EE430 DIGITAL SIGNAL PROCESSING

HOMEWORK 1

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1) $x[n]=x_c(nT_s)=x_c(n/f_s)=4\sin(20\pi n/3+\pi/13)= 4\sin((20\pi/3+k2\pi)n+\pi/13)=$

$=4\sin((20\pi\ f_s /3+k2\pi\ f_s)n/\ f_s +\pi/13)$ where $k \in Z$

Writing in that form in continuous time this signal corresponds to

$x_a(t)= 4\sin((20\pi\ f_s /3+k2\pi\ f_s)t +\pi/13)$ (1)

a) Putting f_s=3 kHz in equation (1) we obtain

$x_a(t)= 4\sin(20000\pi\ t+k2\pi \text{ x } 3000t +\pi/13)= 4\sin(2\pi(10000+3000k)\ t +\pi/13)$ (2)

From equation (2) it can be seen that frequencies in the form of f=10000+3000k Hz gives us the same discrete time signal for integer k.

f=10000+3000k Hz=….4, 7, 10, 13, 16 kHz

b) We know x[n] is equal to

$x[n]= 4\sin(20000\pi n/\ f_s +\pi/13)$ for any sampling frequency f_s. We want it to be equal to

$4\sin(20\pi n/3+\pi/13)= 4\sin((20\pi/3+ k2\pi)n+\pi/13)= 4\sin((20000\pi/\ f_s)n+\pi/13)$

Thus

$20000\pi/\ f_s=20\pi/3+ k2\pi$

Multiplying both sides with $3f_s/2\pi$ we get

$30000=10\ f_s+3k \text{ x } f_s \rightarrow f_s=30000/(10+3k)$ Hz

Hence every sampling frequency in the form of $f_s=30000/(10+3k)$ Hz gives the same discrete signal for this case where k is an integer.

2) If fundamental period exists, it is $N=k2\pi/\omega_0$ where ω_0 is the radial frequency of signal's continuous counterpart and k is an integer. If integer N values can be found, then the discrete signal is periodic and the smallest positive N is the fundamental period.

$\sin(1.74\pi n + 3.1) \rightarrow \omega_0=1.74\pi$ rad/sample

$N= k2\pi/1.74\pi=k \text{ x } 100/87$

for k=87 N=100 samples

$\sin(1.74\pi n + 3.1\pi) \rightarrow \omega_0 = 1.74\pi$ rad/sample

$N = k2\pi/1.74\pi = k \times 100/87$

for k=87 N=100 samples

$\cos(15.74\pi n + 3\pi/8) \rightarrow \omega_0 = 1.74\pi$ rad/sample

$N = k2\pi/15.74\pi = k \times 100/787$

for k=787 N=100 samples

$\cos(\sqrt{\pi} n) \rightarrow \omega_0 = \sqrt{\pi}$ rad/sample

$N = k2\pi/\sqrt{\pi} = k \times 2\sqrt{\pi}$

In that case N cannot be an integer for integer k values so this sequence is not periodic.

$\cos(\pi\sqrt{\pi} n) \rightarrow \omega_0 = \pi\sqrt{\pi}$ rad/sample

$N = k2\pi/\pi\sqrt{\pi} = k \times 2/\sqrt{\pi}$

In that case N cannot be an integer for integer k values so this sequence is not periodic.

$\cos(\pi\sqrt{2}\, n) \rightarrow \omega_0 = \pi\sqrt{2}$ rad/sample

$N = k2\pi/\pi\sqrt{2} = k \times \sqrt{2}$

In that case N cannot be an integer for integer k values so this sequence is not periodic.

3) $\cos(\pi n)$ is the highest frequency sinusoidal DT signal. It is the $(-1)^n$ sequence where the successive n values come between -1 and 1. This means it has a fundamental period of 2 which is the lowest possible period. Lowest period also refers to highest frequency.

We can also think $\cos(2\pi n)$ as the highest frequency signal as its fundamental period is 1. However, this signal produces constant values of 1. Thus, it is not an actual sinusoidal in general sense and we take $\cos(\pi n)$ as the highest frequency sinusoidal DT signal.

