Fırat Bozkan KILIÇ
DSP ÖDEV #1

1) $x_c(t) = 4\sin(20000\pi t + \frac{\pi}{13})$ is sampled with $F_s = 3kHz$.

$x[n] = x_c[n.T_s]$ where $T_s = \frac{1}{F_s}$

$x[n] = x_c[\frac{n}{F_s}] = 4\sin\left(\frac{20\pi n}{3} + \frac{\pi}{13}\right)$ sin value doesn't change if we add $k2\pi$.

$x'[n] = 4\sin\left(\left(\frac{20\pi}{3} + k2\pi\right)n + \frac{\pi}{13}\right) = 4\sin\left(\left(\frac{20\pi}{3}f_s + k2\pi f_s\right)\frac{n}{f_s} + \frac{\pi}{13}\right)$ are all same with $x[n]$.

continuous time of those signals are $t = \frac{n}{f_s}$, where $f_s = 3kHz$

$$x'(t) = 4\sin(\underbrace{20000\pi t + k2\pi.3000t}_{2\pi(10000+3000k)t} + \frac{\pi}{13})$$

$w_0 = 2\pi f_0 \Rightarrow f = \frac{w_0}{2\pi} = 10000 + 3000k$. each sinusoidal CT signal with this frequency where k is an integer will yield same DT signal when sampled with 3kHz rate.

b) $x_c(t)$ sampled with F_s is $x[n] = x_c(nT_s) = x_c(\frac{n}{F_s})$

$x[n] = 4\sin\left(20000\pi.\frac{n}{F_s} + \frac{\pi}{13}\right)$ should be equal to $x'[n] = 4\sin\left(\frac{20\pi n}{3} + \frac{\pi}{13}\right)$ in part a.

$$4\sin\left[\frac{20\pi n}{3} + \frac{\pi}{13}\right] = 4\sin\left(\left(\frac{20\pi}{3} + k.2\pi\right)n + \frac{\pi}{13}\right).$$

$$\overset{1}{\cancel{2}}0000\cancel{\pi}\frac{\cancel{n}}{f_s} + \cancel{\frac{\pi}{13}} = \left(\frac{\overset{10}{\cancel{20}}}{3}\cancel{\pi} + k\cancel{2\pi}\right)\cancel{n} + \cancel{\frac{\pi}{13}}$$

$$\frac{10000}{f_s} = \frac{10}{3} + k$$

$$10000 = \frac{10 f_s}{3} + k f_s.$$

$$\frac{10000}{\frac{10}{3}+k} = f_s = \frac{30000}{10+3k}.$$

for all sampling frequencies in this form, resultant DT signal is the same.

2) Period is $N = \frac{k 2\pi}{w_0}$ where k is the smallest number (integer.) making $\frac{2\pi}{w_0}$ integer.

$\sin(1.74\pi n + 3.1) \rightarrow N = k \frac{2\cancel{\pi}}{1.74\cancel{\pi}} = k \frac{100}{87} \Rightarrow N = 100$ samples.

$\sin(1.74\pi n + 3.1\pi) \rightarrow$ same as above.

$\cos\left(15.74\pi n + \frac{3\pi}{8}\right) = \cos\left(7.(2\pi n) + 1.74\pi n + \frac{3\pi}{8}\right) \rightarrow$ same as above.

$\cos(\sqrt{\pi} n)$: $N = k . \frac{2\cancel{\pi}}{\sqrt{\cancel{\pi}}} = 2\sqrt{\pi} k$. There is no such integer k value that make N integer $\Rightarrow$ non-periodic.

$\cos(\pi \sqrt{\pi} n)$: $N = k \frac{2\pi}{\pi\sqrt{\pi}}$; non-periodic. (same argument as above.

$\cos(\pi\sqrt{2} n)$: $N = k \frac{2\cancel{\pi}}{\cancel{\pi}\sqrt{2}} = k\sqrt{2}$: non-periodic.

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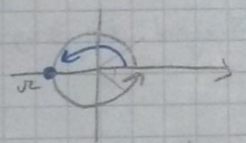
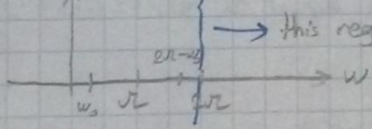
3) It is π. DT signals don't change when we add/subtract $k2\pi$, So, 2π length of w spans whole freq range.



