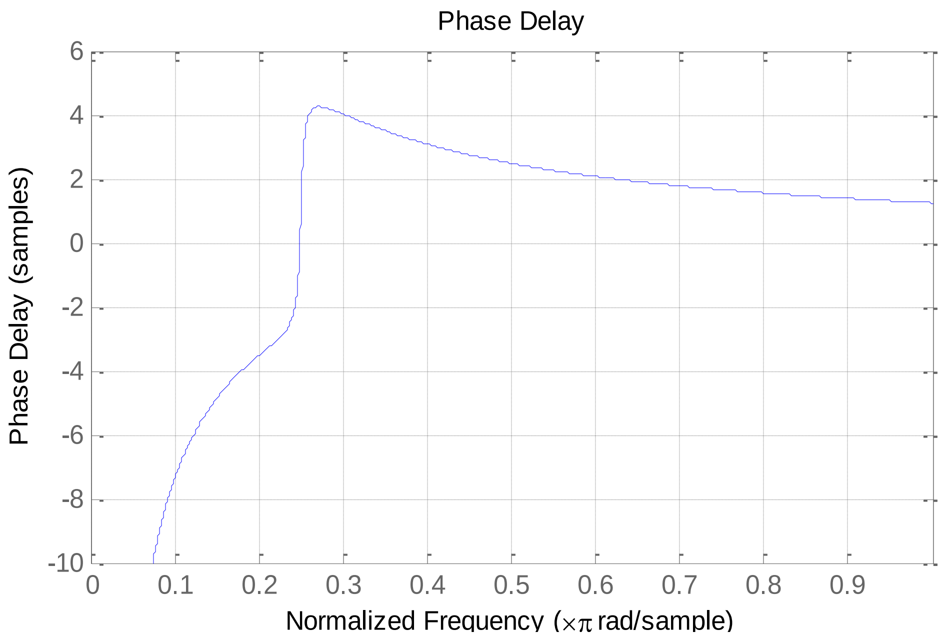
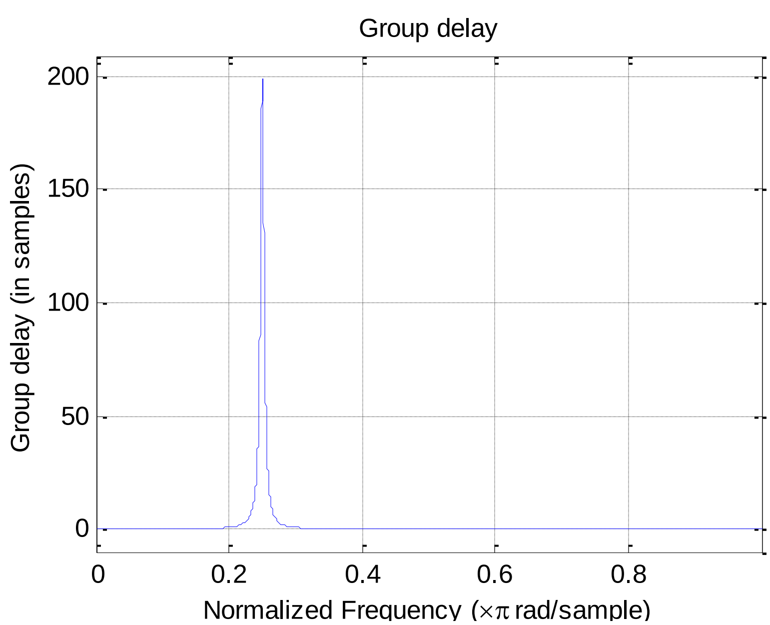
EE 430 Section 2 HW5 Answers Part-1

(For any questions contact Erdal Epçaçan, epcacan@metu.edu.tr, D-122)

1) The magnitude response is zero at $\omega = \theta$. $180°$ phase shift occurs when ω pass through θ.
2)
3)
   a.