4) **Linearity:**

$x_1 \to y_1$ $x_2 \to y_2$ $x_3 \to y_3$

$x_3[n]=ax_1[n]+bx_2[n]$

$$y_3[n]= \begin{cases} x_3[\frac{n}{2}], \text{ n even} \\ \\ (x_3[\frac{n-1}{2}]+ x_3[\frac{n+1}{2}])/2, \text{ n odd} \end{cases} = \begin{cases} ax_1[\frac{n}{2}]+bx_2[\frac{n}{2}], \text{ n even} \\ \\ (ax_1[\frac{n-1}{2}]+bx_2[\frac{n-1}{2}]+ax_1[\frac{n+1}{2}]+bx_2[\frac{n+1}{2}])/2, \text{ n odd} \end{cases} =$$

$$= ay_1[n]+by_2[n]$$

Since $y_3[n]= ay_1[n]+by_2[n]$ system is linear.

Time-invariance:

Assume $x \to y$, $x_1 \to y_1$ and $x_1[n]=x[n-n_0]$.

$$y_1[n]= \begin{cases} x_1[\frac{n}{2}]= x[\frac{n}{2}- n_0]=x[\frac{n-2n_0}{2}], \text{ n even} \\ \\ (x_1[\frac{n-1}{2}]+ x_1[\frac{n+1}{2}])/2=(x[\frac{n-1}{2}-n_0]+ x[\frac{n+1}{2}-n_0])/2=(x[\frac{n-1-2n_0}{2}]+ x[\frac{n+1-2n_0}{2}])/2, \text{ n odd} \end{cases}$$

$$y[n-n_0]= \begin{cases} x[\frac{n-n_0}{2}], \text{ n even} \\ \\ (x[\frac{n-n_0-1}{2}]+ x[\frac{n-n_0+1}{2}])/2, \text{ n odd} \end{cases}$$

$y_1[n] \neq y[n-n_0]$. Thus system is time-varying.

Hence this system is linear and time-varying.

5) These systems are not LTI so we must apply the general definitions of causality and stability.

$$y[n] = 2^{\delta[n+1]} + x[n-3] \Rightarrow$$

Output of this system is only dependent on past values of the input. There is a n+1 parameter in the impulsive term but it is not related with input. Thus system is causal.

Assume B is a finite value and $|x[n]|<B$.

$$|y[n]|=|2^{\delta[n+1]}+x[n-3]| \leq |2^{\delta[n+1]}|+|x[n-3]| < |2^{\delta[n+1]}|+B$$

$2^{\delta[n+1]}$ is a bounded term also. It is equal to 1 most of the time and equal to 2 when n=-1. Thus |y[n]| is bounded for bounded inputs. The system is stable.

Hence this system is causal and stable.

$$y[n] = \begin{cases} [-\delta[n-1]] + x[n-3] & n > 0 \\ \\ 2^{n}[n-3] & n \leq 0 \end{cases} \quad \Rightarrow$$

In order to proceed further we should write the input-output relation of y[n] explicitly for positive n.

For n=1:

y[1]= $y[-\delta[0]] + x[-2]$= $y[-1] + x[-2]$= $2^{-1}x[-4] + x[-2]$

For n>1

y[n]=y[0]+x[n-3]= $2^{0}x[-3]$+x[n-3]=$x[-3]$+x[n-3]

For n≤0, it is clear that output is dependent on past inputs. For n=1 output is dependent on the input values at n=-4 and n=-2 which are both past times for n=1. For n>1 x[n-3] term clearly refers to a past input. There is also a term which is dependent on the input at n=-3. It is a past value for the case of n>1. Hence there is no problem. Outputs are always dependent on past input values for all intervals. System is causal.

Assume B is finite and |x[n]|<B.

For n≤0

$|y[n]|=|2^{n}||x[n-3]|<B$

Since we are dealing with negative n values 2^{n} can be equal to 1 at most. We also know that |x[n-3]|<B so |y[n]| is less than B and bounded for n≤0.

For n=1

$|y[1]|=| 2^{-1}x[-4] + x[-2]| \leq 2^{-1}|x[-4]|+| x[-2]|<1.5B$

y[1] is a bounded value.

