Middle East Technical University
Dept. of EEE

HW 3

EE 430 Digital Signal Processing – 2014 Fall

Due: Nov. 21, 2014

Submit for problems 2, 3, 6, 7, 8.

1) Let $x[n] = \delta[n] + 3\delta[n-1] + \delta[n-2]$.

a. Plot $x[n]$ and its periodic extension, $\tilde{x}[n]$, for $N = 3$ and $N = 5$.

$$\tilde{x}_N[n] = \sum_{k=-\infty}^{\infty} x[n-kN] = x\left[((n))_N\right]$$

b. Find the Discrete Fourier Series (DFS) coefficients, $\tilde{X}_3[k]$, of $\tilde{x}_3[n]$. Write the DFS representation of $\tilde{x}_3[n]$.

c. Find the Discrete Fourier Series (DFS) coefficients, $\tilde{X}_5[k]$, of $\tilde{x}_5[n]$. Write the DFS representation of $\tilde{x}_5[n]$.

d. Find the DTFT, $X(e^{j\omega})$, of $x[n]$. Plot its magnitude and phase.

e. Verify that $\tilde{X}_3[k] = X(e^{j\omega})\big|_{\omega=k\frac{2\pi}{3}}$ and $\tilde{X}_5[k] = X(e^{j\omega})\big|_{\omega=k\frac{2\pi}{5}}$, i.e., uniformly spaced samples of DTFT of $x[n]$. Show these samples on the magnitude and phase plot of $X(e^{j\omega})$.

f. Compute the sample values of $\tilde{x}_3[n]$ and $\tilde{x}_5[n]$ using their DFS representations and their DFS coefficients you found in parts (b) and (c), respectively.

2)

a. Find the 3-point and 5-point DFTs ($X_3[k]$ and $X_5[k]$) of $x[n]$ given in Question-1.

b. What is the relationship between $X_3[k]$ and $\tilde{X}_3[k]$, and $X_5[k]$ and $\tilde{X}_5[k]$?

c. How would you find $x[n]$ using its 3-point and 5-point DFTs?

d. Find $X_3[k]$ and $X_5[k]$ using MATLAB.

3) Let $y[n] = \delta[n-2] + 3\delta[n-3] + \delta[n-4]$ and $z[n] = 3\delta[n] + \delta[n-1] + \delta[n-4]$

a. Plot $y[n]$ and $z[n]$.

b. Relate $y[n]$ and $z[n]$ to $x[n]$ of Question-1

c. Find the 5-point DFTs, $Y_5[k]$ and $Z_5[k]$, of $y[n]$ and $z[n]$. Do they have 3-point DFTs? Why?

4) Let $\widetilde{W}[k] = X(e^{j\omega})\big|_{\omega=k\frac{2\pi}{2}}$, i.e., two (uniformly spaced) samples from each period of $X(e^{j\omega})$, DTFT of $x[n]$ in Question-1.

a. Find the periodic sequence $\widetilde{w}[n]$ whose DFS coefficients are $\widetilde{W}[k]$.

b. Find the relationship between $\widetilde{w}[n]$ and $x[n]$.

5) (Generalization of the result in Question-4) Let $x[n]$ be an <u>arbitrary</u> sequence with a DTFT $X(e^{j\omega})$and

$$\widetilde{W}[k] = X(e^{j\omega})\big|_{\omega=k\frac{2\pi}{M}}$$

i.e., M (uniformly spaced) samples from each period of $X(e^{j\omega})$. Also let $\widetilde{w}[n]$ be the periodic sequence whose DFS coefficients are $\widetilde{W}[k]$.

a. Show that

$$\widetilde{w}[n] = \sum_{k=-\infty}^{\infty} x[n-kM]$$

b. Assuming that $x[n]$ has finite length N. Comment on the cases $M \geq N$ and $M < N$.

c. Verification using MATLAB. You may use the following code to verify for different values of M and N.

```
clear all
close all

N = 10;
n = 0:(N-1);
x = 1:N;


M = 3;
% M = 5;
% M = 7;
% M = 10;
% M = 15;

W_M = exp(-j*2*pi/M);

F = W_M .^ n ;

for k = 0:(M-1)
    DFT_matrix(k+1,:) = F.^k;
end

Z = DFT_matrix * x';

z = ifft(Z)
```

6) Let $x[n] = 3\delta[n] - 2\delta[n-1] + \delta[n-2] + \delta[n-3] - 2\delta[n-4] - \delta[n-5]$.