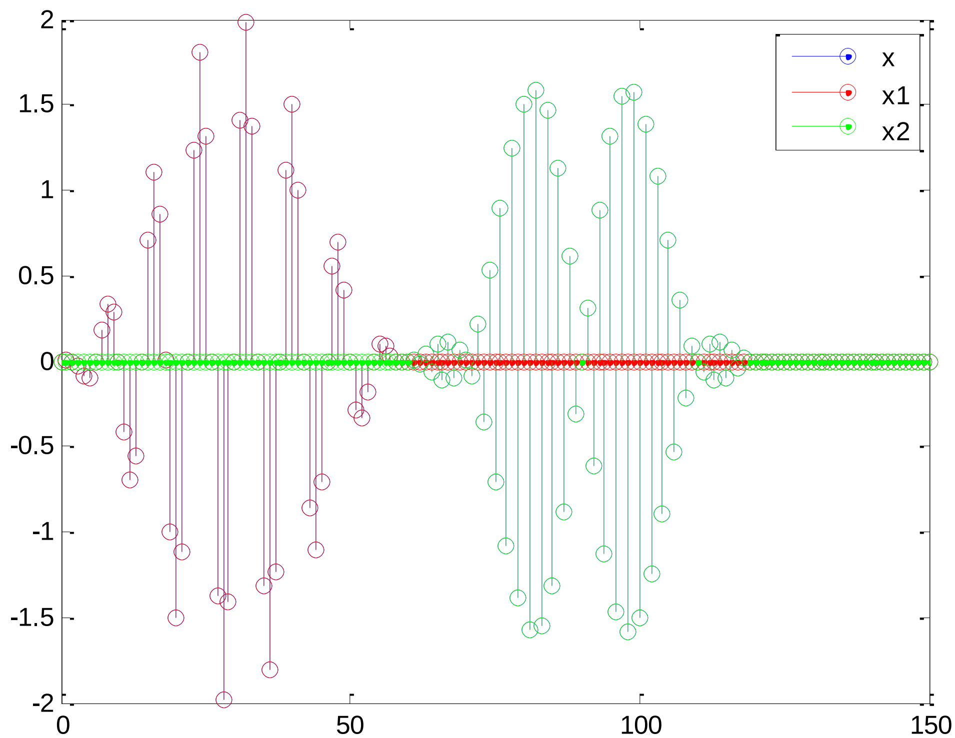


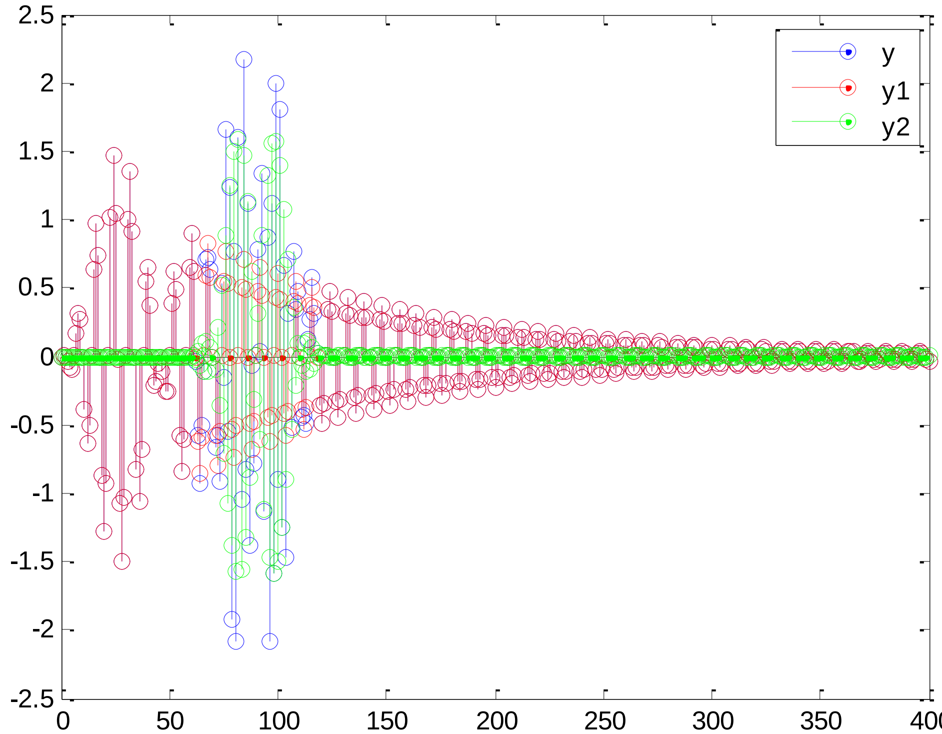
b. Let

$$x[n] = x_1[n] + x_2[n]$$
$$x_1[n] = \left(1 - cos\left(\frac{\pi}{30n}\right)\right) cos\left(\frac{\pi}{4}n\right)(u[n] - u[n-60])$$
$$x_2[n] = \left(1 - cos\left(\frac{\pi}{30}(n-60)\right)\right) cos\left(\frac{19\pi}{20}n\right)(u[n-60] - u[n-120])$$

Since the system is all pass and group delay is almost zero for $x_2[n]$, it will be observed at the output almost with no change. For $x_1[n]$, $cos\left(\frac{\pi}{4}n\right)$ part will decay very slowly since its frequency is very close to the pole angle (frequency).

c.

