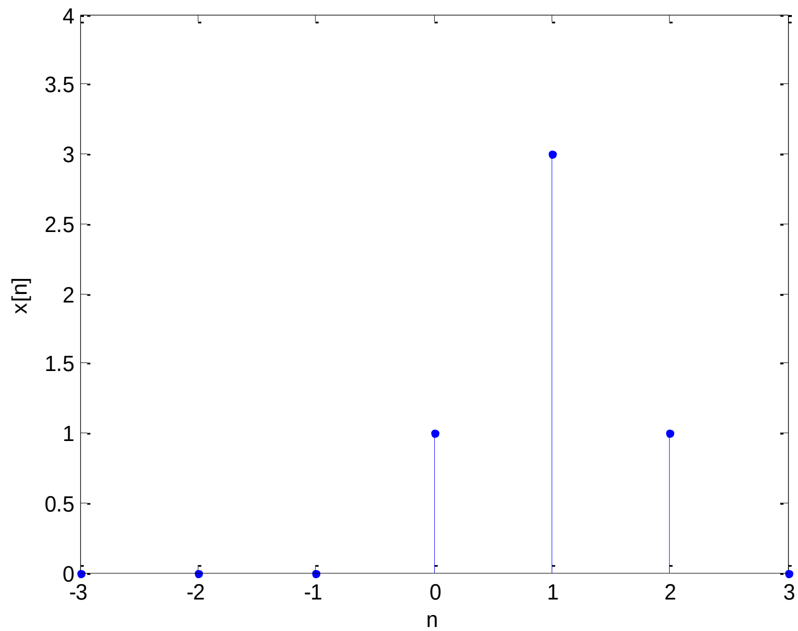
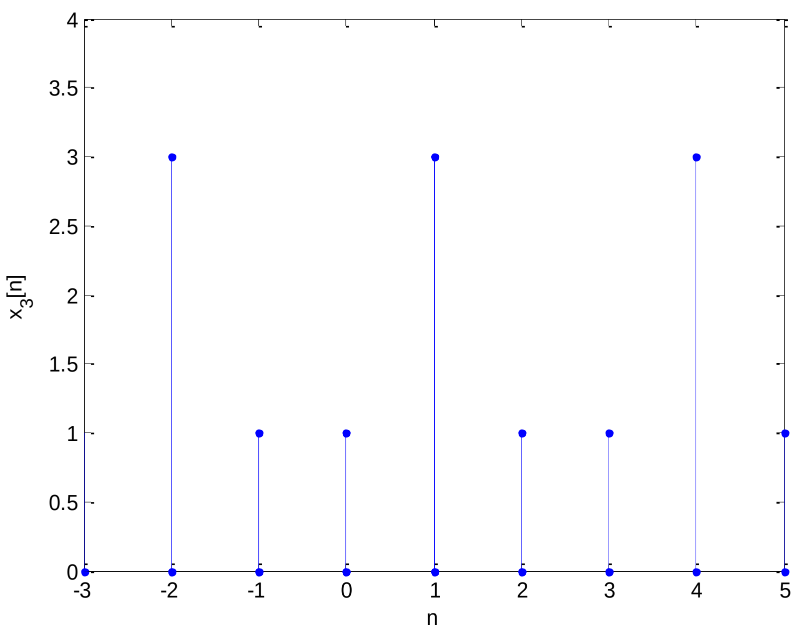
EE 430 Section 2 HW3 Answers

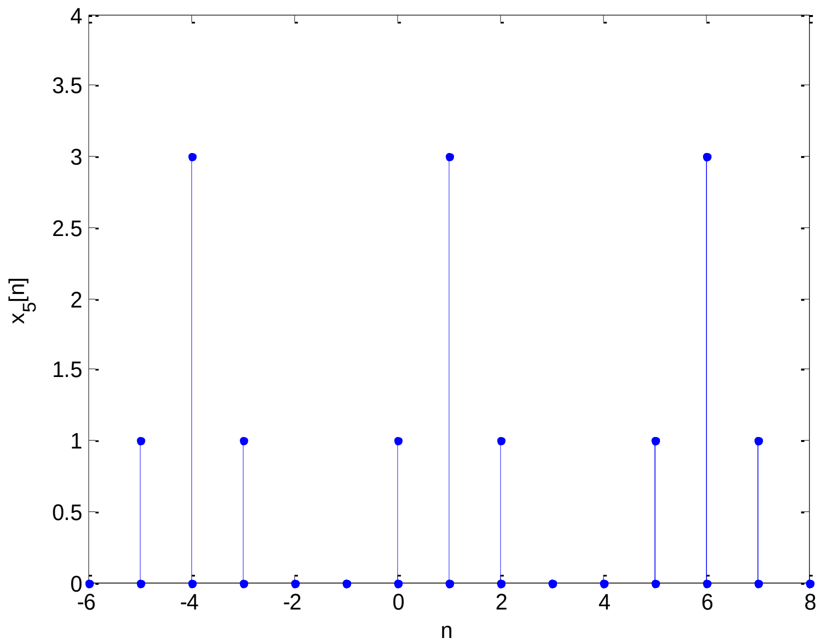
(For any questions contact Erdal Epçaçan, epcacan@metu.edu.tr, D-122)

