EE430 - HW1
Solutions

1)

a) $x[n] = x_c(nT)$

$$= 4.\sin\left(\frac{20000\pi.n}{3\times10^3} + \frac{\pi}{13}\right)$$

$$= 4.\sin\left(\frac{20\pi n}{3} + \frac{\pi}{13}\right) = 4.\sin\left(\frac{2\pi n}{3} + \frac{\pi}{13}\right)$$

$$x[n] = 4\sin\left(\frac{2\pi n}{3} + \frac{\pi}{13} + 2\pi nk\right) \quad k = \ldots -1, 0, 1 \ldots$$

$$\hat{x}(t) = 4\sin\left(\frac{2\pi.t}{3\,T} + \frac{\pi}{13} + 2\pi k.\frac{t}{T}\right) \quad k = \ldots -1, 0, 1 \ldots$$

$$= 4.\sin\left(2\pi\times10^3 t + \frac{\pi}{13} + 6\pi\times10^3 t.k\right) \quad k = \ldots, -1, 0, 1 \ldots$$

$$= \boxed{4\sin\left(2\pi\times10^3 t\,(1+3k) + \frac{\pi}{13}\right) \quad k = \ldots, -1, 0, 1 \ldots}$$

b) $x[n] = 4.\sin\left(\frac{2\pi n}{3} + \frac{\pi}{13}\right) = 4\sin\left(20000\pi\, nT + \frac{\pi}{13}\right)$

$$\frac{2\pi n}{3} + 2\pi nk = 2\pi\times10^4 nT \Rightarrow \frac{1}{3} + k = 10^4.T$$

$$\frac{1+3k}{3.10^4} = T \Rightarrow \boxed{f_s = \frac{3\times10^4}{1+3k}}$$

for $k = \ldots -1, 0, 1, \ldots$

2) • $\sin(1{,}74\pi(n+N)+3{,}1) \overset{?}{=} \sin(1{,}74\pi n + 3{,}1)$

$\Downarrow$

$\sin(1{,}74\pi n + \underbrace{1{,}74\pi . N}_{2\pi k} + 3{,}1)$ $\Rightarrow 2\pi k = 1{,}74\pi . T$

$\frac{2k}{1{,}74} = N \Rightarrow \frac{k}{0{,}87} = N$ → integer (k → integer)

$\boxed{N_{fund} = 100}$

• $\sin(1{,}74\pi n + 3.1\pi) \rightarrow \boxed{N_{fund} = 100}$

• $\cos\left(15{,}74\pi n + \frac{3\pi}{8}\right) \rightarrow \frac{2\pi}{15{,}74\pi} k = \downarrow$ (k integer) integer $\boxed{N fund = 100}$

• $\cos(\sqrt{\pi}\, n) \rightarrow \frac{2\pi}{\sqrt{\pi}} k = N$ $\boxed{\text{not periodic}}$

• $\cos(\pi\sqrt{\pi} n) \rightarrow \frac{2\pi}{\pi\sqrt{\pi}} k = N$ $\boxed{\text{not periodic}}$

• $\cos(\pi\sqrt{2} n) \rightarrow \frac{2\pi}{\pi\sqrt{2}} k = N$ $\boxed{\text{not periodic}}$

there is no integer value k, to obtain integer N.

3) $x[n] = \sum_{k=-\infty}^{+\infty} x[k] . \delta[n-k] \xrightarrow{\text{linear}} y[n] = \sum_{k=-\infty}^{+\infty} x[k] h_k[n]$

$y[n] = \sum_{k=-\infty}^{+\infty} x[k](n-k)\, u[n-k]$

for $x[n-m]$, we want $y_1[n-m] = \sum_{k=-\infty}^{+\infty} x[k-m](n-k)\, u[n-k]$

but we have $y_2[n-m] = \sum_{k=-\infty}^{+\infty} x[k](n-k-m)\, u[n-k-m]$

$$y_1[n-m] = \sum_{k=-\infty}^{n} x[k-m].(n-k) \longrightarrow y_1[n-m]=y_2[n-m]$$

time-invariant system.

$$y_2[n-m] = \sum_{k=-\infty}^{n-m} x[k]\,(n-k-m) = \sum_{k=-\infty}^{n} x[k-m]\,(n-k)$$

4) $a x_1[n] + b x_2[n] \xrightarrow[?]{\text{Linearity}} a y_1[n] + b y_2[n]$

$$a.\ y_1[n] = \begin{cases} a.x_1\left[\frac{n}{2}\right], & n \text{ is even} \\ \dfrac{a x_1\left[\frac{n-1}{2}\right] + a x_1\left[\frac{n+1}{2}\right]}{2}, & n \text{ is odd} \end{cases}$$

$a x_1[n] \xrightarrow{L} a y_1[n]$ ✓

$$b.\ y_2[n] = \begin{cases} b.x_2\left[\frac{n}{2}\right], & n \text{ is even} \\ \dfrac{b\, x_2\left[\frac{n-1}{2}\right] + b.x_2\left[\frac{n+1}{2}\right]}{2}, & n \text{ is odd} \end{cases}$$

$b x_2[n] \xrightarrow{L} b y_2[n]$ ✓

+

$$a.y_1[n] + b y_2[n] = \begin{cases} a.x_1\left[\frac{n}{2}\right] + b x_2\left[\frac{n}{2}\right], & n \text{ is even} \\ \dfrac{a.x_1\left[\frac{n-1}{2}\right] + a x_1\left[\frac{n+1}{2}\right] + b x_2\left[\frac{n-1}{2}\right] + b x_2\left[\frac{n+1}{2}\right]}{2}, & n \text{ is odd} \end{cases}$$

when we take "$a x_1[n] + b x_2[n]$" as a input of the system, the same result can be obtained for "$a y_1[n] + b y_2[n]$".

