Middle East Technical University
Dept. of EEE

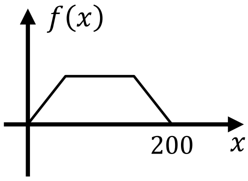
HW 4

EE 430 Digital Signal Processing – 2014 Fall

Due: Dec. 1, 2014

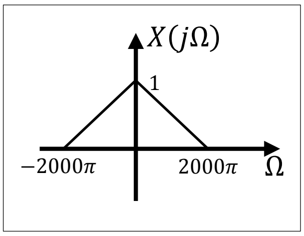
Submit for problems 1,2,3,5,7,8

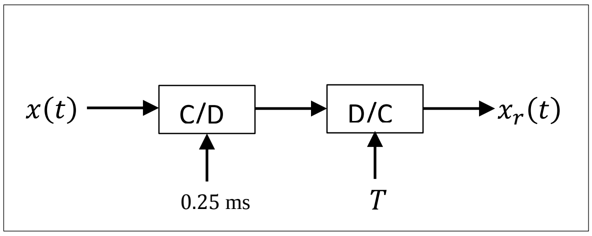
1) A function $f(x)$ is shown in the figure. Plot $g(x) = A\sum_{k=-\infty}^{\infty} f(Ax - k8A)$ for A =50 and A =20. Find the condition on A so that $f(Ax - k8A)$, $k \in Z$, do not overlap.



2) $x(t)$ is a complex valued signal whose CTFT is zero for $\Omega \leq -100\pi$ and $\Omega \geq 1000\pi$. Find the minimum sampling frequency in Hz so that $x(t)$ can be recovered from its samples, $x[n]$.

3) $x(t)$ has a hypothetical CTFT, $X(j\Omega)$, as shown in the figure. $x(t)$ is applied to the system shown below.





Find and plot $X_r(j\Omega)$ for

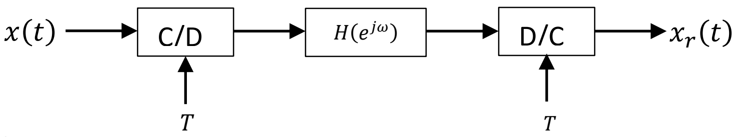
a) $T = 0.75$ ms
b) $T = 0.05$ ms

4) A discrete-time signal, $x[n]$, is obtained by sampling a continuous-time signal, $x_c(t)$, bandlimited to $\frac{\pi}{T}$. Sampling period is T. $x[n]$ is applied to a causal LTI system whose impulse response is

$$h_D[n] = \begin{cases} \dfrac{\sin\left(\frac{\pi}{T}\left(\frac{T}{5} + (n-D)T\right)\right)}{\frac{\pi}{T}\left(\frac{T}{5} + (n-D)T\right)} & n = 0,1,\ldots,2D \\ 0 & o.w. \end{cases}$$

Discuss the relationship of the output, $y[n]$, of this system to $x_c(t)$. May your discussion have any practical implications?

5) It is desired to have an ideal bandpass filter with the system below.



Specifically, it is desired to have

$$H_{eff}(j\Omega) = \frac{X_r(j\Omega)}{X(j\Omega)} = \begin{cases} 1 & 9000\pi \le |\Omega| \le 10000\pi \\ 0 & o.w. \end{cases}$$

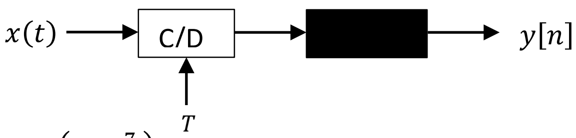
It is known that $H(e^{j\omega})$ is a highpass filter.

a) Find T, specify $H(e^{j\omega})$; are any constraints/conditions required to make the system act as the specified bandpass filter?

