EE430 Digital Signal Processing
HW 1

1. A DT $x[n]$ is obtained by sampling the $x_c(t) = 4\sin(20000\pi t + \frac{\pi}{13})$ at sampling rate of 3 kHz.

   (a) The same DT signal can be constructed by sampling the set of signals in a form
   $$x_c(t) = 4\sin((20000 + 3000k)\pi t + \frac{\pi}{13}) \quad , \quad k = [-6, \infty)$$
   or equivalently,
   $$x_c(t) = 4\sin((2000 + 3000k)\pi t + \frac{\pi}{13}) \quad , \quad k = [0, \infty)$$

   (b) Let $\Omega_0 = 2000$ for our signal, the sampled signal can be expressed as;
   $$x[n] = 4\sin(\Omega_0 n T_s + \pi/3)$$
   it should be equal to the signal sampled at $\widetilde{T}_s \triangleq T_s + \Delta T$
   $$x[n] = 4\sin(\Omega_0 n \widetilde{T}_s + \pi/3) = 4\sin(\Omega_0 n(T_s + \Delta T) + \pi/3)$$
   To satisfy the equation $\Omega_0 n \Delta T$ should be equal to $k2\pi$ and knowing that
   $$\Omega_0 = 2\pi f_0$$
   $$\Delta t = \frac{k}{f_0} = kT_0$$
   $$\widetilde{T}_s = T_s + \Delta T$$
   using the equations above, the new set sampling frequencies that give the same $x[n]$ can be found as follows;
   $$\boxed{f_s' = \frac{f_s f_0}{f_0 + k f_s}} \quad , \quad k = 0, 1, 2...$$
   for our case $f_0 = 1000$ and $f_s = 3000$, from there other sampling frequencies that yield $x[n]$ from $x_c(t)$ can be calculated.

2. For any DT sinusoidal $cos(w_0 n + \phi)$ or complex exponential $e^{w_0 n + \phi}$ to be periodic with N, it has to satisfy the following,
   $$w_0 n = k2\pi$$
   or equivalently,
   $$\boxed{N = \frac{2\pi}{w_0}k} \quad , \quad k, N \in \mathcal{Z}$$
   For the given functions,

- $\sin(1.74\pi n + 3.1)$, periodic with $N_1 = \dfrac{2\pi}{1.74\pi}k = 100$ with $k = 87$
- $\sin(1.74\pi n + 31\pi)$, periodic with $N_2 = \dfrac{2\pi}{1.74\pi}k = 100$ with $k = 87$
- $\cos(15.74\pi n + \dfrac{3\pi}{8})$, periodic with $N_3 = \dfrac{2\pi}{15.74\pi}k = 100$ with $k = 787$
- $\cos(\sqrt{\pi}n)$, not periodic since there is no integer k that makes $N_4 = \dfrac{2\pi}{\sqrt{\pi}}k$ an integer
- $\cos(\pi\sqrt{\pi}n)$, not periodic since there is no integer k that makes $N_5 = \dfrac{2\pi}{\pi\sqrt{\pi}}k$ an integer
- $\cos(\pi\sqrt{2}n)$, not periodic since there is no integer k that makes $N_6 = \dfrac{2\pi}{\pi\sqrt{2}}k$ an integer

3. For any linear system, the output can be calculated as,

$$y[n] = x[n] * h[n] \triangleq \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Response to a shifted response, can be calculated as follows,

$$h_k[n] = (u-k)u[n-k] = \delta[n-k] * h[n]$$

$$\delta[n-k] * h[n] \triangleq \sum_{a=-\infty}^{\infty} \delta[a-k]h[n-a] = h[n-a]$$

$$h[n-a]|_{a=k} \equiv h[n-k] = h_k[n] = (n-k)u[n-k]$$

From there, the impulse response $h[n]$ can be found as,

$$h[n] = nu[n]$$

Thus, $y[n]$ for any input can be found as follows,

$$y[n] = x[n] * h[n] \triangleq \sum_{k=-\infty}^{\infty} x[k](n-k)u[n-k]$$

$$\boxed{y[n] = \sum_{k=-\infty}^{n} x[k](n-k)}$$

To check time-invariance, let us find $y[n-m$ and the output $y_1[n]$ for an input $x_1[n] \triangleq x[n-m]$

$$y[n-m] = \sum_{k=-\infty}^{\infty} x[k]h[n-m-k]$$

$$y_1[n] = x[n-m] * h[n] \triangleq \sum_{k=-\infty}^{\infty} x[k-m]h[n-k]$$

Letting $\widetilde{k} \triangleq k-m$, $k = m + \widetilde{k}$

$$y_1[n] = \sum_{\widetilde{k}=-\infty}^{\infty} x[\widetilde{k}]h[n-m-\widetilde{k}]$$

It can be easily seen that $y[n-m] = y_1[n]$. Thus, the system is **Time-Invariant**.

4. The system basically up-samples the system, by adding the average of two consecutive samples between these samples. An example Input/Output pair for the system can be seen at *Figure 1*.

