EE430
HOMEWORK 4

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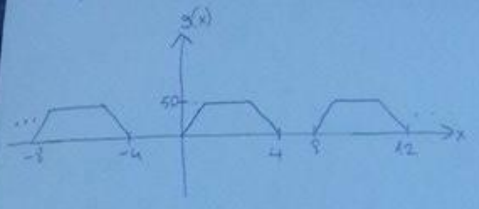
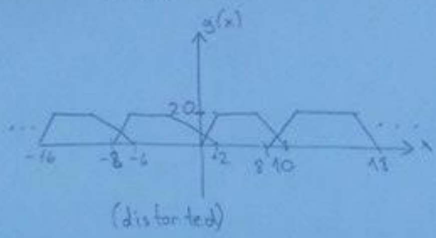
1) $A=50$ $A=20$

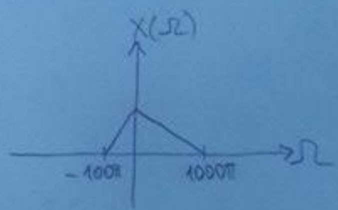


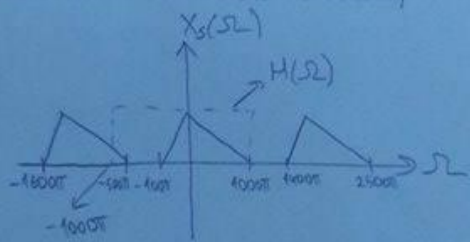


In above figures we can see that the rightmost edge of the signal closest to origin is at $x=\frac{200}{A}$ and the leftmost edge of the signal shifted to its right is at $x=8$. For no overlap $8>\frac{200}{A}$. More generally;

$$|A|>25$$

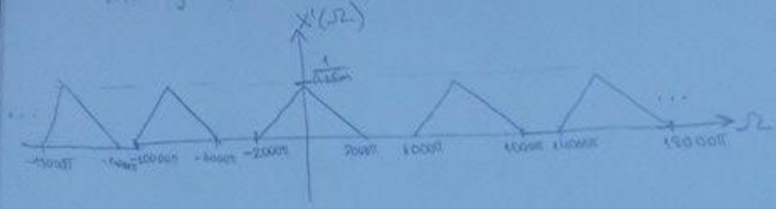
2) Minimum sampling frequency is two times the highest frequency component in the spectrum. In this case it must be 2000π rad/s or 1 kHz. Since the spectrum is not symmetric there can be a misunderstanding that it is 1100π rad/s (550 Hz). Think the below situation;





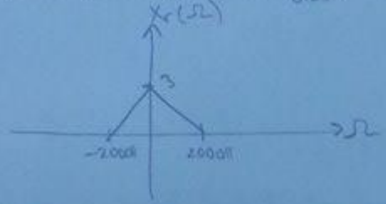
We have an $X(\Omega)$ signal that obeys the situation of the question and it is sampled with 750 Hz which is between 550 Hz and 1 kHz values. There is no aliasing after the sampling. But in order to recover the signal we must pass it through a lowpass filter at the last step. If we assume the filter $H(\Omega)$ has a real impulse response, its spectrum must be symmetric. By that way it also passes a small portion of shifted signal which causes distortion. Hence minimum sampling frequency must be 1 kHz not 550 Hz.

3) 0.25 ms corresponds to 4 kHz or 8000π rad/s. Thus before the lowpass filtering step we have below signal.



