What happens if you pick N uniformly spaced samples of the DTFT, $X(e^{j\omega})$, of an arbitrary sequence $x[n]$ as

$$X[k] = X(e^{j\omega})\Big|_{\omega=k\frac{2\pi}{N}} \quad k = 0, \dots, N-1$$

and obtain a new sequence $\hat{x}[n]$ according to

$$\hat{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{jk\frac{2\pi}{N}n} \ ?$$

i.e. how are $\hat{x}[n]$ and $x[n]$ are related to each other?

Answer formally.

Express your answer verbally.

Give an example for a pair $x[n]$ and $\hat{x}[n]$.