EE 430 Section 1-3 HW5 Solution

(For any questions contact Erdal Epçaçan, epcacan@metu.edu.tr, D-122)

1)

$$H(z) = \frac{(z-1.2e^{j\pi/4})(z-1.2e^{-j\pi/4})(z-1.2e^{j3\pi/4})(z-1.2e^{-j3\pi/4})}{(z-0.8e^{j\pi/6})(z-0.8e^{-j\pi/6})(z-0.8e^{-j5\pi/6})(z-0.8e^{j5\pi/6})}$$

$$H_{min}(z) = \frac{(1-5/6\,e^{j\pi/4}z^{-1})(1-5/6\,e^{-j\pi/4}z^{-1})(1-5/6\,e^{j3\pi/4}z^{-1})(1-5/6\,e^{-j3\pi/4}z^{-1})}{(1-0.8e^{j\pi/6}z^{-1})(1-0.8e^{-j\pi/6}z^{-1})(1-0.8e^{j5\pi/6}z^{-1})(1-0.8e^{-j\frac{5\pi}{6}}z^{-1})}\,K_1$$

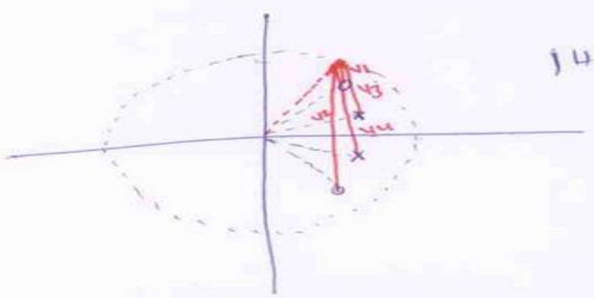
$$H_{ap}(z) = \frac{(z^{-1}-5/6\,e^{j\pi/4})(z^{-1}-5/6\,e^{-j\pi/4})(z^{-1}-5/6\,e^{j3\pi/4})(z^{-1}-5/6\,e^{-j3\pi/4})}{(1-5/6\,e^{j\pi/4}z^{-1})(1-5/6\,e^{-j\pi/4}z^{-1})(1-5/6\,e^{j3\pi/4}z^{-1})(1-5/6\,e^{-j3\pi/4}z^{-1})}$$

b)

$$H_1(z) = K_2\,\frac{(1-5/6\,e^{j3\pi/4}z^{-1})(1-5/6\,e^{-j3\pi/4}z^{-1})}{(1-0.8e^{j\pi/6}z^{-1})(1-0.8e^{-j\pi/6}z^{-1})}$$ → select zeros near π and poles near 0

$$H_2(z) = K_3\,\frac{(1-5/6\,e^{j\pi/4}z^{-1})(1-5/6\,e^{-j\pi/4}z^{-1})}{(1-0.8e^{j\frac{5\pi}{6}}z^{-1})(1-0.8e^{-j\frac{5\pi}{6}}z^{-1})}$$ → zeros near 0, poles near π

c)



$$|H_1(e^{j\pi/8})| = K_2\cdot\frac{V_1\,V_2}{V_3\,V_4}$$

2)

a) Since $G(z)$ will also have a generalized linear phase, the symmetricity should be kept. Then at $n = N/2$ $\cos(w_0(N/2 - k)) = \mp 1$ should satisfy so

$$k = N/2 - \frac{2\pi l}{w_0}, \quad l, k \in \mathbb{Z}$$

b) $y[n] = x[n] * h[n]$

$$= \sum_{m=0}^{N} h[m] \cos(w_0(n-m)), \quad h[n] = h[N-n]$$

$$= \sum_{m=0}^{N/2-1} h[m] \Big(\underbrace{\cos(w_0(n-m)) + \cos(w_0(n-N+m))}_{2\cos(w_0(n-N/2)) \cdot \cos(w_0(m-N/2))} \Big) + h[N/2] \cos(w_0(n-N/2))$$

$$= \cos(w_0(n-N/2)) \left[2 \sum_{m=0}^{N/2-1} h[m] \cos(w_0(m-N/2)) + h[N/2] \right]$$

c) Type-III $\Rightarrow h[n] = -h[N-n]$

$$H(z) = -z^{-N} H(z^{-1})$$

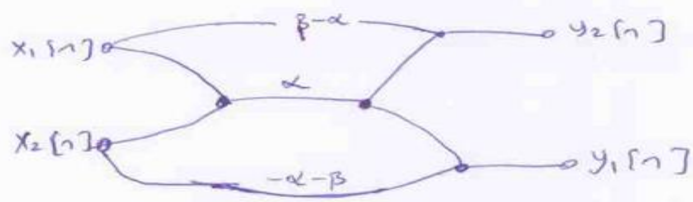
Take $z = 1, -1 \Rightarrow H(1) = 0 = H(-1)$

