Q2) A LTI system transfer function is given as,

$$H(z)=\frac{1+2z^{-2}}{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

a) Plot the pole-zero diagram for H(z).

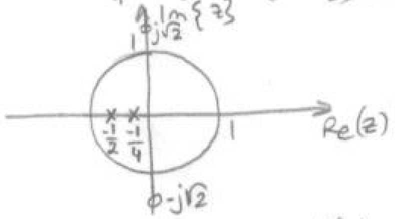
b) Plot the approximate magnitude characteristics of $H(e^{j\omega})$ by indicating $\omega=0$ and $\omega=\pi$ values. Determine the filter type.

c) Let $H(z)=H_{min}(z)H_{ap}(z)$ where $H_{min}(z)$ is the minimum-phase and $H_{ap}(z)$ is the allpass filter. Find $H_{min}(z)$ and $H_{ap}(z)$. Plot their pole-zero diagrams.

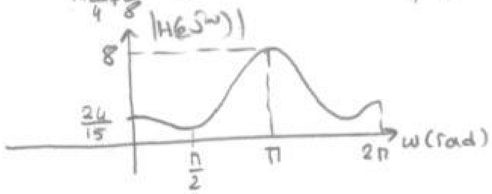
d) H(z) is cascade connected with a suitable filter $H_2(z)$ and $G(z)=H(z)H_2(z)$ is obtained as a FIR generalized linear-phase Type III filter. Find the minimum order G(z) and plot its pole-zero diagram.

a) $H(z)=\dfrac{z^2+2}{z^2+\frac{3}{4}z+\frac{1}{8}}=\dfrac{(z+j\sqrt{2})(z-j\sqrt{2})}{(z+\frac{1}{2})(z+\frac{1}{4})}$ zeros: $z_1=j\sqrt{2}$ $z_2=-j\sqrt{2}$

poles: $z_3=-\frac{1}{2}$ $z_4=-\frac{1}{4}$

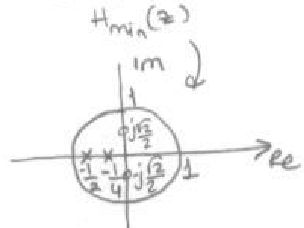


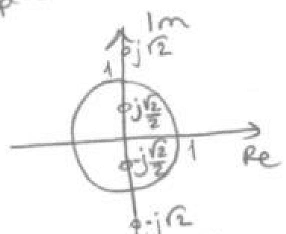
b) $H(e^{j0})=\dfrac{3}{1+\frac{3}{4}+\frac{1}{8}}=\dfrac{24}{15}$ $H(e^{j\pi})=\dfrac{1+2}{1-\frac{3}{4}+\frac{1}{8}}=8$



This is a highpass filter

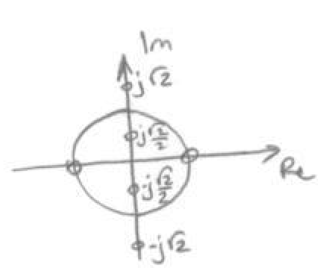
c) $H(z)=\dfrac{1+2z^{-2}}{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}=\underbrace{\dfrac{2+z^{-2}}{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}}_{H_{min}(z)}\cdot\underbrace{\dfrac{1+2z^{-2}}{2+z^{-2}}}_{H_{ap}(z)}$





d) $G(z)=\dfrac{1+2z^{-2}}{1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}\cdot\left(1+\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}\right)\cdot(2+z^{-2})(1+z^{-1})(1-z^{-1})$

$G(z)=(1+2z^{-2})(2+z^{-2})(1+z^{-1})(1-z^{-1})$



Q3)

a) The impulse response of a system, $h[n]$ is given as $h[1]=1$, $h[2]=2$, $h[3]=1$ and 0 for all other values of $h[n]$. Find and <u>plot</u> the magnitude and phase response (ARG[.]) for this system. Comment on the type of this filter.

b) Assume the system in part-a) is cascaded by a real Type-I generalized linear-phase system with length 9, whose zeros are equal to $z_1=-0.5$, $z_2=0.5e^{j\pi/4}$, $z_3=e^{-j3\pi/4}$.

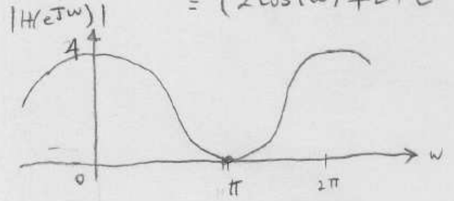
  i. Find the overall pole-zero plot of the cascaded system

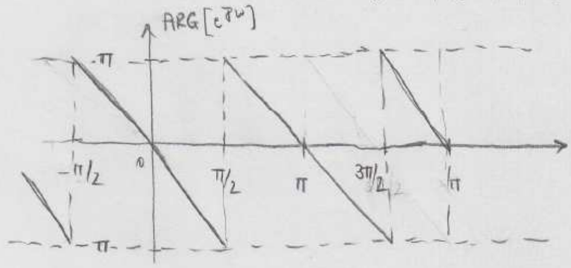
  ii. Find the group delay of the cascaded system.

c) For the system in part-a), select $h[2]$ and $h[3]$ (while remaining all other values unchanged), in such a way that the system is still a generalized linear-phase system and $H(e^{j0})=0$.

a) $H(e^{j\omega}) = \sum_{n=\infty}^{\infty} h(n) e^{-j\omega n} = e^{-j\omega} + 2e^{-j2\omega} + e^{-j3\omega} = e^{-j2\omega}(e^{+j\omega} + e^{-j\omega}) + 2e^{-j2\omega}$