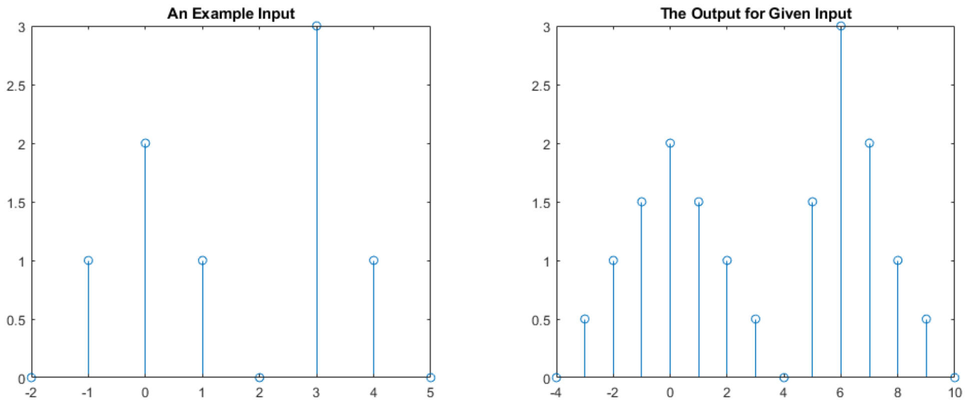

Figure 1: An Example Input/Output Pair

- For linearity, let us check the output $y[n]$ for the input $x[n] = ax_1[n] + bx_2[n]$

$$y[n] = \begin{cases} \dfrac{x[n]}{2} & if \quad n \quad is \quad even \\ \dfrac{x[\frac{n-1}{2}] + x[\frac{n+1}{2}]}{2} & if \quad n \quad is \quad odd \end{cases}$$

$$y[n] = \begin{cases} a\dfrac{x_1[n]}{2} + b\dfrac{x_2[n]}{2} & n \quad is \quad even \\ a\dfrac{x_1[\frac{n-1}{2}] + x_1[\frac{n+1}{2}]}{2} + b\dfrac{x_2[\frac{n-1}{2}] + x_2[\frac{n+1}{2}]}{2} & if \quad n \quad is \quad odd \end{cases}$$

Let us also find $y_1[n]$ and $y_2[n]$ for $x_1[n]$ and $x_2[n]$ respectively,

$$y_1[n] = \begin{cases} \dfrac{x_1[n]}{2} & n \;\; is \;\; even \\ \dfrac{x_1[\frac{n-1}{2}] + x_1[\frac{n+1}{2}]}{2} & if \;\; n \;\; is \;\; odd \end{cases}$$

$$y_2[n] = \begin{cases} \dfrac{x_2[n]}{2} & n \;\; is \;\; even \\ \dfrac{x_2[\frac{n-1}{2}] + x_2[\frac{n+1}{2}]}{2} & if \;\; n \;\; is \;\; odd \end{cases}$$

It can be clearly seen that $y[n] = ay_1[n] + y_2[n]$ for $x[n] = ax_1[n] + bx_2[n]$. Thus, the system is **Linear**.

- For time invariance, let us check $y[n-m]$ and $y_1[n]$ for the $x_1[n] = x[n-m]$

$$y[n-m] = \begin{cases} \dfrac{x[n-m]}{2} & if \;\; (n-m) \;\; is \;\; even \\ \dfrac{x[\frac{n-m-1}{2}] + x[\frac{n-m+1}{2}]}{2} & if \;\; (n-m) \;\; is \;\; odd \end{cases}$$

$$y_1[n] = \begin{cases} \dfrac{x_1[n]}{2} & if \;\; n \;\; is \;\; even \\ \dfrac{x_1[\frac{n-1}{2}] + x_1[\frac{n+1}{2}]}{2} & if \;\; n \;\; is \;\; odd \end{cases}$$

$$y_1[n] = \begin{cases} \dfrac{x[n-m]}{2} & if \;\; n \;\; is \;\; even \\ \dfrac{x[\frac{n-m-1}{2}] + x[\frac{n-m+1}{2}]}{2} & if \;\; n \;\; is \;\; odd \end{cases}$$

Due to condition difference $y[n-m] \neq y_{[}n]$. For different m, the result changes. Thus, the system is **not Time-Invariant**.

5. Let us check stability and causality for the following systems;

   - $$y[n] = 2^{\delta[n+1]} + x[n-3]$$

     The system is **not casual** since the impulse response $h[n] \neq 0$ as $n < 0$;

     $$h[n] = 2^{\delta[n+1]} + \delta[n-3]$$

For BIBO stability, let us assume $|x[n]| < \beta_x < \infty$ and check $|y[n]|$;

$$|y[n]| = |2^{\delta[n+1]} + x[n-3]| = |c + x[n]| = \beta_y < \infty$$

where c and β_y are finite constants, thus, the system is **Stable**.

- $$y[n] = \begin{cases} y[-\delta[n-1]] + x[n-3] & if \quad n > 0 \\ 2^n x[n-3] & if \quad n \leq 0 \end{cases}$$

The system is **Casual** since the impulse response $h[n] = 0$ as $n < 0$;

$$h[n] = \begin{cases} h[-\delta[n-1]] + \delta[n-3] & if \quad n > 0 \\ 2^n \delta[n-3] & if \quad n \leq 0 \end{cases}$$

For BIBO stability, let us assume $|x[n]| < \beta_x < \infty$ and check $|y[n]|$;

$$|y[n]| = \begin{cases} |y[-\delta[n-1]] + x[n-3]| & if \quad n > 0 \\ |2^n x[n-3]| & if \quad n \leq 0 \end{cases}$$

Let us assume $|y[n]| < \beta_z < \infty$. Checking the two conditions, the assumption holds, thus, it can be seen that the system is **Stable**.