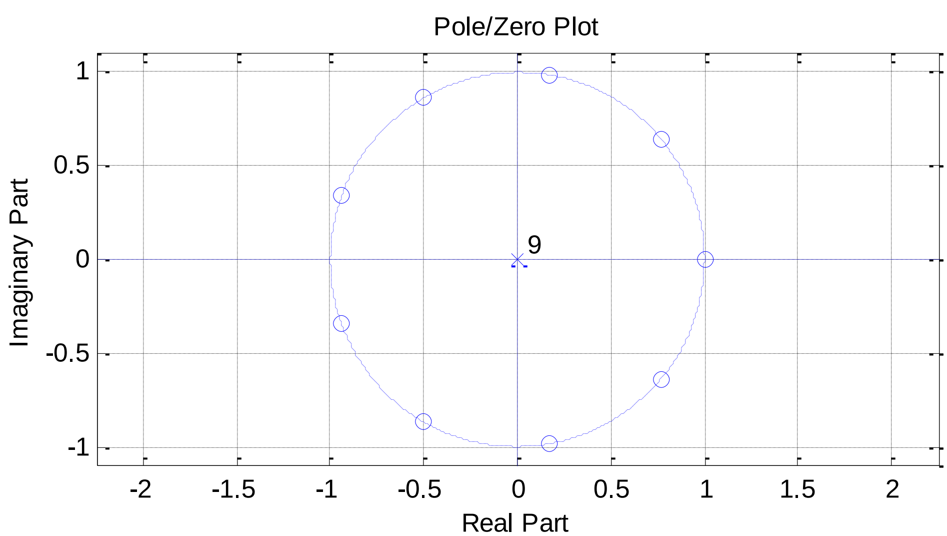


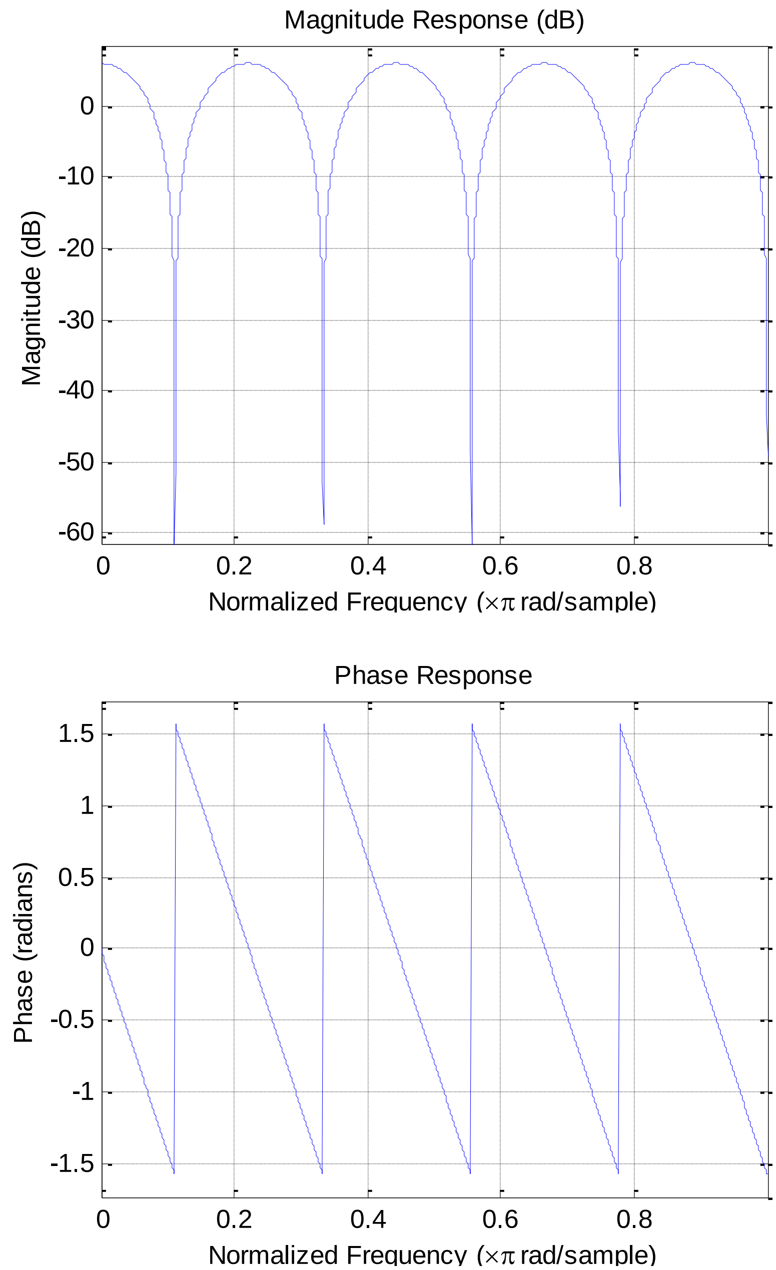


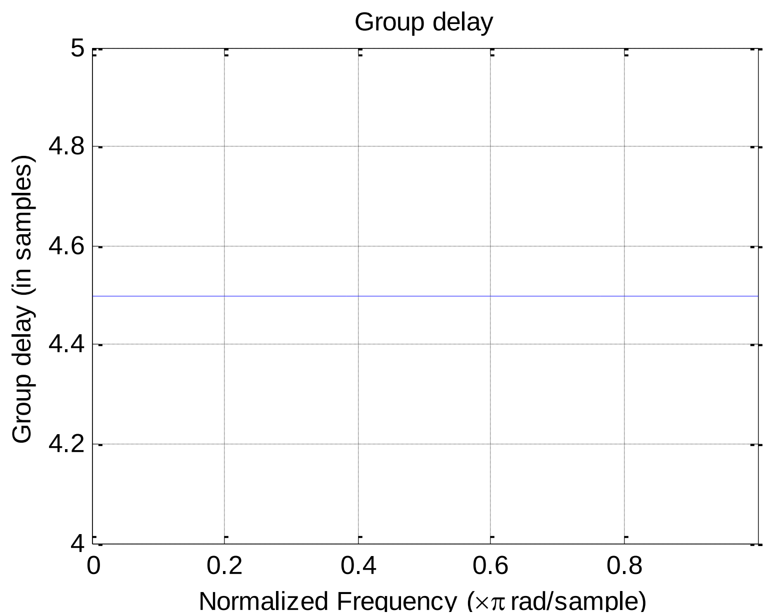
4)

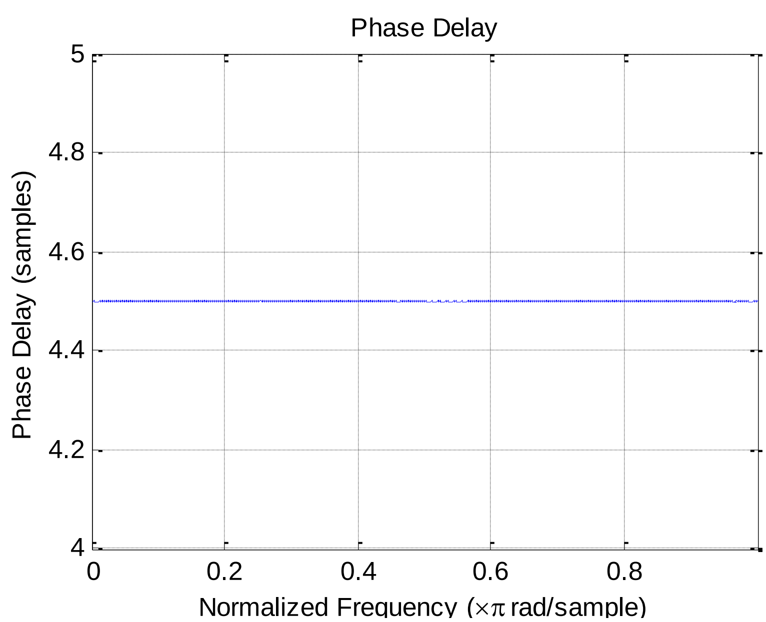
a. $h[n] = \delta[n] - \delta[n-9]$

b.