d) Odd length $\Rightarrow$ Type I or Type III

Type-I $\Rightarrow$ if z_0 is a zero $1/z_0$ is also zero

Type III $\Rightarrow$ If z_0 is a zero $-1/z_0$ is also zero

3) a)



$$y_1[n] = \alpha x_1[n] - \beta x_2[n], \quad y_2[n] = \beta x_1[n] + \alpha x_2[n]$$

b)

$$Y_2(z) = \beta X_1(z) + \alpha X_2(z) = \beta X_1(z) + \alpha z^{-1} Y_2(z)$$

$$\Rightarrow Y_2(z) = \frac{\beta}{1-\alpha z^{-1}} X_1(z)$$

$$Y_1(z) = \alpha X_1(z) - \beta z^{-1} Y_2(z) = \alpha X_1(z) - \beta z^{-1} \frac{\beta}{1-\alpha z^{-1}} X_1(z)$$

$$\Rightarrow \frac{Y_1(z)}{X_1(z)} = \frac{\alpha - \alpha^2 z^{-1} - \beta^2 z^{-1}}{1-\alpha z^{-1}} = H(z) = \frac{\alpha - (\alpha^2+\beta^2) z^{-1}}{1-\alpha z^{-1}}$$

c) $\alpha^2+\beta^2 = 1, \quad |\alpha| < 1$

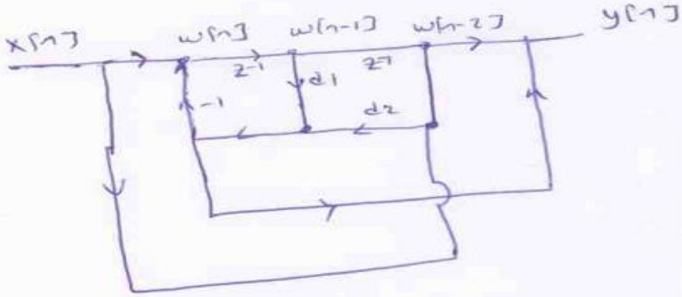
d) $\bar{H}(z) = \dfrac{-\alpha - z^{-1}}{1+\alpha z^{-1}}$

$$H(z) + \bar{H}(z) = \frac{(-2+2\alpha^2) z^{-1}}{1-\alpha^2 z^{-2}} \Rightarrow \Big|_{z=e^{j\omega}} = \frac{(-2+2\alpha^2) e^{-j\omega}}{1-\alpha^2 e^{-2j\omega}} = H_1(e^{j\omega})$$

For $\omega = \pi/2 \Rightarrow H_1(e^{j\pi/2}) = \dfrac{(-2+2\alpha^2) e^{-j\pi/2}}{1+\alpha^2}$

$$\angle H_1(e^{j\pi/2}) = -\pi/2$$

4)



$$W(z) = X[z] - d_1 z^{-1} W(z) - d_2 W(z) z^{-2} - d_2 X(z)$$

$$W(z)\left(1 + d_1 z^{-1} + d_2 z^{-2}\right) = X(z)\left(1 - d_2\right)$$

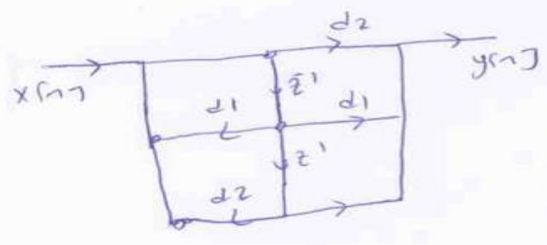
$$Y(z) = z^{-2} W(z) + d_1 z^{-1} W(z) + d_2 z^{-2} W(z) + d_2 X(z)$$

$$= W(z)\left(d_1 z^{-1} + (1+d_2) z^{-2}\right) + d_2 X(z)$$

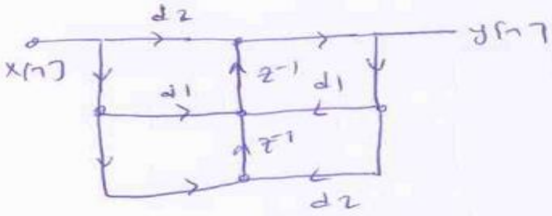
$$= \frac{(1-d_2)\left(d_1 z^{-1} + (1+d_2) z^{-2}\right) X(z)}{1 + d_1 z^{-1} + d_2 z^{-2}} + d_2 X(z)$$

$$Y(z) = \frac{d_2 + d_1 z^{-1} + z^{-2}}{1 + d_1 z^{-1} + d_2 z^{-2}} X(z) \Rightarrow H(z) = \frac{d_2 + d_1 z^{-1} + z^{-2}}{1 + d_1 z^{-1} + d_2 z^{-2}}$$

b)



