allemagne* in 1768–72 were an admirably clear exposition of the basic principles of mechanics, optics, acoustics, and physical astronomy. Not a classroom teacher, Euler nevertheless had a more pervasive pedagogicalinfluence than any modern mathematician. He had few disciples, but he helped to establish mathematical education in Russia.

Euler devoted considerable attention to developing a more perfect theory of lunar motion, which was particularly troublesome, since it involved the so-called three-body problem—the interactions of Sun, Moon, and Earth. (The problem is still unsolved.) His partial solution, published in 1753, assisted the British Admiralty in calculating lunar tables, of importance then in attempting to determine longitude at sea. One of the feats of his blind years was to perform all the elaborate calculations in his head for his second theory of lunar motion in 1772. Throughout his life Euler was much absorbed by problems dealing with the theory of numbers, which treats of the properties and relationships of integers, or whole numbers (0, ±1, ±2, etc.); in this, his greatest discovery, in 1783, was the law of quadratic reciprocity, which has become an essential part of modern number theory.

In his effort to replace synthetic methods by analytic ones, Euler was succeeded by J.-L. Lagrange. But, where Euler had delighted in special concrete cases, Lagrange sought for abstract generality, and, while Euler incautiously manipulated divergent series, Lagrange attempted to establish infinite processes upon a sound basis. Thus it is that Euler and Lagrange together are regarded as the greatest mathematicians of the 18th century, but Euler has never been excelled either in productivity or in the skillful and imaginative use of algorithmic devices (i.e., computational procedures) for solving problems.

Carl B. Boyer

Wikipedia:

Leonhard Euler /ˈɔɪlər/ OY-lər;
[; 15 April 1707 – 18 September 1783)
was a Swiss mathematician, physicist, astronomer, logician and engineer who made important and influential discoveries in many branches of mathematics like infinitesimal calculus and graph theory while also making pioneering contributions to several branches such as topology and analytic number theory. He also introduced much of the modern mathematical terminology and notation, particularly for mathematical analysis, such as the notion of a mathematical function.[3] He is also known for his work in mechanics, fluid dynamics, optics, astronomy, and music theory.[4]

Euler was one of the most eminent mathematicians of the 18th century, and is held to be one of the greatest in history. He is also widely considered to be the most prolific mathematician of all time. His collected works fill 60 to 80 quarto volumes,[5] more than anybody in the field. He spent most of his adult life in Saint Petersburg, Russia, and in Berlin, then the capital of Prussia.

A statement attributed to Pierre-Simon Laplace expresses Euler's influence on mathematics: "Read Euler, read Euler, he is the master of us all."[6][7]

A Fundamental Motivation

EIGENFUNCTIONS OF LTI SYSTEMS

Let the input signal be $e^{j\omega_0 n}$, i.e. a complex exponential with frequency ω_0

$$x[n] = e^{j\omega_0 n} \longrightarrow \boxed{h[n]} \longrightarrow y[n] = ?$$

The output will be

$$y[n] = e^{j\omega_0 n} * h[n]$$

$$= \sum_{k=-\infty}^{\infty} h[k] e^{j\omega_0 (n-k)}$$

$$= e^{j\omega_0 n} \underbrace{\sum_{k=-\infty}^{\infty} h[k] e^{-j\omega_0 k}}_{H(e^{j\omega_0})}$$

$$= e^{j\omega_0 n} \, H\left(e^{j\omega_0}\right)$$

We assumed that

$$\sum_{k=-\infty}^{\infty} h[k] e^{-j\omega_0 k}$$

is finite.

Therefore, the output is the same (<u>except a complex scaling factor</u>) as the input.

The complex scale factor, $H\left(e^{j\omega_0}\right)$, is called the frequency response.

Complex exponentials or real sinusoids are called as the *eigenfunctions* of LTI systems.

Ex:

$$h[n] = \left[1 \quad -1 \quad -\frac{1}{2}\right] \quad and \quad x[n] = e^{j\frac{\pi}{3}n}$$

Type equation here.

FREQUENCY RESPONSE

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} h[k]\, e^{-j\omega k}$$

if converges, is called the "frequency response".

