EE 430 Section 2 HW1 Solutions

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1. $x[n] = x_c(nT) = 4\sin\left(20000\pi nT + \frac{\pi}{13}\right)$, $T = \frac{1}{fs} = \frac{1}{3000}\ s$

$$x[n] = 4\sin\left(\frac{20000\pi n}{3000} + \frac{\pi}{13}\right) = 4\sin\left(\frac{20\pi n}{3} + \frac{\pi}{13}\right) = 4\sin\left(\frac{2\pi n}{3} + \frac{\pi}{13}\right)$$

   a. The frequencies of the continuous-time signals that yield $x[n]$ with the same sampling frequency will be obtained as follows;
   Let the signals be defined as $4\sin\left(f2\pi nT + \frac{\pi}{13}\right)$ then

$$f = 1000, 4000, 7000, \dots$$

   b. Similarly for the this frequency the following sampling frequencies yield the same signal

$$T = \frac{1}{30000}, \frac{4}{30000}, \frac{7}{30000}, \dots$$

2. Remember for periodicity $x[n] = x[n+N]$ then

   $sin(1.74\pi n + 3.1)$, $\sin(1.74\pi(n+N) + 3.1\pi)$ and $\cos\left(15.74\pi(n+N) + \frac{3\pi}{8}\right)$ are periodic with fundamental period 100.

   $\cos(\sqrt{\pi}n), \cos(\pi\sqrt{\pi}n)$ and $\cos(\pi\sqrt{2}n)$ are not periodic.

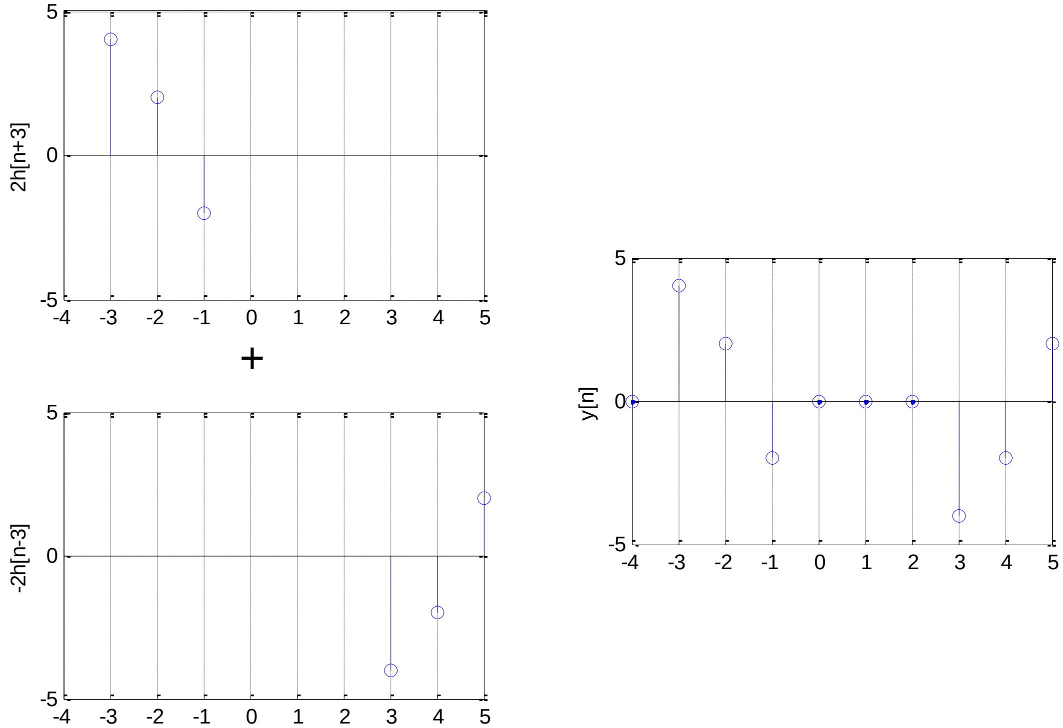
3. The highest frequency for a discrete time sinusoidal signal is π.

4. This is an interpolation operation. It is a linear operation. It is not time-invariant.

5. The first system is causal and stable.

   The second system is both causal and stable system.