For n>1

$|y[n]|=|[-3]+x[n-3]| \leq | x[-3]|+| x[n-3]|<2B$

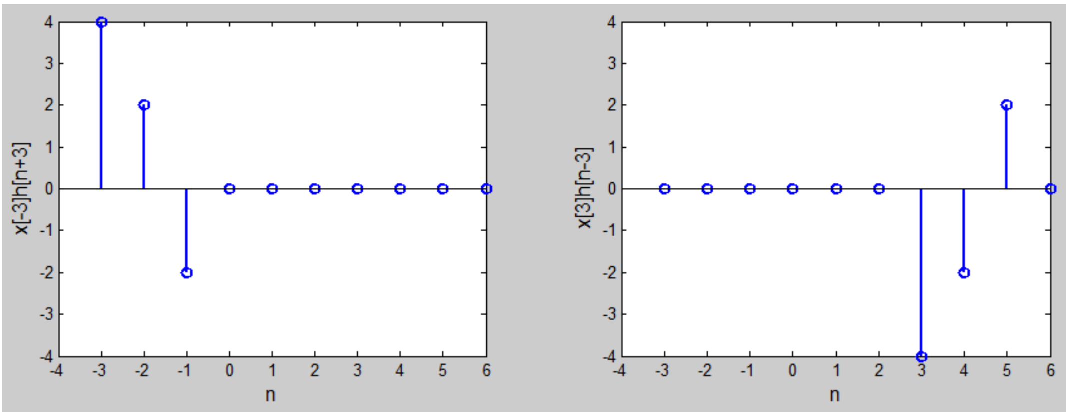
y[n] is bounded for n>1.

Outputs are found to be bounded at every interval for bounded inputs. System is stable.

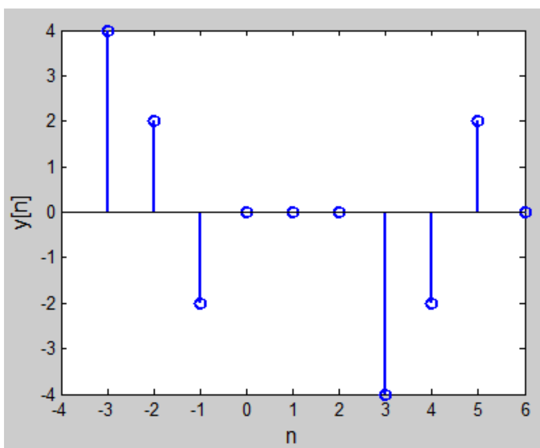
Hence this system is causal and stable.

6) $y[n]=x[n]*h[n]=\sum_{k=-\infty}^{\infty} x[k]h[n-k]=\sum_{k=-\infty}^{\infty} x[n-k]h[k]$

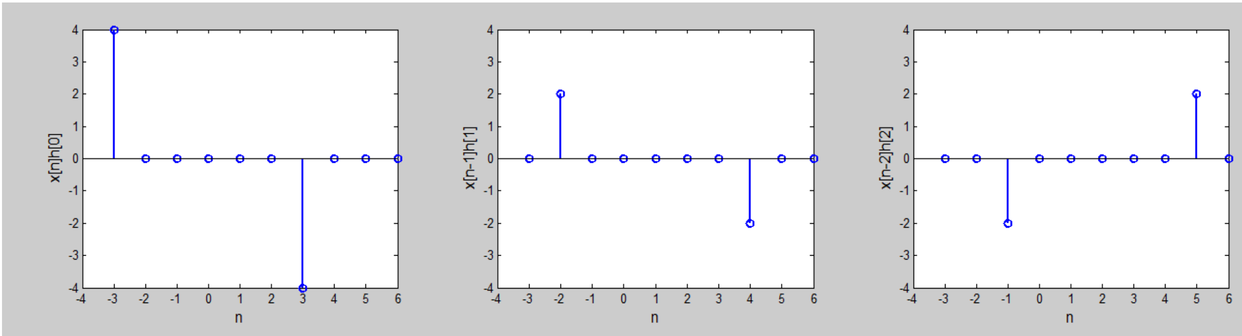
$y[n]=\sum_{k=-\infty}^{\infty} x[k]h[n-k]=x[-3]h[n+3]+x[3]h[n-3]$



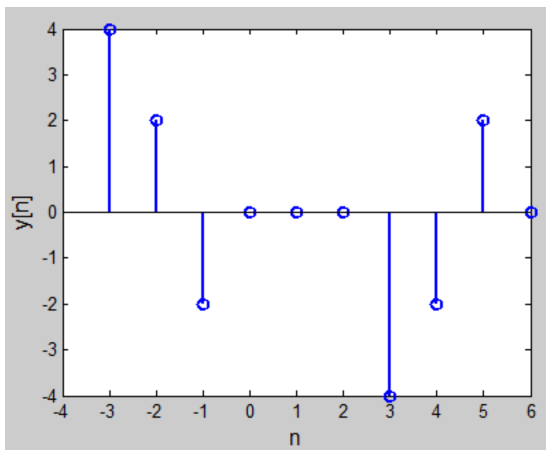
Adding these two signals we find



$y[n]=\sum_{k=-\infty}^{\infty} x[n-k]h[k]$=x[n]h[0]+ x[n-1]h[1]+ x[n-2]h[2]



Adding these three signals we find



The results we obtained by keeping x stationary and shifting h or keeping h stationary and shifting x is identical as we have expected. Since convolution is a commutative operation.