4) c)



d) This is an IIR system

The poles should be inside U. C. that is

root of denominator should be smaller than 1

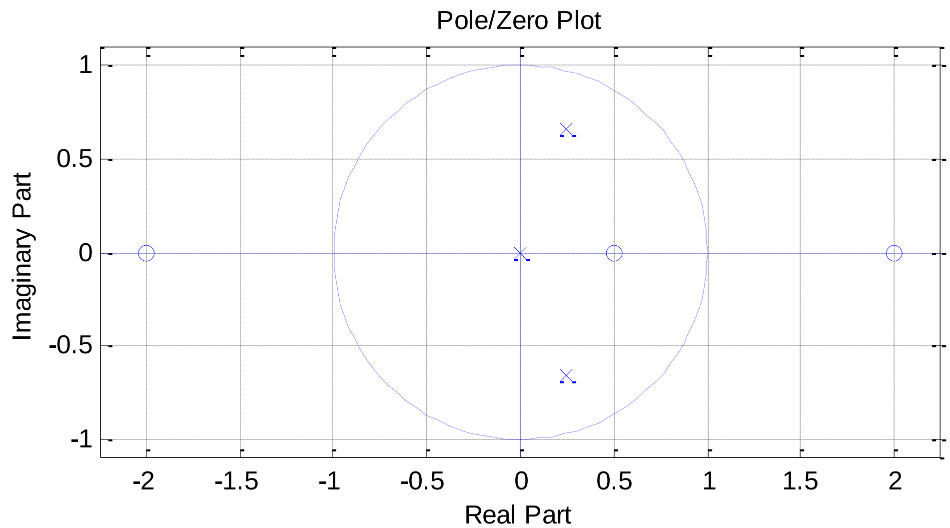
$$\left| \frac{-d_1 \mp \sqrt{d_1^2 - 4d_2}}{2d_1} \right| < 1$$

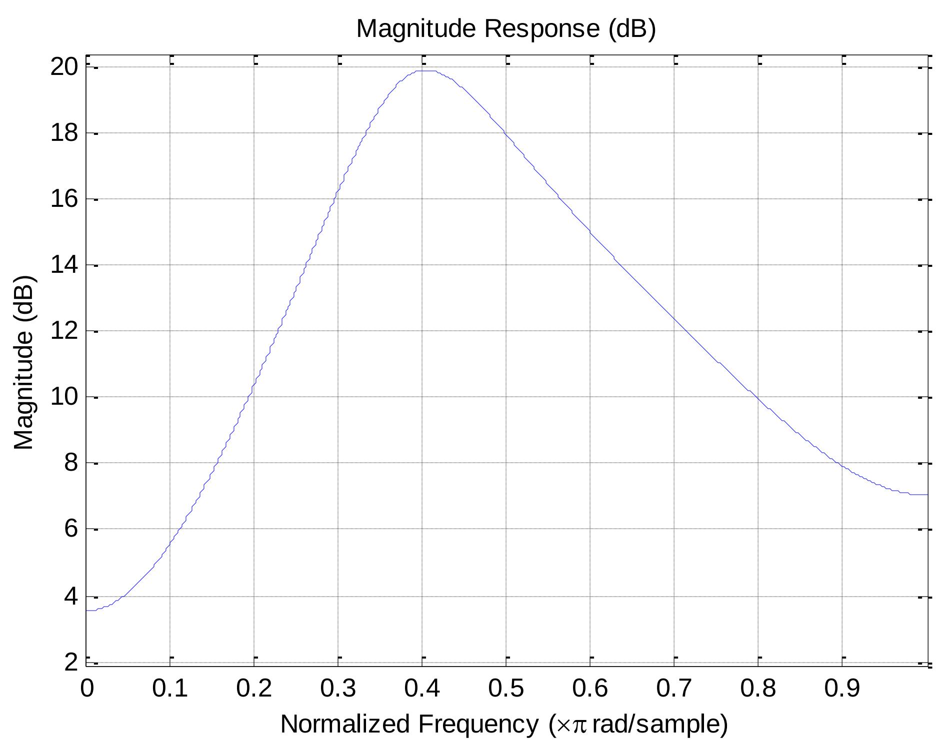
$$\Rightarrow -1 < \frac{-d_1 \mp \sqrt{d_1^2 - 4d_2}}{2d_1} < 1 \Rightarrow -1/2 < \sqrt{\frac{d_1^2 - 4d_2}{4d_1^2}} < 3/2$$