Magnitude Response (dB)
Magnitude (dB)
0
-10
-20
-30
-40
-50
-60
0
0.2
0.4
0.6
0.8
Normalized Frequency (×π rad/sample)
Phase Response
Phase (radians)
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
Normalized Frequency (×π rad/sample)

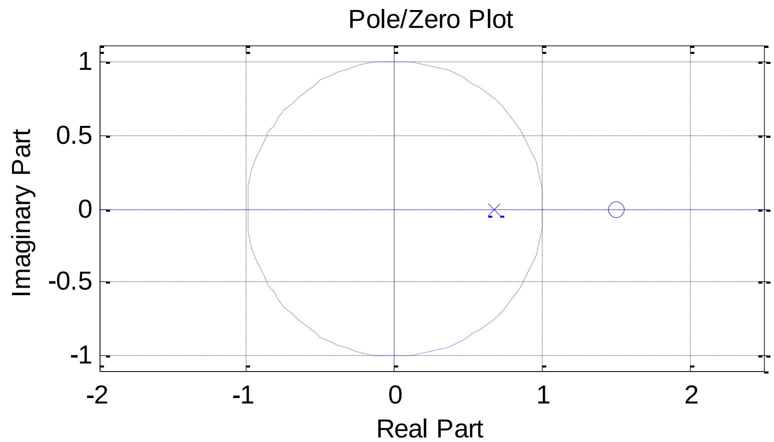

Group delay
Group delay (in samples)
5
4.8
4.6
4.4
4.2
4
0
0.2
0.4
0.6
0.8
Normalized Frequency (×π rad/sample)


Phase Delay
Phase Delay (samples)
5
4.8
4.6
4.4
4.2
4
0
0.2
0.4
0.6
0.8
Normalized Frequency (×π rad/sample)

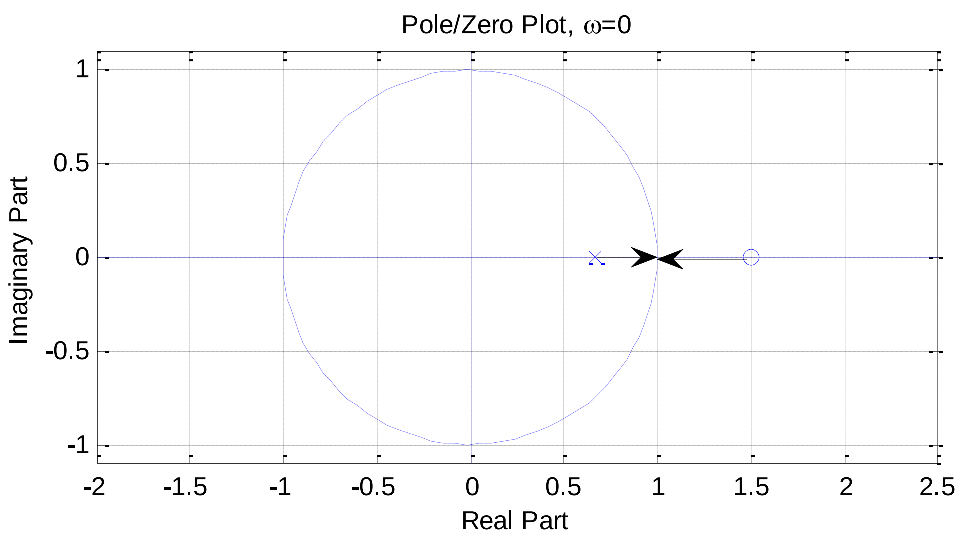
5)

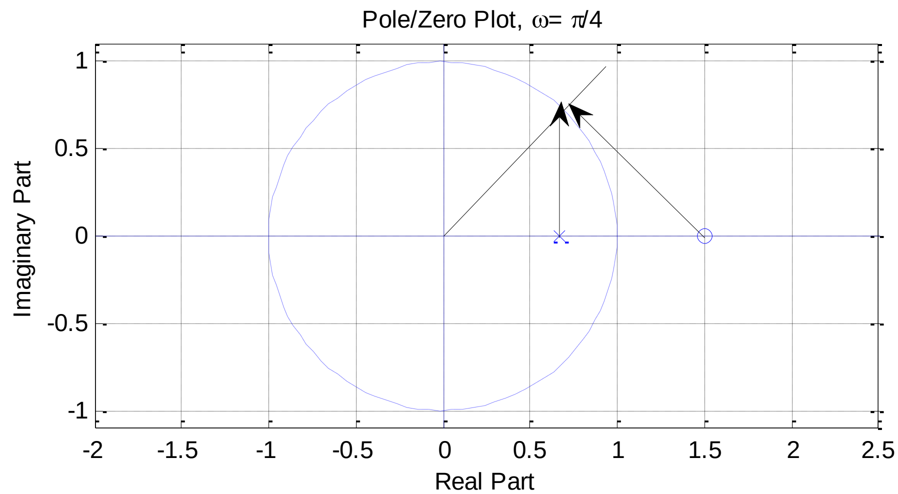
6)

a.


Pole/Zero Plot
Imaginary Part
Real Part
1
0.5
0
-0.5
-1
-2
-1
0
1
2

b.

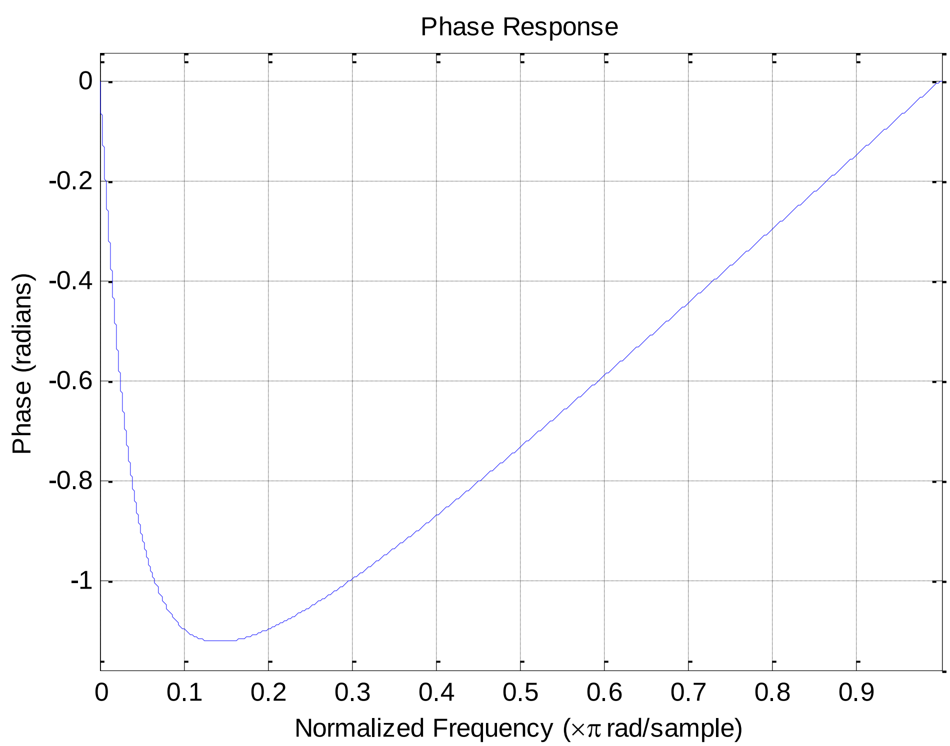

Pole/Zero Plot, ω=0
Imaginary Part
Real Part
1
0.5
0
-0.5
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5