Therefore, the system is <u>linear.</u>

For a system $x[n] \xrightarrow{T} x\left[\frac{n}{2}\right]$

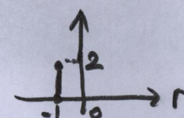
input		output
$x[n-m]$	$\rightarrow$	$x\left[\frac{n}{2}-m\right] = y_1[n-m]$
$x[n-m]$	$\rightarrow$	$x\left[\frac{n-m}{2}\right] = y_2[n-m]$

$y_1[n-m] \neq y_2[n-m]$

the system is NOT time-invariant

Also, the value of n (even or odd) has an effect on the response $\rightarrow$ $\boxed{\text{time-varying system}}$

5) $y[n] = 2^{\delta[n+1]} + x[n-3]$



* For bounded input, $x[n-3]$ is bounded and $2^{\delta[n+1]}$ is always bounded, so the output is bounded → stable system

* We can easily see that the output depends on previous values of input → causal system

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$$y[n] = \begin{cases} y[-\delta[n-1]] + x[n-3] & n > 0 \\ 2^{n}\, x[n-3] & n \leq 0 \end{cases}$$

* $y[0] = 1 . x[-3] \rightarrow y[0]$ is bounded for bounded input.

  $y[1] = y[-1] + x[-2] \rightarrow y[1]$ is bounded

  $y[-1] = \frac{1}{2}\, x[-4] \rightarrow y[-1]$ is bounded

  If $\underset{n>0}{n \neq 1}$, $y[n] = y[0] + x[n-3] \Rightarrow y[n]$ is bounded

  a constant

  If $n \leq 0$, $\boxed{0 < 2^{n} < 1}$ and $\boxed{|x[n]| < M} \Rightarrow y[n]$ is bounded

* The system is stable

* The system is causal (The system output $y[n]$ depends on previous values of $x[n]$)

6) $y[n] = \sum_{k=-\infty}^{+\infty} x[k] \cdot x[n-k] = \sum_{k=-6}^{24} x[k]\, x[n-k]$

The first term $\rightarrow -3\,\delta[n+6]$

Thes last term $\rightarrow -4\,\delta[n-24]$

$-3\,\delta[n+6] * x[n] \Rightarrow$ shift to the left (6) and multiply with -3

The first non-zero term $\boxed{9\,\delta[n+12]}$

$-4\,\delta[n-24] * x[n] \Rightarrow$ shift to the right (24) and mult. with -4

The last non-zero term $\boxed{16\,\delta[n-48]}$

7) $x[n] = u[n] = \delta[n] + \delta[n-1] + \dots \rightarrow \boxed{h[n]} \rightarrow h[n] + h[n-1] + \dots$

$$y[n] = \sum_{k=0}^{\infty} h[n-k] = \sum_{k=0}^{\infty} 3\left(\frac{1}{2}\right)^{n-k} u[n-k] - 2\left(\frac{1}{3}\right)^{n-k-1} u[n-k]$$

$$= \sum_{k=0}^{n} \left(3\left(\frac{1}{2}\right)^{n-k} - 2\left(\frac{1}{3}\right)^{n-k-1}\right)$$

$$= 3\cdot\left(\frac{1}{2}\right)^{n} \cdot \sum_{k=0}^{n} 2^{k} - 2\left(\frac{1}{3}\right)^{n-1} \sum_{k=0}^{n} 3^{k}$$

$$= 3\left(\frac{1}{2}\right)^{n} \cdot \left(2^{n+1}-1\right) - 2\left(\frac{1}{3}\right)^{n-1}\left(\frac{3^{n+1}-1}{2}\right)$$

$$= 6 - 3\left(\frac{1}{2}\right)^{n} - 9 + \left(\frac{1}{3}\right)^{n-1} = \boxed{\left(\frac{1}{3}\right)^{n-1} - 3\left(\frac{1}{2}\right)^{n} - 3}$$

$y[n] = \left(\frac{1}{3}\right)^{n-1} - 3\left(\frac{1}{2}\right)^{n} - 3$

as $n \to \infty$

$\boxed{y[n] = -3}$

8) $h[n] - \frac{1}{2} h[n-1] = \delta[n] - \delta[n-1] + \delta[n-2]$

a)

$h[0] - \frac{1}{2} h[-1] = 1$ $h[-1] - \frac{1}{2} h[-2] = 0$

$h[1] - \frac{1}{2} h[0] = -1$

$h[2] - \frac{1}{2} h[1] = 1$ $h[3] - \frac{1}{2} h[2] = 0$

For a causal LTI system, $h[n]$ should be zero for $n<0$.