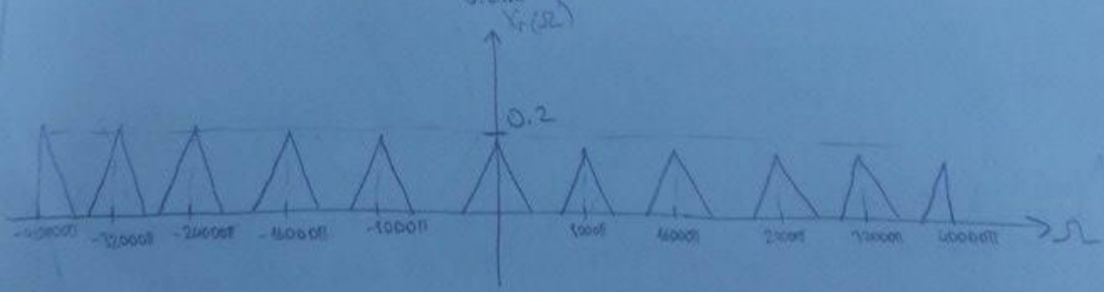
a) 0.75 ms ⇒ 1.33 kHz or 2660π rad/s

The 2660π rad/s only passes baseband signal so we obtain the original signal with a scale factor of $\frac{T}{0.25m} = 3$.

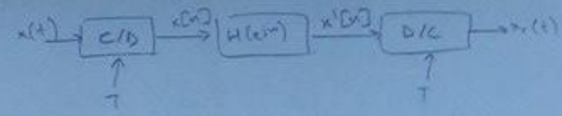


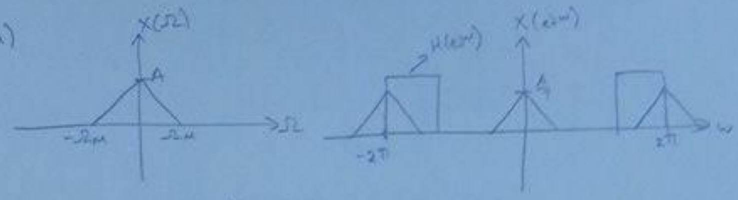
b) 0.05 ms ⇒ 20 kHz or 40000π rad/s.

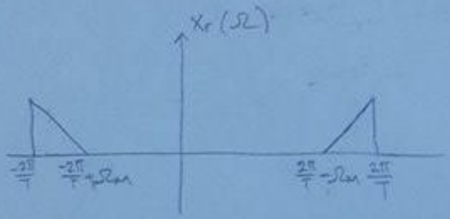
This filter passes baseband signal and high frequency terms until 40000π rad/s with a scale factor of $\frac{T}{0.25m} = 0.2$.



We cannot recover the original signal in this case.

5) 

a) 



Equating $\frac{2\pi}{T} = 10000\pi \Rightarrow \boxed{T = 0.2\text{ ms}}$

Also $\frac{2\pi}{T} - \Omega_M = 9000\pi \Rightarrow \boxed{\Omega_M = 1000\pi \text{ rad/s}}$

Hence sampling period is $T = 0.2$ ms. Our constraint is input signal must be bandlimited to 1000π rad/s or 500 Hz.

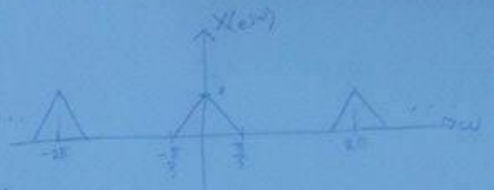
b) $\frac{2\pi}{T} = 1000\pi \Rightarrow \boxed{T = 2\text{ ms}}$ $\quad \frac{2\pi}{T} - \Omega_M = 900\pi \Rightarrow \boxed{\Omega_M = 100\pi \text{ rad/s}}$

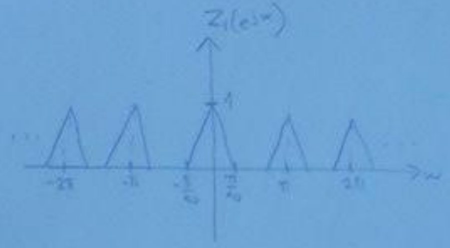
7) a) $Z_1(e^{jw}) = X(e^{jw2})$ $\quad Y_1(e^{jw}) = \frac{1}{5}\sum_{k=0}^{4} Z_1\left(e^{j\left(\frac{w}{5} - k\frac{2\pi}{5}\right)}\right) = \frac{1}{5}\sum_{k=0}^{4} X\left(e^{j\left(\frac{w2}{5} - k\frac{2\pi}{5}\right)}\right)$