It is the complex valued gain of a LTI system to a complex exponential of particular "frequency".

$$
\begin{aligned}
&\dots \\
&+h[-2]e^{j2\omega} \\
&+h[-1]e^{j\omega} \\
&+h[0] \\
&+h[1]e^{-j\omega} \\
&+h[2]e^{-j2\omega} \\
&+\cdots
\end{aligned}
$$

FREQUENCY RESPONSE IS PERIODIC WITH 2π.

$$H\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$$

$$H\left(e^{j(\omega+k2\pi)}\right) = \sum_{n=-\infty}^{\infty} h[n]e^{-j(\omega+k2\pi)n}$$

$$= \sum_{n=-\infty}^{\infty} h[n]\, e^{-j\omega n}\ \underbrace{e^{-jk2\pi n}}_{=1}$$

$$= H\left(e^{j\omega}\right)$$

MAGNITUDE AND PHASE

$$H(e^{j\omega}) = |H(e^{j\omega})| e^{j\measuredangle H(e^{j\omega})}$$

$$= |H(e^{j\omega})| e^{j\,\theta(\omega)}$$

$$\Rightarrow$$

$$H\left(e^{j\omega_0}\right)e^{j\,\omega_0 n} = \left|H\left(e^{j\omega_0}\right)\right|e^{j\,\theta(\omega_0)}e^{j\,\omega_0 n}$$

$$= \left|H\left(e^{j\omega_0}\right)\right|e^{j\left(\omega_0 n+\theta(\omega_0)\right)}$$

$$H\left(e^{j\omega}\right) = H_R\left(e^{j\omega}\right) + jH_I\left(e^{j\omega}\right)$$

$$\left|H\left(e^{j\omega}\right)\right| = \sqrt{H_R^2(e^{j\omega}) + H_I^2(e^{j\omega})} \qquad magnitude$$

$$\measuredangle H\left(e^{j\omega}\right) = \tan^{-1}\frac{H_I\left(e^{j\omega}\right)}{H_R(e^{j\omega})} \qquad phase$$

FOR A REAL LTI SYSTEM, FREQUENCY RESPONSE IS CONJUGATE SYMMETRIC

Real LTI system means impulse response is real valued.

$$H^*\left(e^{j\omega}\right) = \left(\sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}\right)^*$$

$$= \sum_{n=-\infty}^{\infty} h[n]e^{j\omega n}$$

$$= H\left(e^{-j\omega}\right)$$

$$H^*\left(e^{j\omega}\right) = H\left(e^{-j\omega}\right)$$

Ex:

$$h[n] = [1\ 1]$$

Magnitude is even symmetric
Phase is odd symmetric

$$H^*\left(e^{j\omega}\right) = \left(\left|H\left(e^{j\omega}\right)\right|e^{j\theta(\omega)}\right)^*$$
$$= \left|H\left(e^{j\omega}\right)\right|e^{-j\theta(\omega)}$$

$$H\left(e^{-j\omega}\right) = \left|H\left(e^{-j\omega}\right)\right|e^{j\theta(-\omega)}$$

$$\left|H\left(e^{j\omega}\right)\right|e^{-j\theta(\omega)} = \left|H\left(e^{-j\omega}\right)\right|e^{j\theta(-\omega)}$$

THE RESPONSE TO A SINUSOID (REAL LTI SYSTEM)

$$x[n] = \cos(\omega n) \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$x[n] = \cos(\omega n) = \frac{1}{2}\left(e^{j\omega n} + e^{-j\omega n}\right)$$

since LTI

$$y[n] = \frac{1}{2}\left(H\left(e^{j\omega}\right)e^{j\omega n} + H\left(e^{-j\omega}\right)e^{-j\omega n}\right)$$

Hence

$$y[n] = \frac{1}{2}\left|H\left(e^{j\omega}\right)\right|\left(e^{j\theta(\omega)}e^{j\omega n} + e^{-j\theta(\omega)}e^{-j\omega n}\right)$$

$$= \left|H\left(e^{j\omega}\right)\right| \cos\left(\omega n + \theta(\omega)\right)$$

In summary,

$$\cos(\omega n) \xrightarrow{\text{Real LTI system}} \left|H\left(e^{j\omega}\right)\right| \cos\left(\omega n + \theta(\omega)\right)$$

