$\boxed{14}$ $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$ exists for $z = 4e^{j\pi}$

This means that $z = 4e^{j\pi}$ is in ROC of $X(z)$.

$$\left| X(4e^{j\pi}) \right| = \left| \sum_{n=-\infty}^{\infty} x[n] 4^{-n} e^{-j\pi n} \right| < \infty$$

Then $\mathcal{F}\{x[n]4^{-n}\}$ exists. This means $x[n]4^{-n}$ sequence is absolutely summable, i.e.,

$$\sum_{n=-\infty}^{\infty} |x[n]| 4^{-n} < \infty$$

Now consider $x[n]4^{-n} e^{j\frac{5\pi}{7}n}$ sequence. It is absolutely summable since,

$$\sum_{n=-\infty}^{\infty} \left| x[n] e^{j\frac{5\pi}{7}n} \right| 4^{-n} = \sum_{n=-\infty}^{\infty} |x[n]| 4^{-n} < \infty.$$

Then $\left| \sum_{n=-\infty}^{\infty} x[n] 4^{-n} e^{j\frac{5\pi}{7}n} e^{-j\pi n} \right| < \infty$

$$= \left| X(4e^{j\frac{+2\pi}{7}n}) \right| < \infty$$

So $X(z)$ exists for $z = 4e^{j\frac{2\pi}{7}}$,

In general, consider the sequence $y[n] = x[n]4^{-n} e^{j(-\omega+\pi)n}$

It is absolutely summable. So,

$$\left| Y(e^{j\pi}) \right| = \left| \sum_{n=-\infty}^{\infty} x[n] 4^{-n} e^{j(-\omega+\pi)n} e^{-j\pi n} \right| < \infty.$$

$$= \left| \sum_{n=-\infty}^{\infty} x[n] 4^{-n} e^{-j\omega n} \right| = \left| X(4e^{j\omega}) \right| < \infty$$

$\boxed{15}$ Assume that $x[n]=0$ for some $n<n_0$.

Then

$$X(z)=\sum_{n=n_0}^{\infty} x[n]\,z^{-n} \quad \text{and } X(4e^{j\omega}) \text{ exists.}$$

This means $\mathcal{F}\{x[n]4^{-n}\}$ exists $\Longrightarrow$

$$\sum_{n=n_0}^{\infty} |x[n]|\,4^{-n} < \infty \;\equiv\; x[n]4^{-n} \text{ is absolutely summable.}$$

Now consider the sequence $x[n]4.1^{-n}$.

$$\sum_{n=n_0}^{\infty} |x[n]|\,4.1^{-n} = \sum_{n=n_0}^{\infty} |x[n]|\,4^{-n}\left(\frac{4.1}{4}\right)^{-n}$$

$$= \underbrace{\sum_{n=n_0}^{0} |x[n]|\,4^{-n}\left(\frac{4.1}{4}\right)^{-n}}_{=A \;<\infty\; \text{(finite sum)}} + \sum_{n=1}^{\infty} |x[n]|\,4^{-n}\underbrace{\left(\frac{4.1}{4}\right)^{-n}}_{<1 \text{ for } n\geq 1}$$

if $n_0<0$

$$\leq A + \sum_{n=1}^{\infty} |x[n]|\,4^{-n} < \infty$$

So $X(z)$ exists for $z=4.1e^{j\omega}$ $0\leq\omega<2\pi$

Now consider $z=3.9e^{j\omega}$. The sequence $x[n]\,3.9^{-n}$ is not necessarily absolutely summable, since

$$\sum_{n=1}^{\infty} |x[n]|\,4^{-n}\underbrace{\left(\frac{3.9}{4}\right)^{-n}}_{>1 \text{ for } n\geq 1} \text{ is not necessarily finite.}$$

(16) a) $h[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[n]$

i) $x_1[n] = \cos\left(\frac{\pi}{2} n\right)$

$y_1[n]$ does not exist.

ii) $x_2[n] = \cos\left(\frac{\pi}{2} n\right) u[n]$

$$y_2[n] = \cos\left(\frac{\pi n}{2}\right) u[n] + \frac{4}{5} \cos\left(\frac{\pi}{2}(n-1)\right) u[n-1]$$

$$+ \frac{1}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{4}{5}(2)^n u[n]$$

b) $h[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[-n]$

i) $y_1[n] = \frac{8}{5} \cos\left(\frac{\pi}{2} n\right)$

ii) $y_2[n] = \frac{8}{5} \cos\left(\frac{\pi}{2} n\right) + \frac{1}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{4}{5} 2^n u[-n-1]$

c) $h[n] = 5^n u[-n]$

i) $y_1[n] = \frac{5}{\sqrt{26}} \cos\left(\frac{\pi}{2} n + \tan^{-1}\left(\frac{1}{5}\right)\right)$

ii) $$y_2[n] = \frac{25}{26} \cos\left(\frac{\pi}{2} n\right) u[n] - \frac{5}{26} \cos\left(\frac{\pi}{2}(n-1)\right) u[n-1]$$

$$+ \frac{25}{26} (5)^n u[-n-1]$$

d) $h[n] = 3^n u[n]$

i) $y_1[n]$ does not exist.

