Discrete-Time Systems

Classification of Systems

- with memory - memoryless
- linear - nonlinear
- time-invariant – time-varying
- causal-noncausal
- stable-unstable

Discrete-Time Systems

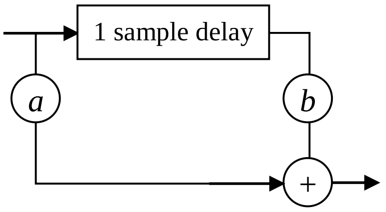
Roughly stated,
A system is a transformation
A system is an input-output relationship

$$x[n] \longrightarrow \boxed{T\{\cdot\}} \longrightarrow y[n]$$

SISO system

<u>Ex</u>: A delay system $y[n] = x[n-\Delta]$

<u>Ex</u>: $y[n] = ax[n] + bx[n-1]$



In general, $y[n] = \sum_{N_1}^{N_2} a_k x[n-k]$

<u>Ex</u>: $y[n] = y[n-1] + x[n] \qquad \Rightarrow \qquad y[n] = \sum_{k=-\infty}^{n} x[k]$

The input-output relationship
- may be linear or nonlinear
- may change in time (time-varying) or not (time-invariant)
- may involve finite or infinite number of input samples

Classification of Systems

- With memory - memoryless
- Linear - nonlinear
- Time-invariant – time-varying
- Causal-noncausal
- Stable-unstable

With memory - memoryless

$y[n] = x[n]$, $y[n] = 3x[n]$, $y[n] = 4^{x[n]}$ are memoryless

whereas

$y[n] = x[n-1]$,

$y[n] = x[n+1]$,

$y[n] = x[n-1] + x[n]$,

$y[n] = y[n-1] + x[n]$ have memory

Linearity

A system, $T\{\bullet\}$, is said to be linear if it satisfies

a) additivity: $T\{x_1[n]+x_2[n]\}=T\{x_1[n]\}+T\{x_2[n]\}$

b) homogeneity: $T\{ax[n]\}=aT\{x[n]\}$

Ex: $y[n]=\sum_{k=-\infty}^{n}x[k]$ linear.

$y[n]=\log|x[n]|$ nonlinear

$y[n]=x[n]+3$ nonlinear

Time-Invariance

Let $y_1[n]=T\{x[n]\}$ and $y_2[n]=T\{x[n-\Delta]\}$ be the outputs of the system to $x[n]$ and $x[n-\Delta]$, respectively.

Then, if $y_2[n]=y_1[n-\Delta]$ the system is said to be time-invariant.