Ex: Let

$$h[n] = \delta[n] + \delta[n-1]$$

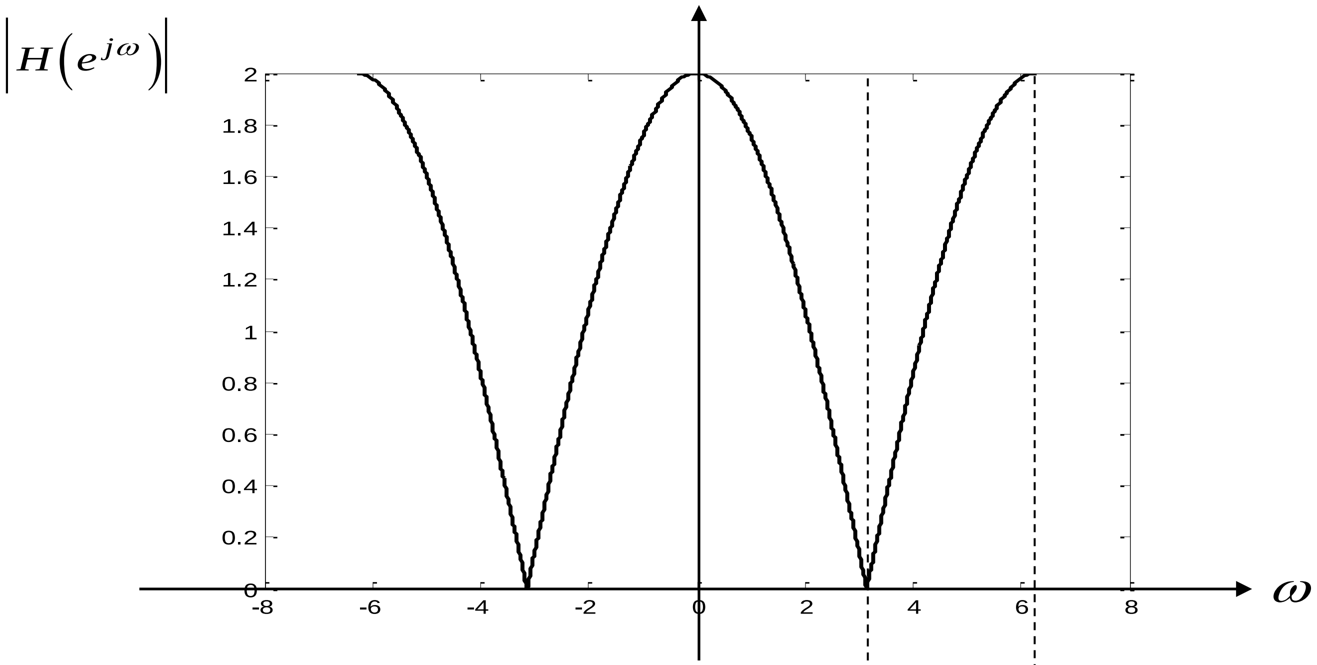
$$H\left(e^{j\omega}\right) = 1 + e^{-j\omega}$$

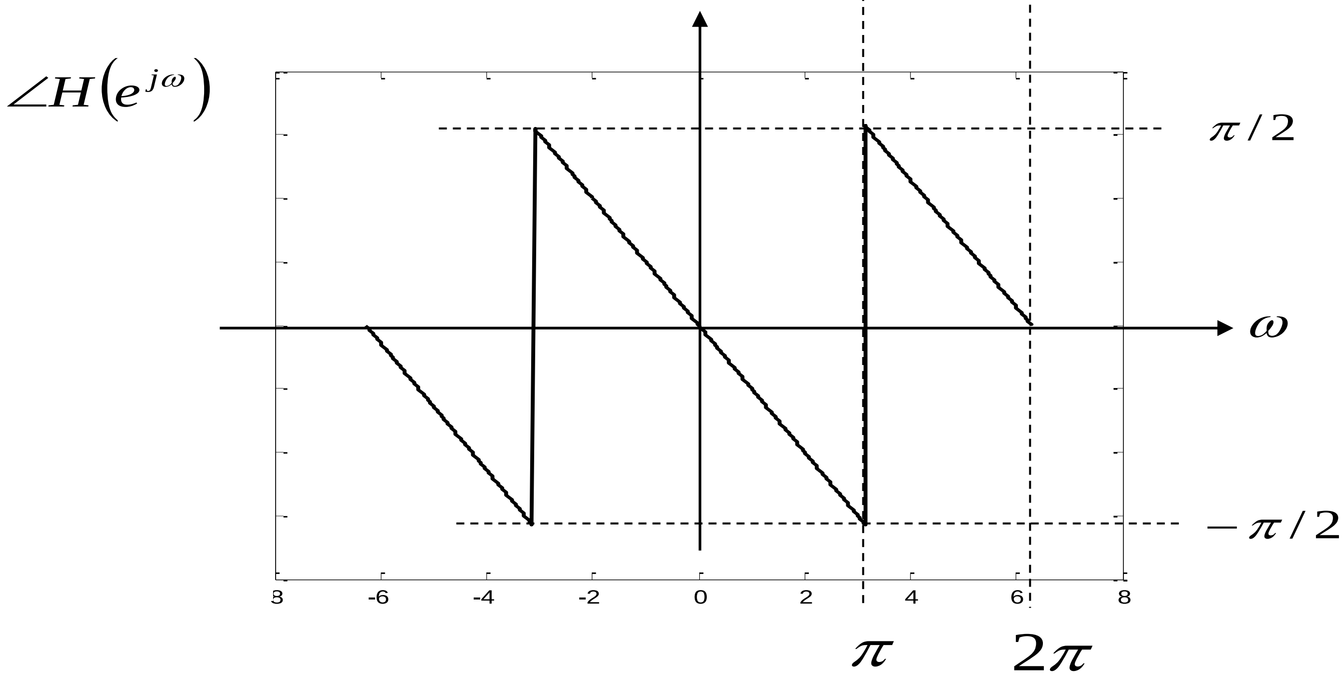
$$= e^{-j\frac{\omega}{2}}\left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}}\right)$$