$\cos(x) = \cos(2\pi - x)$.

π is the highest freq in DT.

Also, we can show it easier in;



$\rightarrow$ this region can be expressed as $(w_0 + k2\pi)$

w_0 & $2\pi - w_0$ has same cosine value, so, $[0, \pi]$ spans the freq range and therefore π is the highest freq.

4)

$$y[n] = \begin{cases} x[\frac{n}{2}] & , n \text{ is even} \\ \dfrac{x[\frac{n-1}{2}] + x[\frac{n+1}{2}]}{2} & , n \text{ is odd.} \end{cases}$$

$y[0] = x[0]$

$y[1] = \dfrac{x[0] + x[1]}{2}$

$y[2] = x[1]$

2 sample difference in y for 1 sample.
This is dilated version of x. It reduces its frequency by half.

Checking linearity and time invariancy

for linearity: $x_1[n] \rightarrow y_1[n]$, $x_2[n] \rightarrow y_2[n]$

$x_1[n] + x_2[n] \rightarrow y_1[n] + y_2[n]$ should be satisfied.

$$y_1[n] = \begin{cases} x_1[\frac{n}{2}] & , n \text{ is even} \\ \dfrac{x_1[\frac{n-1}{2}] + x_1[\frac{n+1}{2}]}{2} & , n \text{ is odd.} \end{cases}$$

$$y_2[n] = \begin{cases} x_2[\frac{n}{2}] & , n = 2k \\ \dfrac{x_2[\frac{n-1}{2}] + x_2[\frac{n+1}{2}]}{2} & , n = 2k-1 \end{cases}$$

$$y_3[n] = \begin{cases} x_1[\frac{n}{2}] + x_2[\frac{n}{2}] & , n = 2k \\ \dfrac{x_1[\frac{n-1}{2}] + x_1[\frac{n+1}{2}]}{2} + \dfrac{x_2[\frac{n-1}{2}] + x_2[\frac{n+1}{2}]}{2} & , n = 2k-1 \end{cases}$$

$y_3[n] = y_1[n] + y_2[n]$ ✓ linear.

for time invariance,

$$\left.\begin{array}{l} x[n] \rightarrow y_1[n] \\ x[n-n_0] \rightarrow y_2[n] \end{array}\right\} \quad y_2[n] = y_1[n-n_0] \text{ must be satisfied.}$$

system is time variant since for $x[n]$, $y_1[1] = \dfrac{x[0] + x[1]}{2}$

$y_1[2] = x[1]$,

shift system with $n_0 = 1$, $x[n-1]: \rightarrow$ $y_2[1] = x[\frac{-1}{2}] + x[\frac{1}{2}] \rightarrow$ not even defined.

system is time varying

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5) causal, stable? causality: no future term. $h[n] = 0$ for $n<0$.

a) $y[n] = 2^{\delta[n+1]} + x[n-3]$

$h[n] = 2^{\delta[n+1]} + \delta[n-3] \rightarrow$ not causal. $h[-1] = 2$

stable, say $x[n] < B_x$ (bounded input. then,

$y[n] < B_y$ because it is sum of two bounded, finite terms.

b) $y[n] = \begin{cases} y[-\delta[n-1]] + x[n-3] & , n>0 \\ 2^n x[n-3] & , n \le 0 \end{cases}$

which can be written as

$y[n] = \begin{cases} y[-1] + x[n-3] & , n=1 \\ x[n-3], & n>0,\ n \neq 1 \\ 2^n x[n-3] & , n \le 0. \end{cases}$

$y[1] = y[-1] + x[-2]$. $\Rightarrow y[1] = \frac{1}{2} x[-4] + x[-2]$.