6. Note that $x[n]$ can be written as $x[n] = 2\delta[n+3] - 2\delta[n-3]$ so $y[n] = h[n] * x[n] = 2h[n+3] - 2h[n-3]$

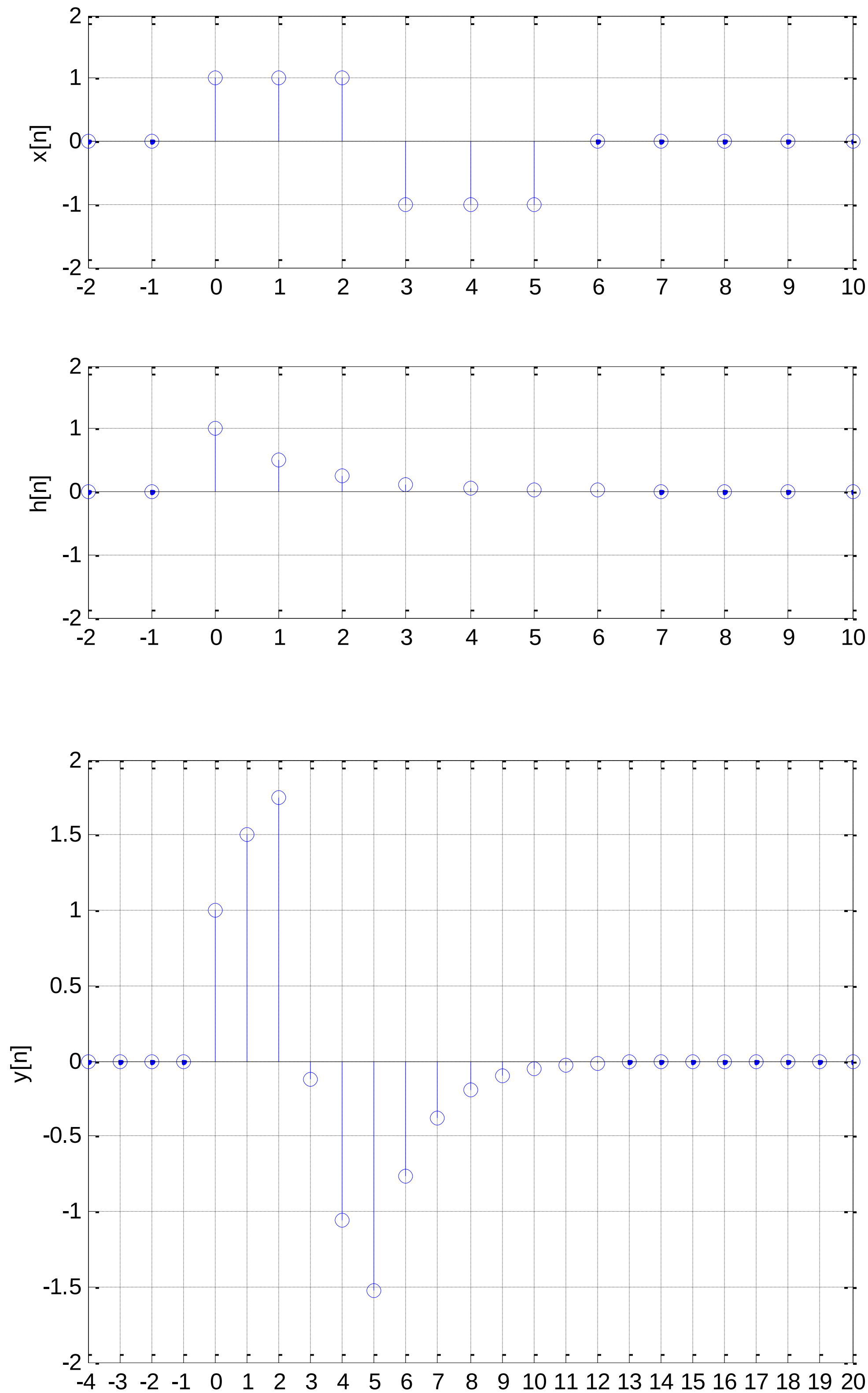


Or using the definition of convolution $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$ the same result will be obtained