6. $$y[n] = x[n] * x[n] = \sum_{k=-\infty}^{\infty} x[k]x[n-k]$$

If the first non-zero element of $x[n]$ is $x[-6] = -3$ and the last non-zero element of $x[n]$ is equal to $x[24] = -4$, the first and last non-zero elements of $y[n]$ will be $y[-12]$ from $(-6 = 6 + n)$ and $y[48]$ from $(24 = -24 + n)$. These values can be calculated from the formula above as,

$$\boxed{y[-12] = x[-6]x[-6] = 9}$$

$$\boxed{y[48] = x[24]x[24] = 16}$$

7. Let us calculate $y[n]$ as $n \to \infty$, given that $x[n] = u[n]$ and $h[n] = 3(\frac{1}{2})^n u[n] + 2(\frac{1}{3})^{n-1} u[n]$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[n] = \sum_{k=-\infty}^{\infty} u[k](3(\frac{1}{2})^{(n-k)} u[n-k] + 2(\frac{1}{3})^{n-k-1} u[n-k])$$

$$y[n] = \sum_{k=0}^{n} 3(\frac{1}{2})^{(n-k)} + 6(\frac{1}{3})^{(n-k)}$$

with simple change of variables, let $m \triangleq n - k$

$$= \sum_{m=n}^{0} 3(\frac{1}{2})^{m} + 6(\frac{1}{3})^{m}$$

or as $n \rightarrow \inf$

$$y[n] = \sum_{m=0}^{\infty} 3(\frac{1}{2})^{m} + 6(\frac{1}{3})^{m}$$

$$\boxed{\lim_{n\rightarrow\infty} y[n] = 3\frac{1}{1-1/2} - 6\frac{1}{1-1/3} = -3}$$

8. Let us analyse the system $y[n] - \frac{1}{2}y[n-1] = x[n] - x[n-1] + x[n-2]$ Assume homogeneous system $y[n] - \frac{1}{2}y[n-1] = 0$ and solve it for finding homogeneous solution $y_h[n]$.

$$y[n] - \frac{1}{2}y[n-1] = 0 \quad , \quad with \quad y_h[n] = Ar^n$$

$$Ar^n - \frac{A}{2}r^{n-1} = 0$$

$$Ar^{n-1}(r - \frac{1}{2}) = 0$$

$$\boxed{r = 1/2}$$

Thus, the homogeneous solution will be in the form of $A(\frac{1}{2})^n$, Let us now find the impulse response of the system $y_1[n] - \frac{1}{2}y_1[n-1] = x[n]$, the homogeneous solution will also satisfy this system and the impulse response will be also in form of $y_h[n]$

$$h_1[n] - \frac{1}{2}h_1[n-1] = \delta[n]$$

We also know that $h_1[n]$ will be in the for $Ar^n u[n]$ since the system is casual. To find A, let us calculate $h_1[n]$ at $n = 0$.

$$h_1[0] = \frac{1}{2}h[-1] + \delta[0] = 1 = A(\frac{1}{2})^{0} = A$$

Thus, we have found the impulse response of the system $y_1[n] - \frac{1}{2}y_1[n-1] = x[n]$ as

$$\boxed{h_1[n] = (\frac{1}{2})^{n} u[n]}$$

(a) Due to lineartity of the system, the impulse response of the system $y[n]-\frac{1}{2}y[n-1]=x[n]-x[n-1]+x[n-2]$ will be superposition of the impulse response $h_1[n]$;

$$h[n]=h_1[n]-h_1[n-1]+h_1[n-2]$$

$$h[n]=\frac{1}{2}^{n}u[n]-\frac{1}{2}^{n-1}u[n-1]+\frac{1}{2}^{n-2}u[n-2]$$

$$h[n]=\frac{1}{2}^{n}(u[n]-2u[n-1]+4u[n-2])$$

$$\boxed{h[n]=\frac{1}{2}(\delta[n]-\delta[n-1]+3u[n-2])}$$

(b) Frequency response $H(e^{jw})$ can be found as;

$$H(e^{jw})=\sum_{n=\infty}^{\infty}h[n]e^{-jwn}$$

$$H(e^{jw})=\sum_{n=\infty}^{\infty}(\frac{1}{2})^{n}e^{-jwn}\delta[n]-\sum_{n=\infty}^{\infty}(\frac{1}{2})^{n}e^{-jwn}\delta[n-1]+3\sum_{n=\infty}^{\infty}(\frac{1}{2})^{n}e^{-jwn}u[n-2]$$

$$H(e^{jw})=1-\frac{1}{2}e^{jw}+3\sum_{n=2}^{\infty}(\frac{e^{-jw}}{2})^{n}$$

$$H(e^{jw})=-2-2e^{-jw}+\frac{6}{2-e^{-jw}}\quad if\quad |\frac{e^{-jw}}{2}|<1$$

$$\boxed{H(e^{jw})=-2-2e^{-jw}+\frac{6}{2-e^{-jw}}\quad if\quad \pi<w<\pi}$$

(c) To use freqz command, we have to compete the Z-transform of the system,

$$Y(z)-\frac{1}{2}Y(z)z^{-1}=X(z)-X(z)z^{-1}+X(z)z^{-2}$$

$$\boxed{H(z)\triangleq\frac{Y(z)}{X(z)}=\frac{1-z^{-1}+z^{-2}}{1-\frac{1}{2}z^{-1}}}$$

The result can be seen at *Figure 2*.