ii) $$y_2[n] = \frac{1}{10} \cos\left(\frac{\pi}{2} n\right) u[n] + \frac{3}{10} \cos\left(\frac{\pi}{2}(n-1)\right) u[n-1]$$

$$+ \frac{9}{10} 3^n u[n]$$

18

a) $x[n] = \delta[n+3] + \delta[n-3]$

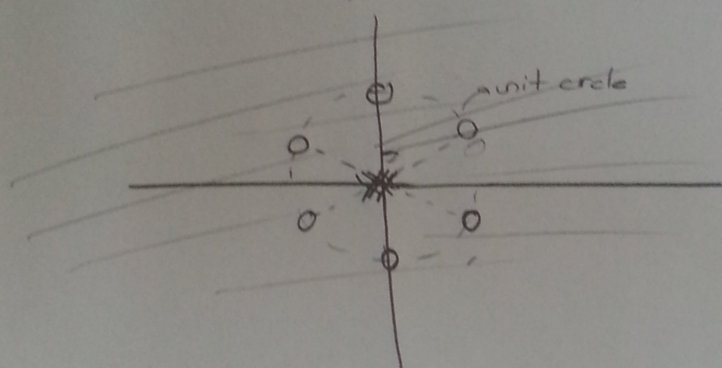
$$X(z) = z^{3} + z^{-3} = z^{3} + \frac{1}{z^{3}} = \frac{z^{6}+1}{z^{3}}$$

The zeros $z^{6}+1=0 \Rightarrow z^{6}=-1$

$z_1 = e^{j\frac{\pi}{6}}$, $z_2 = e^{j\frac{3\pi}{6}}$, $z_3 = e^{j\frac{5\pi}{6}}$, $z_4 = e^{j\frac{7\pi}{6}}$, $z_5 = e^{j\frac{9\pi}{6}}$

$z_6 = e^{\frac{11\pi}{6}}$

There exist 3 poles at $z=0$ and 3 poles at ∞.



ROC:
$0 < |z| < \infty$

b) $x[n] = \delta[n+3]$

$X(z) = z^{3}$: There are 3 zeros at $z=0$ and 3 poles at ∞.

ROC: $|z| < \infty$

c) $x[n] = \delta[n-3]$

$$X(z) = z^{-3} = \frac{1}{z^{3}}$$

There are 3 zeros at ∞ and 3 poles at $z=0$.

ROC: $0 < |z|$

(19) a) $X(z) = \dfrac{z\left(z+\frac{1}{2}\right)}{z-\frac{1}{2}}$ ROC: $\frac{1}{2} < |z| < \infty$

zeros: $z_1 = 0$, $z_2 = \frac{-1}{2}$

poles $p_1 = \frac{1}{2}$, $p_2 = \infty$

b) $y[n] = x[n-5]$

$Y(z) = X(z)\, z^{-5} = \dfrac{\left(z+\frac{1}{2}\right)}{z^4\left(z-\frac{1}{2}\right)}$ ROC: $|z| > \frac{1}{2}$

(Because $y[n]$ is a right sided sequence)

zeros: $z_1 = \frac{-1}{2}$, $z_2 = z_3 = z_4 = z_5 = \infty$ (4 zeros at ∞)

poles: $p_1 = \frac{1}{2}$, $p_2 = p_3 = p_4 = p_5 = 0$ (4 poles at 0)

c) $y[n] = n\,x[n]$

$Y(z) = -z\dfrac{dX(z)}{dz} = \dfrac{-z\left(z-\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)\right)\left(z-\left(\frac{1}{2}-\frac{1}{\sqrt{2}}\right)\right)}{\left(z-\frac{1}{2}\right)^2}$

ROC: $\frac{1}{2} < |z| < \infty$

($n\,x[n]$ is a right sided sequence)

zeros: $z_1 = 0$, $z_2 = \frac{1}{2} + \frac{1}{\sqrt{2}}$, $z_3 = \frac{1}{2} - \frac{1}{\sqrt{2}}$

poles: $p_1 = p_2 = \frac{1}{2}$, $p_3 = \infty$

d) $y[n] = \cos\left(\frac{\pi}{2}n\right)x[n] \longrightarrow$ right sided sequence