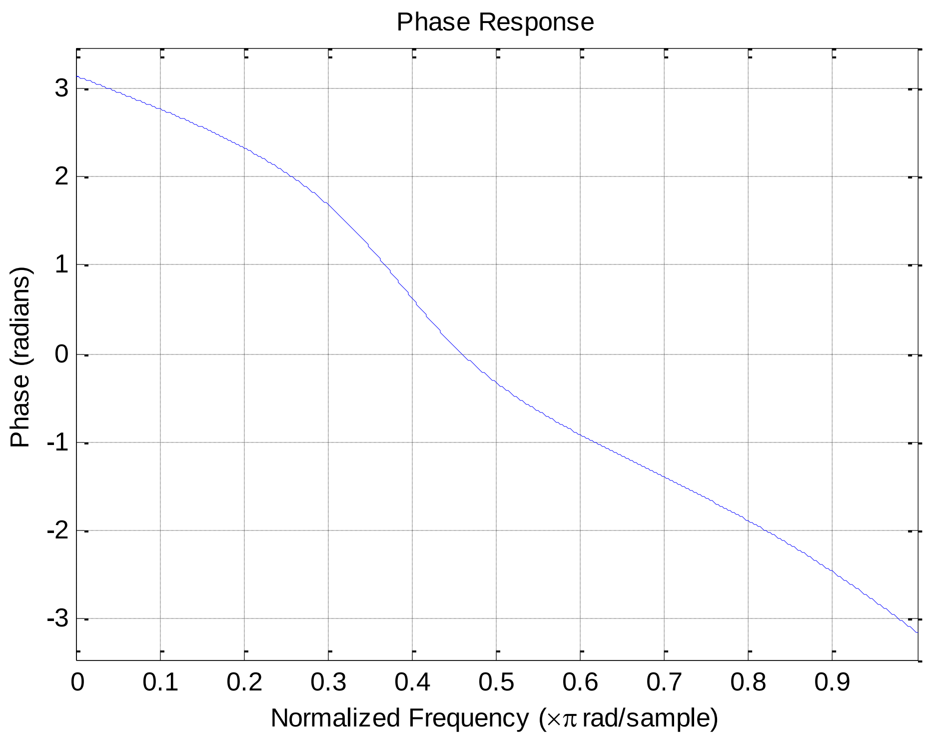
$$\Rightarrow 1/4 < \frac{d_1^2 - 4d_2}{4d_1^2} < 9/4$$

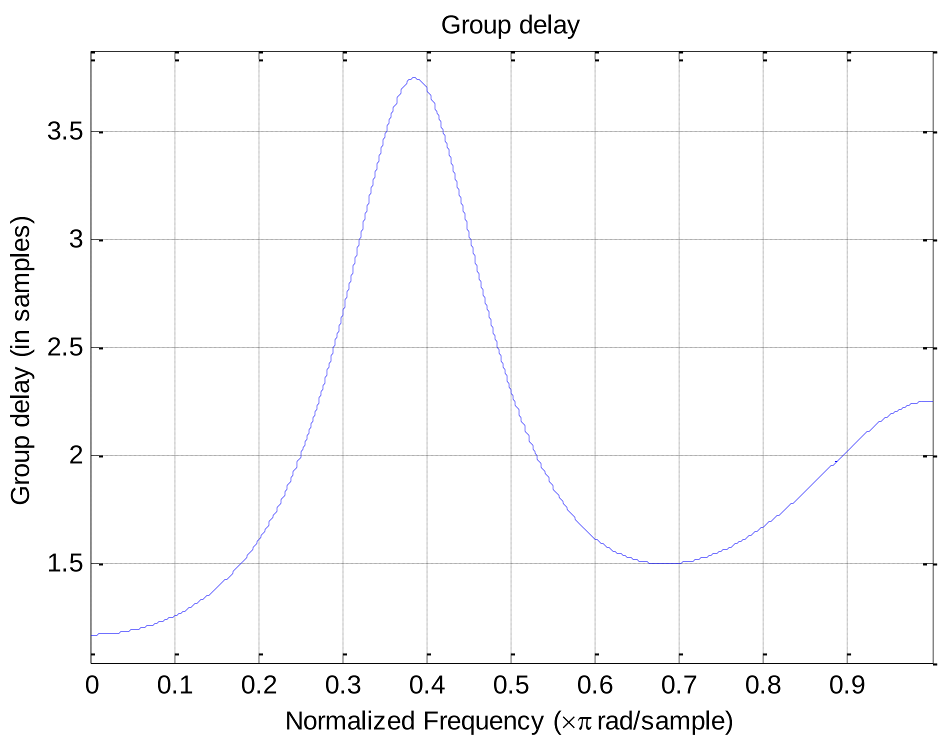
Matlab Part:

a)