$$= e^{-j\frac{\omega}{2}}\ 2\cos\left(\frac{\omega}{2}\right)$$

The magnitude and phase are

$$\left|H\left(e^{j\omega}\right)\right| = \left|2\cos\left(\frac{\omega}{2}\right)\right| \qquad \measuredangle H\left(e^{j\omega}\right) = \begin{cases} -\dfrac{\omega}{2} & \cos\left(\dfrac{\omega}{2}\right) \geq 0 \\ -\dfrac{\omega}{2} \pm \pi & \cos\left(\dfrac{\omega}{2}\right) < 0 \end{cases}$$





```
clear all
close all

b = [1 0 0 0 1];
a = 1;

[H,w] = freqz(b,a,1024, 'whole');
% H = fftshift(H)
% w = w - pi;


plot(w,abs(H))
xlabel('rad/sample')
% plot(w/pi,abs(H))
% xlabel('\times \pi  rad/sample')
ylabel('|H|')
figure
plot(w,angle(H))
xlabel('rad/sample')
ylabel('angle of H, rad')
% plot(w/pi,angle(H)/pi)
% xlabel('\times \pi rad/sample')
% ylabel('angle of H, \times \pi  rad')
```

Let the input be $$x[n] = \cos\left(\frac{\pi}{5}n\right)$$

The output is $$y[n] = 1.9021\cos\left(\frac{\pi}{5}n - \frac{\pi}{10}\right)$$

Since

$$H\left(e^{j\frac{\pi}{5}}\right) = 2\cos\left(\frac{\pi}{10}\right)e^{-j\frac{\pi}{10}}$$
$$= 1.9021e^{-j\frac{\pi}{10}}$$

or

If the input is $x[n] = \cos\left(\frac{6\pi}{5}n\right)$

The output is $y[n] = 0.6180\cos\left(\frac{6\pi}{5}n - \frac{6\pi}{10} + \pi\right)$

Since

$$H\left(e^{j\frac{6\pi}{5}}\right) = 2\cos\left(\frac{6\pi}{10}\right)e^{-j\frac{6\pi}{10}}$$

$$= -0.6180\, e^{-j\frac{6\pi}{10}}$$

$$= 0.6180\, e^{-j\frac{6\pi}{10}+\pi}$$

FOR A REAL LTI SYSTEM, FREQUENCY RESPONSE IS ALSO CONJUGATE SYMMETRIC WRT π

$$H(\omega) = H(\omega + 2\pi) \text{ and } H(\omega) = H^*(-\omega)$$

$$\Rightarrow H^*(-\omega) = H(\omega + 2\pi)$$

$$\omega \rightarrow \omega - \pi$$

$$\Rightarrow H^*(\pi - \omega) = H(\pi + \omega)$$

USEFUL TIPS

$$e^{jx} + e^{jy} = e^{j\frac{x+y}{2}}\left(e^{j\frac{x-y}{2}} + e^{-j\frac{x-y}{2}}\right)$$

$$= 2e^{j\frac{x+y}{2}}\cos\left(\frac{x-y}{2}\right)$$

Special Case: Symmetric coefficients:

$$a + be^{-j\omega} + be^{-j2\omega} + ae^{-j3\omega} = e^{-j\frac{3\omega}{2}}\left(ae^{j\frac{3\omega}{2}} + be^{j\frac{\omega}{2}} + be^{-j\frac{\omega}{2}} + ae^{-j\frac{3\omega}{2}}\right)$$