$y[-1] = \frac{1}{2} x[-4]$

system is causal. system is stable for bounded inputs:

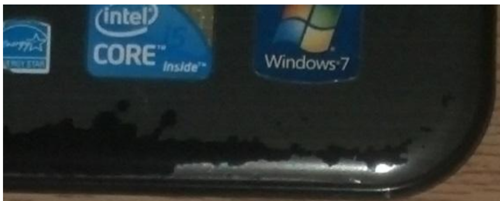
$\rightarrow$ for $n>0$, $n \neq 1$ $y[n] = x[n-3] < B_{y_1}$

$\rightarrow$ for $n=1$, $y[1] = \frac{1}{2} x[-4] + x[-2] < B_{y_2}$ (both bounded)

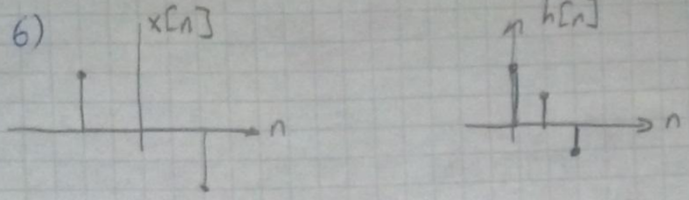
$\rightarrow$ for $n \le 0$, $y[n] = 2^n x[n-3]$. $< B_{y_3}$ (2^n is exponentially decreases, it is bounded between $(0, 1]$

$x[n]$ is bounded as well.

product of two unbounded signals.



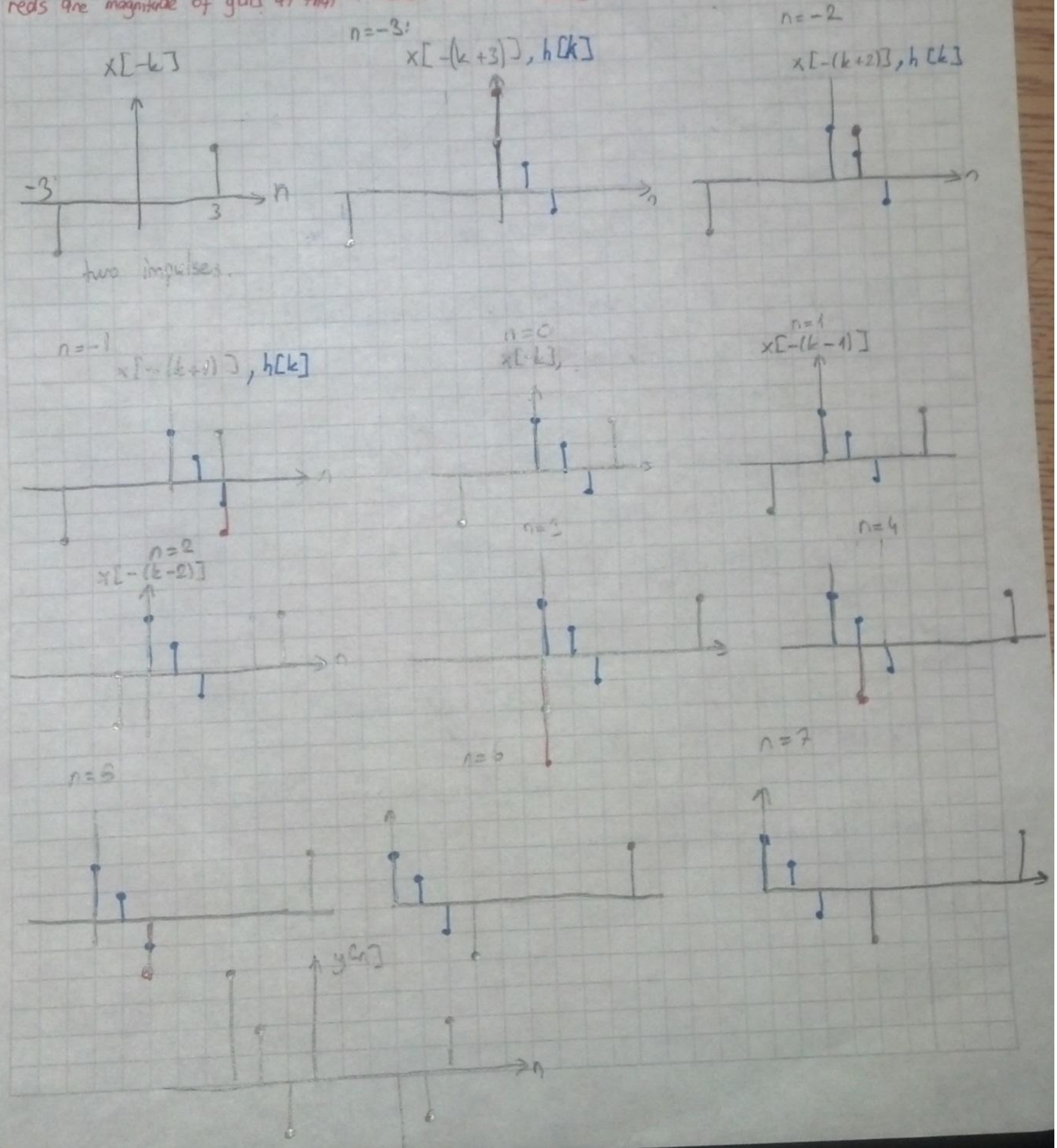
6)