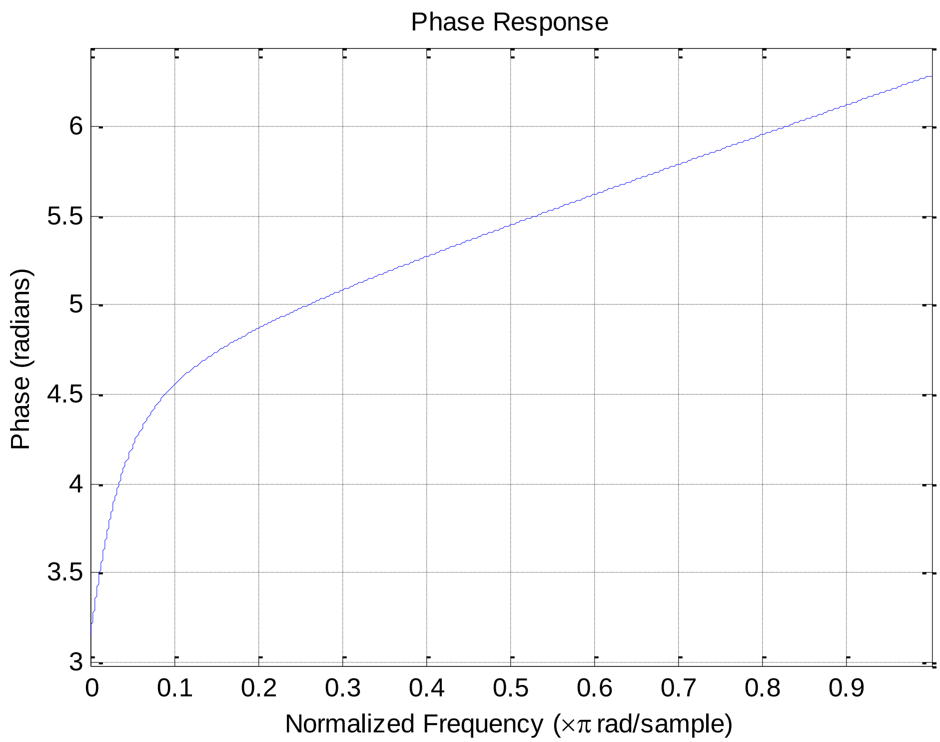

Pole/Zero Plot, ω= π/4
Imaginary Part
Real Part
1
0.5
0
-0.5
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5

c. 0°
d. Yes

7)

8)


Phase Response
Phase (radians)
Normalized Frequency (×π rad/sample)
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9



When the pole is inside the unit circle, the phase of the pole vector increases as ω changes from 0 to π.

When the pole is outside the unit cirle, the phase of the pole vector first decreases as ω changes from 0 to $\omega = \cos^{-1}\frac{9}{10}$, then increases as ω changes from $\omega = \cos^{-1}\frac{9}{10}$ to π.

9) No, they are not.
10) Yes, they are.

11)
12)

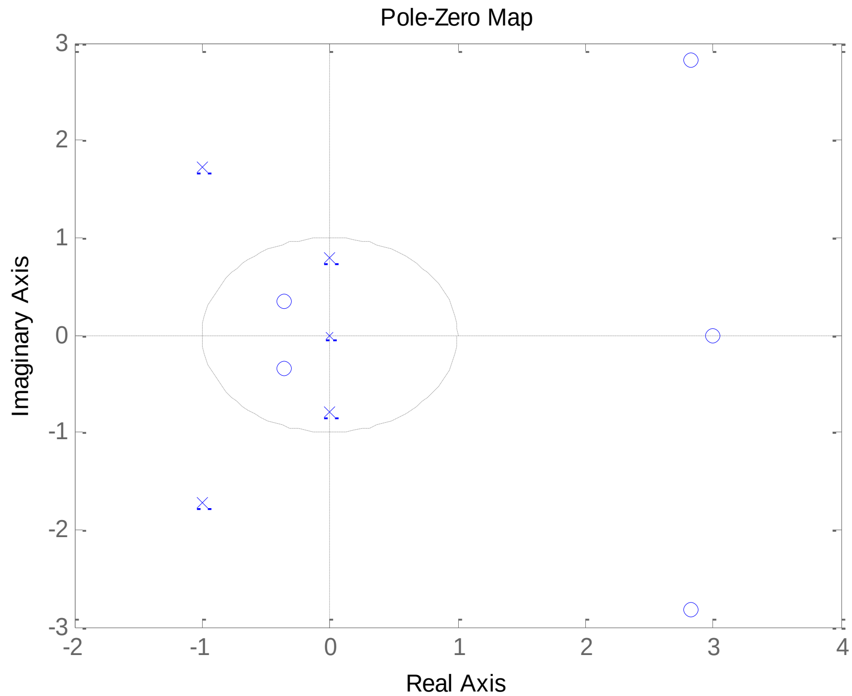
$$H_{all}(z) = \frac{\left(z^{-1} - \frac{1}{2}\right)\ \left(z^{-1} + \frac{1}{2}\right)}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{2}z^{-1}\right)}$$

$$H_{min}(z) = \frac{-4(2 - z^{-1})}{z^{-2} + z^{-1} + 2}$$

13)
14)

$$H(z) = \frac{\prod_{k=1}^{5}(1 - z_k z^{-1})}{\prod_{k=1}^{4}(1 - p_k z^{-1})}$$

a.



b.

$$H_{eg}(z) = H(z)H_{all}(z)$$

Any all pass system multiplied, in Z domain, will have the same magnitude system.

c. For stable and causal realization the poles of the overal system should be inside the unit circle, the poles outside the unit cirle should be elimşnated by allpass system. For real impulse response the zeros and poles of allpass system should be real.