$Y(z) = \dfrac{z^2}{\left(z-\frac{1}{2}j\right)\left(z+\frac{1}{2}j\right)}$

ROC: $|z| > \frac{1}{2}$

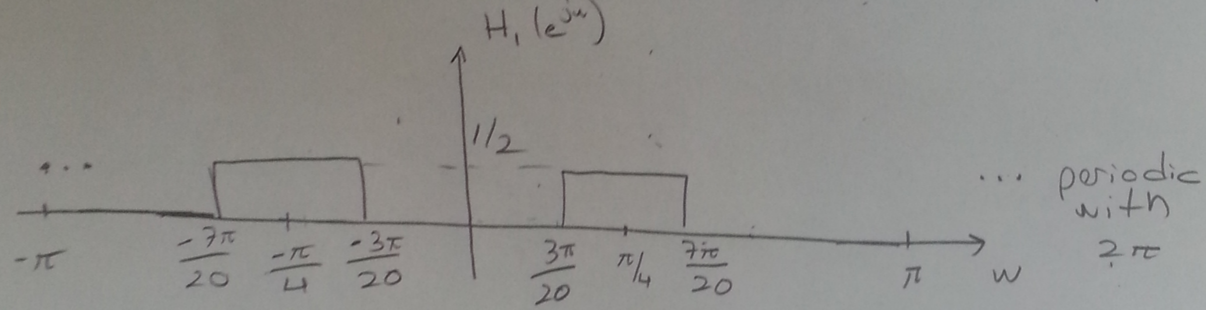
zeros: $z_1 = z_2 = 0$

poles $p_1 = \frac{1}{2}j$, $p_2 = \frac{-1}{2}j$

20)

a) $h[n] = \dfrac{\sin\left(\frac{\pi}{10} n\right)}{\pi n}$ $\Longrightarrow$ The sequence is not causal and the system is not causal.

b) $$H_1(e^{jw}) = \begin{cases} \dfrac{1}{2}, & \dfrac{3\pi}{20} < |w| < \dfrac{7\pi}{20} \\ 0, & \dfrac{7\pi}{20} < |w| \leq \pi \end{cases}$$ with 2π periodicity



(21)

a) $y[n]-\frac{1}{2}y[n-1]=x[n]$

b) $y[n]-\frac{1}{2}y[n-1]=x[n]$

Note that they are the same. Auxiliary conditions determine whether the system is causal or not.

c) $y[n]-\frac{1}{2}y[n-1]=x[n-1]$

d) $16y[n]-8y[n-1]=x[n-4]$

e) $y[n]-\frac{5}{2}y[n-1]+y[n-2]=2x[n]-\frac{5}{2}x[n-1]$

(22) a) ROC: $\frac{1}{2}<|z|<3$

$$h[n]=\frac{4}{15}\left(\frac{1}{2}\right)^{n-1}u[n-1]+\frac{14}{15}3^{n-1}u[-n]$$

b) $$H_1(z)=\frac{z^{-2}\left[\frac{-2}{9}-\frac{4}{9}z^{-1}\right]}{(1-2z^{-1})(1-\frac{1}{3}z^{-1})}=\frac{\frac{-2}{9}(z+2)}{(z-2)(z-\frac{1}{3})z}$$

Zeros: $z_1=-2,\ z_2=z_3=\infty$

poles: $p_1=2,\ p_2=\frac{1}{3},\ p_3=0$

ROC: $\frac{1}{3}<|z|<2$

(23)

$$H(z) = \frac{z + 2 + z^{-1}}{-2z^2 - 4z + 4z^{-1} + 2z^{-2}} = \frac{z^3 + 2z^2 + z}{-2z^4 - 4z^3 + 4z + 2}$$

$$= \frac{z\cancel{(z+1)(z+1)}}{\cancel{(z^2+2z+1)}(-2z^2+2)} = \frac{1/2\, z}{1 - z^2} = \frac{-1/2\, z^{-1}}{(1-z^{-1})(1+z^{-1})}$$

$$= \frac{A}{1-z^{-1}} + \frac{B}{1+z^{-1}}$$

(There is a mistake in the question. Since there are poles on the unit circle, system cannot be stable.)

$$A + B = 0$$

$$A - B = \frac{-1}{2} \Rightarrow A = \frac{-1}{4}, \; B = \frac{1}{4}$$

Assume that ROC: $|z| > 1$

$$H(z) = \frac{-1/4}{1-z^{-1}} + \frac{1/4}{1+z^{-1}}$$

$$h[n] = \frac{-1}{4} u[n] + \frac{1}{4} (-1)^n u[n]$$

(24) From Parseval's relation;

$$\sum_{n=-\infty}^{\infty} \left| \frac{\sin\left(\frac{\pi n}{4}\right)}{\pi n} \right|^2 = \frac{1}{4}$$