Date: 27.12.2016 Time: 15:40 Duration: **110 minutes** Attempt all questions Closed books and notes

Middle East Technical University
Electrical-Electronics Engineering Department

EE 430 Digital Signal Processing

Midterm Examination III

CLOSED BOOKS
110 MINUTES

SOLUTIONS

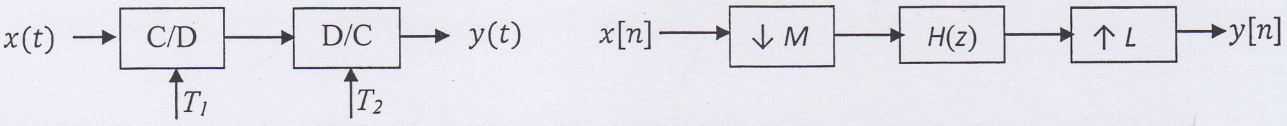
LASTNAME	
NAME	
STUDENT ID:	

Question	Grade
Q1 (25pts)	
Q2 (25pts)	
Q3 (25pts)	
Q4 (25pts)	
TOTAL	

Warning: Plagiarism is defined as the action of using or copying someone else's idea or work and pretending that you thought of it, or created it. Cheating is defined as lying or behaving dishonestly in order to reach your goal. In grading the exam papers in this course, occurrences of plagiarism and cheating will be seriously dealt with, leading to punishment through disciplinary procedures as indicated in University Catalog.

I have read and fully understood the warning, and I pledge to comply with the exam rules. SIGNATURE :

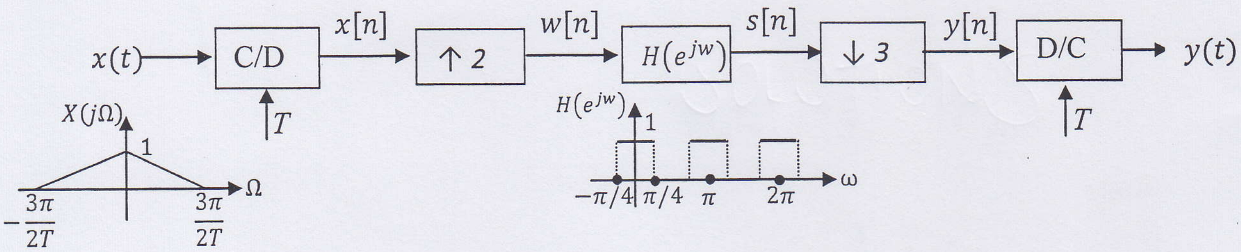
Q1) Consider the following systems. Assume there is no aliasing at the input.



a) Explain the relation between T_1 and T_2 such that the overall system between $x(t)$ and $y(t)$ is
(i) linear (ii) time-invariant

Explain the relation between L and M such that the overall system between $x[n]$ and $y[n]$ is
(i) linear (ii) time-invariant