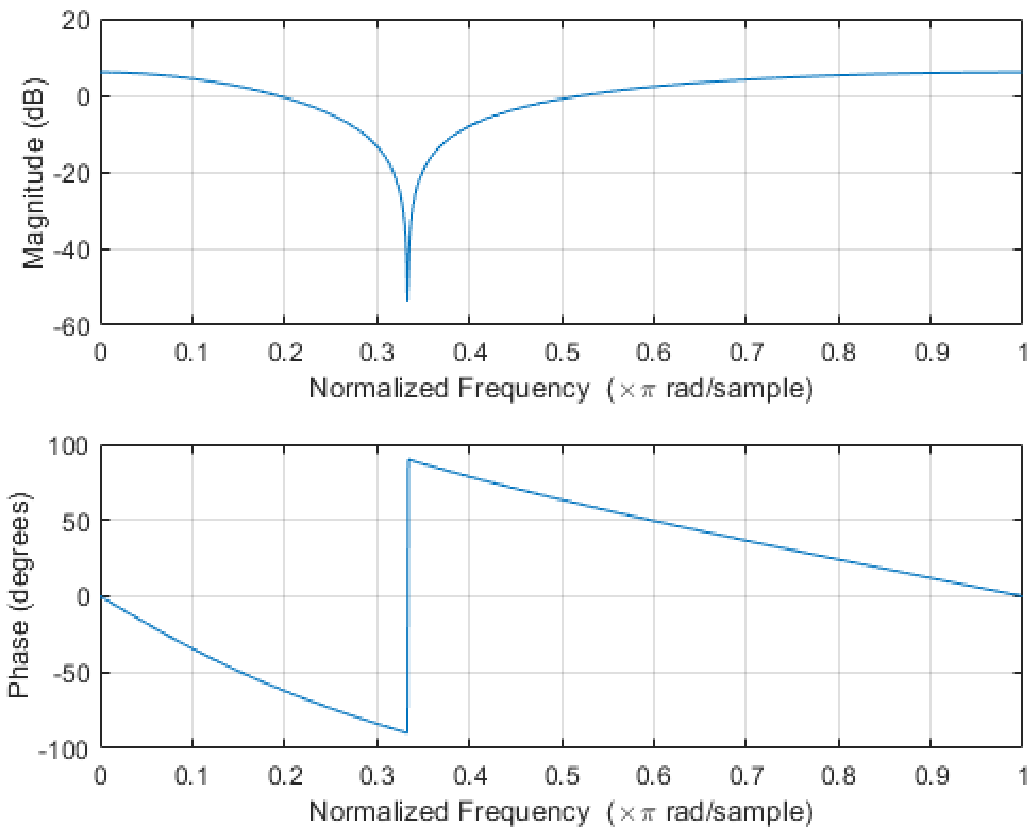

Figure 2: Magnitude and Phase of Frequency Response

```
a=[1 -1 1];
b=[1 -1/2];
freqz(a,b,1024)
```

(d) Since we have $H(e^{jw})$, we can find $y[n]$ from

$$Y(e^{jw}) = X(e^{jw})H(e^{jw})$$

instead of convolution, for that we need to find $X(e^{jw})$ first. Given that $x[n] = \cos(\frac{\pi}{3}n) + \sin(\frac{\pi}{2}n + \frac{\pi}{4})$

$$x[n] = \cos(\frac{\pi}{3}n) + \sin(\frac{\pi}{2}n)\cos(\frac{\pi}{4}) + \cos(\frac{\pi}{2}n)\sin(\frac{\pi}{4}))$$

$$x[n] = \cos(\frac{\pi}{3}n) + \sin(\frac{\pi}{2}n)\frac{1}{\sqrt{2}} + \cos(\frac{\pi}{2}n)\frac{1}{\sqrt{2}})$$

$$X(e^{jw}) = \pi\left[\delta[w-\frac{\pi}{3}] + \delta[w+\frac{\pi}{3}]\right] - \frac{j\pi}{\sqrt{2}}\left[\delta[w-\frac{\pi}{2}] - \delta[w+\frac{\pi}{2}]\right]$$
$$+\frac{\pi}{\sqrt{2}}\left[\delta[w-\frac{\pi}{2}] + \delta[w+\frac{\pi}{2}]\right]$$

$$X(e^{jw}) = \pi\left[\delta[w-\frac{\pi}{3}] + \delta[w+\frac{\pi}{3}]\right] + \frac{\pi}{\sqrt{2}}(1-j)\left[\delta[w-\frac{\pi}{2}]\right]$$
$$+\frac{\pi}{\sqrt{2}}(1+j)\left[\delta[w+\frac{\pi}{2}]\right]$$

$$Y(e^{jw}) = X(e^{jw})H(e^{jw})$$

$$Y(e^{jw}) = \pi\left[-2-2e^{-j\pi/3} + \frac{6}{2-e^{-j\pi/3}}\right]\delta[w-\frac{\pi}{3}]$$
$$+\pi\left[-2-2e^{-j\pi/3} + \frac{6}{2-e^{-j\pi/3}}\right]\delta[w+\frac{\pi}{3}]$$
$$+\frac{\pi}{\sqrt{2}}(1-j)\left[-2-2e^{-j\pi/2} + \frac{6}{2-e^{-j\pi/2}}\right]\delta[w-\frac{\pi}{2}]$$
$$+\frac{\pi}{\sqrt{2}}(1+j)\left[-2-2e^{j\pi/2} + \frac{6}{2-e^{j\pi/2}}\right]\delta[w+\frac{\pi}{2}]$$

by some computation, the term above becomes

$$\boxed{Y(e^{jw}) = \frac{\pi}{\sqrt{2}}\left[\frac{6}{5} + j\frac{2}{5}\right]\delta[w-\frac{\pi}{2}] + \frac{\pi}{\sqrt{2}}\left[\frac{6}{5} - j\frac{2}{5}\right]\delta[w+\frac{\pi}{2}]}$$

the $y[n]$ can be found to be as

$$y[n] = \frac{\pi}{\sqrt{2}}\left[\frac{6}{5} + j\frac{2}{5}\right]\frac{1}{2\pi}e^{j\frac{\pi}{2}n} + \frac{\pi}{\sqrt{2}}\left[\frac{6}{5} - j\frac{2}{5}\right]\frac{1}{2\pi}e^{-j\frac{\pi}{2}n}$$

by also some computation, the term above becomes

$$\boxed{y[n] = \frac{1}{\sqrt{2}}\left[\frac{3}{5} + j\frac{1}{5}\right]e^{j\frac{\pi}{2}n} + \frac{1}{\sqrt{2}}\left[\frac{3}{5} - j\frac{1}{5}\right]e^{-j\frac{\pi}{2}n}}$$