15)
16)

$$H_{lin}(e^{j\omega}) = A(e^{j\omega})e^{j\omega\alpha}$$

$A(e^{j\omega}) > 0$ and real valued

$$H_{glin}(e^{j\omega}) = A(e^{j\omega})e^{j\omega\alpha + j\beta}$$

$A(e^{j\omega})$ real valued but bipolar

17) No, they should obey some symmetry properties due to their special form given in previous question
18) Biliniear transformation may yield a lower order design.
19)
20) Yes, they may, impulse invariance is many-to-one mapping due to aliasing. Ex.
21)

7.7

$$0.99 \leq \left|H_d(e^{j\omega})\right| \leq 1.01\,, \qquad 0 \leq |\omega| \leq 0.20\pi$$

$$\left|H_d(e^{j\omega})\right| \leq 0.01\,, \qquad 0.22\pi \leq |\omega| \leq \pi$$

7.13

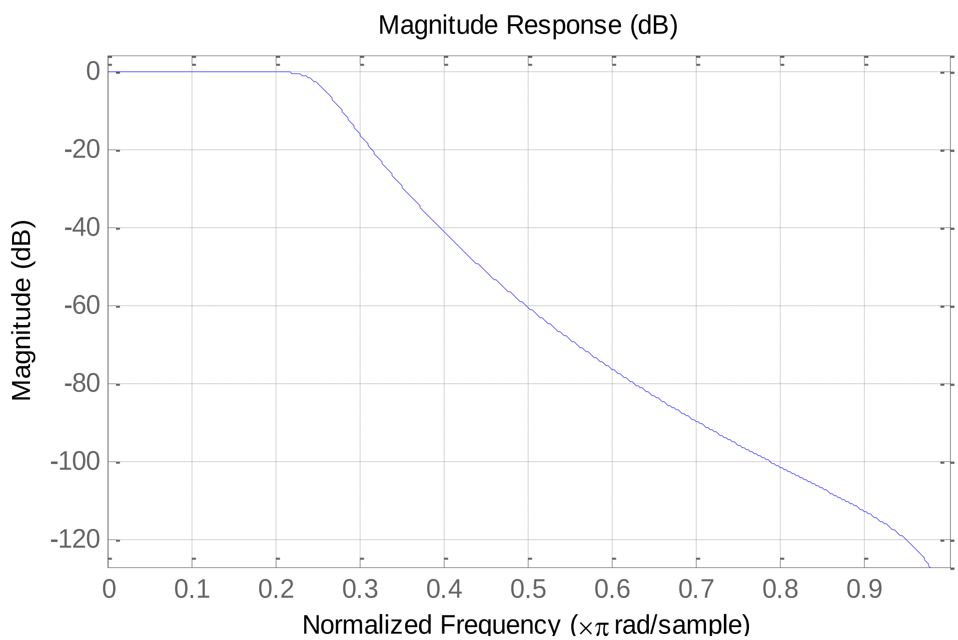
$T = 50\ \mu s$, T is unique

7.14

$T = 1.46\ ms$, T is unique

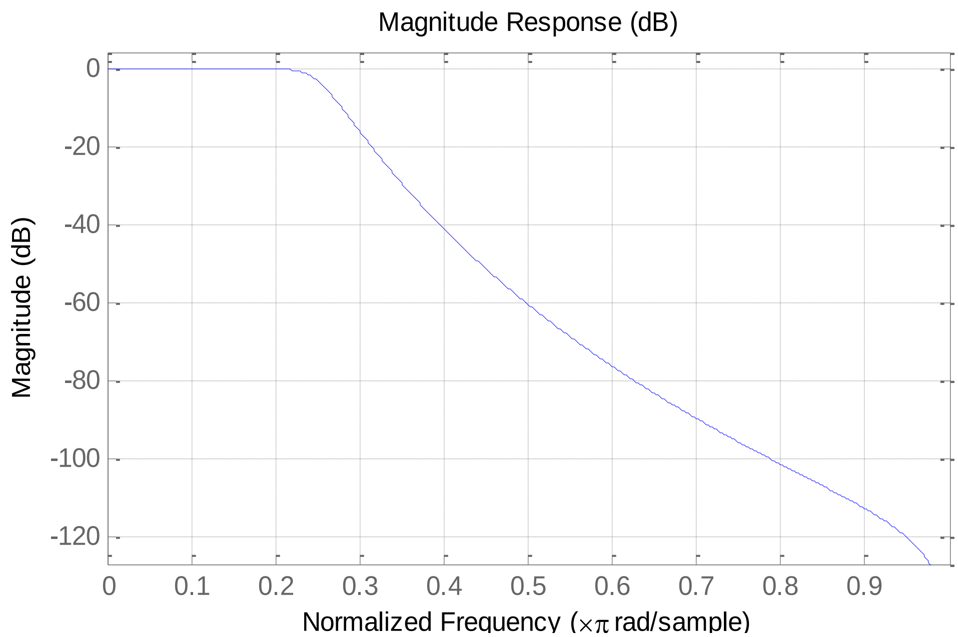
22) Hanning M=80, Hamming M = 80, Blackman M = 120
23)
24) Impulse invariance

$$b = [0.0000 \quad 0.0073 \quad 0.0467 \quad 0.0282 \quad 0.0016 \quad 0]$$
$$a = [1.0000 \quad -2.5558 \quad 2.9455 \quad -1.8133 \quad 0.5862 \quad -0.0787]$$