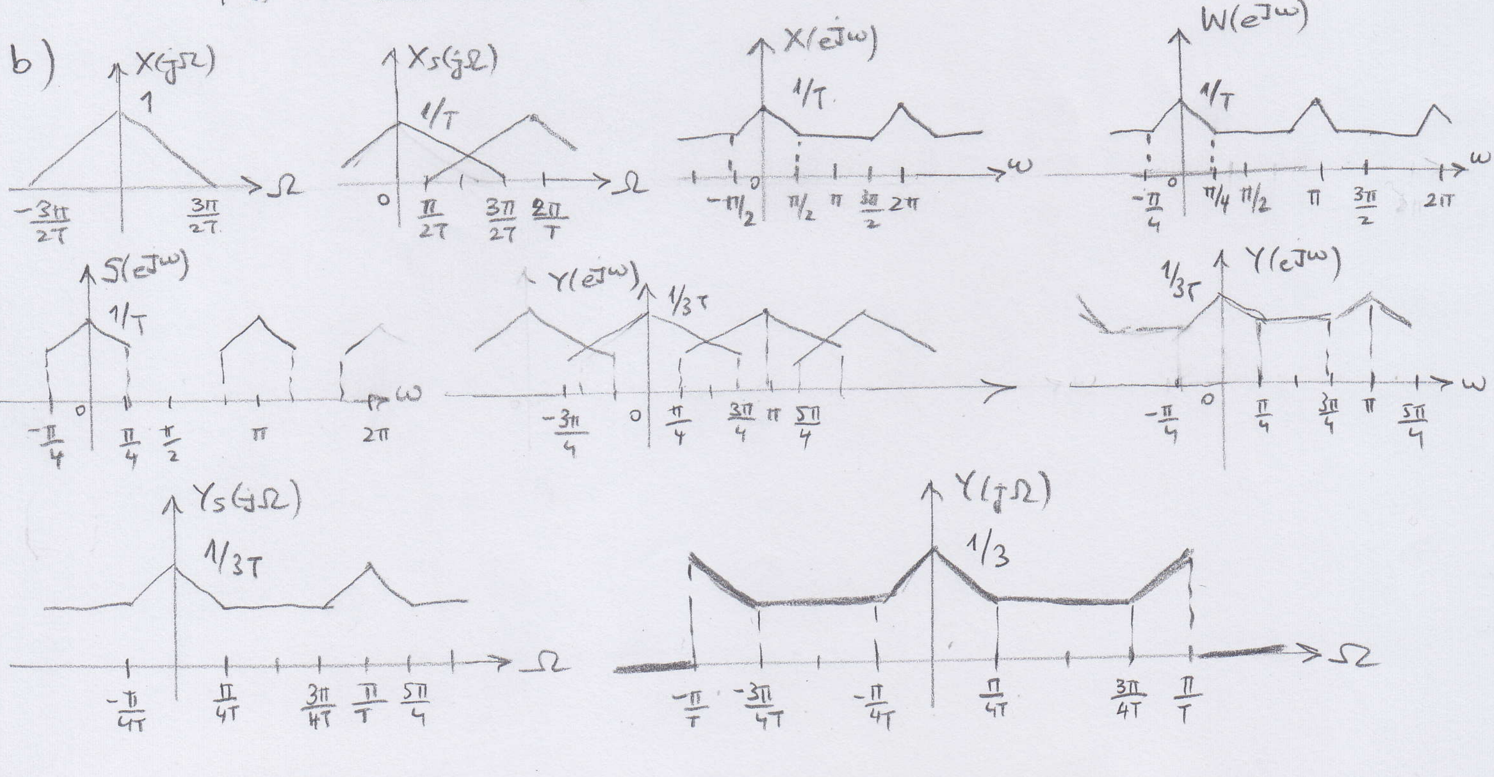
b) For the following system, if $X(j\Omega)$ and $H(e^{jw})$ are given as below, <u>plot in detail</u> $X(e^{jw})$, $W(e^{jw})$, $S(e^{jw})$, $Y(e^{jw})$ and $Y(j\Omega)$.



a) (i) Linearity is satisfied independent of T_1 & T_2. C/D & D/C blocks are linear.
(ii) If $T_1 \neq T_2$, shifts at the input & output will be effected differently.
For time-invariance $\Rightarrow T_1 = T_2$.

(i) Linearity is satisfied independent of L & M. C/D & D/C blocks are linear.
(ii) If $L \neq M$, shifts at the input & output will be effected differently.
For time-invariance $\Rightarrow L = M$.

b)



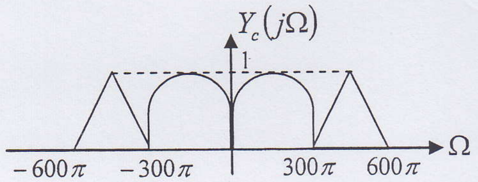
Q2) Consider a continuous-time signal $y_c(t)$ whose CTFT, $Y_c(j\Omega)$, is shown below. It is desired to process $y_c(t)$ by a discrete-time (DT) system and obtain another continuous-time signal, $w(t)$, whose CTFT, $W(j\Omega)$, is also given below.

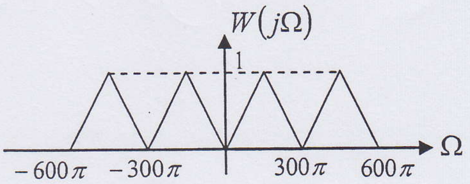
In order to obtain the desired system, only the following components are provided:

- a single pair of ideal C/D and D/C converter (both operate only at the sampling rates of 600 samples/sec.)
- a single downsampler/compressor (to half rate; i.e. $\downarrow 2$)
- a single upsampler/expander (to an unknown rate)
- a single ideal DT filter.

a) Considering the given components and CTFT plots, comment on the upsampling rate for the upsampler/expander. Since the DT filter has <u>constant gain for its passbands</u>, comment on how to convert the input spectra. $Y_c(j\Omega)$, to the desired result, $W(j\Omega)$.

b) Design a system that results with $W(j\Omega)$ from the input $Y_c(j\Omega)$ by only using the component above. Plot its block diagram, indicating all relevant parameters (filter gain, cutoff frequencies, upsampling rate, etc.). <u>Plot</u> the spectra of the signals after every component in the block diagram.