7)

i) $y[-n] = \sum_{k=-\infty}^{\infty} x[k]h[-n-k]$ (3)

Let x[-n]=a[n] and h[-n]=b[n].

$x[-n] * h[-n] = a[n] * b[n] = \sum_{k=-\infty}^{\infty} a[k]b[n-k] = \sum_{k=-\infty}^{\infty} x[-k]h[-n+k] =$

$= \sum_{m=\infty}^{-\infty} x[m]h[-n-m] = \sum_{k=-\infty}^{\infty} x[k]h[-n-k]$ (4)

Since the results of equations (3) and (4) are equal then we can conclude that

$y[-n]= x[-n] * h[-n]$

ii) y[n-4]= $\sum_{k=-\infty}^{\infty} x[k]h[n-4-k]$=$\sum_{k=-\infty}^{\infty} x[n-4-k]h[k]$ (5)

Let x[n-4]=a[n] and h[n-4]=b[n].

x[n-4] $*$ h[n]= a[n] $*$ b[n]= $\sum_{k=-\infty}^{\infty} a[n-k]h[k]$=$\sum_{k=-\infty}^{\infty} x[n-4-k]h[k]$ (6)

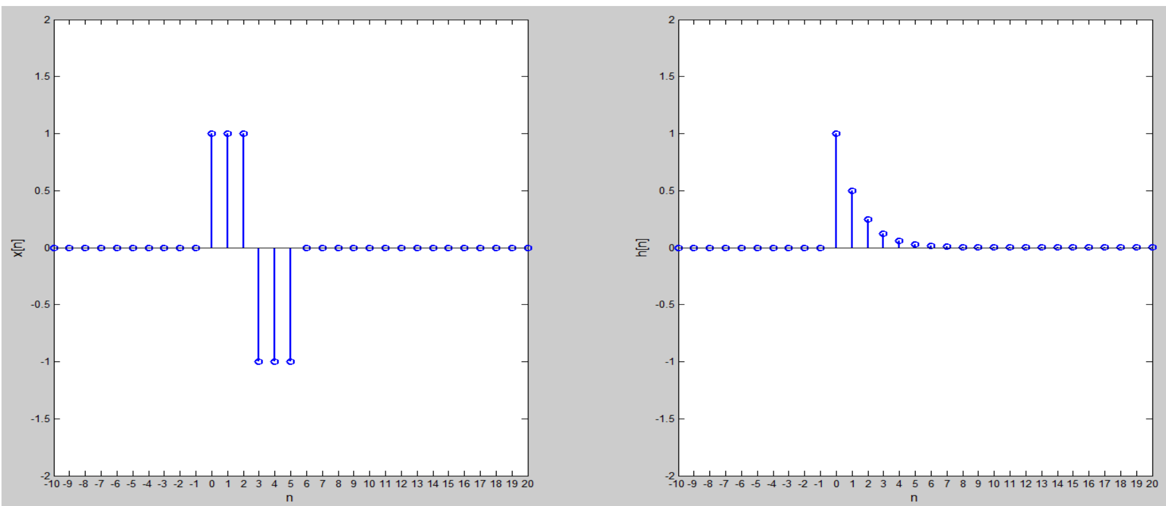
x[n] $*$ h[n-4]= x[n] $*$ b[n]= $\sum_{k=-\infty}^{\infty} x[k]b[n-k]$=$\sum_{k=-\infty}^{\infty} x[k]h[n-4-k]$ (7)

Results of equations (5), (6) and (7) are all equal or commuted versions of each other which does not change the result for convolution. Hence we can conclude that

y[n-4]= x[n-4] $*$ h[n]= x[n] $*$ h[n-4]

8)

