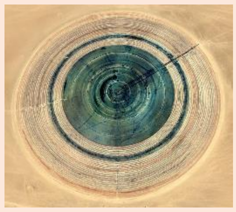
Middle East Technical University
Dept. of EEE

HW 5

EE 430 Digital Signal Processing – 2014 Fall



Due: Dec. 22, 2014

Submit for problems 0, 2, 5, 7, 11, 13, 15, 23, 25, 29

0) A photograph of what is in the title?

Transform Analysis of LTI Systems

1) The system function, $H(z)$, of a LTI system has a zero at $z = e^{j\theta}$. Describe, verbally and graphically, how the magnitude and phase functions behave around $\omega = \theta$ rads.

2) Can a LTI system have identical phase delay and group delay functions? Yes or no, explain your claim. Give examples.

3)

a) Plot the phase delay and group delay functions of a first order allpass system with a pole at $0.99e^{j\frac{\pi}{4}}$.

b) The following signal is applied to the allpass system.

$$x[n] = \left(1 - \cos\left(\frac{\pi}{30}n\right)\right)\cos\left(\frac{\pi}{4}n\right)(u[n] - u[n-60]) + \left(1 - \cos\left(\frac{\pi}{30}(n-60)\right)\right)\cos\left(\frac{19\pi}{20}n\right)(u[n-60] - u[n-120])\,.$$

Can you write the output signal approximately?

c) Find the output using MATLAB and compare to your answer in part (b).

4) Let $H(z) = 1 - z^{-9}$.

a) Find the impulse response of this system.

b) Draw the pole zero diagram.

c) Plot the magnitude, phase, phase delay and group delay functions.

d) Is this system invertible?

5) Plot the magnitude, phase, phase delay and group delay functions of the following systems roughly using zero and pole vectors.

$$H_1(z) = \frac{\left(1 - 0.95e^{j\frac{\pi}{4}}z^{-1}\right)\left(1 - 0.95e^{-j\frac{\pi}{4}}z^{-1}\right)}{\left(1 - 0.95e^{j\left(\frac{\pi}{4}+\frac{\pi}{20}\right)}z^{-1}\right)\left(1 - 0.95e^{-j\left(\frac{\pi}{4}+\frac{\pi}{20}\right)}z^{-1}\right)}$$

$$H_2(z) = \frac{\left(1 - 0.95e^{j\frac{\pi}{4}}z^{-1}\right)\left(1 - 0.95e^{-j\frac{\pi}{4}}z^{-1}\right)}{\left(1 - 0.95e^{j\left(\frac{\pi}{4}-\frac{\pi}{20}\right)}z^{-1}\right)\left(1 - 0.95e^{-j\left(\frac{\pi}{4}-\frac{\pi}{20}\right)}z^{-1}\right)}$$

6) Consider the first order allpass system function,

$$\frac{z^{-1} - \frac{2}{3}}{1 - \frac{2}{3}z^{-1}} \qquad (*)$$

a) Draw the pole-zero diagram.
b) Show the pole and zero vectors and their angles for $\omega = 0$ and $\omega = \frac{\pi}{4}$.
c) Find the phase of the frequency response at $\omega = 0$ using the pole-zero vectors.
d) Find the phase of the frequency response at $\omega = 0$ using $(*)$. Does it agree with the answer of part-(c) ?

7) Plot the phase functions of $1 - 0.9z^{-1}$ and $1 - \frac{10}{9}z^{-1}$, and describe their difference.

8) Plot the phase functions of $\frac{1}{1-0.9z^{-1}}$ and $\frac{1}{1-\frac{10}{9}z^{-1}}$, and describe their difference.

9) Are allpass systems invertible?

10) Are minimum phase systems invertible?

11) Can you make a minimum-phase – allpass decomposition for $(z) = \frac{(1-2z^{-1})(z+2)\left(z-\frac{1}{2}\right)}{1-\frac{1}{2}z^{-1}+\frac{1}{2}z^{-2}}$? If yes, find the minimum-phase and allpass functions.

12) Can you make a minimum-phase – allpass decomposition for $(z) = \frac{(1-2z^{-1})(z+2)}{\left(1-\frac{1}{2}z^{-1}+\frac{1}{2}z^{-2}\right)\left(z+\frac{1}{2}\right)}$? If yes, find the minimum-phase and allpass functions.