convolution operation is:

$$x[n] * h[n] \triangleq \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[n-k]h[k]$$

reds are magnitude of y[n] at that n instant written above



2) Period is

sin (1.2

6) 7)

i) $y[-n] \overset{?}{=} x[-n] * h[-n]$

$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$y[-n] = \sum_{k=-\infty}^{\infty} x[k]\, h[-n-k]$ *

Let: $x[-n] = k[n]$, $h[-n] = \ell[n]$

$x[-n] * h[-n] = k[n] * \ell[n] = \sum_{k=-\infty}^{\infty} k[k]\, \ell[n-k]$

make substitutions:

$= \sum_{k=-\infty}^{\infty} x[-k]\, h[-n+k]$

Let $-k = a$

$= \sum_{a=-\infty}^{\infty} x[a]\, h[-n-a]$ which is equal to *

ii) $y[n-4] = x[n-4] * h[n] = x[n] * h[n-4]$

Let $x[n-4] = m[n]$

$h[n-4] = n[n]$

$x[n-4] * h[n] = m[n] * h[n] = \sum_{k=-\infty}^{\infty} m[k]\, h[n-k]$

$= \sum_{k=-\infty}^{\infty} x[k-4]\, h[n-k]$

$\ell = k-4$

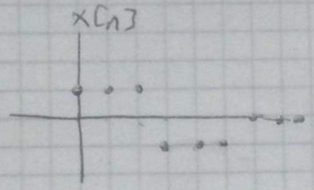
$\sum_{\ell=-\infty}^{\infty} x[\ell]\, h[n-4-\ell] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-4-k]$

$\Rightarrow y[n-4] = x[n-4] * h[n]$

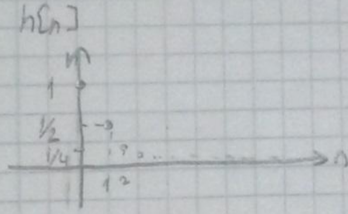
proving the other side is similar.

8) $x[n] = u[n] - 2u[n-3] + u[n-6]$, $h = \left(\frac{1}{2}\right)^n u[n]$

Let's draw:



$= \delta[n] + \delta[n-1] + \delta[n-2] - \delta[n-3] - \delta[n-4] - \delta[n-5]$



$y[n] = x[n] * h[n]$

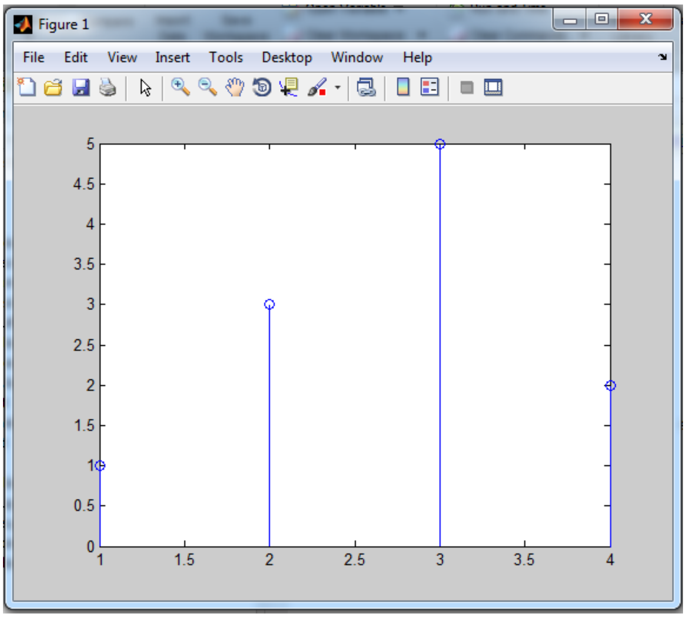
$$y[n] = \left(\frac{1}{2}\right)^n u[n] + \left(\frac{1}{2}\right)^{n-1} u[n-1] + \left(\frac{1}{2}\right)^{n-2} u[n-2]$$