Bilinear

$$b = 0.0027 \quad 0.0134 \quad 0.0268 \quad 0.0268 \quad 0.0134 \quad 0.0027$$
$$a = 1.0000 \quad -2.5922 \quad 3.0272 \quad -1.8779 \quad 0.6112 \quad -0.0825$$

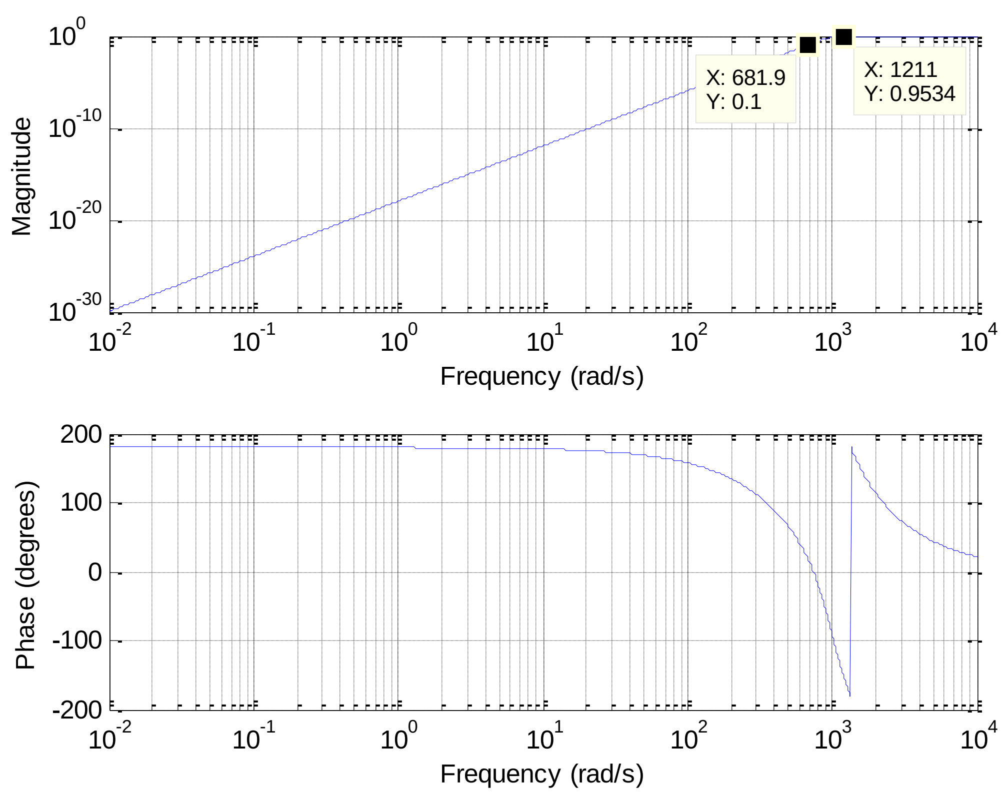


25)

26)

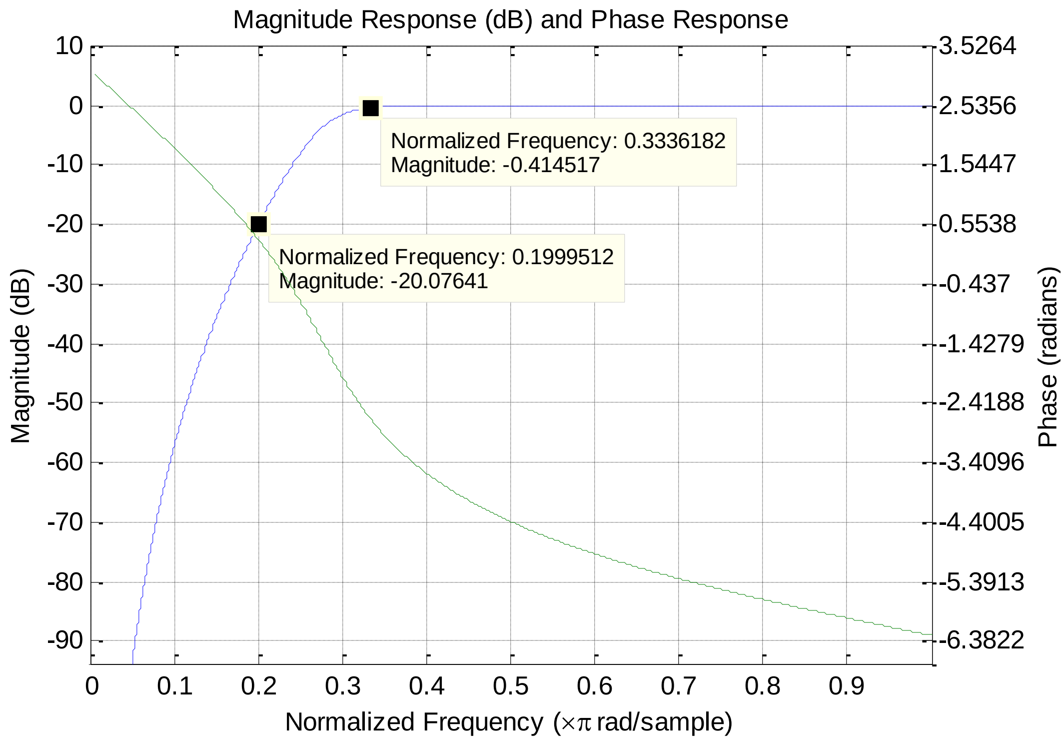
a. This is a butterworth filter

b.



c.