$$= 2e^{-j\frac{3\omega}{2}}\left(b\cos\left(\frac{\omega}{2}\right) + a\cos\left(\frac{3\omega}{2}\right)\right)$$

in general, let $a_k = a_{N-k} \qquad k = 0,1,\ldots,N$

$$\sum_{k=0}^{N} a_k e^{-jk\omega} = \begin{cases} 2\, e^{-j\frac{N}{2}\omega} \displaystyle\sum_{k=1}^{\frac{N+1}{2}} a_k \cos\left(\left(k-\frac{1}{2}\right)\omega\right) & \text{if } N \text{ is odd} \\ e^{-j\frac{N}{2}\omega}\left(a_{\frac{N}{2}} + 2\displaystyle\sum_{k=1}^{\frac{N}{2}} a_k \cos(k\omega)\right) & \text{if } N \text{ is even} \end{cases}$$

or, if $a_k = -a_{N-k} \qquad k = 0,1,\dots,N$

$$\sum_{k=0}^{N} a_k e^{-jk\omega} = \begin{cases} j\,2\,e^{-j\frac{N}{2}\omega} \displaystyle\sum_{k=1}^{\frac{N+1}{2}} a_k \sin\left(\left(k-\frac{1}{2}\right)\omega\right) & \text{if } N \text{ is odd} \\ j\,2\,e^{-j\frac{N}{2}\omega} \left(\displaystyle\sum_{k=1}^{\frac{N}{2}} a_k \sin(k\omega)\right) & \text{if } N \text{ is even} \end{cases}$$

Note that $a_{\frac{N}{2}} = 0$ when N is even.

FREQUENCY RESPONSE OF PURE DELAY

A pure delay:

$$y[n] = x[n - \Delta]$$

Let

$$x[n] = e^{j\omega n}$$

Then

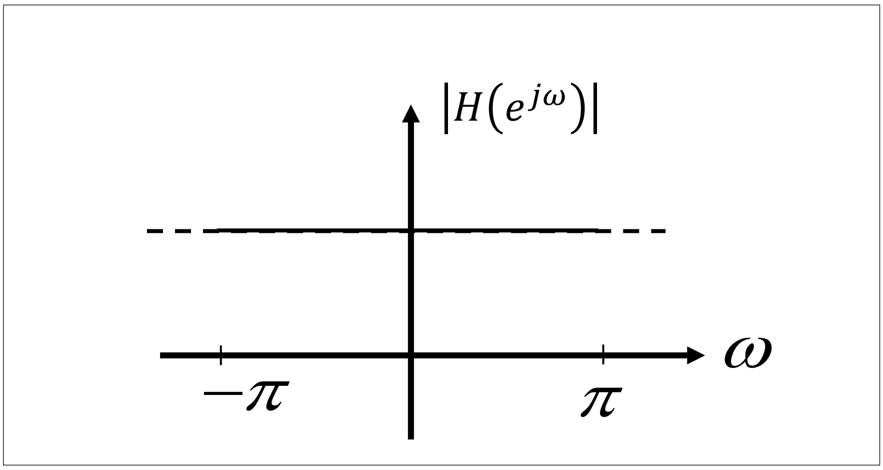
$$y[n] = e^{j\omega(n-\Delta)}$$

$$= e^{-j\omega\Delta} e^{j\omega \mathrm{n}}$$

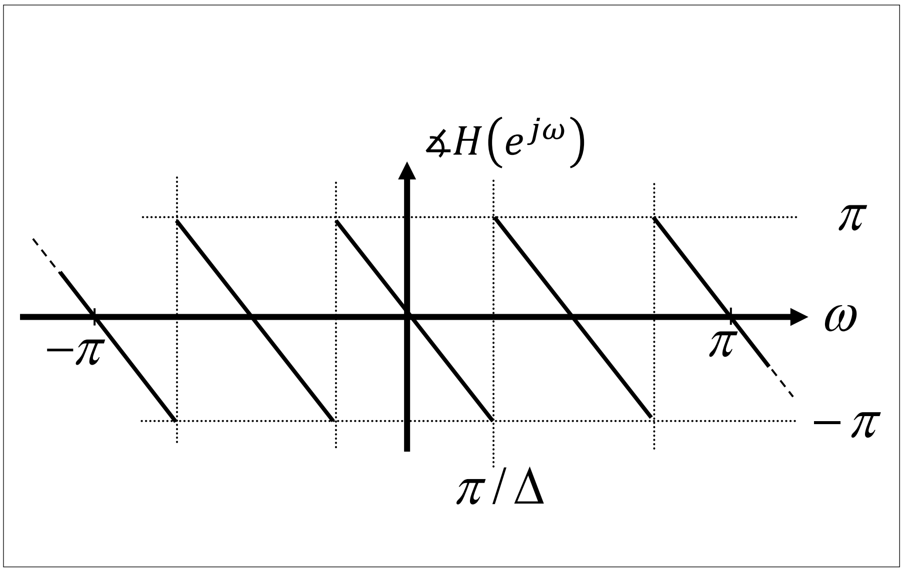
$$\Rightarrow H\left(e^{j\omega}\right) = e^{-j\omega\Delta}$$

