DISCRETE-TIME SYSTEMS

Classification of Systems

- with memory - memoryless
- linear - nonlinear
- time-invariant – time-varying
- causal-noncausal
- stable-unstable

A Quotation from a Recent Research Paper:

Null Space Component Analysis for Noisy Blind Source Separation

“The solutions for the BSS problem were investigated under various source signal mixing models. Initially, linear instantaneous (**memoryless**) mixing models were used [3], followed by linear **convolution** mixing models [4]. More recently, nonlinear mixing models [5, 6, 7], bounded component analysis [8, 9], and the sparsity-based approach [10, 11] have been exploited.”

DISCRETE-TIME SYSTEMS

A system is a transformation of signals

A system is an input-output relationship

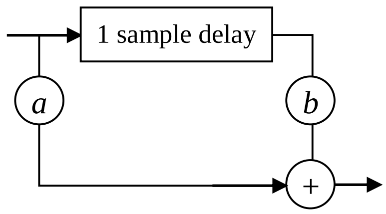
$$x[n] \longrightarrow \boxed{T\{\cdot\}} \longrightarrow y[n]$$

A SISO system

Ex: A delay system $y[n] = x[n - \Delta]$

Ex: A FIR (Finite Impulse Response) system

$$y[n] = ax[n] + bx[n-1]$$



In general,

$$y[n] = \sum_{N_1}^{N_2} a_k x[n-k]$$

Ex: An IIR (Infinite Impulse Response) system

$$y[n] = y[n-1] + x[n]$$

Equivalently,

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

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WITH MEMORY - MEMORYLESS

$$y[n] = x[n], \quad y[n] = 3x[n], \quad y[n] = 4^{x[n]}$$

are memoryless

whereas

$$y[n] = x[n-1],$$

$$y[n] = x[n+1],$$

$$y[n] = x[n-1] + x[n],$$

$$y[n] = y[n-1] + x[n]$$

have memory

You have heard or you will hear about “dynamic systems”; they have memory.

LINEARITY

A system, $T\{\bullet\}$, is said to be linear if it satisfies

a) additivity: $T\{x_1[n]+x_2[n]\}=T\{x_1[n]\}+T\{x_2[n]\}$

b) homogeneity: $T\{ax[n]\}=aT\{x[n]\}$

Ex: $y[n]=\sum_{k=-\infty}^{n}x[k]$ linear.

$y[n]=\log|x[n]|$ nonlinear

$y[n]=x[n]+3$ nonlinear

TIME-INVARIANCE

Let $y_1[n] = T\{x[n]\}$ and $y_2[n] = T\{x[n-\Delta]\}$ be the outputs of the system to *x*[*n*] and *x*[*n*-*Δ*], respectively.

Then, if $y_2[n] = y_1[n-\Delta]$ the system is said to be time-invariant.

Ex: (compressor/downsampler) $y[n] = x[Mn]$ *M*: integer

$$\rightarrow \boxed{\downarrow M} \rightarrow$$

Is it time-invariant?