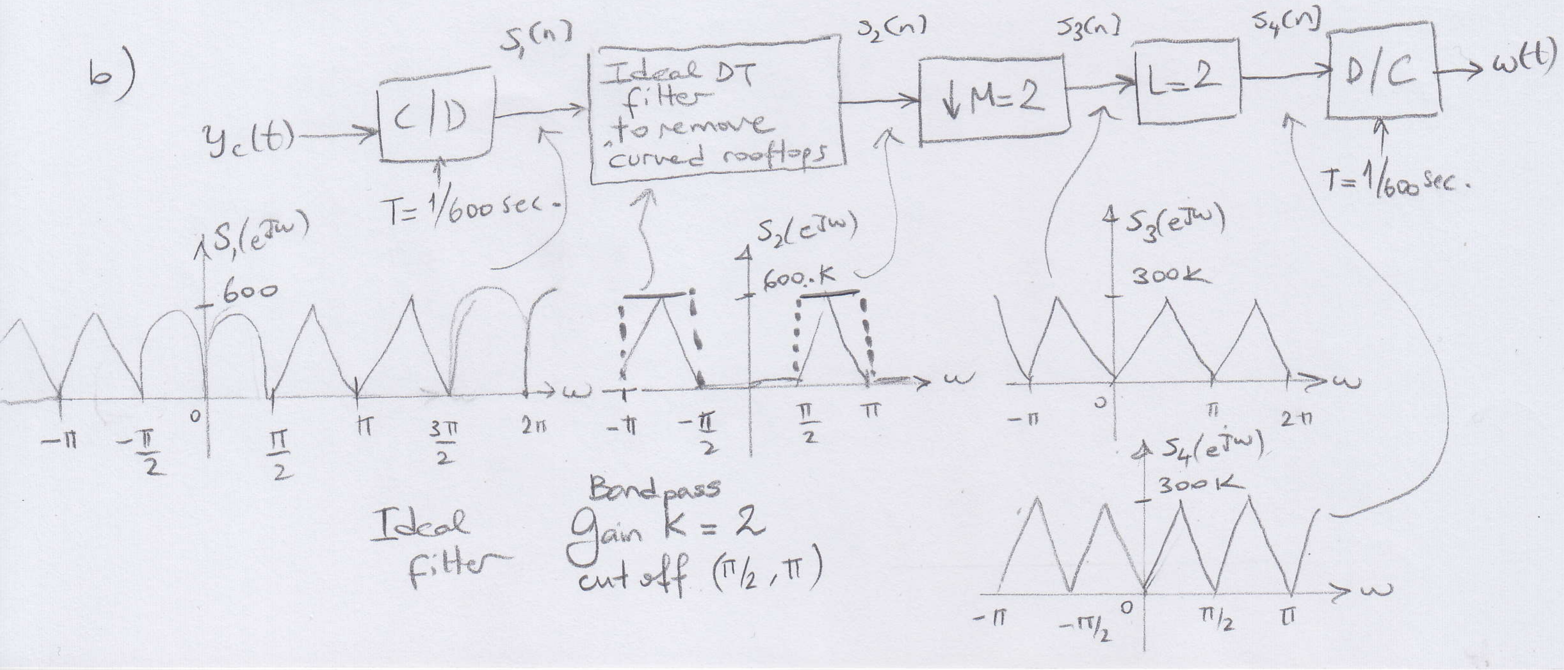




a) Upsampling Rate: Since "rooftop waveforms" have the same bandwidth, and sampling rates for C/D & D/C are same $\Rightarrow L=2$ to make input/output time scales remain same.

Conversion of Spectra: Since ideal filter has constant gain, "curved rooftop waveform" can not be converted into "triangular rooftop". Hence they should be supressed.

b)

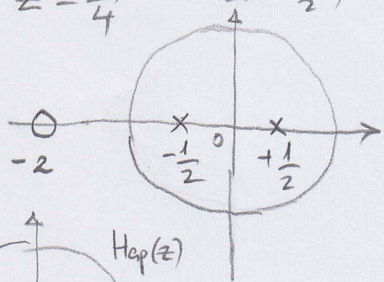


Q3) Consider the following LCCDE representing an LTI system whose impulse response is $h[n]$:

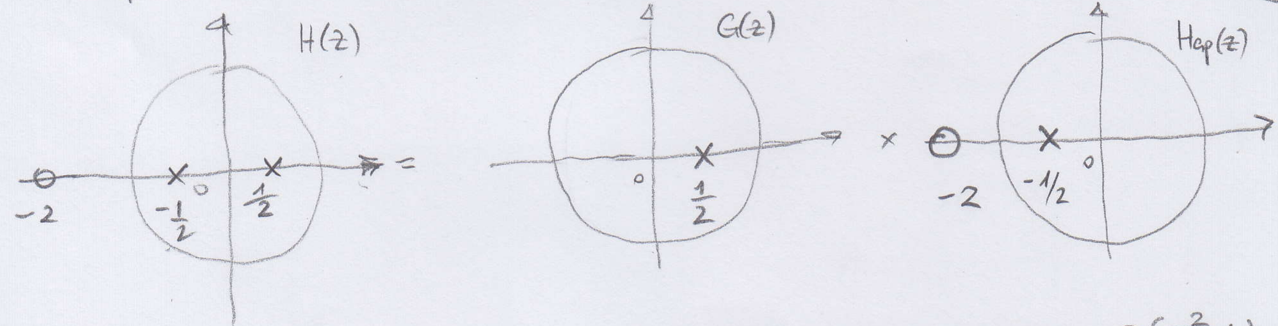
$$y[n] - 0.25\,y[n-2] = x[n-1] + 2x[n-2]$$

a) Find the poles and zeros of the system function, $H(z)$.
b) Explain how to convert this system into a minimum-phase system, $G(z)$, without changing its magnitude response. Draw pole-zero plot for $G(z)$.
c) The minimum-phase system, $G(z)$ in part-b is cascased by another system, $W(z)$, whose system function is equal to $W(z)=1+z^{-2}$. Approximately plot the magnitude response of the overall system, $G(z)W(z)$.
d) For the system, $W(z)$, find and plot the principal phase ($ARG[W(e^{jw})]$), determine its group delay.

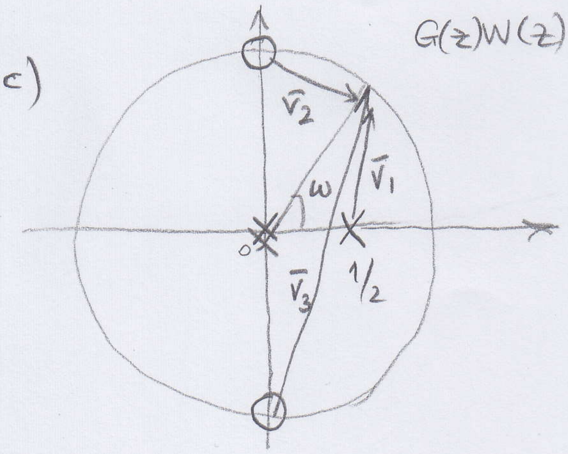
a) $Y(z)\left[1-\frac{1}{4}z^{-2}\right] = X(z)\left[z^{-1}+2z^{-2}\right] \Rightarrow H(z) = \frac{z^{-1}+2z^{-2}}{1-\frac{1}{4}z^{-2}} = \frac{z+2}{z^2-\frac{1}{4}} = \frac{z+2}{(z-\frac{1}{2})(z+\frac{1}{2})}$



b) Decompose $H(z)$ into all-pass & minimum phase:



c)

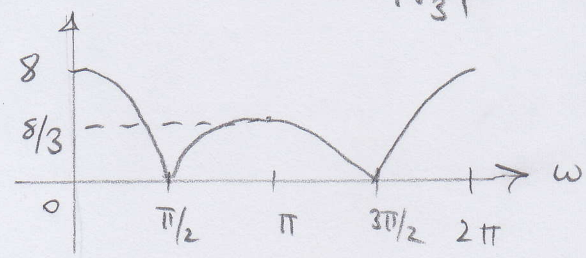


$G(z)W(z) = \frac{2}{z-\frac{1}{2}}(1+z^{-2}) = \frac{2(z^2+1)}{(z-\frac{1}{2})z^2}$

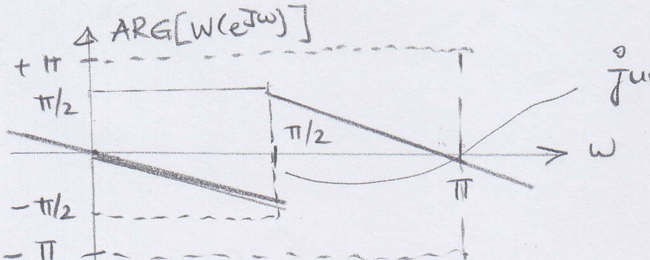
$|G(e^{j\omega})W(e^{j\omega})| = 2.\frac{|\bar{V}_1|.|\bar{V}_2|}{|\bar{V}_3|}$