$h[0] = 1 \quad h[1] = -\frac{1}{2} \quad h[2] = \frac{3}{4}$

$h[n] = \frac{3}{4}\left(\frac{1}{2}\right)^{n-2}$ for $n \geq 2$

$h[n] = \frac{3}{4}\left(\frac{1}{2}\right)^{n-2} \cdot u[n] - 2\delta[n-1] - 2\delta[n]$

b) $Y(e^{j\omega}) - \frac{1}{2} Y(e^{j\omega}) \cdot e^{-j\omega} = X(e^{j\omega})\left[1 - e^{-j\omega} + e^{-j2\omega}\right]$

$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1 - e^{-j\omega} + e^{-j2\omega}}{1 - \frac{1}{2} e^{-j\omega}}$ $\left(h[n] = \left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{2}\right)^{n-1} u[n-1] + \left(\frac{1}{2}\right)^{n-2} u[n-2]\right)$

d) $H(e^{j\frac{\pi}{3}}) = \frac{1 - e^{-j\frac{\pi}{3}} + e^{-j\frac{2\pi}{3}}}{1 - \frac{1}{2} e^{-j\frac{\pi}{3}}}$ $H(e^{j\frac{\pi}{2}}) = \frac{1 - e^{-j\frac{\pi}{2}} + e^{-j\pi}}{1 - \frac{1}{2} e^{-j\frac{\pi}{2}}}$

$= \frac{1 - \frac{1}{2} + j\frac{\sqrt{3}}{2} - \frac{1}{2} - j\frac{\sqrt{3}}{2}}{\ldots} = 0$ $= \frac{j}{1 + \frac{1}{2}j} = |H(e^{j\frac{\pi}{2}})| \cdot e^{j\measuredangle H(e^{j\frac{\pi}{2}})}$

$\boxed{y[n] = |H(e^{j\frac{\pi}{2}})| \sin\left(\frac{\pi}{2}n + \frac{\pi}{4} + \measuredangle H(e^{j\frac{\pi}{2}})\right)}$

e) $H^*(e^{j(2\pi-\omega)}) = H^*(e^{-j\omega})$ → periodic with 2π

$$= F\{h^*[n]\}$$

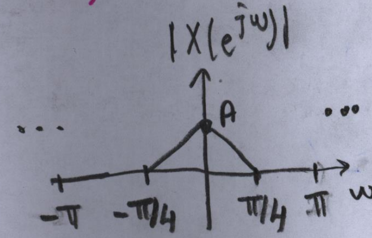
$h[n] = h^*[n]$ → for real valued $h[n]$

$$(H^*(e^{j(2\pi-\omega)}) = 1-e^{-j\omega}+e^{-j2\omega} / (1-\tfrac{1}{2}e^{-j\omega}))$$

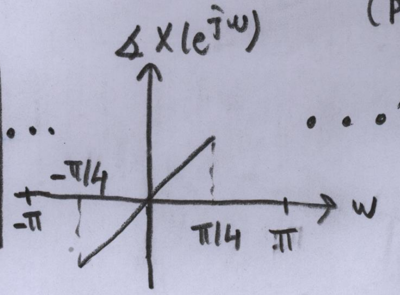
g) For real valued $x[n]$ → $|X(e^{j\omega})|$ is an even function.

↳ $\measuredangle X(e^{j\omega})$ is an odd function

a)



$$\left[\begin{array}{l} |b| = |a| \\ 2b = \frac{\pi}{2} \\ b = \pi/4 \quad a = -\pi/4 \end{array}\right]$$

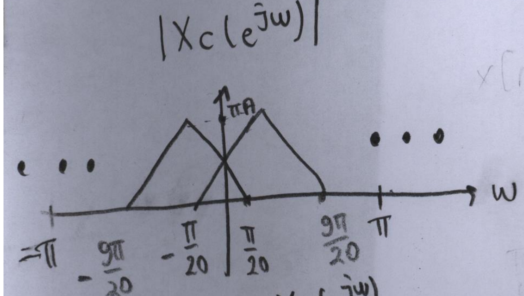


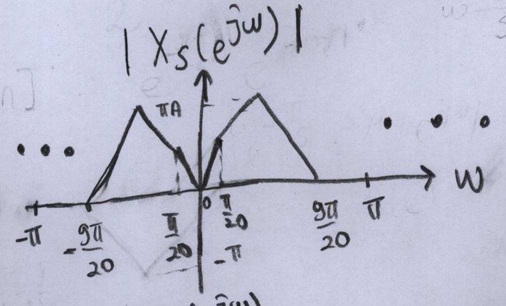
(Periodic with 2π)

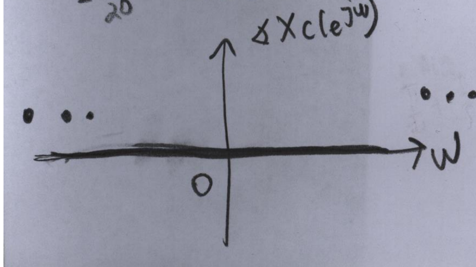
b) $X_c(e^{j\omega}) = \pi\left[X(e^{j(\omega-\frac{\pi}{5})}) + X(e^{j(\omega+\frac{\pi}{5})})\right]$