$$\left|H\left(e^{j\omega}\right)\right| = 1 \qquad \measuredangle H\left(e^{j\omega}\right) = -\omega\Delta$$

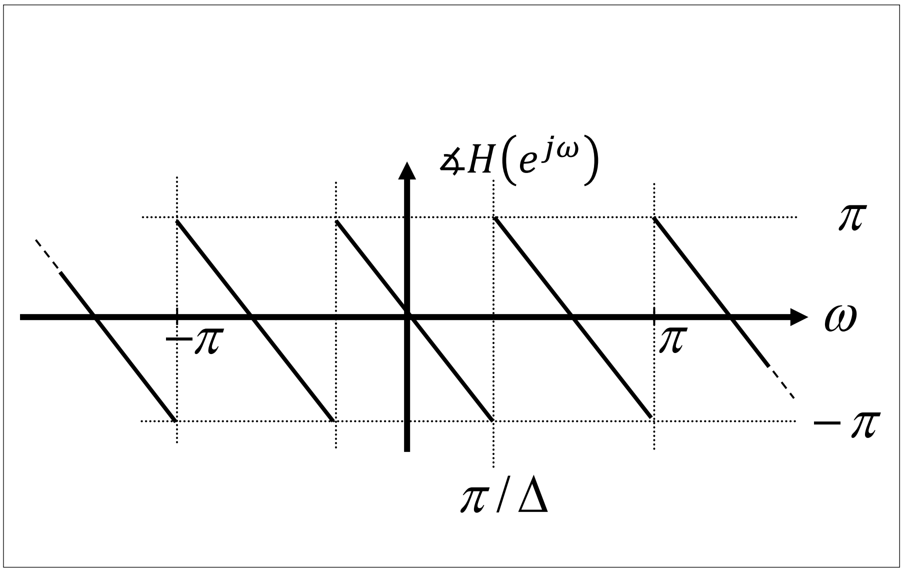
Frequency response magnitude and phase of pure delay



$\Delta = 4$



$\Delta = 3$



It can also be computed as,

$$h[n] = \delta[n - \Delta]$$

$$H\left(e^{j\omega}\right) = \sum_{n=-\infty}^{\infty} \delta[n - \Delta]\, e^{-j\omega n}$$

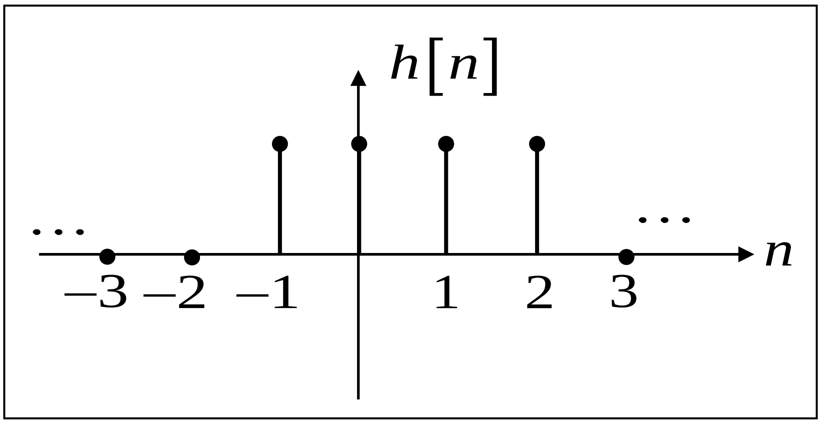
$$= e^{-j\omega\Delta}$$

LINEAR PHASE SYSTEMS

The phase of the frequency response of a pure delay system is a “**linear”** function of frequency.

Such systems are called “**linear phase**” systems.