1.

a.







b.

$$\tilde{X}_3[k] = \sum_{n=0}^{2} x[n]e^{-\frac{j2\pi kn}{3}}$$

$$\tilde{X}_3[0+3k] = 5$$
$$\tilde{X}_3[1+3k] = -1 - 1.7321j$$
$$\tilde{X}_3[2+3k] = -1 + 1.7321j \qquad for -\infty < k < \infty$$
$$x_3[n] = \frac{1}{3}\sum_{k=0}^{2} \tilde{X}_3[k]e^{\frac{j2\pi kn}{3}}$$

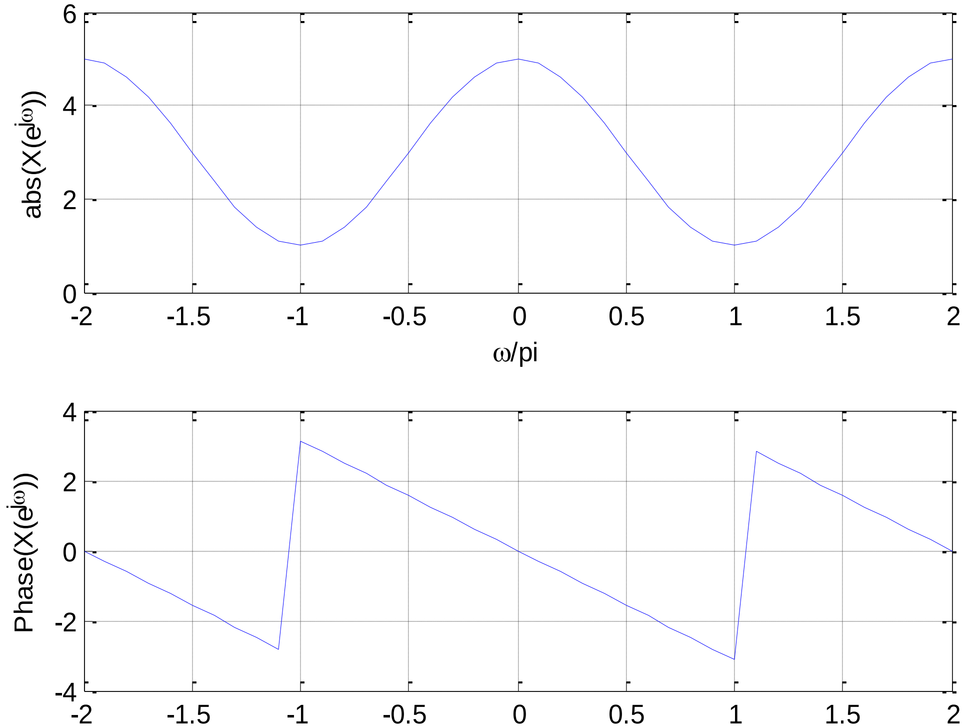
c.