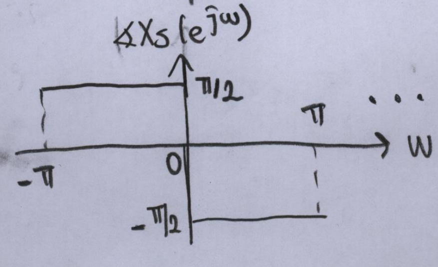
$X_s(e^{j\omega}) = \frac{\pi}{j}\left[X(e^{j(\omega-\frac{\pi}{5})}) - X(e^{j(\omega+\frac{\pi}{5})})\right]$

c) If $X(e^{j\omega})$ is real valued → $\measuredangle X(e^{j\omega}) = 0$









10) 2.55)

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] . e^{-j\omega n}$$

- $X(e^{j0}) = \sum_{n=-\infty}^{+\infty} x[n] = 8$
- $X(e^{j\pi}) = \sum_{n=-\infty}^{+\infty} x[n] \, (-1)^n = 2-(6) = -4$

- $x[n+2] \Rightarrow$ real and even function

  $X(e^{j\omega}).e^{j2\omega} \Rightarrow$ real valued $\boxed{\measuredangle X(e^{j\omega}) = -2\omega}$

- $x[n] = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} X(\omega).e^{j\omega n} \, d\omega \Rightarrow \underbrace{x[0]}_{1} = \frac{1}{2\pi} \int_{\langle 2\pi \rangle} X(\omega) \, d\omega$

$$\boxed{\int_{\langle 2\pi \rangle} X(\omega) \, d\omega = 2\pi}$$

- $X(e^{-j\omega}) \xrightarrow{F^{-1}} \boxed{x[-n]}$

- $Re\{X(e^{j\omega})\} = \frac{X(e^{j\omega}) + X^*(e^{j\omega})}{2}$

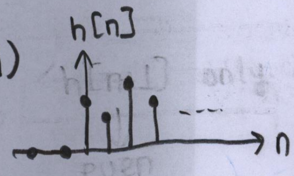
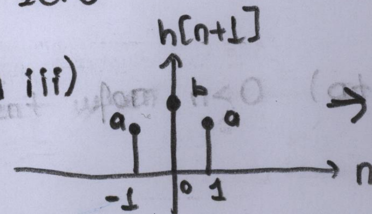
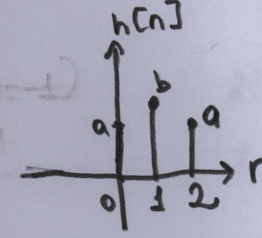
$$\xrightarrow{F^{-1}} \frac{x[n] + x^*[-n]}{2} = \boxed{\frac{x[n] + x[-n]}{2}}$$

11) 2.60) a) ii) $H(e^{j\omega}) = H^*(e^{-j\omega}) \Rightarrow h[n] = h^*[n]$
(real valued)

iii) $H(e^{j\omega}) \cdot e^{j\omega}$ is real $\Rightarrow \measuredangle H(e^{j\omega}) = -\omega$

$h[n+1] \Rightarrow$ real and even

i) for $n<0$, $h[n]$ is zero

i)  $\rightarrow$ i) and iii)  $\rightarrow$ 

(only 3 nonzero components)

11) 2.60) b)

iv) $\frac{1}{2\pi} \int_{\langle 2\pi \rangle} H(e^{j\omega}) = \boxed{h[0] = 2 = h[2]}$

from (10)

v) $H(e^{j\pi}) = \sum_{n=0}^{2} h[n](-1)^n = 0$ (from (10))

$= h[0] - h[1] + h[2] = 0$

$= 4 - h[1] = 0$

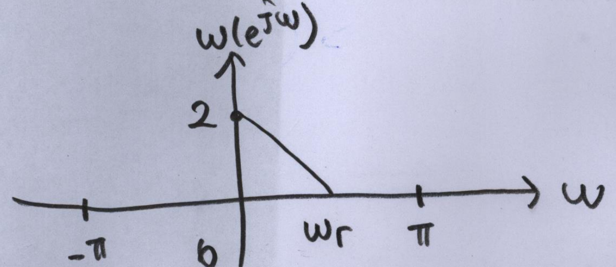
$\boxed{h[1] = 4}$

12) 2.65) $Y(e^{j\omega}) = \begin{cases} -jX(e^{j\omega}) & , 0<\omega<\pi \\ +jX(e^{j\omega}) & , -\pi<\omega<0 \end{cases}$

$$W(e^{j\omega}) = X(e^{j\omega}) + \underbrace{j\,Y(e^{j\omega})}$$

$$= \begin{cases} X(e^{j\omega}) & , 0<\omega<\pi \\ -X(e^{j\omega}) & , -\pi<\omega<0 \end{cases}$$

