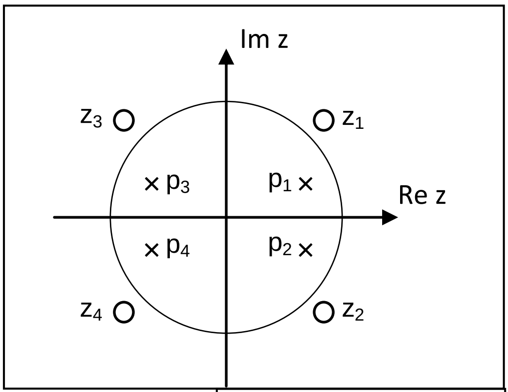
<u>**Due Date**: 22 December 2014, Monday (10:30).</u>

Homework 5

1) Consider a (causal-stable) LTI system with system function $H(z)$ whose pole-zero diagram is shown in the figure.



$$p_1 = 0.8\, e^{j\frac{\pi}{6}} = p_2^*$$

$$p_3 = 0.8\, e^{j\frac{5\pi}{6}} = p_4^*$$

$$z_1 = 1.2\, e^{j\frac{\pi}{4}} = z_2^*$$

$$z_3 = 1.2\, e^{j\frac{3\pi}{4}} = z_4^*$$

a) Express $H(z)$ as $H(z) = H_{\min}(z)\, H_{ap}(z)$ where $H_{\min}(z)$ and $H_{ap}(z)$ are minimum-phase and all-pass system functions, respectively. <u>Show your work!</u>

b) Find $H_1(z)$ and $H_2(z)$ such that $H_{\min}(z) = H_1(z)\, H_2(z)$ and $H_1(z)$ and $H_2(z)$ have, roughly, low-pass and high-pass frequency responses, respectively. <u>Show your work!</u>

c) Find $\left|H_1(e^{j\frac{\pi}{3}})\right|$ using a purely geometric approach on the pole-zero plot of $H_1(z)$. <u>Show your work!</u>

2)

a) Let $H(z) = \sum_{n=0}^{N} h[n] z^{-n}$ be a generalized linear-phase filter of Type-I. Define $g[n] = h[n]\cos(w_0[n-K])$ where K is an integer and w_0 is a rational multiple of π . Find K such that $G(z)$ has generalized linear-phase.

b) Let $x[n] = \cos(w_0 n)$ be the input to a Type-I generalized linear-phase filter of length N+1. Find an expression (in terms of the filter coefficients $h[n]$, w_0 and N) for the output of this filter, $y[n]$, in its <u>simplest</u> form.

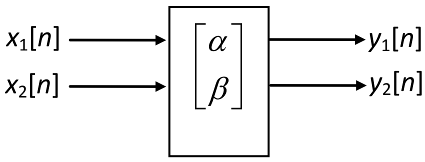
c) Assume that $H_1(z)$ is a Type-III filter. <u>Prove</u> that it has two zeros on the real axis. Find the locations of these zeros as well.

d) Assume that $H_2(z)$ is a complex coefficient generalized linear-phase filter of length 3. Plot all the possible cases of its zero plots.

3) The following *complex multiplier* block diagram is given operating based on the relation:

$$(x_1[n] + j\, x_2[n])\, (\alpha + j\beta) = y_1[n] + j\, y_2[n] \quad (j = \sqrt{-1}).$$

In this equation, α and β are real numbers, whereas $x_1[n]$, $x_2[n]$, $y_1[n]$ and $y_2[n]$ are all real signals.



a) Plot signal flow graph representation of the system above, such that <u>only 3</u> multipliers are used.

b) Given an extra constraint, $x_2[n] = y_2[n-1]$, determine transfer function $H(z) = Y_1(z)/X_1(z)$.

c) Find the conditions on α and β, such that $H(z)$ is a <u>stable all-pass</u> function.

d) Let $\overline{H}(z)$ denote a similar system to the all-pass system, $H(z)$, in part (c) with the only difference that the parameter α is replaced by $-\alpha$ for the figure above. Determine the phase angle value of the system $H(z) + \overline{H}(z)$ at the particular frequency, $w = \pi/2$.

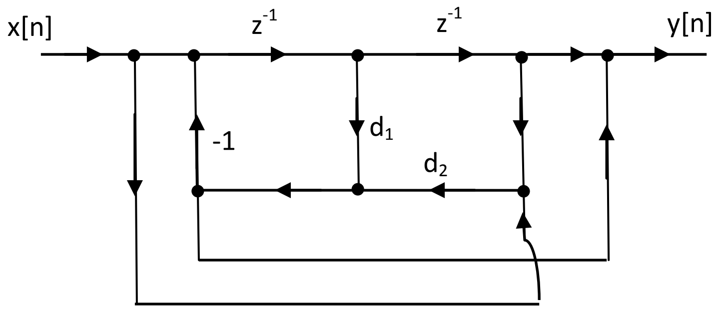
4) Given the following flow graph,

a) Find the transfer function, H(z).

b) Plot the direct form II structure.

c) Plot the transposed direct form II structure.

d) Determine if this structure corresponds to an IIR or FIR system. Find d_1 and d_2 such that the system is casual and stable.



5) Matlab Part:

Consider the following system:

$$H(z) = \frac{(1-2z^{-1})(z+2)\left(z-\frac{1}{2}\right)}{1-\frac{1}{2}z^{-1}+\frac{1}{2}z^{-2}}$$

a) Draw the pole-zero diagram of $H(z)$, plot the magnitude, phase and group-delay of $H(z)$

b) Express this system as $H(z) = H_{min}(z)H_{ap}(z)$

c) Plot the magnitude, phase and group-delay of $H_{min}(z)$ and $H_{ap}(z)$

d) Let

$$H_i(z) = \frac{1}{H_{min}(z)}$$

Plot the magnitude, phase and group-delay of $H_T(z) = H_i(z)H(z)$. Comment on your results.