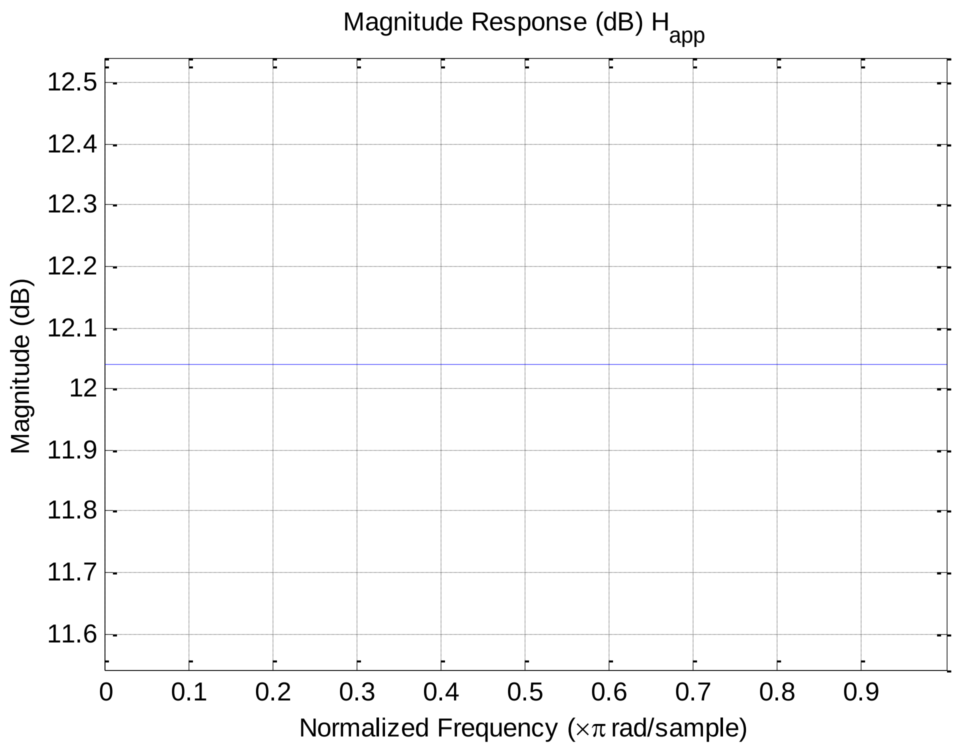


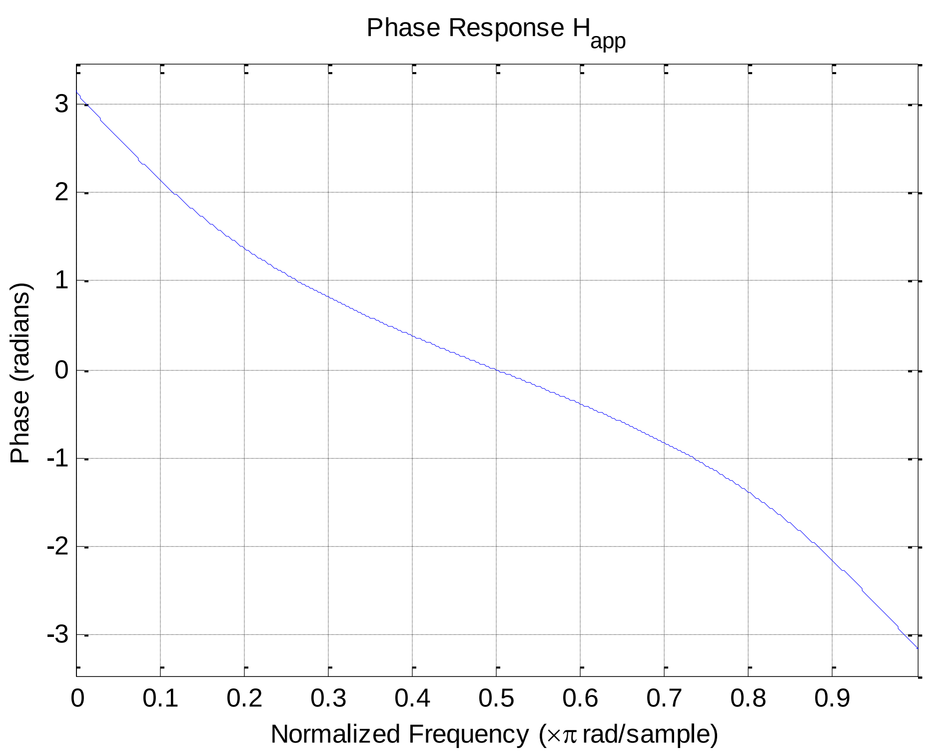


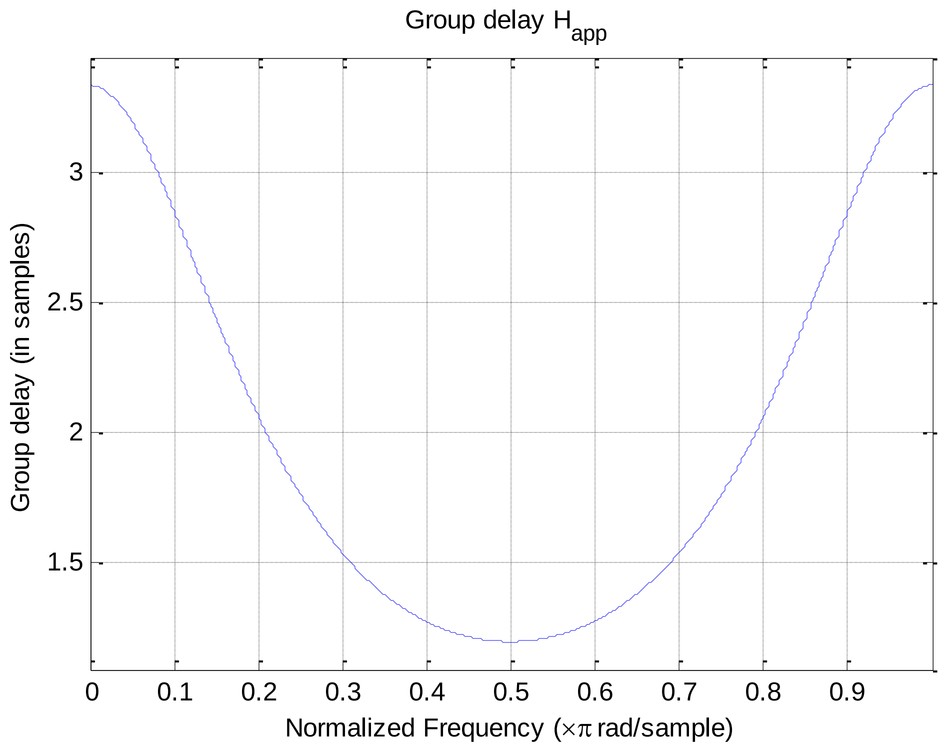

Phase Response
Phase (radians)
3
2
1
0
-1
-2
-3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Normalized Frequency (×π rad/sample)