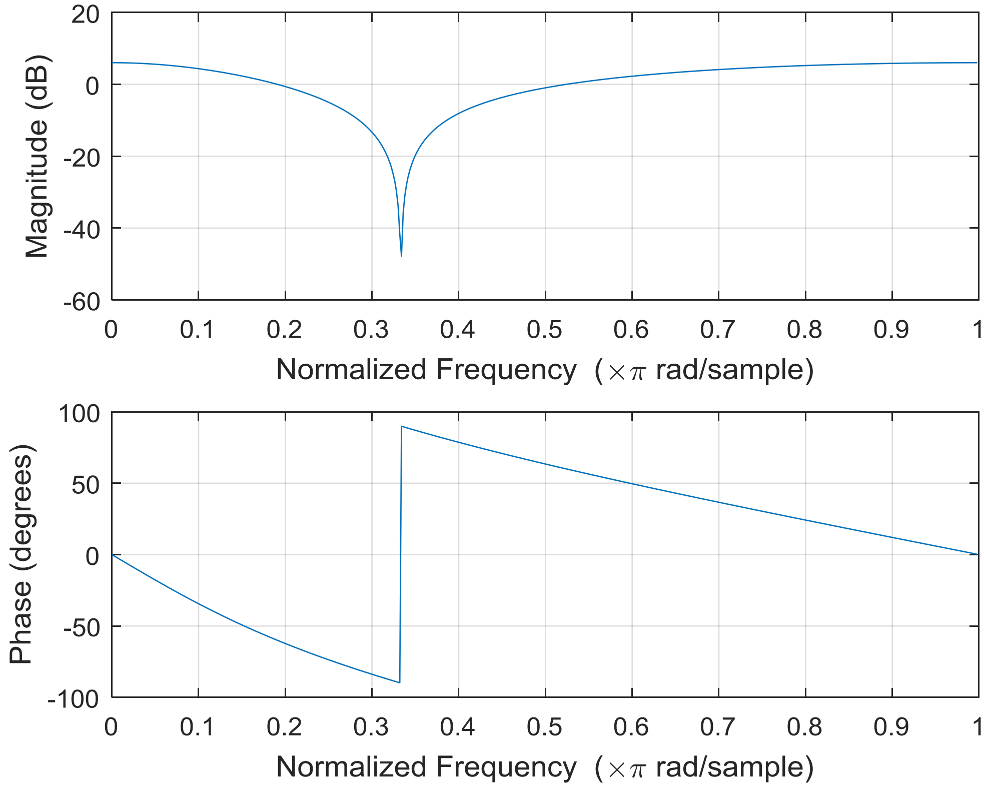
$$W(e^{j\omega}) = \begin{cases} 2X(e^{j\omega}) & , 0<\omega<\pi \\ 0 & , -\pi<\omega<0 \end{cases}$$



MATLAB PART

8) c)

```
>> freqz([1 -1 1], [1 -1/2])
```

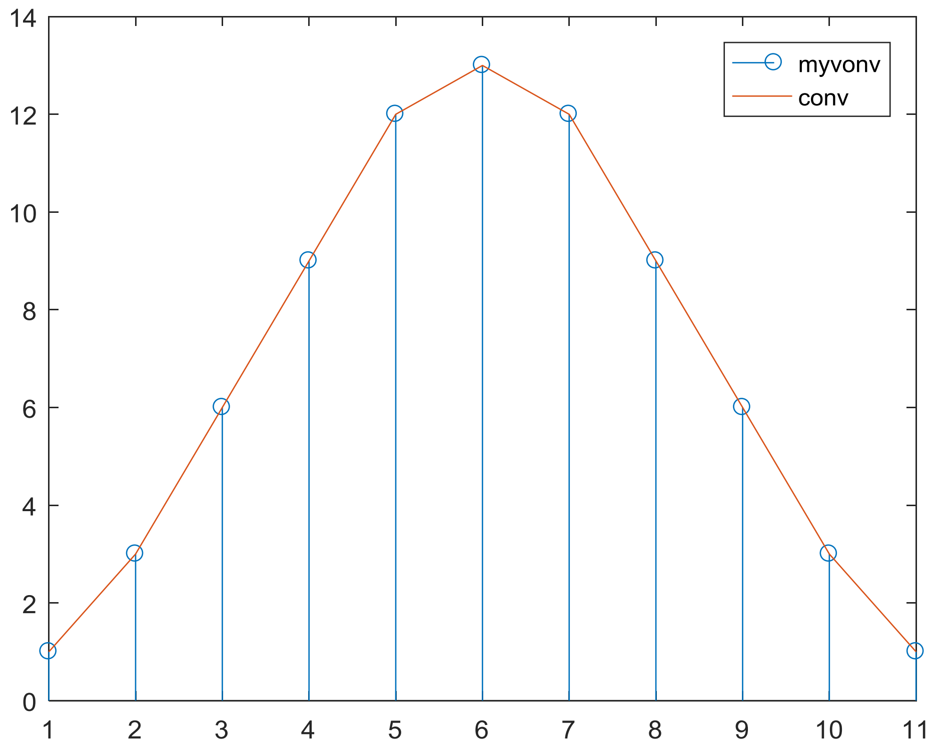


13)

a)