Ex: Frequency response of a “MOVING AVERAGE” system.



$$y[n] = \frac{1}{4}(x[n+1] + x[n] + x[n-1] + x[n-2])$$

$$= \frac{1}{4}\sum_{k=-1}^{2} x[n-k]$$

$$h[n] = \frac{1}{4}\sum_{k=-1}^{2} \delta[n-k]$$

$$H\left(e^{j\omega}\right) = \frac{1}{4}\left(e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega}\right)$$

$$= \frac{1}{4}e^{j\omega}\left(1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega}\right)$$

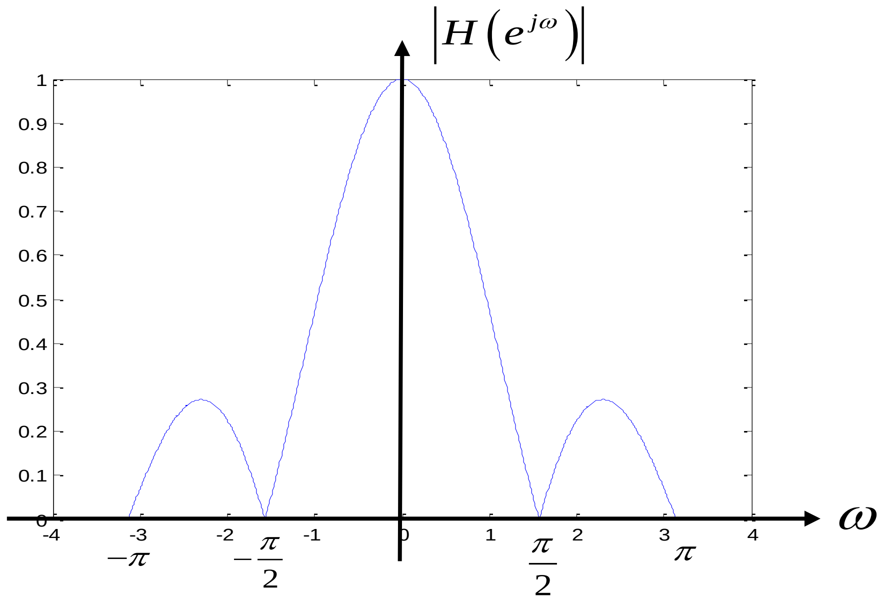
$$= \frac{1}{4}e^{j\omega}\sum_{n=0}^{3} e^{-j\omega n}$$

$$= \frac{1}{4}\ e^{j\omega}\ \frac{1 - e^{-j4\omega}}{1 - e^{-j\omega}}$$

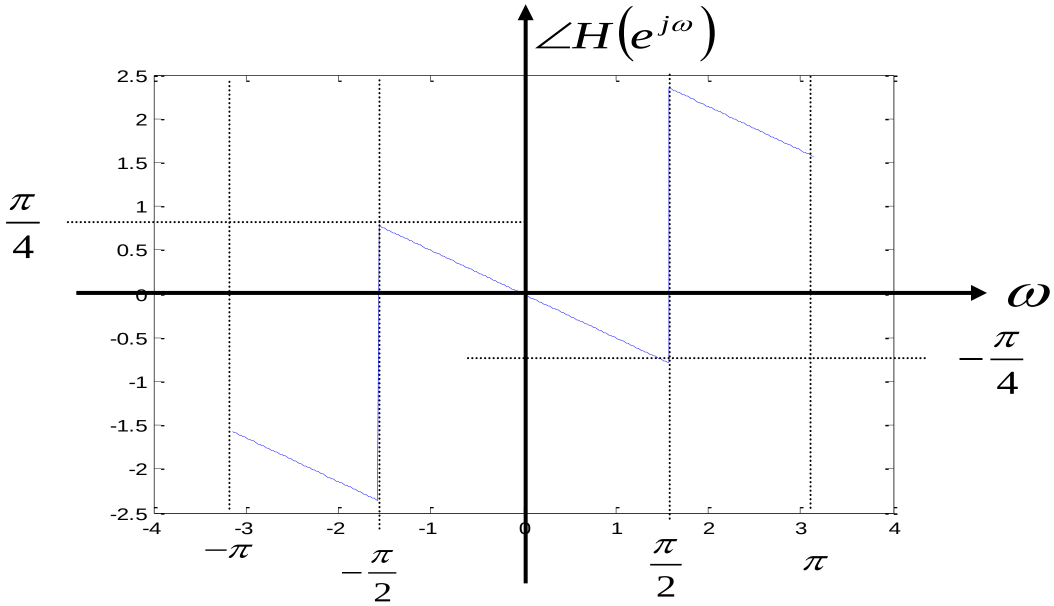
$$= \frac{1}{4}\quad e^{j\omega}\ \ \frac{e^{-j2\omega}}{e^{-j\frac{\omega}{2}}}\quad \frac{e^{j2\omega} - e^{-j2\omega}}{e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}}$$

$$= \frac{1}{4}\quad e^{-j\frac{\omega}{2}}\qquad \frac{\sin(2\omega)}{\sin\left(\frac{\omega}{2}\right)}$$

Is it a linear phase system?