$$- \left(\frac{1}{2}\right)^{n-3} u[n-3] - \left(\frac{1}{2}\right)^{n-4} u[n-4] - \left(\frac{1}{2}\right)^{n-5} u[n-5]$$

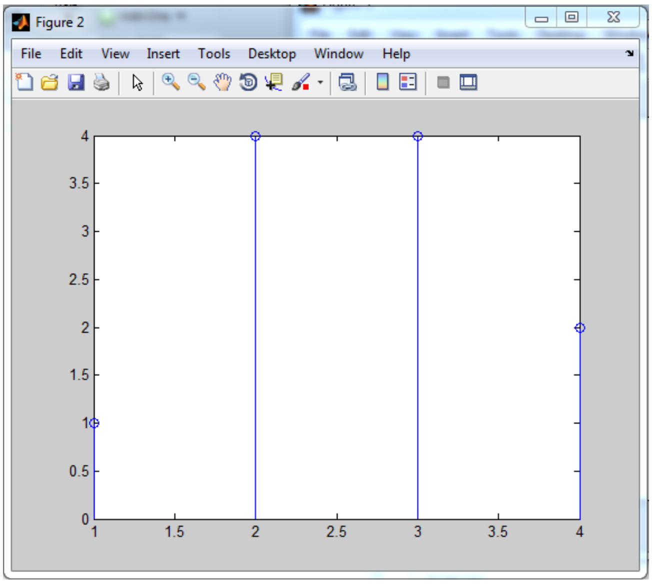
done.

QUESTION 9:

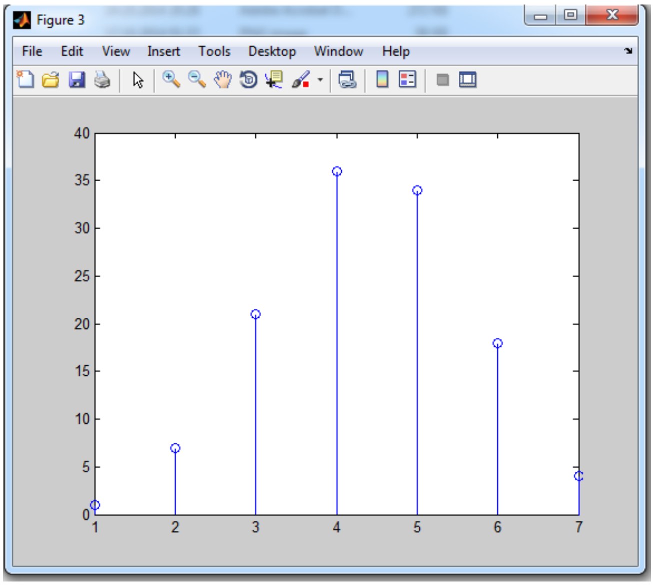
Arbitrary x signal : [1 3 5 2]



Arbitrary h signal: [1 4 4 2]



Result: y signal, the output. It came out as expected in terms of lenght (n+m-1) and magnitude (higher values at middle since both signals are positive valued).



Question 10:

a)

```
>> pol1 = [ 0 1 3 2]

pol1 =

     0     1     3     2

>> roots(pol1)

ans =

    -2
    -1

>> pol2 = [ 3.5 2 1 1 3 1];
>> conv(pol1,pol2)

ans =

         0    3.5000   12.5000   14.0000    8.0000    8.0000   12.0000    9.0000    2.0000

>>
```

Multiplication of 3rd and 5th order polinomials resulted in an 8th order polynomial.

b)

c)

10-c: $y[n] = 1\ 1\ 2\ 3\ 4\ {-1}\ 5$, length of 7 ; $x[n] = [1\ 2\ 3\ 4\ 5]$ length of 5.

find $h[n]$.