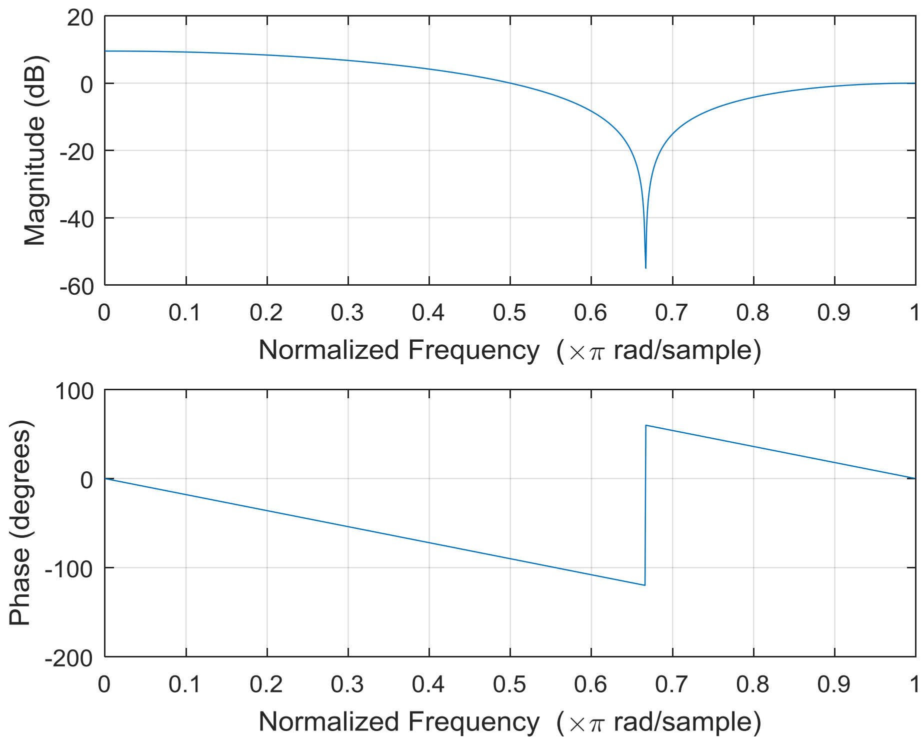
```
function y=myconv(x,h)
y=zeros(1,length(h)+length(x)-1);
for kk=1:length(h)
    y(kk:kk+length(x)-1)=y(kk:kk+length(x)-1)+h(kk)*x;
end
end
```

b)



