

1) A discrete-time signal $x[n]$ is obtained by sampling a continuous-time sinusoidal signal

$$x_c(t) = 4 \sin\left(20000\pi t + \frac{\pi}{13}\right)$$

at a sampling rate of 3 kHz.

- Describe the set of all other continuous-time sinusoidal signals (their frequencies) that yield $x[n]$ when sampled at 3 kHz.
- Describe the set of all other sampling frequencies that yield $x[n]$ from $x_c(t)$.

2) Among the following sinusoidal signals, identify periodic ones and find their fundamental periods.

$$\sin(1.74\pi n + 3.1), \sin(1.74\pi n + 3.1\pi), \cos\left(15.74\pi n + \frac{3\pi}{8}\right), \cos(\sqrt{\pi}n), \cos(\pi\sqrt{\pi}n), \cos(\pi\sqrt{2}n)$$

3) What is the “highest frequency” discrete-time sinusoidal signal? Why?

4) What does the following system do? Is it linear, time-invariant? Prove your answer.

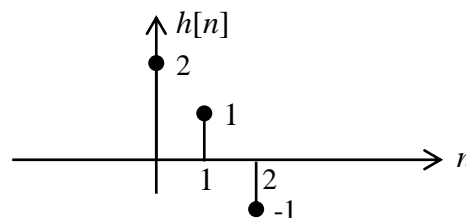
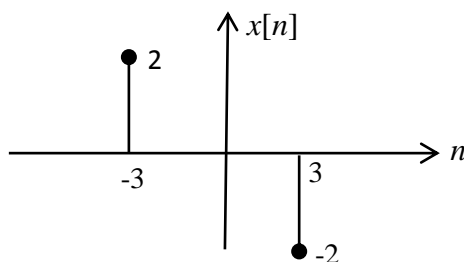
$$y[n] = \begin{cases} x\left[\frac{n}{2}\right] & \text{if } n \text{ is even} \\ \frac{x\left[\frac{n-1}{2}\right] + x\left[\frac{n+1}{2}\right]}{2} & \text{if } n \text{ is odd} \end{cases}$$

5) Are the following systems causal, stable? Justify/discuss/prove.

$$y[n] = 2^{\delta[n+1]} + x[n-3]$$

$$y[n] = \begin{cases} y[-\delta[n-1]] + x[n-3] & n > 0 \\ 2^n x[n-3] & n \leq 0 \end{cases}$$

6) Evaluate (graphically) the convolution of $x[n]$ and $h[n]$ according to the two interpretations of convolution.



7) Show that if $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$ then the following hold:

$$i) y[-n] = x[-n] * h[-n]$$

ii) $y[n - 4] = x[n - 4] * h[n] = x[n] * h[n - 4]$

8) $x[n] = u[n] - 2u[n - 3] + u[n - 6]$ and $h[n] = \left(\frac{1}{2}\right)^n u[n]$

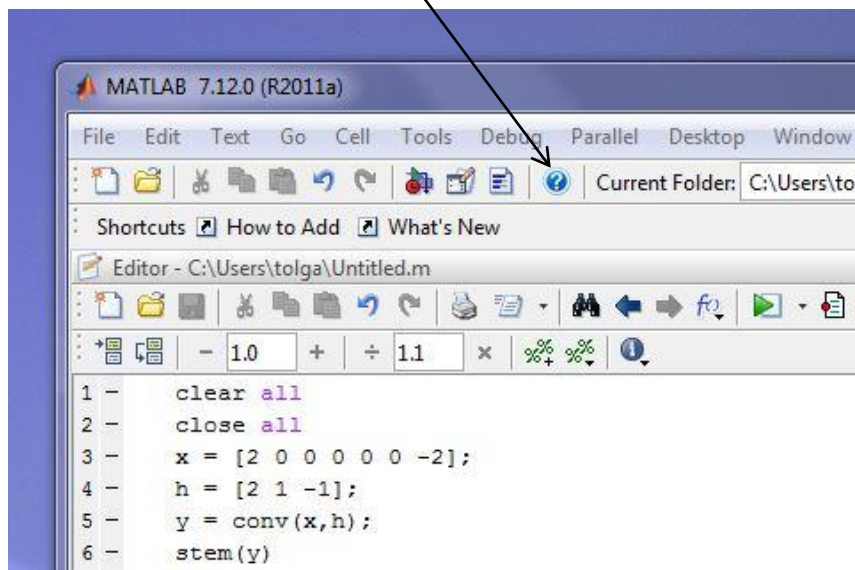
i) Plot $x[n]$ and $h[n]$.

ii) Find $y[n] = x[n] * h[n]$.

9) MATLAB - Generate two arbitrary sequences $x[n]$ and $h[n]$ as vectors and convolve them using “conv” command. Plot $x[n]$, $h[n]$ and their convolution using “stem” command. Is your plot as expected?

Type “help conv” in MATLAB command window to read about the command.

You can find more by visiting the “conv” page in the help menu.



10)

a) Find the result of multiplying two polynomials (one 3rd order, one 5th order) using “conv” command of MATLAB.

b) Read the description of “deconv” command of MATLAB.

c) The output of a LTI system is the sequence $[1 \ 1 \ 2 \ 3 \ 4 \ -1 \ 5]$ for $n = 0, 1, \dots, 6$ when its input is $[1 \ 2 \ 3 \ 4 \ 5]$ $n = 0, 1, \dots, 4$. Find the impulse response of the system. Show your computation. Verify your result using the “deconv” command of MATLAB.

d) In part-b, modify the output as $[1 \ 2 \ 2 \ 3 \ 4 \ -1 \ 5]$ for $n = 0, 1, \dots, 6$ and obtain the result of “deconv” command for the same input. How do you interpret the result?