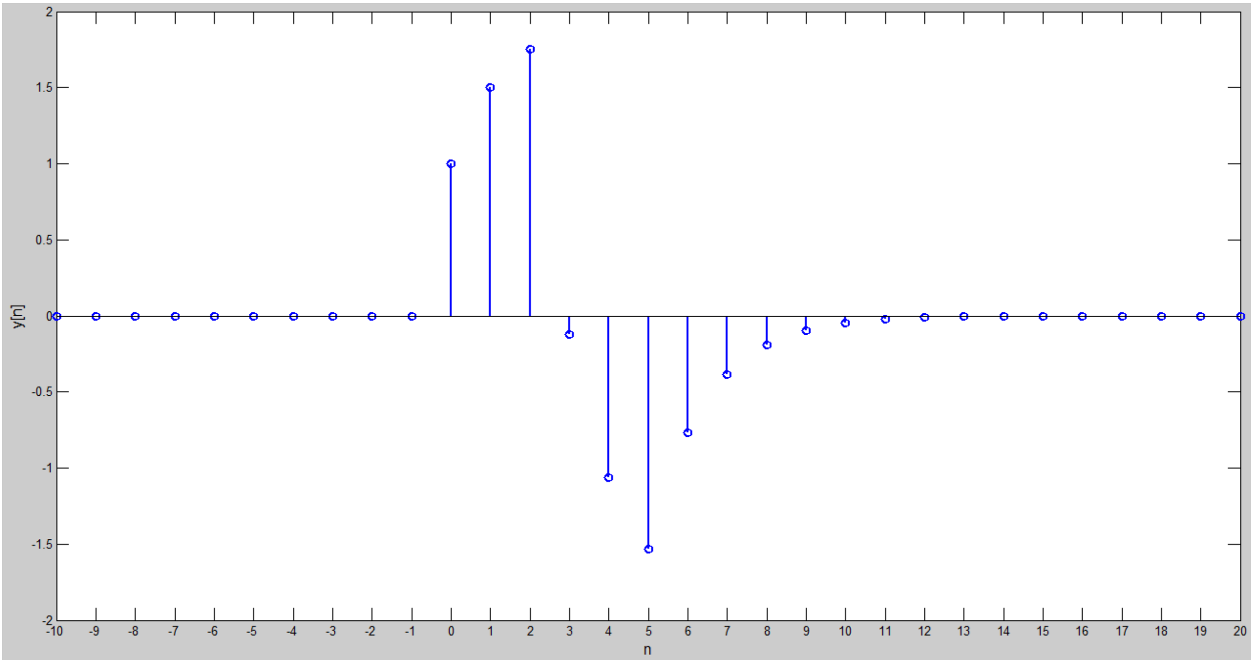
i)



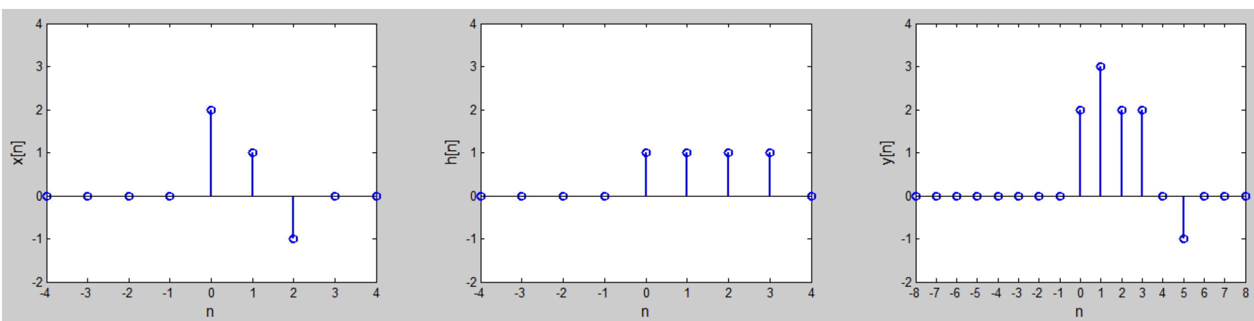
As can be seen from the above figure x[n] is equal to 1 for n=[0 1 2] and equal to -1 for n=[3 4 5]. It is zero otherwise. h[n] is equal to zero for negative n's. It becomes 1 at n=0 and decays exponentially after on.

ii) $y[n]=x[n] * h[n]=\sum_{k=-\infty}^{\infty} x[k]h[n-k]=\sum_{k=0}^{2} h[n-k]$ - $\sum_{k=3}^{5} h[n-k]=$