i. Bilinear transformation



$$\mathrm{T} = 1/1048, \qquad \omega_s \approx \frac{\pi}{5}$$

ii. Impulse invariance

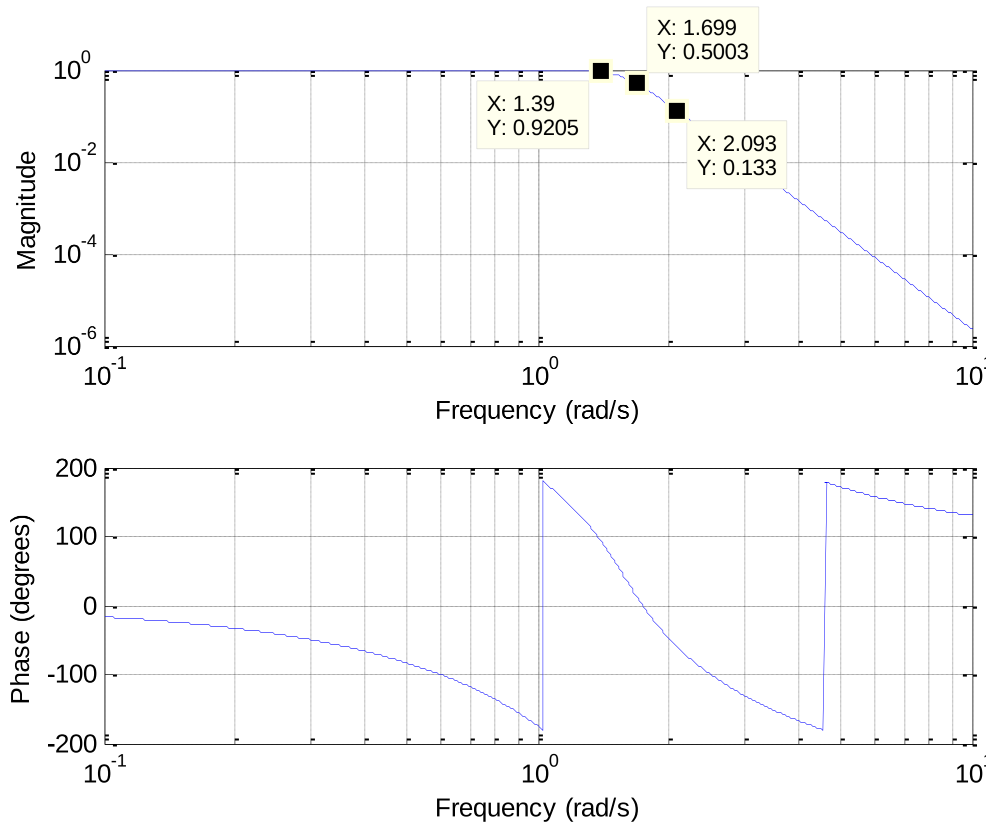
$$\mathrm{T} = \frac{1}{1156}$$

Poles:

$$0.5362 + 0.5929i$$
$$0.5362 - 0.5929i$$
$$0.4441 + 0.3115i$$
$$0.4441 - 0.3115i$$
$$0.4228 + 0.0963i$$
$$0.4228 - 0.0963i$$

27)

a. Can be designed with a butterworth filter of order 7 or higher and a cutt of frequecny of 1.69 rad/s



$$H(s) = \frac{23.6}{s^7 + 7.059\,s^6 + 24.92\,s^5 + 56.55\,s^4 + 88.84\,s^3 + 96.57\,s^2 + 67.51\,s + 23.6}$$

b. Poles:

$$0.0278 + 0.7045i$$

$$0.0278 - 0.7045i$$

$$0.1262 + 0.3537i$$

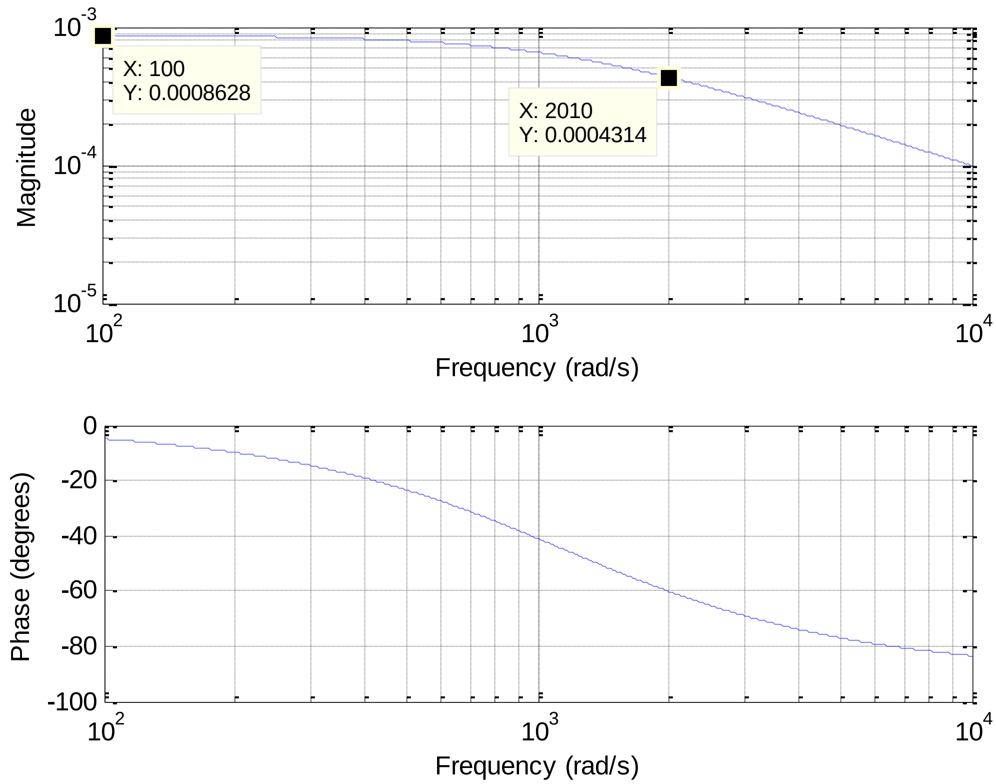
$$0.1262 - 0.3537i$$

$$0.1886 + 0.1530i$$

$$0.1886 - 0.1530i$$

$$0.2079 + 0.0000i$$

28)



$$\omega = \Omega T = 2010\frac{1}{1000} \approx \frac{1}{2}$$