b) Answer part (a) to have

$$H_{eff}(j\Omega) = \frac{X_r(j\Omega)}{X(j\Omega)} = \begin{cases} 1 & 900\pi \le |\Omega| \le 1000\pi \\ 0 & o.w. \end{cases}$$

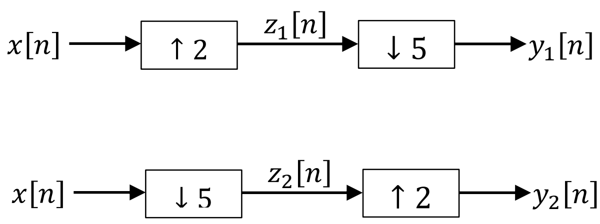
6) Consider the following system. $x(t)$ is bandlimited to $\frac{\pi}{T}$. Black box contains upsamplers, downsaplers, ideal lowpass filters and delay elements.



a) It is desired to have $y[n] = x\left(t - \frac{7}{13}\right)_{t=n\frac{T}{3}}$. Design the system in the black box.

b) How would you answer if it is desired to have $y[n] = x\left(t - \frac{7}{13}\right)_{t=n3T}$?

7) Consider the following two systems:



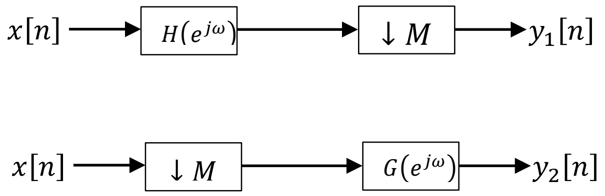
a) Express $Z_1(e^{j\omega})$ and $Y_1(e^{j\omega})$ in terms of $X(e^{j\omega})$.

b) Express $Z_2(e^{j\omega})$ and $Y_2(e^{j\omega})$ in terms of $X(e^{j\omega})$.

c) Are $y_1[n]$ and $y_2[n]$ the same? Prove your claim.

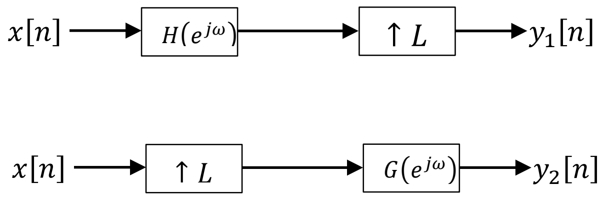
d) Does your answer to part-(c) change if the downsampling factor is 4?

8) Consider the two systems:



Assuming that $x[n]$ is bandlimited to $\frac{\pi}{M}$, can you find a relationship between $H(e^{j\omega})$ and $G(e^{j\omega})$ so that the two systems are equivalent, i.e., $y_1[n] = y_2[n]$? Prove your claim.

9) Consider the two systems:



Can you find a relationship between $H(e^{j\omega})$ and $G(e^{j\omega})$ so that the two systems are equivalent, i.e., $y_1[n] = y_2[n]$? Prove your claim.

10) (**MATLAB**) The distortion resulting from the sampling and reconstruction of a finite duration sinusoidal signal will be analyzed. Let $x[n]$ be obtained by sampling $x_c(t) = \cos(2000\pi t)$ at $f_s = 3$ kHz. The sampling duration is 5 msec.

$$x[n] = \begin{cases} \cos\left(\frac{2\pi}{3}n\right) & n = 0,1,\ldots,15 \\ 0 & o.w. \end{cases}$$

$y(t)$ is obtained from $x[n]$ by using an ideal C/D converter. Therefore

$$y(t) = \sum_{n=0}^{15} x[n] \frac{\sin\left(\frac{\pi(t-nT)}{T}\right)}{\frac{\pi(t-nT)}{T}}$$

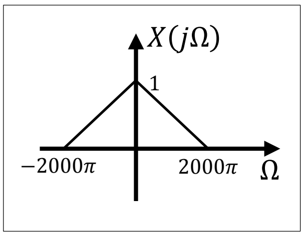
$$= \sum_{n=0}^{15} \cos\left(\frac{2\pi}{3}n\right) \frac{\sin\left(3000\pi\left(t-\frac{n}{3000}\right)\right)}{3000\pi\left(t-\frac{n}{3000}\right)}$$