(e) Let us analyse $H(e^{jw})$ found,

$$H(e^{jw}) = -2 - 2e^{-jw} + \frac{6}{2-e^{-jw}}$$

$$H^*(e^{j(2\pi-w)}) = \left(-2 - 2e^{-j(2\pi-w)} + \frac{6}{2 - e^{-j(2\pi-w)}}\right)^*$$

$$H^*(e^{j(2\pi-w)}) = \left(-2 - 2e^{-j2\pi}e^{jw)} + \frac{6}{2 - e^{-j2\pi}e^{jw)}}\right)^*$$

$$H^*(e^{j(2\pi-w)}) = \left(-2 - 2e^{jw)} + \frac{6}{2 - 2e^{jw)}}\right)^*$$

$$\boxed{H^*(e^{j(2\pi-w)}) = -2 - 2e^{-jw)} + \frac{6}{2 - 2e^{-jw)}}}$$

which obliviously equal to the $H(e^{jw})$ as asked in the question.

9. (a) Since $x[n]$ is a real sequence, magnitude of its frequency response must be even symmetric and phase plot of its frequency response must be odd symmetric.

   (b) With given $x[n]$, $x_c[n]$ and $x_s[n]$, the DTFTs can be found by convolution in the frequency domain,

$$x_c[n] = \cos(\frac{\pi}{5}n)x[n]$$

$$X_c(e^{jw}) = X(e^{jw}) * \mathcal{F}\{\cos(\frac{\pi}{5}n)\}$$

$$\mathcal{F}\{\cos(\frac{\pi}{5}n)\} = \pi\left[\delta[w - \frac{\pi}{5}] + \delta[w + \frac{\pi}{5}]\right]$$

$$\boxed{X_c(e^{jw}) = \pi\left[X(e^{j(w+\pi/5)}) + X(e^{j(w-\pi/5)})\right]}$$

$$x_s[n] = \sin(\frac{\pi}{5}n)x[n]$$

$$X_s(e^{jw}) = X(e^{jw}) * \mathcal{F}\{\sin(\frac{\pi}{5}n)\}$$

$$\mathcal{F}\{\sin(\frac{\pi}{5}n)\} = -j\pi\left[\delta[w - \frac{\pi}{5}] - \delta[w + \frac{\pi}{5}]\right]$$

$$\boxed{X_s(e^{jw}) = -j\pi\left[X(e^{j(w+\pi/5)}) - X(e^{j(w-\pi/5)})\right]}$$

   (c)

10. It is known that, the DTFT of $x[n]$ can be calculated as follows;

$$X(e^{jw}) = \sum_{-\infty}^{\infty} x[n]e^{-jwn}$$

   (a)

$$\boxed{X(e^{jw})|_{w=0} = \sum_{-\infty}^{\infty} x[n] = 6}$$

(b)

$$X(e^{jw})|_{w=\pi} = \sum_{-\infty}^{\infty} x[n]e^{-j\pi n}$$

$$\boxed{X(e^{jw})|_{w=\pi} = \sum_{-\infty}^{\infty} x[n](-1)^n = 2}$$

(c) Since x[n] is symmetric about n=2, the signal has linear phase

$$X(e^{jw}) = A(w)e^{-j2w}$$

$A(w)$ is a zero phase(real) function of w. Thus,

$$\underline{/X(e^{jw})} = -2w \quad , \quad -\pi \leq w \leq \pi$$

(d) Knowing that $\int_{\infty}^{\infty} X(e^{jw}e^{-jwn})dw = 2\pi x[n]$, for $n = 0$, equations becomes what we desired

$$\boxed{\int_{\infty}^{\infty} X(e^{jw})dw = 2\pi x[0] = 4}$$

(e) Assume $x_1[n]$ whose DTFT is $X(e^{-jw})$.

$$X(e^{jw}) = \sum_{-\infty}^{\infty} x[n]e^{jwn} = \sum_{-\infty}^{\infty} x[-n]e^{-jwn} = \sum_{-\infty}^{\infty} x_1[n]e^{-jwn}$$

Thus, it can be seen that,

$$\boxed{x_1[n] = x[-n]}$$

$x[-n]$ can be seen from *Figure 3*.

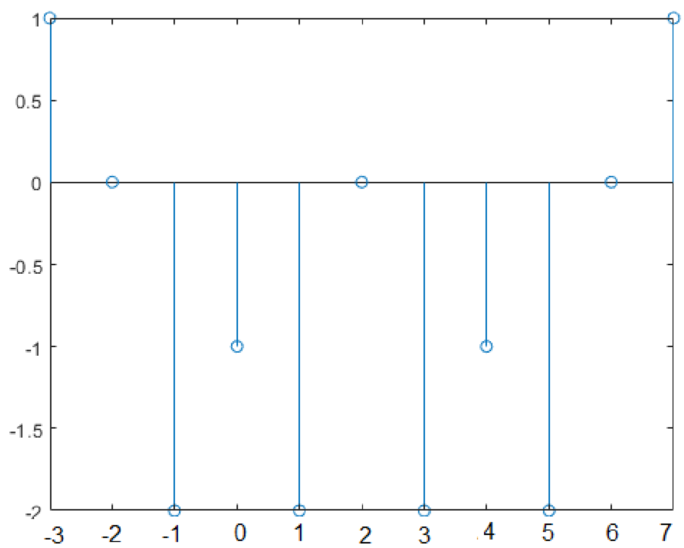

Figure 3: $x[-n]$

(f) Remembering $X(e^{jw}) = A(w)e^{-j2w}$, real part of $X(e^{jw})$ can be written as

$$X_R(e^{jw}) = A(w)\cos(2w)$$

$$X_R(e^{jw}) = \frac{1}{2}A(w)\left(e^{j2w} + e^{-j2w}\right)$$

$$x_R[n] = \mathcal{F}^{-1}\{X_R(e^{jw})\}$$

$$x_R[n] = \frac{1}{2}a[n+2] + \frac{1}{2}a[n-2]$$

$$\boxed{x_R[n] = \frac{1}{2}x[n+4] + \frac{1}{2}x[n]}$$

$x_R[n]$ can be seen from *Figure 4*.