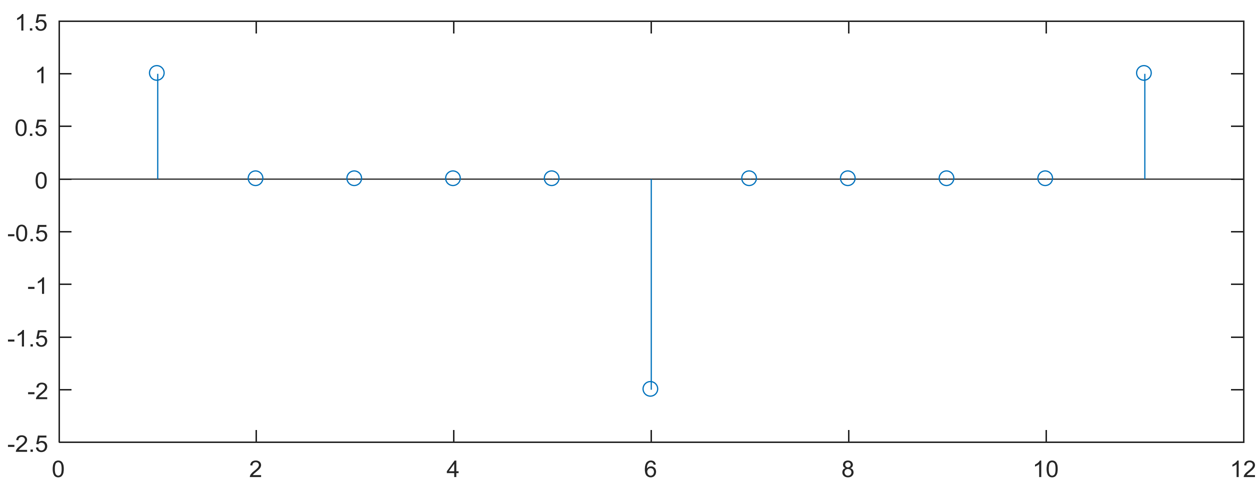
c)

```
>> freqz(h,1,1024)
```

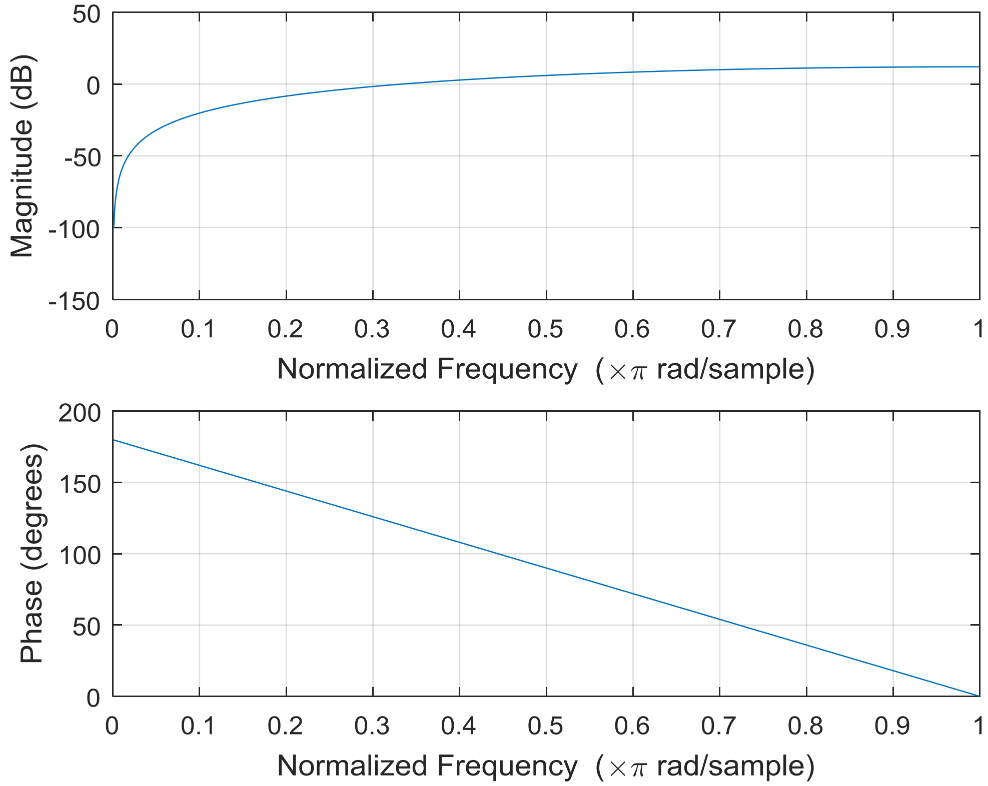


It is a lowpass filter and it is a moving average filter. Hence, it smoothes the original signal.

d)

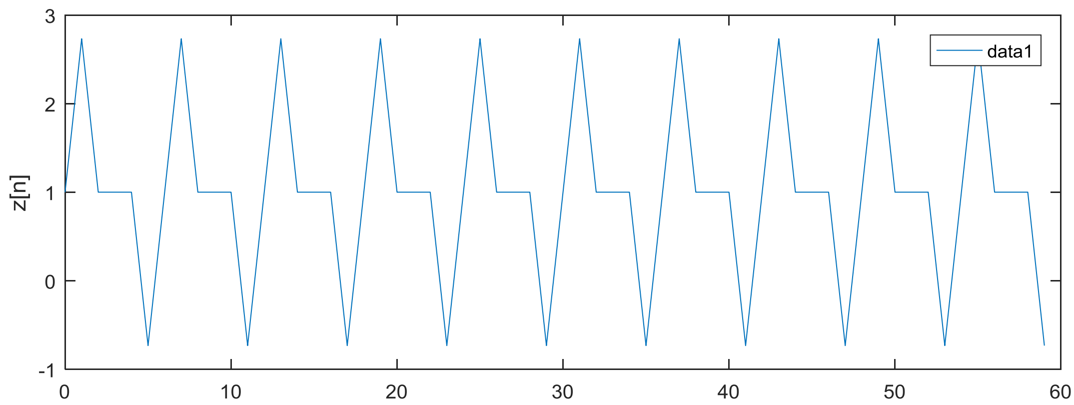


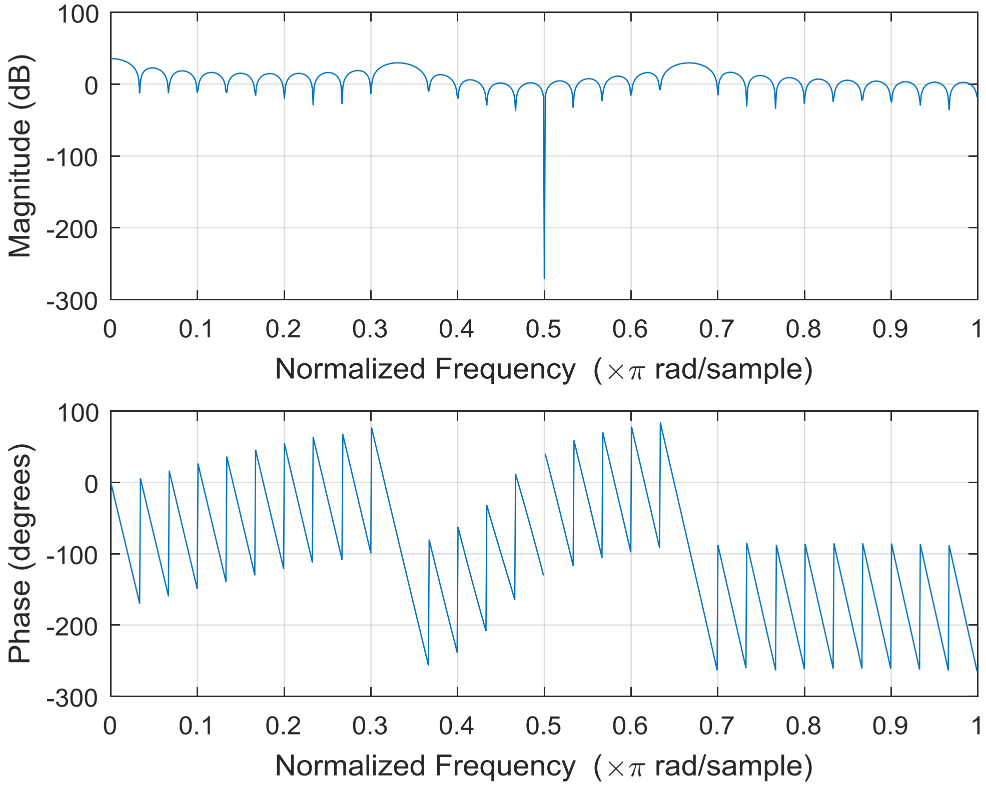
e)



It is a highpass filter and the filter extracts the high frequency component of the input signal. Notice the sharp changes in the output signal.

