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Section: 2

EE430 - HW3

2) a) $x[n] = \delta[n] + 3\delta[n-1] + \delta[n-2]$

$$X_3[k] = \sum_{n=0}^{2} x[n] e^{-j\frac{2\pi}{3}kn} = 1 + 3e^{-j\frac{2\pi}{3}k} + e^{-j\frac{4\pi}{3}k}, \text{ for } k=0,1,2$$

$$0, \text{ elsewhere}$$

$$X_5[k] = \sum_{n=0}^{4} x[n] e^{-j\frac{2\pi}{5}kn} = 1 + 3e^{-j\frac{2\pi}{5}k} + e^{-j\frac{4\pi}{5}k}, \text{ for } k=0,1,2,3,4$$

$$0, \text{ elsewhere}$$

b) $$\tilde{X}_3[k] = \sum_{n=-\infty}^{\infty} X_3[k-3n] = X_3[((k))_3]$$

$$\tilde{X}_5[k] = \sum_{n=-\infty}^{\infty} X_5[k-5n] = X_5[((k))_5]$$

c) $$x[n] = \begin{cases} \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn}, & n = 0,1,2 \cdots N-1 \\ 0, & \text{else} \end{cases}$$

$$\Rightarrow x[n] = \frac{1}{3}\sum_{k=0}^{2} X[k] e^{j\frac{2\pi}{3}kn}, \text{ for } n=0,1,2$$

$$0, \text{ else}$$

$$= \frac{1}{3}\sum_{k=0}^{2} \left(1 + 3e^{-j\frac{2\pi}{3}k} + e^{-j\frac{4\pi}{3}k}\right) e^{j\frac{2\pi}{3}kn}$$

$$= \frac{1}{3}\sum_{k=0}^{2} e^{j\frac{2\pi}{3}kn} + 3e^{j\frac{2\pi}{3}k(n-1)} + e^{j\frac{2\pi}{3}k(n-2)}$$

$$= \frac{1}{3}\left[X_3[0] + X_3[1] e^{j\frac{2\pi}{3}n} + X_3[2] e^{j\frac{4\pi}{3}n}\right], \text{ for } n=0,1,2$$

$$0, \text{ else}$$

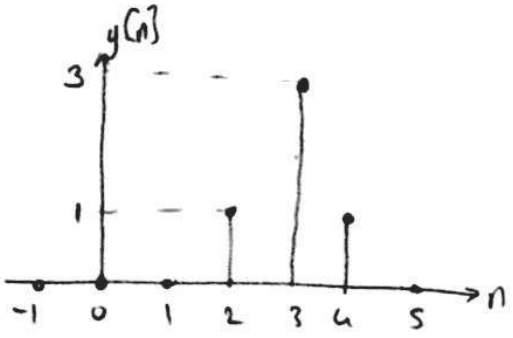
in the same way.: $$x[n] = \frac{1}{5}\left[X_5[0] + X_5[1] e^{j\frac{2\pi}{5}n} + X_5[2] e^{j\frac{4\pi}{5}n} + X_5[3] e^{j\frac{6\pi}{5}n} + X_5[4] e^{j\frac{8\pi}{5}n}\right]$$

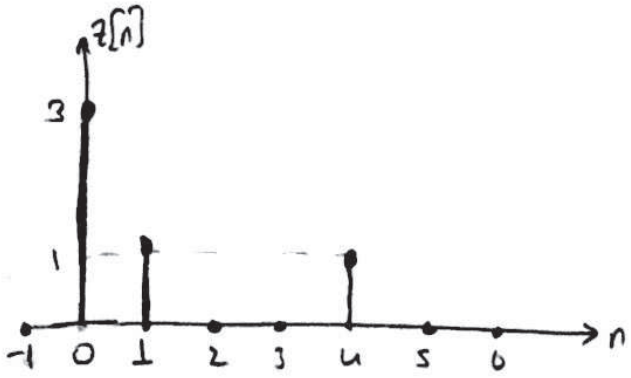
for $n=0,1,2,3,4$

0, else

d) Matlab

3) a)





b) $y[n] = x[n-2]$ $\quad z[n] = x[((n+1))_5]$

c) $y[k] = X[k].e^{-j\frac{4\pi}{5}k} \Rightarrow y[n] = x[n-2] = x[((n-2))_5]$ for this case

$z[k] = X[k]\, e^{j\frac{2\pi}{5}k}$

$z[n]$ and $y[n]$ do not have 3-point DFTs since they are length of 5.