You do not need to compute any DFTs in parts (a)-(c)!

a) Let $W_3[k] = X(e^{j\omega})|_{\omega=k\frac{2\pi}{3}}$, k=0,1,2. Find the sequence $w_3[n]$ whose 3-point DFT is $W_3[k]$.

b) Let $W_5[k] = X(e^{j\omega})|_{\omega=k\frac{2\pi}{5}}$, k=0,1,2,3,4. Find the sequence $w_5[n]$ whose 5-point DFT is $W_5[k]$.

c) Let $W_8[k] = X(e^{j\omega})|_{\omega=k\frac{2\pi}{8}}$, k=0,1,2,3,4,5,6,7. Find the sequence $w_8[n]$ whose 8-point DFT is $W_8[k]$.

Let $h[n] = 2\delta[n] - 1\delta[n-1]$ be the impulse response of a LTI system.

You do not need to compute any DFTs in parts (d)-(f)!

d) Let $H_3[k]$ be the 3-point DFT of $h[n]$. Find the sequence $y_3[n]$ whose 3-point DFT is $W_3[k]H_3[k]$.

e) Let $H_5[k]$ be the 5-point DFT of $h[n]$. Find the sequence $y_5[n]$ whose 5-point DFT is $W_5[k]H_5[k]$.

f) Let $H_8[k]$ be the 8-point DFT of $h[n]$. Find the sequence $y_8[n]$ whose 8-point DFT is $W_8[k]H_8[k]$.

g) Describe the relationships between the sequence $y[n] = x[n] * h[n]$ and the sequences $y_3[n], y_5[n], y_8[n]$.

7) Let $x[n]$ a be sequence of length N; N is even. Let $X[k]$ be its N-point DFT.

a) Show that $X[k]$ can be written as

$$X[k] = E\left[((k))_{\frac{N}{2}}\right] + e^{-jk\frac{2\pi}{N}} O\left[((k))_{\frac{N}{2}}\right] \qquad k = 0,1,\dots,N-1$$

where $E[k]$ and $O[k]$ are the $\frac{N}{2}$-point DFTs of $e[n] = x[2n]$ and $o[n] = x[2n+1]$, respectively.

b) Assume that $x[n]$ is real.

   a. Count the number of real multiplications and real additions in the direct computation of $X[k]$.

   b. Count the number of real multiplications and real additions in the computation of $X[k]$ according to the right hand side of the above expression.

   c. Compare the numbers of arithmetic operations in these two cases.

8) Let $h[n] = 2\delta[n] - \delta[n-1] + \delta[n-2]$ be the impulse response of a LTI system and

$$x[n] = [1\ 2\ 3\ 4\ -1\ -2\ -3\ -4\ 1\ 2\ 3\ 4] \quad 0 \le n < 11$$

be an input to this system. The output $y[n]$ will be found by using the overlap-add method. Take the length, L, of the input segments as $L = 4$.

a) How many point DFTs will be used in this computation?

b) How many input segments are there? Write all of them.

c) Find the response of the system to the individual input segments (use MATLAB; find DFTs, multiply them and then take inverse DFT).

d) Obtain the whole output sequence by using the responses to input segments.

9) Overlap-save type method will be used for the setting in Question-8.

Take the length, L, of the input segments as $L = 6$. Use 7-point DFTs. Answer parts (b)-(d).