$$\tilde{X}_5[k] = \sum_{n=0}^{4} x[n]e^{-\frac{j2\pi kn}{5}}$$

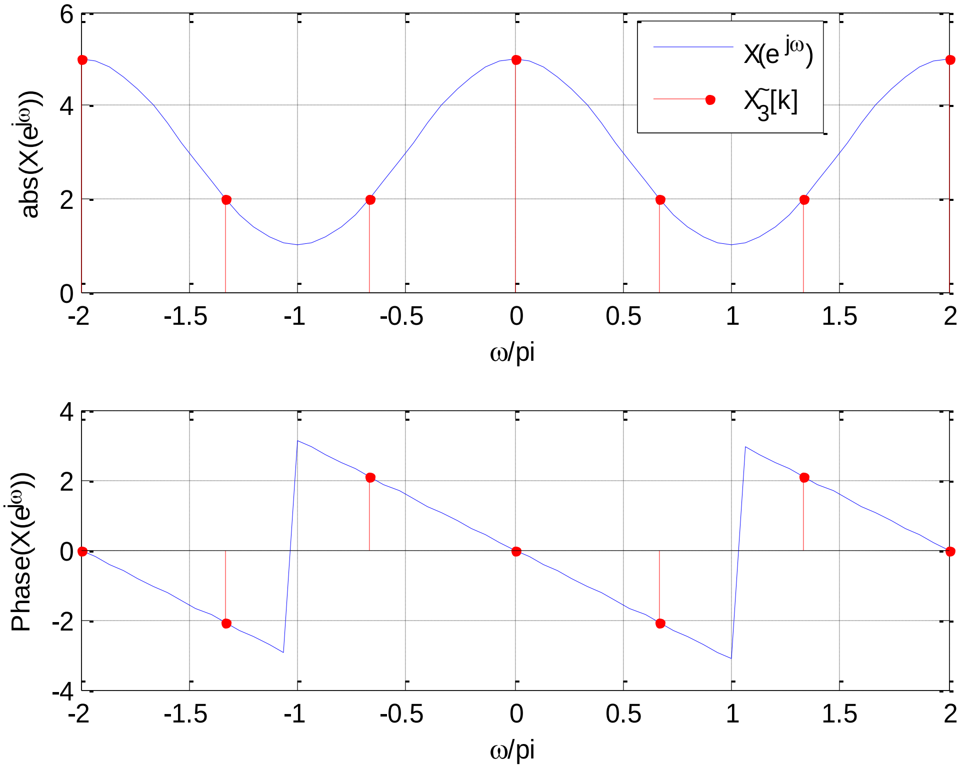
$$\tilde{X}_5[0+5k] = 5$$
$$\tilde{X}_5[1+5k] = 1.118 - 3.441j$$
$$\tilde{X}_5[2+5k] = -1.118 - 0.8123j$$
$$\tilde{X}_5[3+5k] = -1.118 + 0.8123j$$
$$\tilde{X}_5[4+5k] = 1.118 + 3.441j \quad for -\infty < k < \infty$$
$$x_5[n] = \frac{1}{5}\sum_{k=0}^{4} \tilde{X}_5[k]e^{\frac{j2\pi kn}{5}}$$

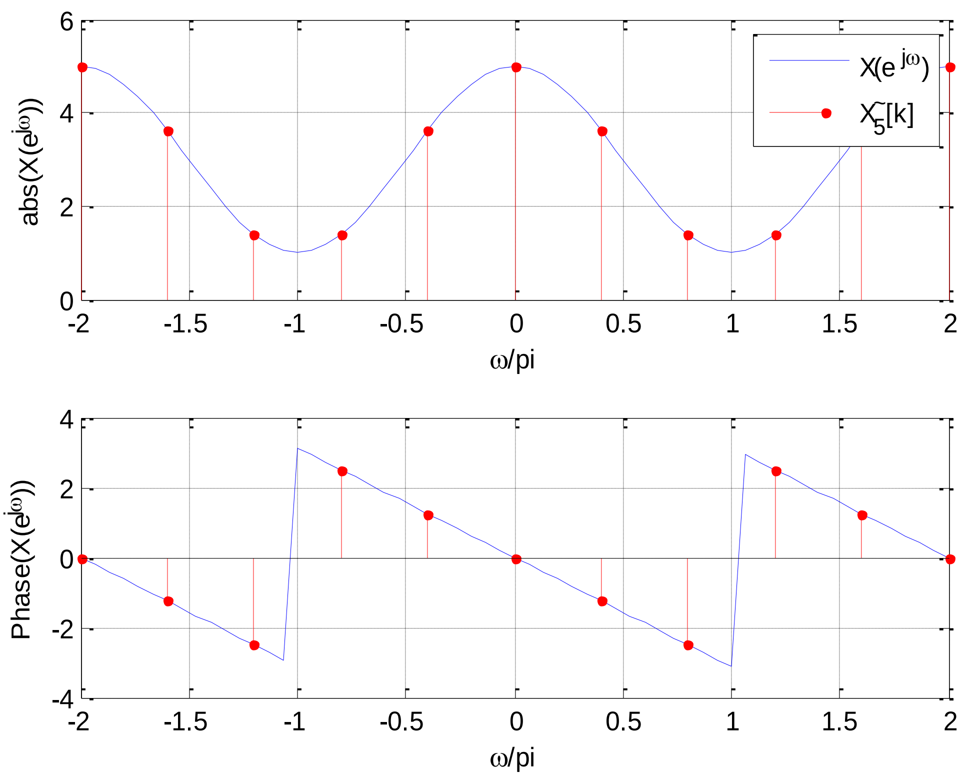
d.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} = e^{-j\omega}(3 + 2cos\omega)$$



e.