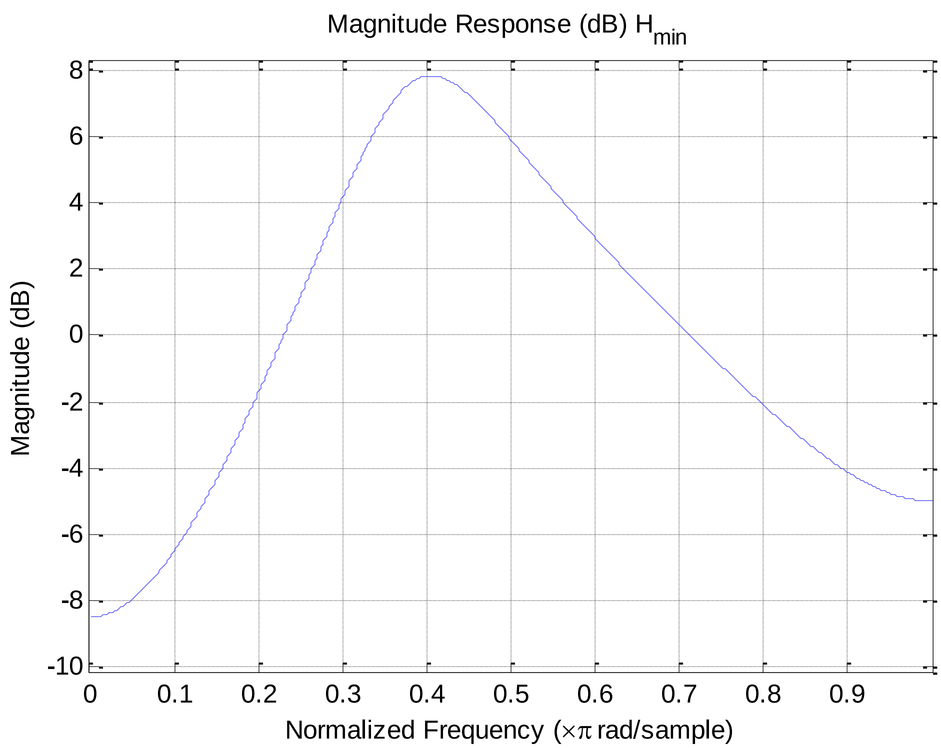

Group delay
Group delay (in samples)
3.5
3
2.5
2
1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Normalized Frequency (×π rad/sample)

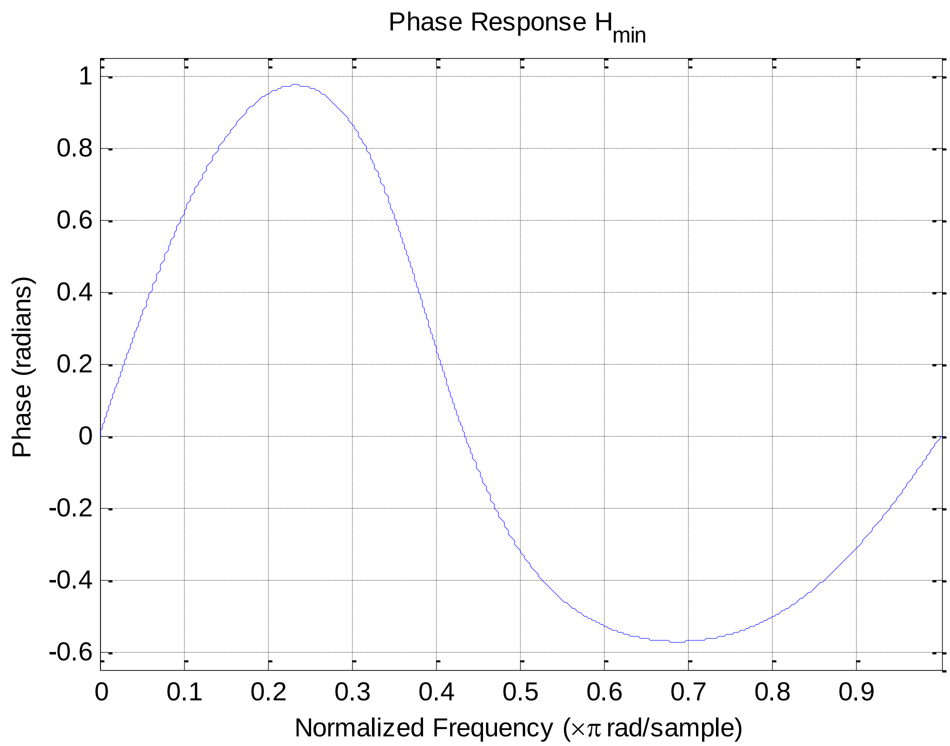
b) & c)