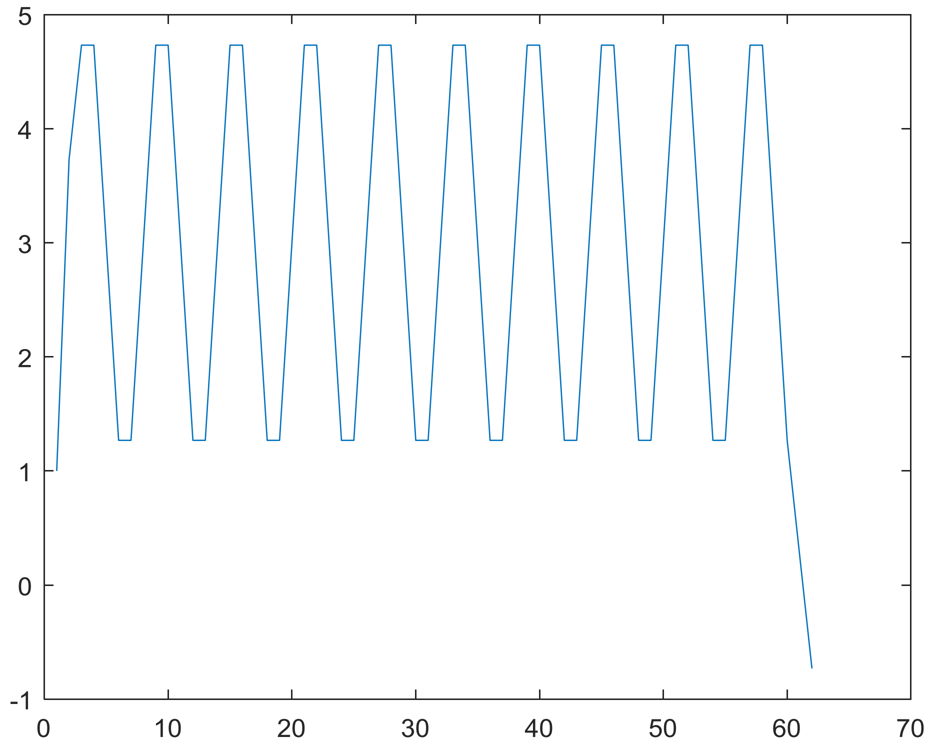
f)

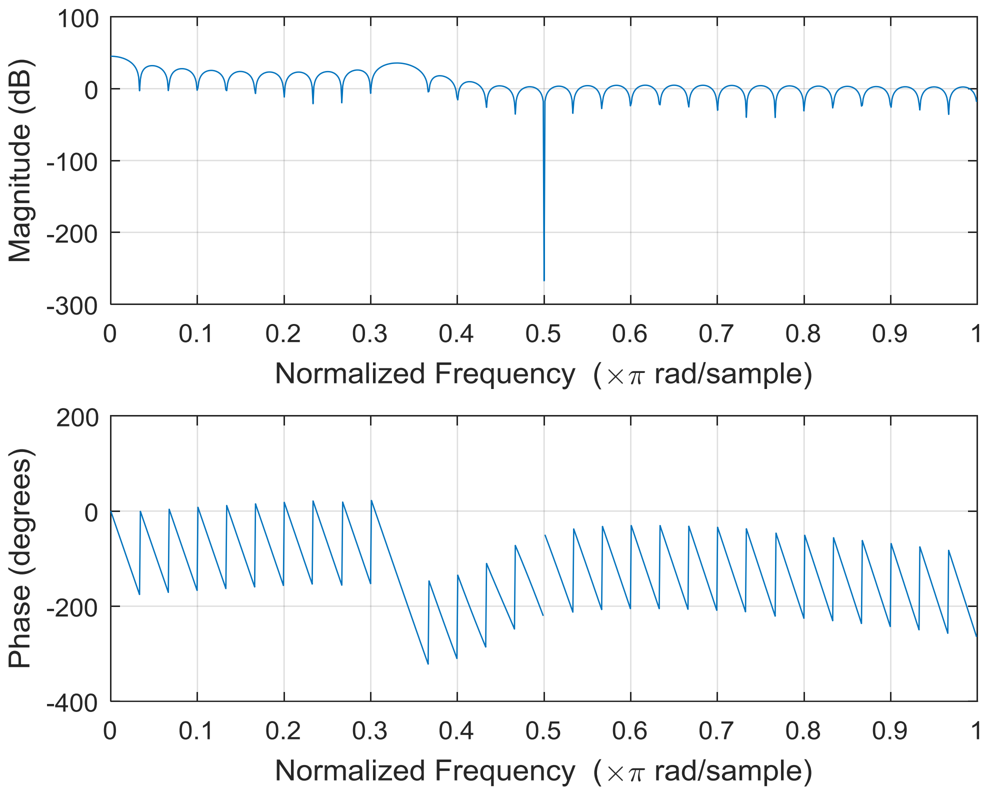




We observe peaks in the magnitude graph at the 0, pi/3, and 2pi/3 freqeuncies which are the frequencies of the three sinosoidal components of z[n].

g)





Since the filter has a zero at 2pi/3 frequency, the correspondind sinusoidal component of z[n] is eliminated.