f. Refer to part a,b,c

2.

a.

$$X_3[k] = \sum_{n=0}^{2} x[n]e^{-\frac{j2\pi kn}{3}}\ , \ \ 0 \le k \le 2$$
$$X_3[0] = 5$$
$$X_3[1] = -1 - 1.7321j$$
$$X_3[2] = -1 + 1.7321j$$
$$X_3[k] = 0\ for\ k \neq 0,1,2$$

$$X_5[k] = \sum_{n=0}^{4} x[n]e^{-\frac{j2\pi kn}{5}}\ , \ \ 0 \le k \le 4$$
$$X_5[0] = 5$$
$$X_5[1] = 1.118 - 3.441j$$
$$X_5[2] = -1.118 - 0.8123j$$
$$X_5[3] = -1.118 + 0.8123j$$
$$X_5[4] = 1.118 + 3.441j$$
$$X_5[k] = 0\ for\ k \neq 0,1,2,3,4$$

b.

$$\tilde{X}_3[k] = X_3\left[\left((k)\right)_3\right]$$
$$\tilde{X}_5[k] = X_5\left[\left((k)\right)_5\right]$$

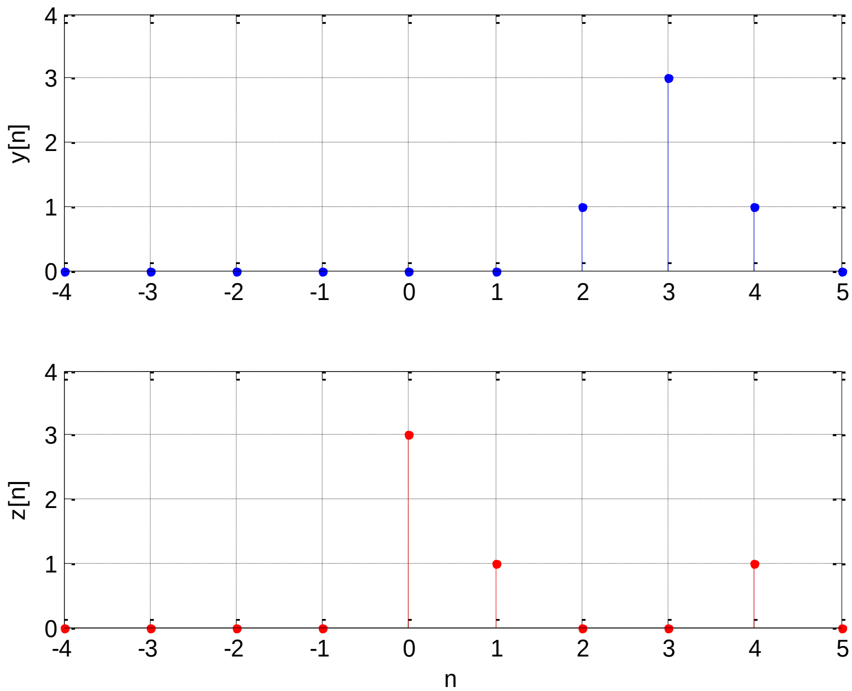
c.

d.

```
x = [1 3 1];
X3 = fft(x,3)
X5 = fft(x,5)
```

3.

a.



b.

$$y[n] = x[n-2]$$
$$z[n] = x\left[\left((n+1)\right)_5\right]$$

c. Using the properties of DFT and part b

$$Y_5[0] = 5$$
$$Y_5[1] = -2.9271 + 2.1266j$$
$$Y_5[2] = 0.4271 - 1.3143j$$
$$Y_5[3] = 0.4271 + 1.3143j$$
$$Y_5[4] = -2.9271 - 2.1266j$$
$$Y_5[k] = 0 \; for \; k \neq 0,1,2,3,4$$