Magnitude Response (dB) H_{app}
Magnitude (dB)
12.5
12.4
12.3
12.2
12.1
12
11.9
11.8
11.7
11.6
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Normalized Frequency (×π rad/sample)

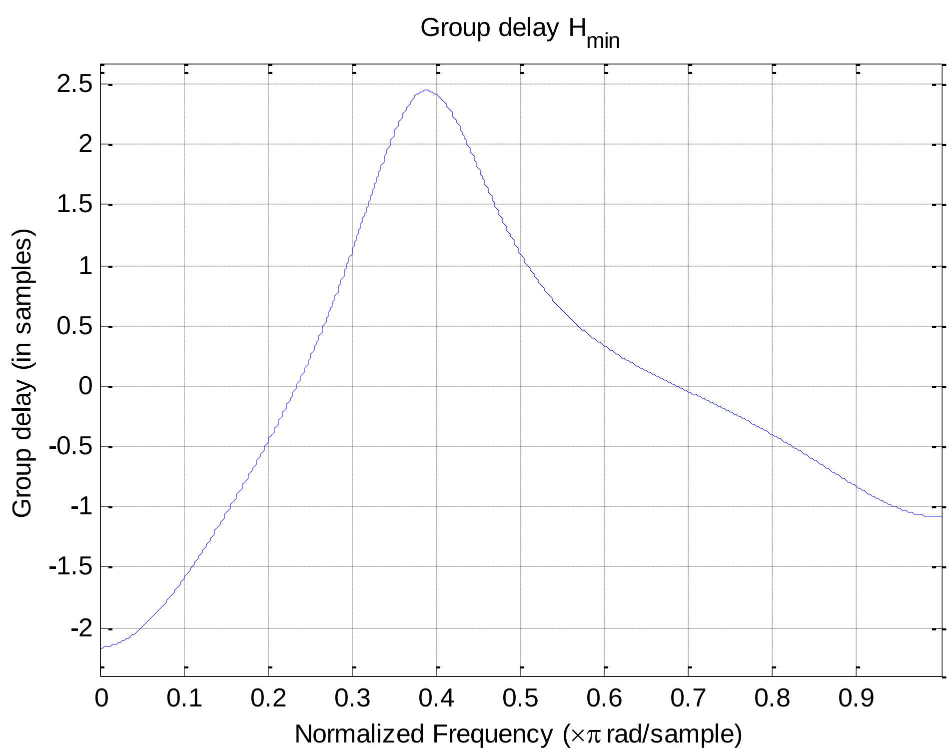

Phase Response H_{app}
Phase (radians)
3
2
1
0
-1
-2
-3
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Normalized Frequency (×π rad/sample)


Group delay H_{app}
Group delay (in samples)
3
2.5
2
1.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Normalized Frequency (×π rad/sample)


Magnitude Response (dB) H_{min}
Magnitude (dB)
8
6
4
2
0
-2
-4
-6
-8
-10
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Normalized Frequency (×π rad/sample)