$= (2\cos(\omega) + 2)\, e^{-j2\omega}$





$h[n]$ is a M even symmetric $\Rightarrow$ Type I

$\begin{pmatrix} h[0] = h[4] \\ h[1] = h[3] \end{pmatrix}$ $M = 4$

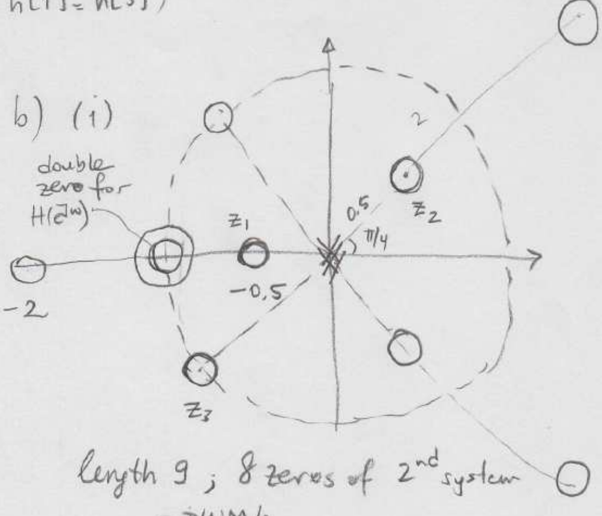
$H(z) = z^{-1} + 2z^{-2} + z^{-3}$

$= (z^2 + 2z + 1)/z^3$

$= (z+1)^2/z^3$

double zero at $z=-1$

b) (i)



(ii) Group delay = grd$[H(e^{j\omega})]$ + grd$[G(e^{j\omega})]$

$= 2 + 4$

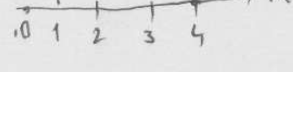
$= 6$

length 9 ; 8 zeros of 2^{nd} system

$e^{-j\omega M/2}$

c) To guarantee $H(e^{j0})=0$ system, assume a Type III system (M even anti sym.)

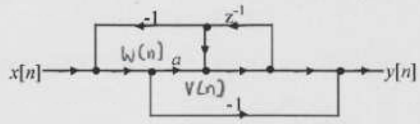
$\Rightarrow$



$\Rightarrow$ $h[1] = -h[3]$ & $h[2] = 0$

$h[3] = -1$

Q4) The following signal flow graph representation is given for a system whose transfer function is $H(z)$.



a) Determine $H(z)$ in terms of the given system parameters, indicating its pole and zeros on the z-plane. Comment on the type of this IIR filter.

b) Obtain and draw the transpose of the above representation.

c) Let $G(z)$ be equal to $G(z)=\dfrac{1-z^{-1}}{1+0.5z^{-1}}$ and assume $G(z)$ is cascaded by itself. Draw the resulting signal flow graph representation in direct form-I and modify it in such a way that 3 delay elements are utilized in this cascaded system.

a)
$$\left.\begin{aligned} v[n] &= a\,w[n]+v[n-1] \\ w[n] &= x[n]-v[n-1] \\ y[n] &= v[n]-w[n] \end{aligned}\right\}$$

$V(z)=aW(z)+z^{-1}V(z) \Rightarrow V(z)(1-z^{-1})=aW(z)$

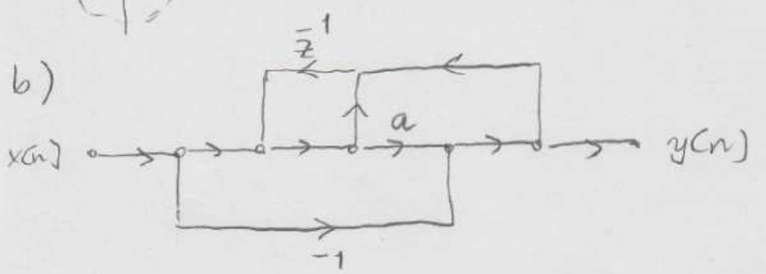
$W(z)=X(z)-z^{-1}V(z) \Rightarrow W(z)=X(z)-\dfrac{az^{-1}}{(1-z^{-1})}W(z)$

$Y(z)=V(z)-W(z)$ $\qquad W(z)\left(\dfrac{1-z^{-1}+az^{-1}}{1-z^{-1}}\right)=X(z)$

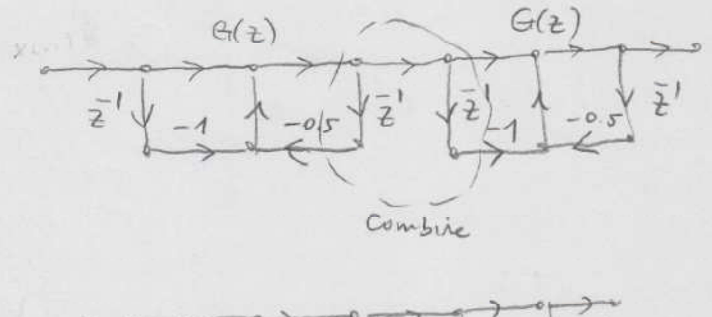
$$\Rightarrow Y(z)=\left(\frac{a-(1-z^{-1})}{1-z^{-1}}\right)W(z) \Rightarrow Y(z)=\frac{(a-1)+z^{-1}}{1-(a-1)z^{-1}}X(z)$$

All-pass system

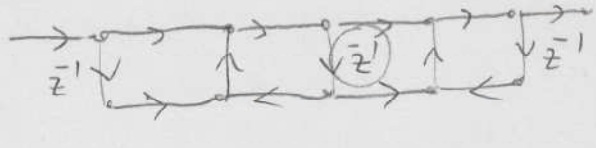
b)



c)



: 4 delay elements



: 3 delay elements.