7. Hint: Let $x'[n] = x[-n]$, $h'[n] = h[-n]$ and find $y'[n] = x'[n] * h'[n]$

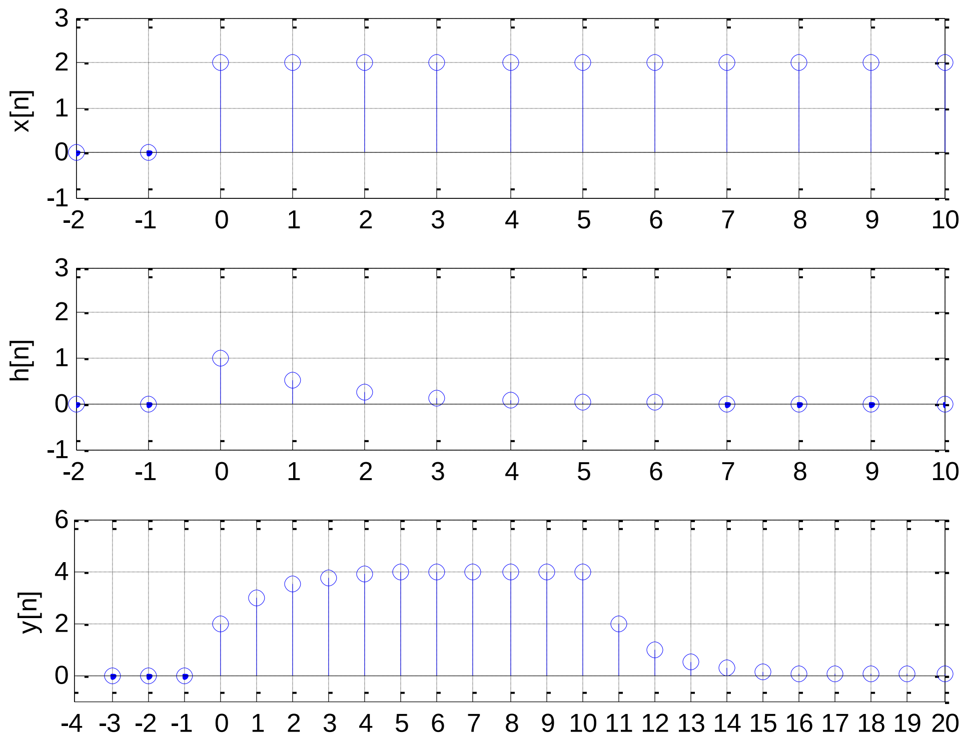
8. Using MATLAB

```
n = -2:10;
u = @(k)(n>=k);
x = u(0)-2*u(3)+u(6);
h = (1/2).^n.*u(0);
subplot(2,1,1);  stem(n,x);  set(gca,'XLim',[-2  10],'YLim',  [-2  2],'XTick',n);
ylabel('x[n]');grid
subplot(2,1,2);  stem(n,h);  set(gca,'XLim',[-2  10],'YLim',  [-2  2],'XTick',n);
ylabel('h[n]');grid

figure;
stem(-4:20,conv(x,h));set(gca,'XLim',[-4   20],'YLim',   [-2   2],'XTick',-4:20);
ylabel('y[n]');grid
```


x[n]
h[n]
y[n]

9. The code and graph is given below and the result of the convolution is as expected

```
n = -2:10;
u = (n>=0);
x = 2*u;
h = (1/2).^n.*u;

subplot(3,1,1);   stem(n,x);   set(gca,'XLim',[-2  10],'YLim',   [-1  3],'XTick',n);
ylabel('x[n]');grid
subplot(3,1,2);   stem(n,h);   set(gca,'XLim',[-2  10],'YLim',   [-1  3],'XTick',n);
ylabel('h[n]');grid
subplot(3,1,3);stem(-4:20,conv(x,h));set(gca,'XLim',[-4  20],'YLim',  [-1  6],'XTick',-
4:20); ylabel('y[n]');grid
```



10.

a. Multiplication of two polynomials means convolution of their coefficients. So multiplication of the following polynomials

$$a(x) = 2x^3 + 3x^2 + 1$$
$$b(x) = x^5 + 2x^4 + x^3 + 2x^2 + 3x + 5$$

Is obtained as follows

a = [2 3 0 1];

b = [1 2 1 2 3 5];

conv(a,b)

2 7 8 8 14 20 17 3 5

b.

c.

```
y =  [1 1 2 3 4 -1 5];
x =  [1 2 3 4 5];
[h r] = deconv(y,x)

h =

     1    -1     1


r =

     0     0     0     0     0     0     0
```

d. h =

1 0 -1

r =

0 0 0 1 2 3 10

In the first case there is no remainder so $conv(h[n], x[n])$ gives $y[n]$ correctly but in the second case there is a remainder so to find $y[n]$ correctly r should be added to convolution output that is $conv(h[n], x[n]) + r$. This result is because $deconv$ command of MATLAB use long division (polynomial division).