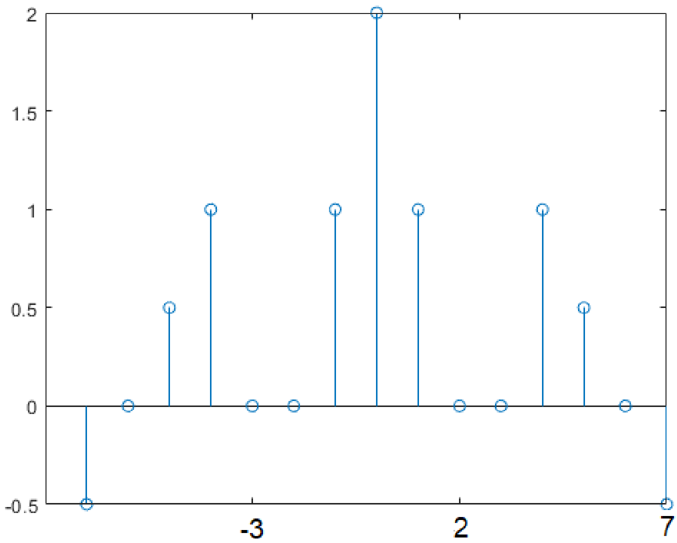

Figure 4: $x_R[n]$

11. (a) From the given informations

- $h[n] = 0$ for $n < 0$, causality,
- $h[n]$ is real, FT is conjugate symmetric,
- $h[n + 1$ is even, real FT

From above, , it can be understood that, $h[n]$ is 3-length long, thus, it is finite-duration.

(b) Notice that, $h[n]$ has three elements and symmetric, so, let us assume $h[0] = h[2] = a$ and $h[1] = b$.

From extra informations

- $2a^2 + b^2 = 2$, Parseval's Theorem
- $2a - b = 0$

Solving the equations found above, $\boxed{a = \frac{\pm 1}{\sqrt{3}}}$ and $\boxed{b = \frac{\pm 2}{\sqrt{3}}}$.

$$\boxed{h[0] = h[2] = \frac{\pm 1}{\sqrt{3}} \quad , \quad h[1] = \frac{\pm 2}{\sqrt{3}}}$$

12. It can be observed from the question that $X(e^{jw})$ is real and

$$Y(e^{jw}) = \begin{cases} -jX(e^{jw}) & , \quad 0 < w < \pi \\ +jX(e^{jw}) & , \quad -\pi < w < 0 \end{cases}$$

It is also given that $w[n] = x[n] + jy[n]$, thus,

$$W(e^{jw}) = +jY(e^{jw})$$

$$\boxed{W(e^{jw}) = \begin{cases} 2X(e^{jw}) \quad , \quad 0 < w < \pi \\ 0 \quad , \quad -\pi < w < 0 \end{cases}}$$

$W(e^{jw})$ can be seen from *Figure 3.*

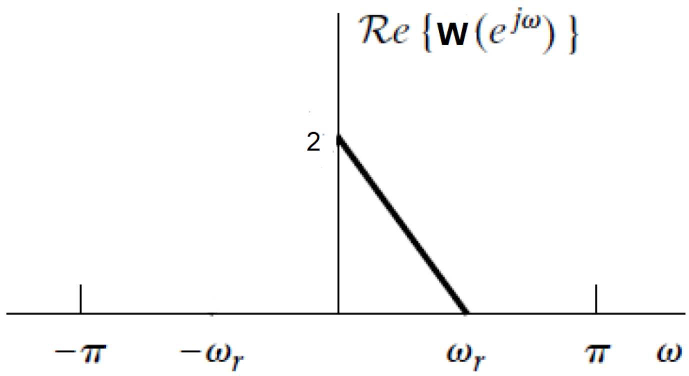

Figure 5: $W(e^{jw})$

13. [MATLAB]

(a)

```matlab
function y  = myconv (x,h)
    i=1;
    for i=1:10
        y(i)=0;
        for k=1:numel(x)
            if (i+1-k)<=0
                y(i)=y(i)+(x(k)*0);
            else
                if (i+1-k)>numel(h)
                    y(i)=y(i)+(x(k)*0);
                else
                    y(i)=y(i)+(x(k)*h(i+1-k));
                    k=k+1;
                end
            end
        end
        i=i+1;
    end
end
```

(b) The result of 'conv' function can be seen at *Figure 6*.

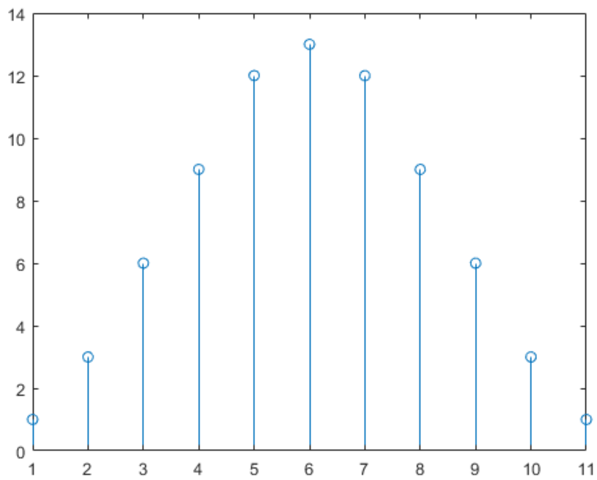

Figure 6: The result of 'conv' function

```
x = [1:5 4:-1:1]
h = [1 1 1]

y=myconv(x,h)

y2=conv(x,h)

stem(y2)
```