6) $x[n] = 3\delta[n] - 2\delta[n-1] + \delta[n-2] + \delta[n-3] - 2\delta[n-4] - \delta[n-5]$

a) $W_3[k] = X(e^{j\omega})\Big|_{\omega=\frac{2\pi k}{3}}$ $k=0,1,2 \Rightarrow W_3[k]$ is 3 point DFT of $x[n]$ but $x[n]$ is of 6 in length.

$\Rightarrow$ we find $w[n]$ by overlaping the remaining 3 value.

that is;

$$\left.\begin{array}{l} w[0] = x[0] + x[3] \\ w[1] = x[1] + x[4] \\ w[2] = x[2] + x[5] \\ w[n] = 0 \quad \text{else} \end{array}\right\} \quad w[n] = 4\delta[n] - 4\delta[n-1]$$

b) in a similar way: $w[n] = 2\delta[n] - 2\delta[n-1] + \delta[n-2] + \delta[n-3] - 2\delta[n-4]$

c) $w[n] = x[n] = 3\delta[n] - 2\delta[n-1] + \delta[n-2] + \delta[n-3] - 2\delta[n-4] - \delta[n-5]$

d) $h[n] = 2\delta[n] - 8[n-1]$

$y_3[k] = H_3[k] W_1[k] \Rightarrow y[n] = h[n] \circledast w[n] \Rightarrow$

$$\begin{matrix}[4 & -4 & 0] \\ [2 & -1 & 0]\end{matrix} \circledast \begin{matrix}8 & -8 & 0 \\ 0 & -4 & 4 \\ + & 0 & 0 & 0\end{matrix}$$

$y[n] \leftarrow \boxed{8 \quad -12 \quad 4}$

$$\Rightarrow y[n] = 8\delta[n] - 12\delta[n-1] + 4\delta[n-2]$$

e) $w[n] = [2 \quad -2 \quad 1 \quad 1 \quad -2]$

$h[n] = [2 \quad -1 \quad 0 \quad 0 \quad 0] \circledast$

$$\begin{matrix}4 & -4 & 2 & 2 & -4 \\ + & 2 & -2 & 2 & -1 & -1 \\ \hline 6 & -6 & 4 & 1 & -5\end{matrix}$$

$$\Rightarrow y[n] = 6\delta[n] - 6\delta[n-1] + 4\delta[n-2] + \delta[n-3] - 5\delta[n-4]$$

f) $w[n] = [3 \quad -2 \quad 1 \quad 1 \quad -2 \quad -1 \quad 0 \quad 0] \circledast$

$h[n] = [2 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$

$$\begin{matrix}6 & -4 & 2 & 2 & -4 & -2 & 0 & 0 \\ + & 0 & -3 & 2 & -1 & -1 & 2 & 1 & 0 \\ \hline 6 & -7 & 4 & 1 & -5 & 0 & 1 & 0\end{matrix}$$

$$\Rightarrow y[n] = 6\delta[n] - 7\delta[n-1] + 4\delta[n-2] + \delta[n-3] - 5\delta[n-4] + \delta[n-6]$$

g) $x[n]$ is of length 6 and $h[n]$ is of length 2

$\Rightarrow$ N-point circular convolution = linear convolution ; if $N \geq 7$

$\Rightarrow y[n] = y_8[n] \neq y_3[n] \neq y_5[n]$

7) a) $X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}$; $k = 0, 1, 2 \ldots N-1$ where $W_N = e^{-j\frac{2\pi}{N}}$