$\omega=0 \Rightarrow |\,| = \frac{2(\sqrt{2})^2}{1/2} = 8$

$\omega=\pi \Rightarrow |\,| = \frac{2(\sqrt{2})^2}{3/2} = 8/3$



d) $W(z) = 1+z^{-2} \Rightarrow W(e^{j\omega}) = 1+e^{-j2\omega} = e^{-j\omega}(e^{+j\omega}+e^{-j\omega}) = 2\cos(\omega)e^{-j\omega}$



jump due to zero at $\omega = \mp\pi/2$.

$grd[W(e^{j\omega})] = -\frac{d(-\omega)}{d\omega} = +1.$

Q4) [This question has independent parts]

a) Assume a rational system function, *H*(*z*), whose coefficients of the polynomials in both numerator and denominator are <u>real</u>.

$$H(z) = \frac{N(z)}{D(z)}$$

If *H*(*z*) is known to be <u>all-pass with unity gain</u>, write *N*(*z*) in terms of *D*(*z*) and *z*.

b) Let the magnitude squred response of an LTI system be equal to

$$|H(e^{jw})|^2 = \frac{\frac{13}{9} - \frac{4}{3}\cos(w)}{\frac{25}{16} - \frac{6}{4}\cos(w)}$$

Find *H*(*z*), if this system is known to be minimum phase. Draw pole-zero plot for *H*(*z*). Explain whether *H*(*z*) represents an FIR or an IIR system.

a) $$H(z) = \frac{\sum_{k=0}^{N} a_k z^{-k}}{\sum_{k=0}^{N} b_k z^{-k}} = \frac{N(z)}{D(z)}$$

$$\Downarrow$$

$$H^*\left(\frac{1}{z^*}\right) = \frac{\sum_{k=0}^{N} a_k z^{+k}}{\sum_{k=0}^{N} b_k z^{+k}} = \frac{N(z^{-1})}{D(z^{-1})}$$

For all pass systems,

$$H(z)\,H^*(1/z^*) = 1$$

$$\frac{N(z)}{D(z)} \cdot \frac{N(z^{-1})}{D(z^{-1})} = 1 \Rightarrow \begin{array}{l} D(z) = N(z^{-1}).K \\ N(z) = D(z^{-1}).K \end{array}$$

Since all pass H(z) satisfies

$$H(z) = \frac{\prod_{k}^{N}(z^{-1} - d_k^*)}{\prod_{k}^{N}(1 - d_k z^{-1})} \Rightarrow K = z^{-2N}$$

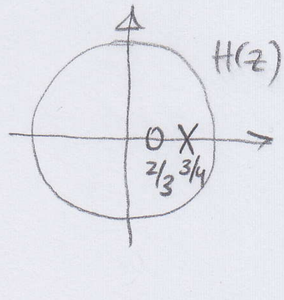
b) $$|H(e^{jw})|^2 = |H(e^{jw})H^*(e^{jw})| = H(e^{jw})H(e^{-jw}) \text{ with real coff.}$$

$$\Rightarrow |H(e^{jw})|^2 = \frac{\frac{13}{9} - \frac{4}{3}\cos w}{\frac{25}{16} - \frac{6}{4}\cos w} = \frac{\frac{13}{9} - \frac{4}{3}\left(\frac{e^{jw} + e^{-jw}}{2}\right)}{\frac{25}{16} - \frac{6}{4}\left(\frac{e^{jw} + e^{-jw}}{2}\right)} = \frac{\left(1 - \frac{2}{3}e^{+jw}\right)\left(1 - \frac{2}{3}e^{-jw}\right)}{\left(1 - \frac{3}{4}e^{+jw}\right)\left(1 - \frac{3}{4}e^{-jw}\right)}$$

Since H(z) is minimum phase, poles & zeros should be within unitcircle.

$$\text{If } H(e^{jw}) = \frac{1 - \frac{2}{3}e^{-jw}}{1 - \frac{3}{4}e^{-jw}} \Rightarrow H(z) = \frac{1 - \frac{2}{3}z^{-1}}{1 - \frac{3}{4}z^{-1}}$$

$$\Rightarrow H(e^{-jw}) = \frac{1 - \frac{2}{3}e^{+jw}}{1 - \frac{3}{4}e^{+jw}}$$



It has a pole outside origin, not cancelled by a zero

$\Rightarrow$ IIR.