(c) Magnitude and phase response of the filter $h[n]$ can be seen at *Figure 7.*

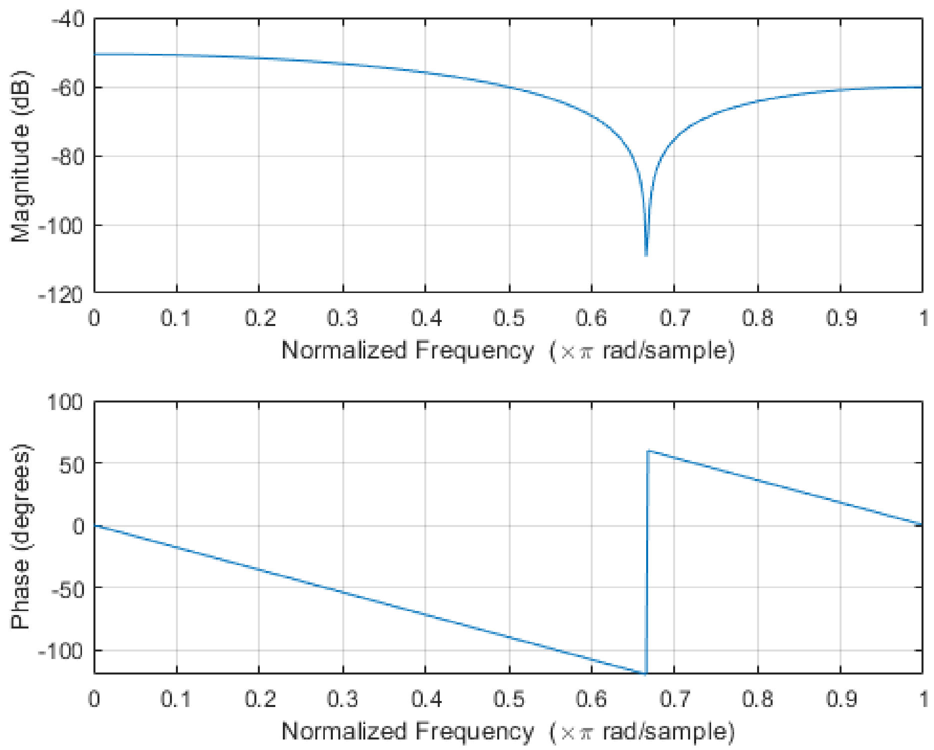

Figure 7: Magnitude and phase response of the filter $h[n]$

```
freqz(h,1024)
```

(d) The result for the $h_2[n] * x[n]$ can be seen at *Figure 8*.

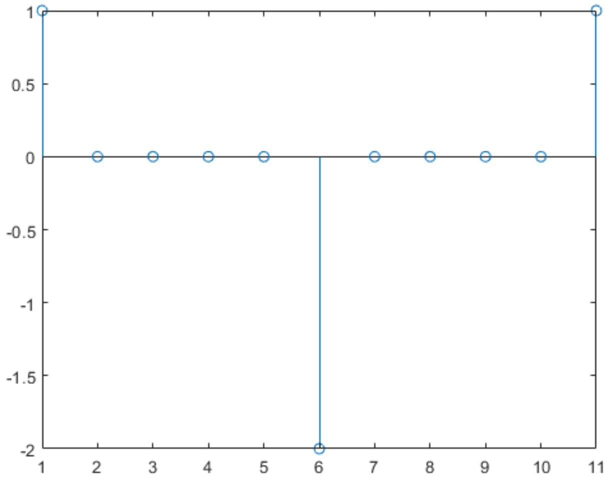

Figure 8: $h_2[n] * x[n]$

```
h2=[1 -2 1]

y3=conv(x,h2)

stem(y3)
```

(e) Magnitude and phase response of the filter $h_2[n]$ can be seen at *Figure 9.*

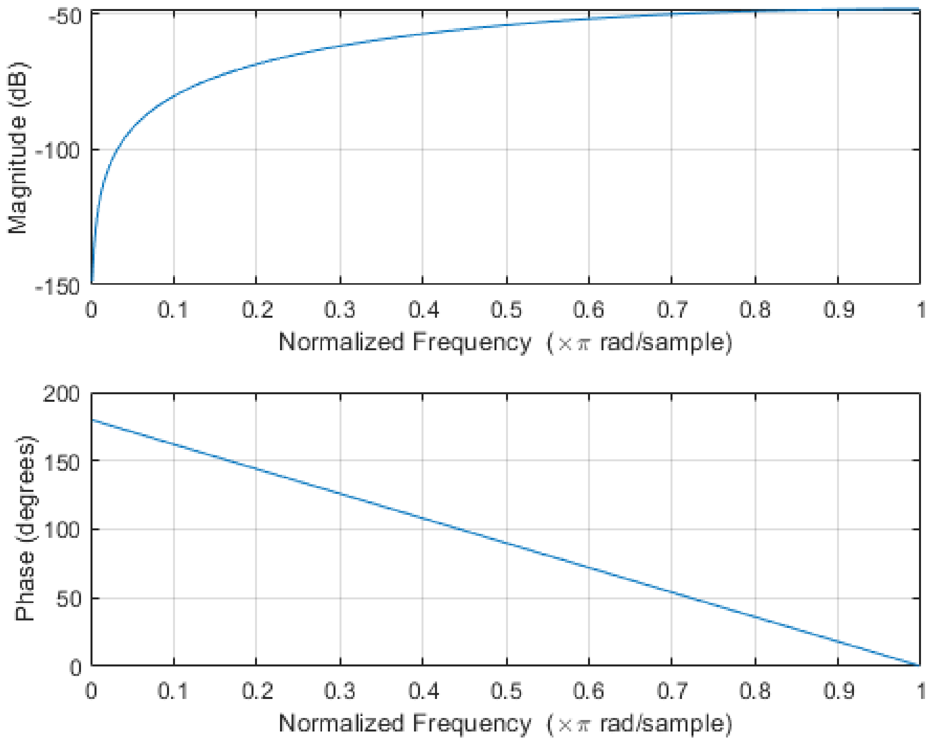

Figure 9: Magnitude and phase response of the filter $h_2[n]$

```
freqz(h2,1024)
```

