HW 6

Middle East Technical University
Dept. of EEE

EE 430 Digital Signal Processing – 2014 Fall

Due:

No submission!

Computation of DFT

1) Consider a sequence whose length is a power-of-three. In this case, decimation-in-time can be applied by forming subsequences of one-third length until all subsequences have only three elements. Derive the algorithm for a sequence of length 9. Draw the signal flow graph.

2)

a) Show that $W_N^m = W_N^{m \pm pN}$ and $W_N^m = \left(W_N^{N-m}\right)^*$, $m = 1,2,\ldots,N-1$ where $W_N = e^{-j\frac{2\pi}{N}}$.

a) Let $x[n]$ be a 5-point <u>real</u> sequence. Write all of the <u>REAL</u> arithmetic operations (in correct sequence) that form a minimum set to compute the 5-point DFT of $x[n]$. **Hint**: For real sequences $X^*\left(e^{j\omega}\right) = X\left(e^{-j\omega}\right)$.

3) Find a way to perform multiplication of two complex numbers by 3 real multiplications and 5 real additions.

4) Solve problems 9.6., 9.9., 9.17. in the textbook.

Implementation Structures for Discrete-Time Systems

5) Find the 8-bit quantized two's-complement representation, $\bar{a}$, of $a = 2.819432011$ for the following scale factors

a) $X_m = 10$
b) $X_m = 3$
c) $X_m = 1$

Calculate $|\bar{a} - a|$ in each case. Which value of X_m yields the minimum absolute error?

Repeat for $a = -2.819432011$.

6) What is the dynamic range (the ratio of the maximum to minimum value) of a $B + 1$ bit representation? Find for $B = 7$ and $B = 20$.

7) Examine Table 6.5 in the textbook. It contains the coefficients of a FIR filter of length 28.

a) What is the maximum-to-minimum ratio (in absolute values) of the unquantized coefficients?
b) What are the dynamic ranges of 8, 13, 14, 16 bit representations?
c) Explain why the four sets of filter coefficients (corresponding to different quantization grades) have different scales.
d) Effectively, what is the length of the filter when its coefficients are quantized using 8 bits?

8) Consider a signal processing system which takes a continuous-time signal as an input, processes the signal in discrete-time and generates a continuous-time output signal. Describe all quantization operations that may be encountered in this system.

9) Why do we study different implementation structures?

10) Does changing the order of cascades matter in a cascade implementation? Why?

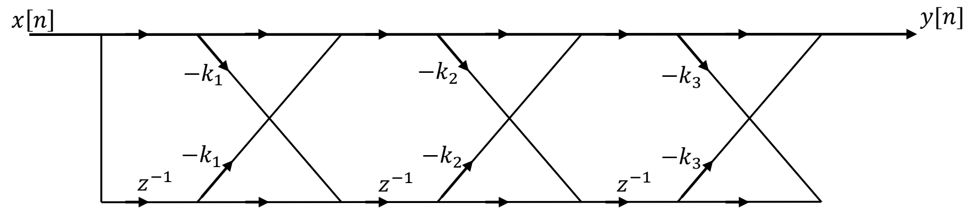
11) You may use the "zp2tf", "zp2sos", "residuez" or other similar functions of MATLAB to answer the following. Let

$$H(z) = \frac{\left(1-0.9e^{j\frac{\pi}{6}}z^{-1}\right)\left(1-0.9e^{-j\frac{\pi}{6}}z^{-1}\right)(1+2z^{-1})}{\left(1-0.9e^{j\frac{4\pi}{6}}z^{-1}\right)\left(1-0.9e^{-j\frac{4\pi}{6}}z^{-1}\right)\left(1-0.9e^{j\frac{5\pi}{6}}z^{-1}\right)\left(1-0.9e^{-j\frac{5\pi}{6}}z^{-1}\right)}.$$

a) Draw the Direct Form I and Direct Form II implementation structures and their transposed forms
b) Draw a cascade form implementation structure. How many different cascade forms (with real coefficients) does this system function yield?
c) Draw a parallel form implementation structure. How many different parallel forms (with real coefficients) does this system function yield?

12)

a) Find the system function of the following "lattice" structure with three stages. Find the impulse response of this system in terms of "*k*-coefficients". Is it FIR or IIR?



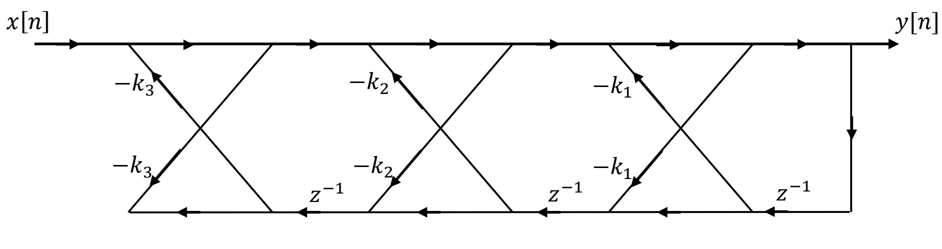
b) Using the algorithm given in Figure 6.38 of the textbook, find the impulse response of the system whose "*k*-coefficients" are $k_1 = 2, k_1 = -2, k_1 = 1, k_1 = 3$.
c) Consider the system function

$$H(z) = -3 + 2z^{-1} - 2z^{-2} + z^{-3}.$$

What is the impulse response of this system? Find the "*k*-coefficients" using the algorithm given in Figure 6.39 of the textbook

13)

a) Find the system function of the following "lattice" structure with three stages. Is it FIR or IIR?



b) Above structure is famous for its nice property about the relationship between "*k*-coefficients" and the stability of the system. What is this property? (Read the related section in the textbook.)