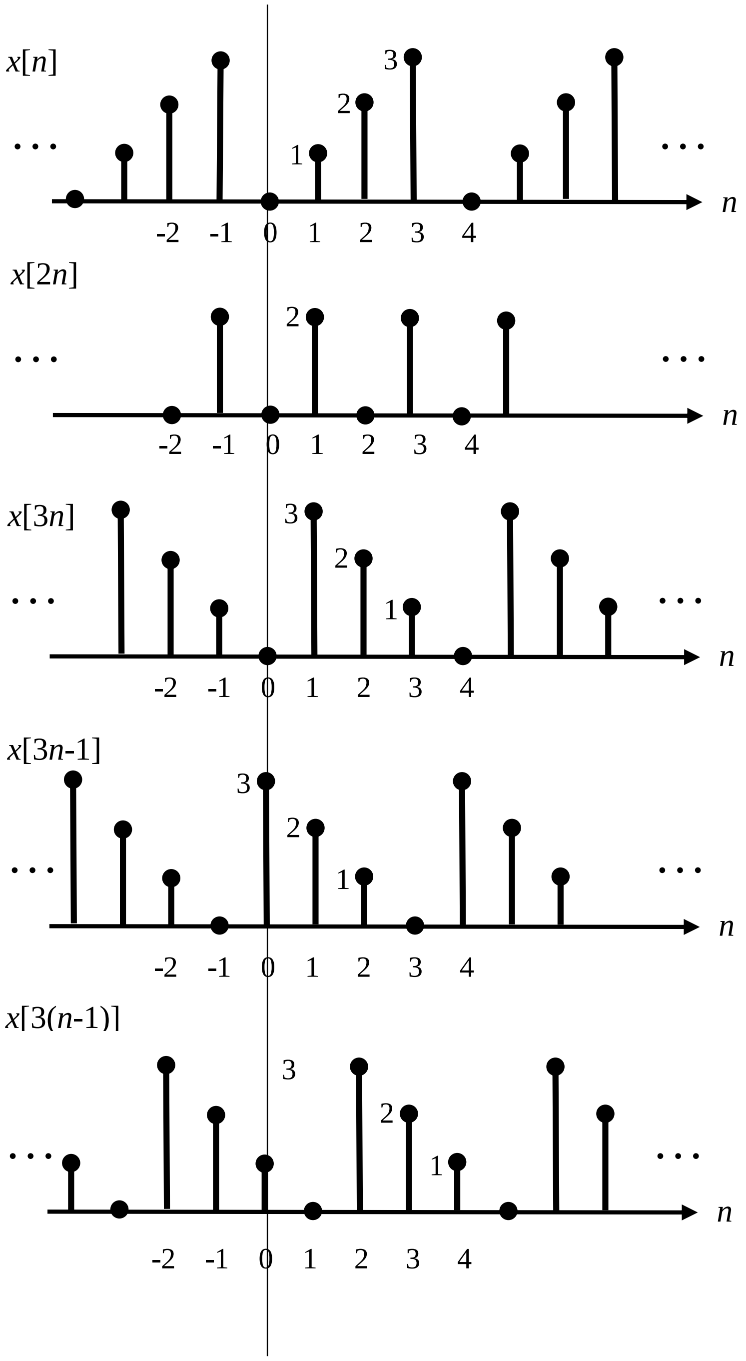
Following the above definition $y_1[n] = x[Mn]$, $y_2[n] = x[Mn-\Delta]$

$\Rightarrow y_2[n] \neq y_1[n-\Delta] = x[Mn - M\Delta]$

So, the system is time-varying.

Show that it is linear! (exercise)

Example: Downsampler is time-varying



$M = 2$

$M = 3$

The response to $x[n-1]$ when $M = 3$

$x[3n]$ delayed by 1 sample

Ex: (expander/upsampler) $y[n]=\begin{cases} x\left[\dfrac{n}{L}\right] & n=kL \\ 0 & n\neq kL \end{cases}$; k, L: integer $\longrightarrow \boxed{\uparrow L} \longrightarrow$

Is it time-invariant?

$$\boxed{y_1[n]=\begin{cases} x\left[\dfrac{n}{L}\right] & n=kL \\ 0 & n\neq kL \end{cases}} \qquad \boxed{y_2[n]=\begin{cases} x\left[\dfrac{n}{L}-\Delta\right] & n=kL \\ 0 & n\neq kL \end{cases}}$$

$$\Rightarrow\ y_2[n]\neq y_1[n-\Delta]=\begin{cases} x\left[\dfrac{n-\Delta}{L}\right] & n-\Delta=kL \\ 0 & n-\Delta\neq kL \end{cases}$$

So, the system is time-varying

Show that it is linear! (exercise)

CAUSALITY

A system is said to be causal if the two output signals $y_1[n]$ and $y_2[n]$ (due to two input signals $x_1[n]$ and $x_2[n]$) satisfy

$$y_1[n] = y_2[n] \qquad n \le n_0$$

whenever

$$x_1[n] = x_2[n] \qquad n \le n_0$$

Ex: $y[n] = x[n+1] - x[n]$ noncausal

$y[n] = x[n-1] - x[n]$ causal

$y[n] = x[n] + 5$ causal

STABILITY (BIBO)

A system is said to be BIBO stable if "bounded inputs yield bounded outputs.", i.e.,

$$|x[n]| \leq B_x < \infty \qquad \Rightarrow \qquad |y[n]| \leq B_y < \infty$$

for arbitrary finite B_x and B_y .

Ex: $$y[n] = \sum_{k=-\infty}^{n} x[k] = y[n-1] + x[n]$$

UNSTABLE

For example, for $x[n] = u[n]$ the output is $y[n] = \begin{cases} n+1 & n \geq 0 \\ 0 & n < 0 \end{cases}$

Bounded input does not yield bounded output.