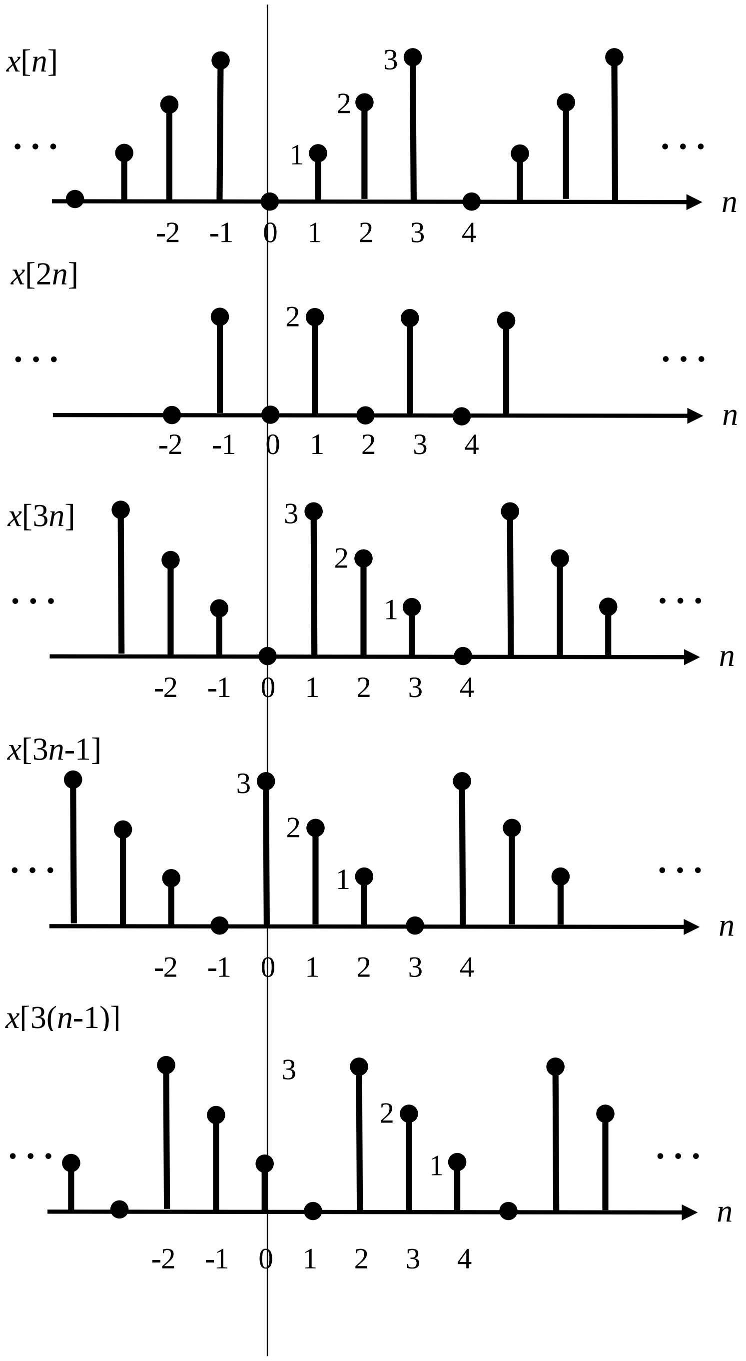
Ex: (compressor/downsampler) $y[n]=x[Mn]$ *M*: integer $\rightarrow\boxed{\downarrow M}\rightarrow$

Following the above definition $y_1[n]=x[Mn]$, $y_2[n]=x[Mn-\Delta]$

$\Rightarrow y_2[n]\neq y_1[n-\Delta]=x[Mn-M\Delta]$

So, the system is time-varying. Show that it is linear! (exercise)

Example: Downsampler is time-varying



$M = 2$

$M = 3$

The response to $x[n-1]$
when $M = 3$

$x[3n]$ delayed by 1 sample

Ex: (expander/upsampler) $y[n]=\begin{cases} x\left[\dfrac{n}{L}\right] & n=kL \\ 0 & n\neq kL \end{cases}$; k, L: integer $\longrightarrow \boxed{\uparrow L} \longrightarrow$

$$\boxed{y_1[n]=\begin{cases} x\left[\dfrac{n}{L}\right] & n=kL \\ 0 & n\neq kL \end{cases}} \qquad \boxed{y_2[n]=\begin{cases} x\left[\dfrac{n}{L}-\Delta\right] & n=kL \\ 0 & n\neq kL \end{cases}}$$

$$\Rightarrow\ y_2[n]\neq y_1[n-\Delta]=\begin{cases} x\left[\dfrac{n-\Delta}{L}\right] & n-\Delta=kL \\ 0 & n-\Delta\neq kL \end{cases}$$

So, the system is time-varying

Show that it is linear! (exercise)

Causality: A system is said to be causal if the two output signals $y_1[n]$ and $y_2[n]$ (due to two input signals $x_1[n]$ and $x_2[n]$) satisfy

$$y_1[n] = y_2[n] \qquad n \le n_0$$

whenever

$$x_1[n] = x_2[n] \qquad n \le n_0$$

Ex:

$y[n] = x[n+1] - x[n]$ noncausal

$y[n] = x[n-1] - x[n]$ causal

$y[n] = x[n] + 5$ causal

Stability: (BIBO)

A system is said to be BIBO stable if "bounded inputs yield bounded outputs.", i.e.,

$$|x[n]| \leq B_x < \infty \qquad \Rightarrow \qquad |y[n]| \leq B_y < \infty$$

for arbitrary finite B_x and B_y .

Ex: $y[n] = \sum_{k=-\infty}^{n} x[k] = y[n-1] + x[n]$ UNSTABLE

For example, for $x[n] = u[n]$ the output is $y[n] = \begin{cases} n+1 & n \geq 0 \\ 0 & n < 0 \end{cases}$

Bounded input does not yield bounded output.