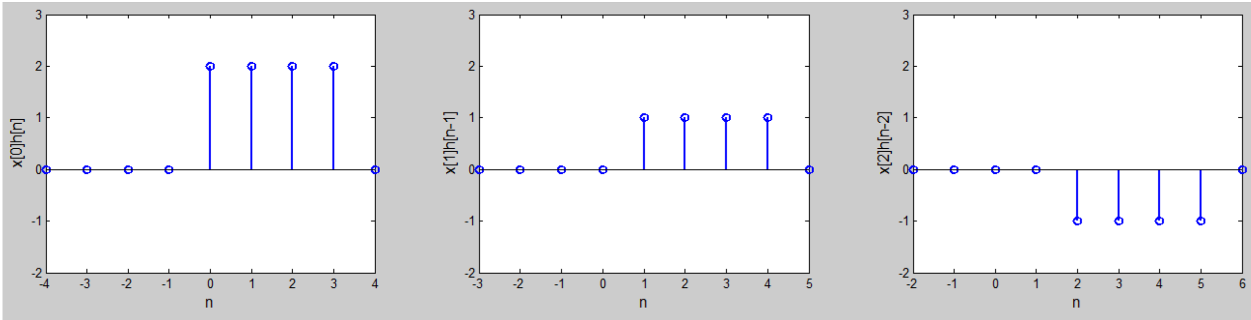
$= h[n]+h[n-1]+h[n-2]-h[n-3]- h[n-4]- h[n-5]$



9) The sequences I have generated are x=[2 1 -1] for n=0:2 and h=[1 1 1 1] for n=0:3. They are zero out of these intervals.



Resulting plots are shown in above figure. y[n] is the sequence [2 3 2 2 0 -1] for n=0:5. Its interval is as we have expected. Its boundaries must be the sum of the boundaries of x and h. In order to see if the overall result is correct, it is best to check the elements of the convolution one-by-one. The related plots are given below.



The sum of these three plots gives the y[n] in the first plot so the results are consistent with what we expect.

10)

a) $p_1(x)=x^3+3x^2+1$ (3rd order) $p_2(x)=x^5+2x^4+x+3$ (5th order)

$p_1 \text{ x } p_2=(x^3+3x^2+1)(x^5+2x^4+x+3)=x^8+2x^7+x^4+3x^3+3x^7+6x^6+3x^3+9x^2+x^5+2x^4+x+3=$

$=x^8+5x^7+6x^6+x^5+3x^4+6x^3+9x^2+x+3$

This is the result we expect. The MATLAB results are given below.

```
>> p1

p1 =

     1     3     0     1

>> p2

p2 =

     1     2     0     0     1     3

>> y=conv(p1,p2)

y =

     1     5     6     1     3     6     9     1     3
```

The elements of matrix y are equal to the coefficient of the polynomial I have evaluated. Hence, it is consistent.

c) This can most easily be verified by the polynomial approach. Since x is the name of one of the functions we are interested in let's choose t as our polynomial parameter.

$y(t)=t^6+t^5+2t^4+3t^3+4t^2-t+5$ $x(t)=t^4+2t^3+3t^2+4t+5$

$h(t)=y(t)/x(t)=(t^6+t^5+2t^4+3t^3+4t^2-t+5)/(t^4+2t^3+3t^2+4t+5)=t^2-t+1$

This is the result we expect. In MATLAB we must see the sequence [1 -1 1] after applying deconv operation.

```
>> y

y =

     1     1     2     3     4    -1     5

>> x

x =

     1     2     3     4     5

>> h=deconv(y,x)

h =

     1    -1     1
```

MATLAB results are consistent with the calculations.

d) $y(t)=t^6+2t^5+2t^4+3t^3+4t^2-t+5$ $\quad x(t)=t^4+2t^3+3t^2+4t+5$

$h(t)=y(t)/x(t)=(t^6+2t^5+2t^4+3t^3+4t^2-t+5)/(t^4+2t^3+3t^2+4t+5)=$

$=t^2-1+(t^3+2t^2+3t+10)/(t^4+2t^3+3t^2+4t+5)$

This time the resultant polynomial has a remainder term. Let's see what MATLAB will give.

```
>> y

y =

     1     2     2     3     4    -1     5

>> x

x =

     1     2     3     4     5

>> h=deconv(y,x)

h =

     1     0    -1
```

In our resultant polynomial there was a square term with coefficient 1, no t term with order 1, a constant term -1 and the remainder. As can be seen from the figure MATLAB gave the coefficients correctly but gave no information about the remainder. This shows deconv command is not always reliable if the division is not exact.