$\left|H\left(e^{j\omega}\right)\right|$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-4
-3
-2
-1
0
1
2
3
4
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
ω



$\angle H\left(e^{j\omega}\right)$
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
$\frac{\pi}{4}$
$-\frac{\pi}{4}$
-4
-3
-2
-1
0
1
2
3
4
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
ω

Note that, according to the “useful tip”, the frequency response above can also be written as

$$
\begin{aligned}
H\left(e^{j\omega}\right) &= \frac{1}{4}\left(e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega}\right) \\
&= \frac{1}{4}e^{j\omega}\left(1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega}\right) \\
&= \frac{1}{4}e^{j\omega} \quad e^{-j\frac{3}{2}\omega}\left(e^{j\frac{3}{2}\omega} + e^{j\frac{1}{2}\omega} + e^{-j\frac{1}{2}\omega} + e^{-j\frac{3}{2}\omega}\right) \\
&= \frac{1}{2} \quad e^{-j\frac{\omega}{2}} \quad \left(\cos\left(\frac{\omega}{2}\right) + \cos\left(\frac{3\omega}{2}\right)\right)
\end{aligned}
$$

Therefore we also have

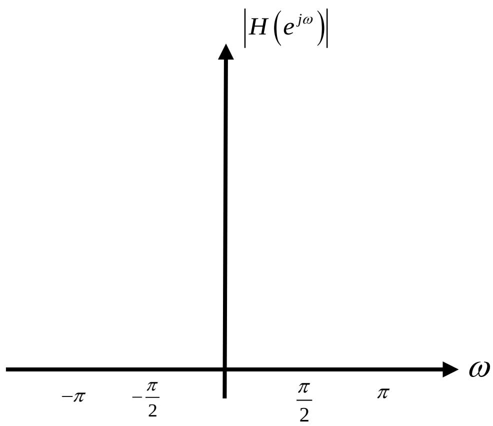
$$
\frac{\sin(2\omega)}{\sin\left(\frac{\omega}{2}\right)} = 2\ \left(\cos\left(\frac{\omega}{2}\right) + \cos\left(\frac{3\omega}{2}\right)\right)
$$

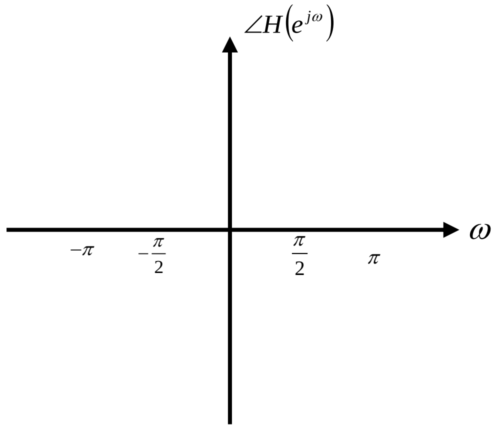
To see:

```
clear all; close all;
w = linspace(0,pi,1000);
x = 0.25 * (sin(2*w)) ./ (sin(w/2));  y = 0.5 * (cos(w/2) + cos(3 * w/2));
plot(w,x,'r'); hold; plot(w,y,'k');
```

Exercise: Using the results of last two examples, find and plot the frequency response of a LTI system whose impulse response is

$$h[n] = \frac{1}{4}\sum_{k=3}^{6} \delta[n-k]$$





“SUDDENLY” APPLIED COMPLEX EXPONENTIAL INPUTS

Study! Textbook Section 2.6.2, “Suddenly” Applied Complex Exponential Inputs

$$x[n] = e^{j\omega n}u[n]$$

LTI Sytem

$$\longrightarrow$$

$$y[n] = e^{j\omega n}\sum_{k=-\infty}^{n} h[k]e^{-j\omega k}$$

$$= \underbrace{e^{j\omega n}H\left(e^{j\omega}\right)}_{\substack{y_{ss}[n] \\ steady\ state}} - e^{j\omega n}\sum_{k=n+1}^{\infty} h[k]e^{-j\omega k}$$