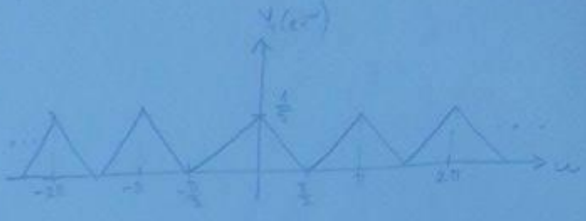
b) $Z_2(e^{jw}) = \frac{1}{5}\sum_{k=0}^{4} X\left(e^{j\left(\frac{w}{5} - k\frac{2\pi}{5}\right)}\right)$ $\quad Y_2(e^{jw}) = Z_2(e^{jw2}) = \frac{1}{5}\sum_{k=0}^{4} X\left(e^{j\left(\frac{w2}{5} - k\frac{2\pi}{5}\right)}\right)$

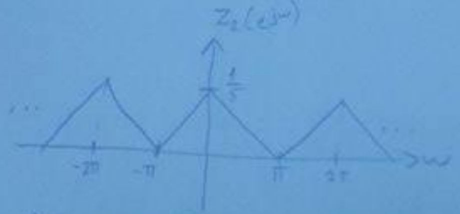
c) As can be seen in parts a and b $Y_1(e^{jw})$ and $Y_2(e^{jw})$ are same. Hence $y_1[n]$ and $y_2[n]$ are the same

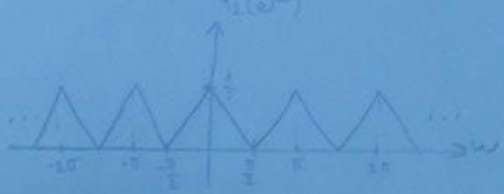
c) Assume we have a proper signal which means it is bandlimited to $\frac{\pi}{5}$.



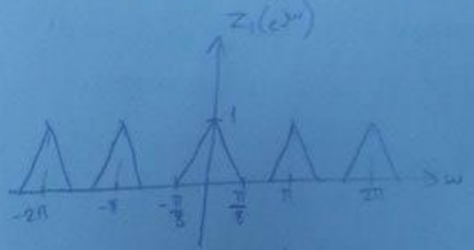


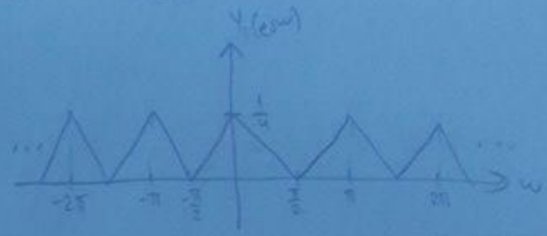


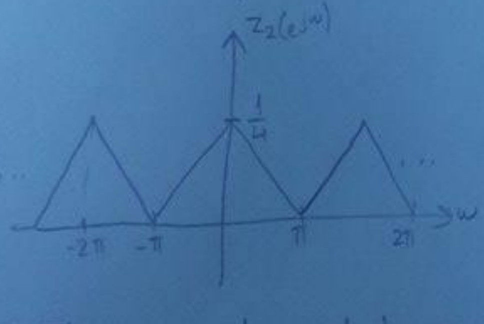


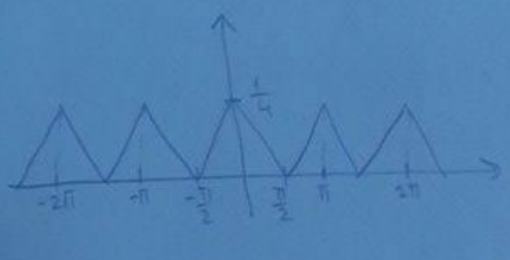


d) Assume $x[n]$ is bandlimited to $\frac{\pi}{4}$ in this case









The answer does not change since there is no filtering order does not matter in this case.