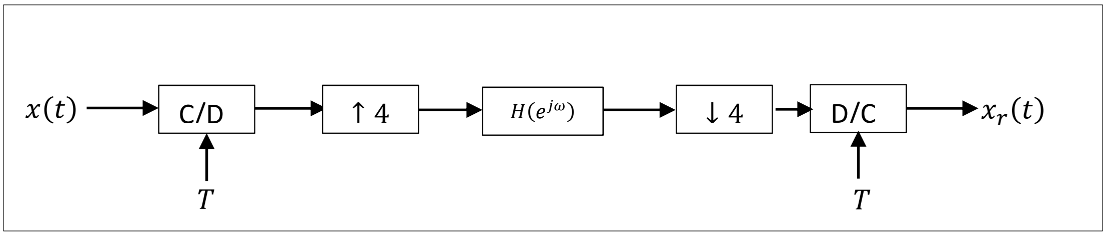
a) Plot $x_c(t)$ and $y(t)$ for $-5\, msec \leq t \leq 10\, msec$ in the same figure.
b) Plot the absolute error $e(t) = |x_c(t) - y(t)|$ for $-5\, msec \leq t \leq 10\, msec$.
c) How does the error behave in $0\, msec \leq t \leq 5\, msec$ interval? Zoom into this interval. Observe the peaky behavior. Where does the smallest peak of $e(t)$ occur? What is this smallest peak value?
d) Change sampling duration to 15 msec. Answer the questions in part-(c) using an observation interval of $-5\, msec \leq t \leq 20\, msec$. What is your conclusion?
e) For the original sampling duration of 5 msec and observation interval of $-5\, msec \leq t \leq 10\, msec$, increase the sampling rate to $f_s = 3$ kHz. Answer the questions in part-(c). What is your conclusion?

You may use the code below:

```
% bandllimited interpolation error analysis

clear all
close all

DelT = 5;                % sampling duration of signal in msec
t    = -5:0.001:10;      % time interval in which we plot our results in msec
t    = t * 0.001;

f    = 1000;             % frequency of continuous time sinuoid to be sampled
Omega = 2 * pi * f ;
f_s  = 3000          ;     % sampling frequency
T_s  = 1/f_s       ;
N    = round(DelT*0.001/T_s)
k    = 0:N;
x    = cos(Omega*k*T_s); % samples of the continuous time sinuoid

% computing the sinc signals in the summation
for n = 0:N
    h_r(n+1,:) = x(n+1) * sin(f_s*pi*(t-n*T_s))./(f_s*pi*(t-n*T_s));
    h_r(n+1,find(isnan(h_r(n+1,:)))) = x(n+1) ;
end

x_r = sum(h_r);    % reconstructed cont-time signal

plot(t,cos(Omega*t),'k','LineWidth',3)
hold
plot(t,x_r,'r--','LineWidth',1.5)

figure
plot(t,abs(x_r - cos(Omega*t)),'r','LineWidth',1.5)
```

11) Consider the system below and the spectrum of its input signal:





a) Is the discrete-time system including the upsampler, $H(e^{j\omega})$ and downsampler a LTI system? Hence, can one claim the existence of an $H_{eff}(j\Omega)$ for an arbitrary $H(e^{j\omega})$? Justify your answer.

b) Assuming that T fulfills the Nyquist rate requirement, can one propose a constraint on $H(e^{j\omega})$ to have an $H_{eff}(j\Omega)$?

c) Now, assume that $H(e^{j\omega}) = 0, \quad \frac{\pi}{12} \leq |\omega| \leq \pi$. What is the set of T values for which one can find $H_{eff}(j\Omega)$?