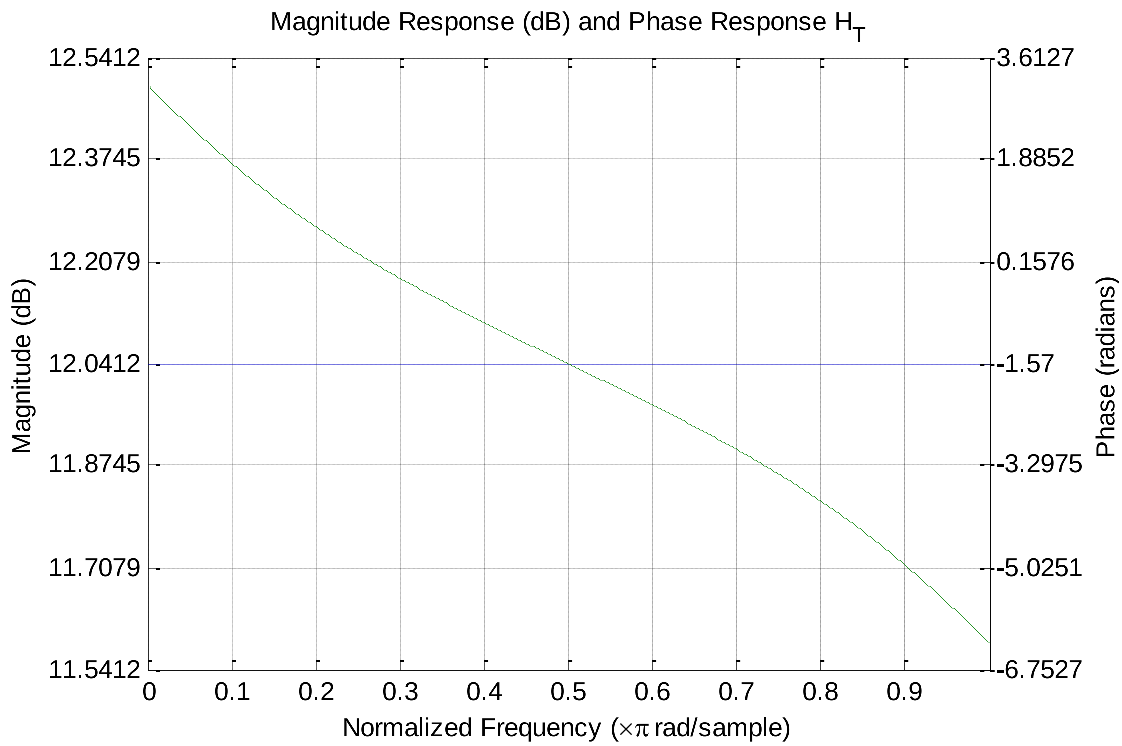
Phase Response H_{min}
Phase (radians)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Normalized Frequency (×π rad/sample)

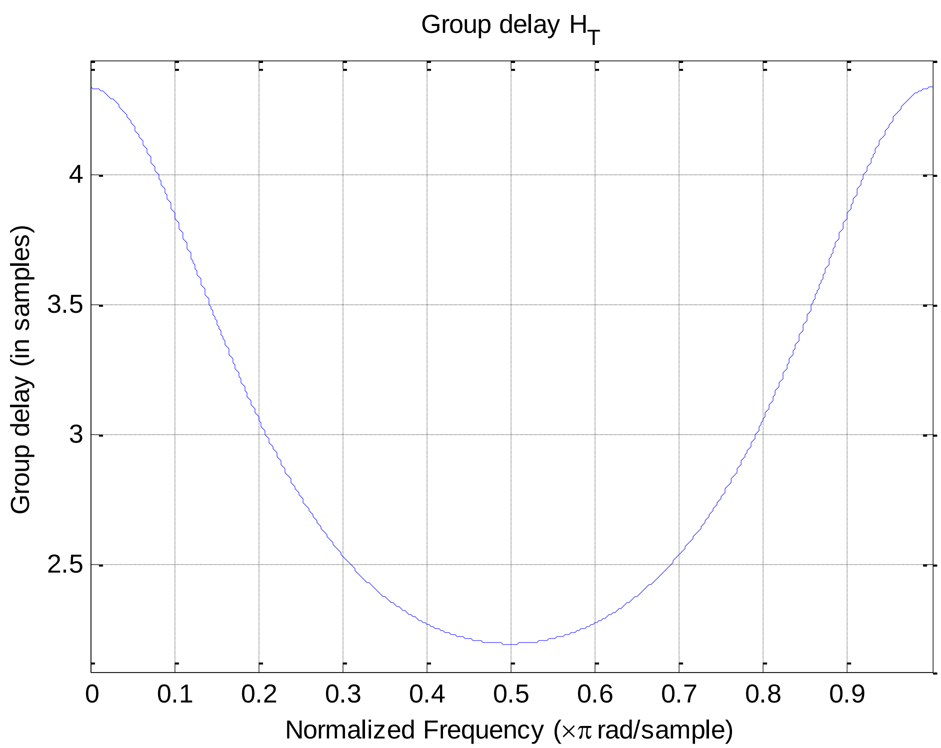


Group delay H_{min}
Group delay (in samples)
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Normalized Frequency (×π rad/sample)

c)

d)


Magnitude Response (dB) and Phase Response H_T
Magnitude (dB)
12.5412
12.3745
12.2079
12.0412
11.8745
11.7079
11.5412
Phase (radians)
3.6127
1.8852
0.1576
-1.57
-3.2975
-5.0251
-6.7527
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Normalized Frequency (×π rad/sample)


Group delay H_T
Group delay (in samples)
4
3.5
3
2.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Normalized Frequency (×π rad/sample)