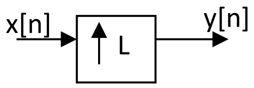
<u>**Due Date**: 15 December 2014, Monday (12:00).</u>

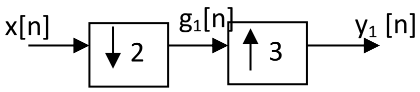
Homework 4

1) a) Consider the up-sampling operation

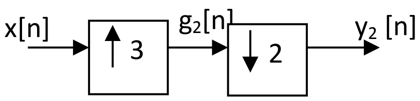


i)Determine if it is a linear system or not and prove your claim.
ii) Determine if it is a time-invariant system or not and prove your claim.
b) Repeat part **a** for down-sampling operation.

c)Consider the following rate change operations. Write $G_1(e^{j\omega})$, $Y_1(e^{j\omega})$.



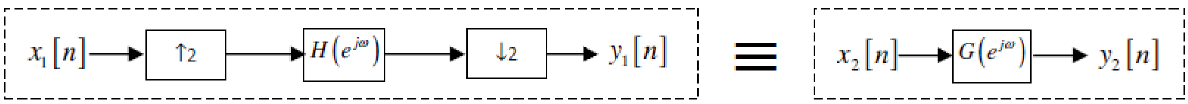
d) Consider the following rate change operations. Write $G_2(e^{j\omega})$, $Y_2(e^{j\omega})$.



e)Show that $y_2[n]=y_1[n]$. Find the general rule for this equality.

f) The DT sequence x[n] is the samples of analog signal $x_c(t)$ with a sampling period of T. Plot the DT system structure for obtaining $x_1[n]$ from x[n] such that $x_1[n]$ samples are 2/3 samples delayed.

2) The two systems in the following figure are equivalent.



Find $Y_1(e^{j\omega})$ and $Y_2(e^{j\omega})$ and show that they are the same for $x_1[n]=x_2[n]$. In other words, prove that the system on the left is equivalent to a LTI system.

3)In this problem, you are required to write a bandpass sampling question and solve it. In other words, plot a bandpass spectrum for $X(e^{j\omega})$ by carefully indicating all the critical frequency points. Then find the minimum sampling frequency, Ω_s. You should design your spectrum such that the sampling frequency is not greater than half of the Nyquist Rate.

4) i)List the differences between C/D and A/D converters.

ii)List the differences between D/C and D/A converters.

iii)Find the minimum number of bits for an A/D converter such that SNR>48dB.

iv)Describe the distortion introduced due to the use of pulse in D/A conversion instead of impulse. Plot the frequency response of the compensated reconstruction filter.

5) Consider a LTI filter,

$$H(z)=1-re^{j\theta}z^{-1}$$

a)r=1, $\theta=\pi/4$. Plot approximately the magnitude and phase response for $H(e^{j\omega})$.

b) Repeat a) for 1/H(z).

c)A real coefficient, causal and stable allpass filter has a zero at r=2, $\theta=\pi/4$. Find and plot the pole-zero plot of this allpass filter. Also write H(z).

6) The following frequency response for a minimum-phase filter is given

$$\left|H\left(e^{j\omega}\right)\right|^2 = \frac{1}{\frac{5}{4} - \cos(\omega)}$$

Find this minimum-phase filter H(z).

7) Consider a LTI system,

$$H(z) = \frac{1+2z^{-1}}{1+\frac{1}{4}z^{-1}}$$

It is possible to decompose this filter into its minimum-phase and allpass components, i.e. $H(z) = H_{\min}(z)H_{ap}(z)$. Find and write $H_{min}(z)$ and $H_{ap}(z)$. Plot their pole-zero plots as well.

8) a) Write the linear phase relations in time and and z-domain for all four types of generalized linear phase filters (Ex: h[n]=h[M-n]).

b)Find and write a minimum order Type III generalized linear-phase filter.

c)Given the generalized linear-phase lowpass filter h[n]=[1 2 4 2 1], $0 \le n \le 4$, decompose this filter into minimum-phase, and allpass parts.

MATLAB PART

1) This problem investigates the aliasing effect for a sinusoid. Consider a sinusoid,

$$x(t) = Sin(2\pi\ f_0 t + \theta)$$

We can sample it by $f_s=1/T_s$ to obtain the discrete-time signal,

$$x[n] = Sin(2\pi \frac{f_0}{f_s} n + \theta)$$

Take sampling frequency as 8kHz.

a) Take f_0=300 Hz and take samples over an interval of 10ms. θ can be arbitrary. Plot the discrete-time sequence by using *stem* (you can also try plot command) command.
b) Now make a series of plots as in (a) by using *subplot* by taking f_0 as 100, 400, 600. Explain what happens when the frequency of the sinusoid increases while the sampling frequency is kept the same.
c) Now choose f_0 as 7525, 7650, and 7900 Hz and make a series of plots. Note that apparent frequency of the sinusoid is decreasing. Explain this.

d) Now change f_0 from 32100 to 32475Hz in 125 Hz steps. Predict in advance whether the apparent frequency will be decreasing or increasing. Then plot your results.

2) In this part, you will implement an ideal lowpass filter and reconstruct from the samples of a DT signal. When a *sinc* function is used for this purpose, analog signal can be obtained from the samples as,

$$x_r(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin(\pi(t-nT_s)/T_s)}{\pi(t-nT_s)/T_s}$$

where T_s is the reconstruction sampling period.

a) Consider $x[n]=\delta[n]$. Use T_s=10 sec.. Find and plot $x_r(t)$ for t=-5 to t=5 (take enough samples for a good plot). You should observe a sinc function. Change T_s=1000 sec., and observe and note if there is any difference.

b) Use the signal in problem 1.a. and choose a suitable T_s. Take 26 samples (x[n], n=0,1,…,25) and obtain x(t). Does the x(t) you found in MATLAB matches with the x(t) calculated from the mathematical expression everywhere?

c) Do the same thing in b) by considering f_0=5000 Hz.