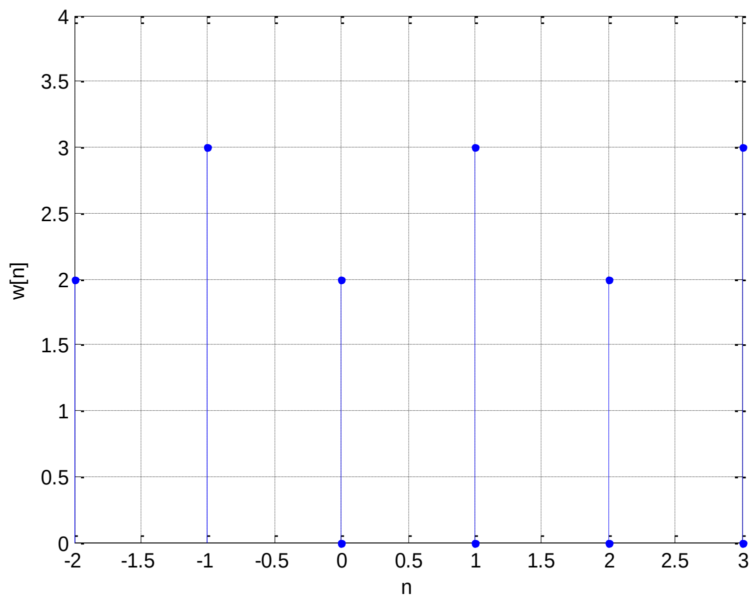
$$Z_5[0] = 5$$
$$Z_5[1] = 3.618$$
$$Z_5[2] = 1.382$$
$$Z_5[3] = 1.382$$
$$Z_5[4] = 3.618$$
$$Z_5[k] = 0 \; for \; k \neq 0,1,2,3,4$$

No they cannot have 3 point DFTs

4.

a. and b. Sampling in frequency means periodizing in time;

$$\tilde{w}[n] = \sum_{r=-\infty}^{\infty} x[n-2r]$$



5.

a. Hint : Consider the IDFT representation of impulse functions shifted by M ($\sum_{r=-\infty}^{\infty} \delta[n - rM]$)

b. When $M \geq N$ then there will be no overlapping between shifted versions of the x[n] however when M<N there will be overlapping and one period of w[n] will no longer be equal to x[n]

c.

6.

a.

$$\widetilde{w}_3[n] = \sum_{k=-\infty}^{\infty} x[n-3k]$$

$$w_3[n] = \begin{cases} \widetilde{w}_3[n] & n = 0,1,2 \\ 0 & o.w. \end{cases} = 4\delta[n] - 4\delta[n-1]$$

b.

$$\widetilde{w}_5[n] = \sum_{k=-\infty}^{\infty} x[n-5k]$$

$$w_5[n] = \begin{cases} \widetilde{w}_5[n] & n = 0,1,2,3,4 \\ 0 & o.w. \end{cases} = 2\delta[n] - 2\delta[n-1] + \delta[n-2] + \delta[n-3] - \delta[n-4]$$

c. M = 8> length of x[n] so $w_8[n] = x[n]$

d. Let $y[n] = x[n] * h[n] = [\,6\ \ -7\ \ 4\ \ 1\ \ -5\ \ 0\ \ 1\,]\ for\ n = 0{:}6$

$$\tilde{y}_3[n] = \sum_{k=-\infty}^{\infty} y[n-3k]$$

$$y_3[n] = \begin{cases} \tilde{y}_3[n] & n = 0,1,2 \\ 0 & o.w. \end{cases} = \delta[n] - 12\delta[n-1] + 4\delta[n-2]$$

e.

$$\tilde{y}_5[n] = \sum_{k=-\infty}^{\infty} y[n-5k]$$

$$y_5[n] = \begin{cases} \tilde{y}_5[n] & n = 0,1,2 \\ 0 & o.w. \end{cases} = 6\delta[n] - 6\delta[n-1] + 4\delta[n-2] + \delta[n-3] - 5\delta[n-4]$$

f. M = 8> length of y[n] so $y_8[n] = y[n]$

7.

a. Hint: In DFT equation of x[n] separate n=even and n = odd terms i.e.;

$$X[k] = \sum_{n=even} x[n]e^{-\frac{j2\pi kn}{N}} + \sum_{n=odd} x[n]e^{-\frac{j2\pi kn}{N}}$$

b.

i. For each k =0:N-1 there are 2N real multiplication and 2(N-1) addition therefore total of $2N^2$ real multiplications and 2N(N-1) real additions

ii. E[k] and O[k] : $\frac{N^2}{2}$ real multiplications and N(N/2-1) real additions

X[k] : 4N real multiplication and 4N real addition to get X[k] from the equation given in part a

Total : $N^2 + 4N$ real multiplications and $N^2 + 2N$ real additions

iii. For N >4 the arithmetic operations are smaller in part ii.