13) Define phase-lag and phase-delay functions.

14) Let

$$H(z) = \frac{\sum_{k=1}^{5}(1 - z_k\, z^{-1})}{\sum_{k=1}^{4}(1 - p_k\, z^{-1})}$$

$$z_1 = z_2^* = 0.5e^{j\frac{3\pi}{4}} \qquad z_3 = z_4^* = 4e^{j\frac{\pi}{4}} \qquad z_3 = 3 \qquad p_1 = p_2^* = 0.8e^{j\frac{\pi}{2}} \qquad p_3 = p_4^* = 2e^{j\frac{2\pi}{3}}$$

a) Plot the pole-zero diagram of $H(z)$.
b) Find the system functions of other systems that have the same magnitude response. Write their system functions. Plot their pole-zero diagrams.
c) Which of them have stable-causal realizations? Which of them have real impulse responses?

15) Let

$$H(z) = \frac{(1 - z_1\, z^{-1})(1 - z_1^*\, z^{-1})}{(1 - p_1\, z^{-1})(1 - p_1^*\, z^{-1})}$$

$$z_1 = 1.2e^{j\frac{3\pi}{4}} \qquad p_1 = 0.5e^{j\frac{\pi}{2}}$$

a) Find the impulse response, $h[n]$.
b) Find the impulse response of the minimum phase “counterpart”, $h_{min}[n]$.
c) Plot the phase-lag functions of $H(z)$ and $H_{min}(z)$, the minimum-phase system function associated with $H(z)$.
d) Plot the group-delay functions related to $H(z)$ and $H_{min}(z)$.
e) Verify the “minimum phase-lag” property of minimum-phase systems.
f) Verify the “minimum group-delay” property of minimum-phase systems.
g) Verify the “minimum energy-delay” property of minimum-phase systems.

16) What is a linear phase system? What is a generalized linear-phase system? Clarify their difference.

17) May a linear-phase system have an arbitrary impulse response? May a generalized linear-phase system have an arbitrary impulse response? Explain.

Filter Design

18) Suppose you have a set of specifications to design a discrete-time filter by transforming a continuous-time filter. May it be the case that one of the transformations (impulse invariance, bilinear) yields a lower order design? Explain your answer.

19) Regarding filter design by transformation, convince yourself that the specific value of sampling period is not important when the design specifications are given in discrete-time. You may refer to problem 7.2. of the textbook.

20) May different continuous-time filters yield the same discrete-time filter when transformed by impulse invariance method? If NO, why? If YES, give an example.

21) Solve problems 7.7., 7.13., 7.14. of the textbook.

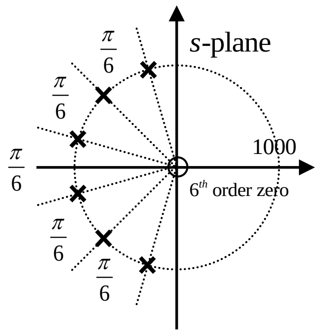
22) Solve problem 7.15. of the textbook.

23) Can an arbitrary function, $s = f(z)$, be used as a variable transformation to transform a continuous-time filter to a discrete-time one?

24) Design two Butterworth filters, one by impulse invariance and the other by bilinear transformation, to meet the specifications given in problem 7.15 of the textbook. First convert the specifications in accordance with the comments in problem7.3.

25) Does one obtain a discrete-time allpass filter if she/he transforms continuous-time allpass filter by bilinear transformation?

26) Given the pole-zero diagram of a continuous-time LTI lowpass filter.



The passband and stopband of the filter are defined as the intervals in which $0.953463 < |H(j\Omega)| < 1$ and $|H(j\Omega)| < 0.1$, respectively.

a) Specify the type of this filter and find its relevant parameters.
b) Find the passband and stopband edge frequencies (Ω_p and Ω_s, respectively) of the filter.
c) It is required to obtain a discrete-time filter using the continuous-time filter given above. The passband edge frequency of the discrete-time filter is desired to be $\frac{\pi}{3}$.
   i- Find the sampling rate, *T*, and write the bilinear transformation expression that relates *s* to *z*. Find the stopband edge frequency of the discrete-time filter. (You are not required to finalize the arithmetic expressions at a single number.)
   ii- Find the sampling rate, *T*, and find the poles of the discrete-time filter that would be obtained by using the impulse invariance method.