(f) Magnitude and phase response of $z[n]$ can be seen at *Figure 11.*

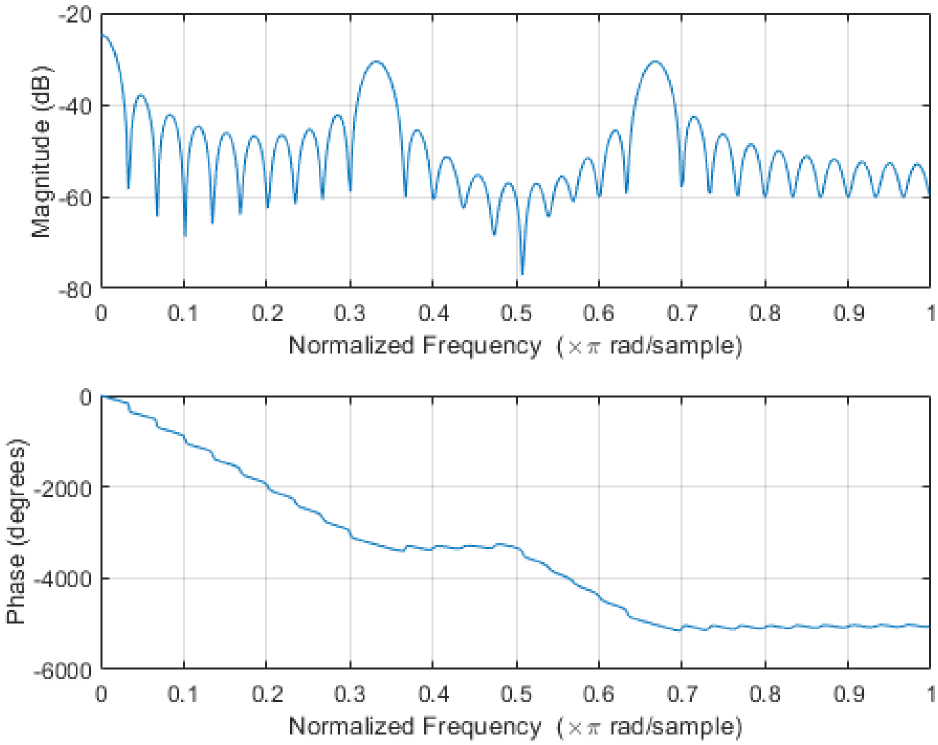

Figure 10: Magnitude and phase response of the $z[n]$

```
i=1;
z(1)=0;
while i<60
    if i < 1
        z(i)=0;
    else
        z(i)=1+sin(pi/3*i)+sin(2*pi/3*i)
    end
    i=i+1;
end

freqz(z,1024)
```

(g) Magnitude and phase response of $y[n] = z[n] * h[n]$ can be seen at *Figure 11.*

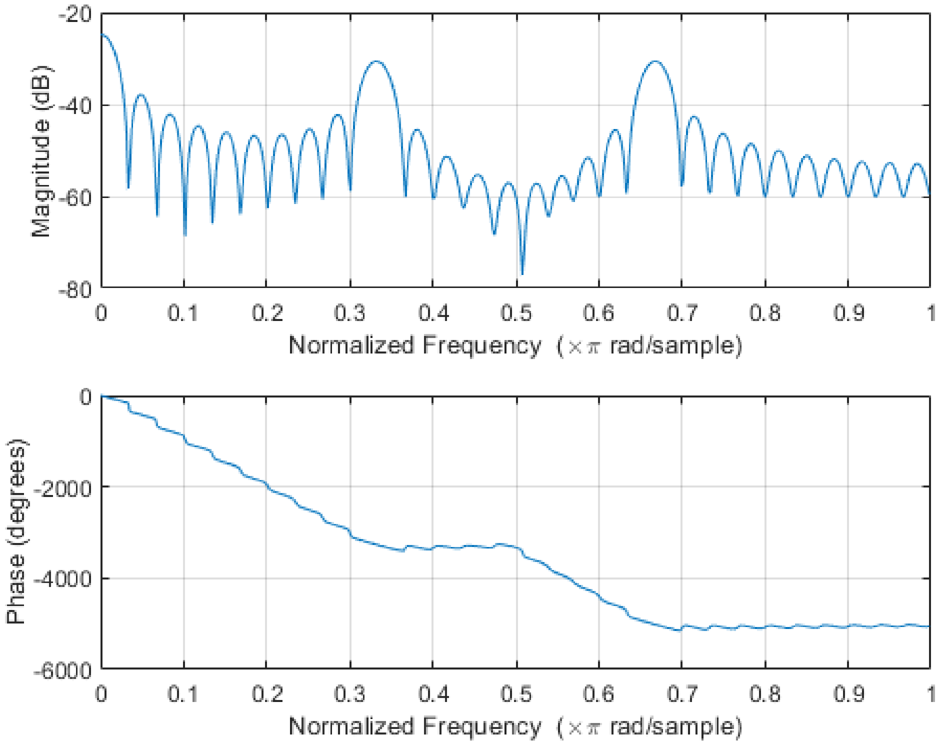

Figure 11: Magnitude and phase response of the $y[n]$

```
y=conv(z,h)

freqz(y,1024)
```