8.

a. L = 4 (length of segments), P= 3(length of h[n]) then $N = 4 + 3 - 1 = 6$

b. 3 segments :

$$x_0 = [1\ 2\ 3\ 4],\ \ x_1 = [-1 -2 -3 -4],\ \ x_2 = [1\ 2\ 3\ 4]$$

c.

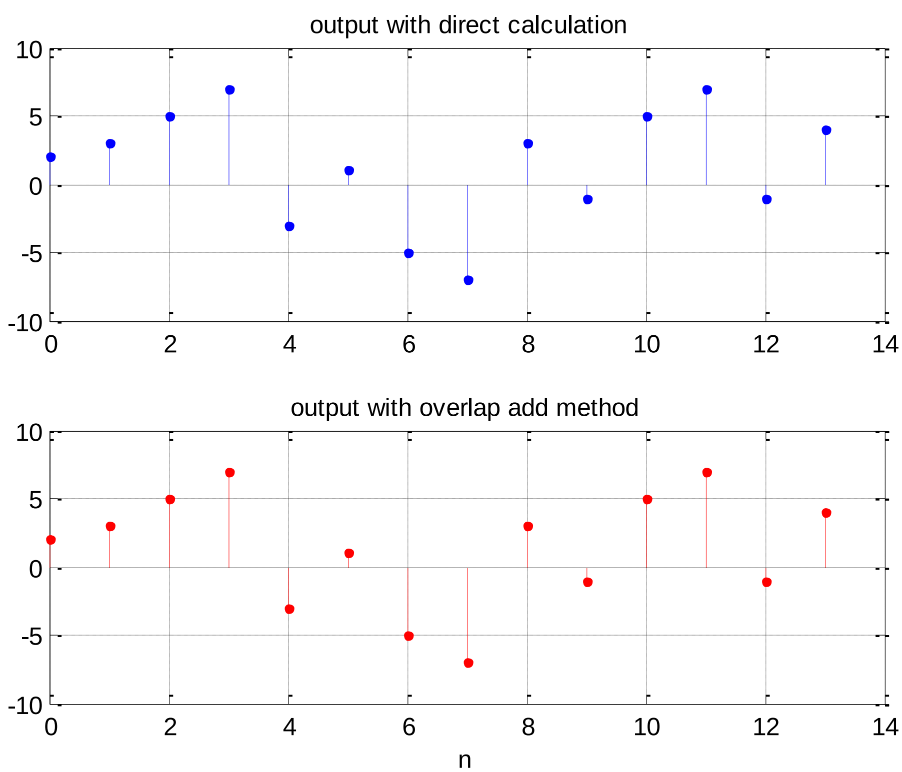
```
x = [1 2 3 4 -1 -2 -3 -4 1 2 3 4];
lx = length(x);
h = [ 2 -1 1];
lh = length(h);
L = 4;
N = L+lh-1;
```

```
y = conv(x,h); % output with direct calculation

Y = zeros(lx/L,lx+lh-1);

for k = 1:lx/L
    Y(k,(k-1)*L+1:k*L+lh-1) = ifft(fft(h,N).*fft(x((k-1)*L+1:k*L),N));
end


y1 = sum(Y,1); %output with overlap add method
subplot(211)
stem(0:length(y)-1,y,'.')
grid on
subplot(212)
stem(0:length(y1)-1,y1,'r.')
grid on
xlabel('n')
```



9.

a. 3 segments

$$x_0 = [0\ 1\ 2\ 3\ 4 - 1],\ \ x_1 = [\ 4\ \ - 1 - 2 - 3 - 4\ 1],\ \ x_2 = [\ -4\ 1\ 2\ 3\ 4\ 0\ ]$$

b.

```
x0 =[0 1 2 3 4 -1];
x1 =[ 4 -1 -2 -3 -4 1];
x2= [ -4 1 2 3 4 0];
N = 7;
```

```
y0 = ifft(fft(x0,N).*fft(h,N))
y1 = ifft(fft(x1,N).*fft(h,N))
y2 = ifft(fft(x2,N).*fft(h,N))
```

y0	-1	2	3	5	7	-3	5								
y1					9	-6	1	-5	-7	3	-5				
y2									-8	6	-1	5	7	-1	4
y		2	3	5	7	-3	1	-5	-7	3	-1	5	7	-1	4