$$\frac{1}{(0.953463)^2} \cong 1.1 \qquad \frac{1}{(0.953463)^4} \cong 1.21 \qquad \frac{1}{(0.953463)^{12}} \cong 1.7716$$

$$\sqrt[12]{0.7716} \cong 0.9786 \qquad \sqrt[4]{0.7716} \cong 0.9372 \qquad \sqrt[2]{0.7716} \cong 0.8784$$

$$\sqrt[12]{0.21} \cong 0.8780 \qquad \sqrt[4]{0.21} \cong 0.6769 \qquad \sqrt[2]{0.21} \cong 0.4583$$

$\sqrt[12]{0.1} \cong 0.8254$ $\sqrt[4]{0.1} \cong 0.5623$ $\sqrt[2]{0.1} \cong 0.3162$

$\sqrt[12]{99} \cong 1.4666$ $\sqrt[4]{99} \cong 3.1543$ $\sqrt[2]{99} \cong 9.9499$

$\sqrt[12]{9999} \cong 2.1544$ $\sqrt[4]{9999} \cong 10$ $\sqrt[2]{9999} \cong 100$

$\sqrt[12]{10^{12}-1} \cong 10$ $\sqrt[4]{10^{12}-1} \cong 10^3$ $\sqrt[2]{10^{12}-1} \cong 10^6$

27)

a) Design a continuous-time Butterworth filter (causal and stable) satisfying the following specifications

i. $|H(j\Omega)|^2 \geq 0.92$ for $0 \leq \Omega \leq \frac{\pi}{3}$

ii. $|H(j\Omega)|^2 \leq 0.151$ for $\frac{2\pi}{3} \leq \Omega$

Determine Ω_c (cut-off) frequency and N (order) of the filter. Find $H(s)$.

b) What are the poles of the discrete-time filter obtained by the impulse invariance method?

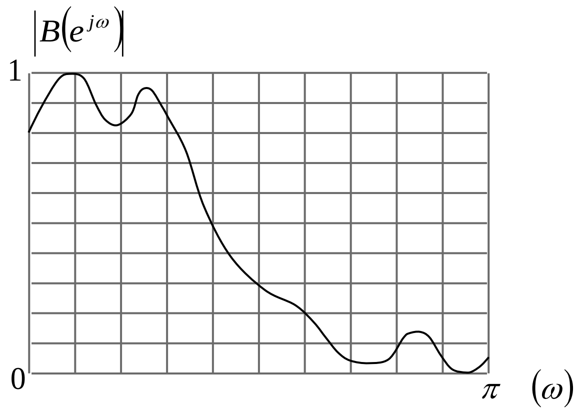
28) Consider a CT filter with the transfer function, $H(s) = \frac{1}{s+\frac{2000}{\sqrt{3}}}$. Using bilinear transformation, this filter is transformed into a DT filter that works at a sampling rate of 1000 Hz. Find the DT frequency at which the magnitude of the DT filter drops to half (½) of its maximum value.

29)

a- Let the curve in the figure be the frequency magnitude response, $|B(e^{j\omega})|$, of a "poorly designed" lowpass filter for the given passband and stopband edge frequencies $\frac{3\pi}{10}$ rad. and $\frac{6\pi}{10}$ rad., respectively. It is known that this bad design satisfies the given specifications exactly. What are the given specifications ($|H(e^{j\omega})|$) of this design? i.e. find the values of A, B, C and D below.

$$A \leq |H(e^{j\omega})| \leq B \qquad 0 \leq \omega \leq \frac{3\pi}{10}$$

$$C \leq |H(e^{j\omega})| \leq D \qquad \frac{6\pi}{10} \leq \omega \leq \pi$$



b- Suppose that you are required to design a "better" discrete-time filter based on a Butterworth template having the specifications you found in part-a. Impulse invariance technique will be used in this design task.

i- Find the specifications, all other relevant parameters and pole locations of the corresponding continuous-time Butterworth filter.

ii- Describe how to transform the continuous-time filter to its discrete-time counterpart using the impulse invariance method.

c- Find the specifications of the continuous-time Butterworth filter in case you use bilinear transformation instead of the impulse invariance method.