$$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]\, h[k]$$

$h[n]$ has a length of 3: $h[0], h[1], h[2]$

$y[0] = x[0] \cdot h[0] = 1$

$y[1] = x[1]\,h[0] + x[0]\,h[1] = 1$; $x[1] = 2$, $h[0] = 1$, $x[0] = 1$ → $h[1] = -1$

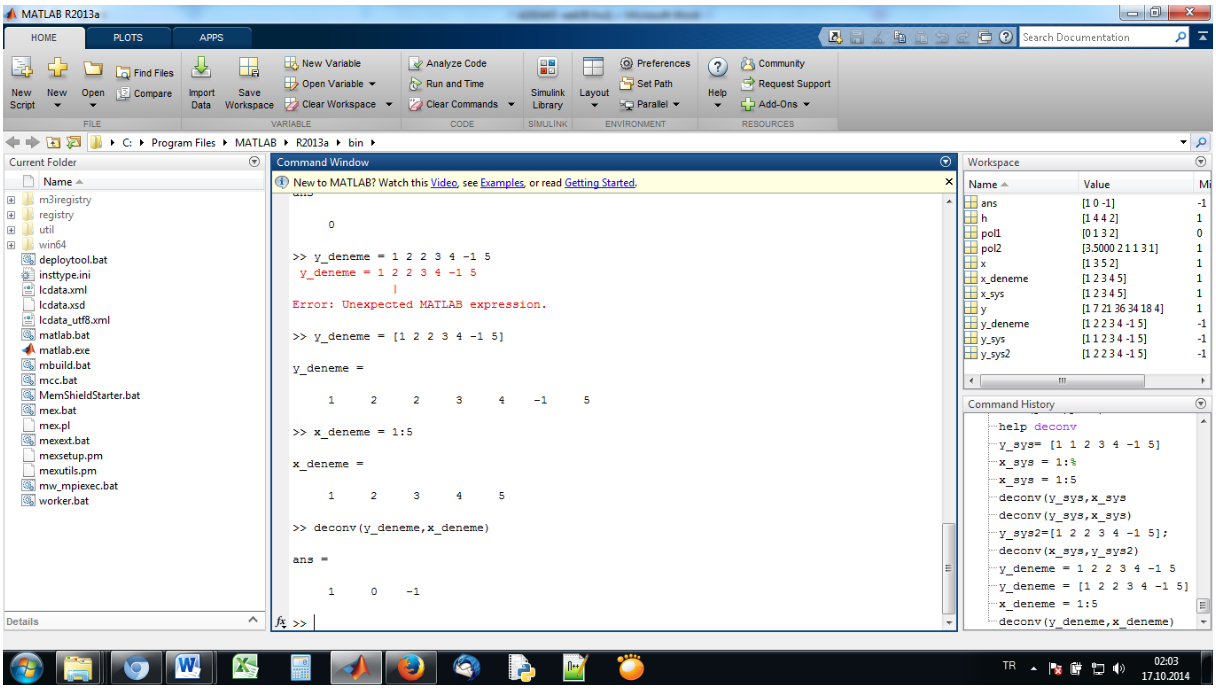
$y[2] = x[2]\,h[0] + x[1]\,h[1] + x[0]\,h[2] = 2$; $3 \cdot 1 + 2 \cdot (-1) + 1 \cdot h[2]$ → $h[2] = 1$

$$\boxed{h[n] = \delta[n] - \delta[n-1] + \delta[n-2]}$$

this is my hand calculation. MATLAB found the same result.

```
Command Window
New to MATLAB? Watch this Video, see Examples, or read Getting Started.

y_sys =

     1     1     2     3     4    -1     5

>> x_sys = 1:%
 x_sys = 1:%
           |
Error: Expression or statement is incomplete or incorrect.

>> x_sys = 1:5

x_sys =

     1     2     3     4     5

>> deconv(y_sys,x_sys
 deconv(y_sys,x_sys
                  |
Error: Expression or statement is incorrect--possibly unbalanced (, {, or [.

>> deconv(y_sys,x_sys)

ans =

     1    -1     1

fx >> 
```

d)



This time, MATLAB calculated itwrong because the polynomial division for finding transfer function has a remainder term. (Q+R form where r is remainder). So, we have to be careful using deconv.