$$= \sum_{n:\text{even}} x[n] W_N^{kn} + \sum_{n:\text{odd}} x[n] W_N^{kn} = \sum_{m=0}^{\frac{N}{2}-1} \underbrace{x[2m]}_{e[n]} W_N^{2mk} + \sum_{m=0}^{\frac{N}{2}-1} \underbrace{x[2m+1]}_{o[n]} W_N^{(2m+1)k}$$

$$= \underbrace{\sum_{m=0}^{\frac{N}{2}-1} e[n] W_{\frac{N}{2}}^{mk}}_{\frac{N}{2}\text{ point DFT of } e[n]} + W_N^k \underbrace{\sum_{m=0}^{\frac{N}{2}-1} o[n] W_{\frac{N}{2}}^{mk}}_{\frac{N}{2}\text{ point DFT of } o[n]}$$

here we used $W_N^{2km} = W_{\frac{N}{2}}^{km}$

$$\Rightarrow X[k] = E\left[((k))_{\frac{N}{2}}\right] + e^{-j\frac{2\pi}{N}k} O\left[((k))_{\frac{N}{2}}\right] \text{ ; for } k = 0, 1, 2 \ldots N-1$$

b)

i) $X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} = \sum_{n=0}^{N-1} x[n]\left(\cos\left(\frac{2\pi}{N}kn\right) - j\sin\left(\frac{2\pi}{N}kn\right)\right)$ } here we are given that $x[n]$ is a real sequence

Direct computation involves $2N$ real multiplications and $2N-2$ real additions for each k.

$\Rightarrow$ For all values of $X[k]$; direct computation involves $2N^2$ real multiplications and $2N^2-2N$ real additions

ii)

$$X[k] = E[((k))_{\frac{N}{2}}] + W_N^k O[((k))_{\frac{N}{2}}]$$

we know that $E[k] = E\left[k+\frac{N}{2}\right]$, $O[k] = O\left[k+\frac{N}{2}\right]$ } periodicity with $\frac{N}{2}$.

and we also know that $W_N^{k+\frac{N}{2}} = -W_N^k$

$\Rightarrow$ we can divide $X[k]$ into two as follows:

$$X[k] = E[k] + W_N^k O[k] \quad \text{for } k = 0,1,2 \ldots \frac{N}{2}-1$$

$$X\left[k+\frac{N}{2}\right] = E[k] - W_N^k O[k] \quad \text{for } k = 0,1,2 \ldots \frac{N}{2}-1$$

$\Rightarrow$ We need to do $2\left(\frac{N}{2}\right)^2$ real multiplications and $2\left(\frac{N}{2}\right)^2 - 2\left(\frac{N}{2}\right)$ real additions for $E[k]$ and $O[k]$. Also $W_N^k \cdot O[k]$ involves $4(N)$ real multiplications and $2(N)$ real additions. To find $E[k] + W_n^k O[k]$, we also need to do $2(N)$ real additions.

Method is called Split-Radix FFT algorithm

Computation involves total of
$2\left(\frac{N}{2}\right)^2 + 2\left(\frac{N}{2}\right)^2 + 4(N)$
$= N^2 + 4N$ real mults.
and N^2+2N real additions

iii) Second computation method is better especially for large N's (includes less comps)

8) $h[n] = 2\delta[n] - \delta[n-1] + \delta[n-2]$ $x[n] = [1\ 2\ 3\ 4\ -1\ -2\ -3\ -4\ 1\ 2\ 3\ 4]$ $0 \le n < 11$

a) $P=3$ $L=4$ $\Rightarrow$ $N = 3+4-1 = 6$-point DFT, we will use.

b) input is length of 12 $\Rightarrow$ there are 3 input segments $\Rightarrow$ $x_1 = [1\ 2\ 3\ 4]$
$x_2 = [-1\ -2\ -3\ -4] = -x_1$
$x_3 = [1\ 2\ 3\ 4] = x_1$

c) $y_1 = [1\ 2\ 3\ 4] \circledast [2\ -1\ 1\ ] = [2\ 3\ 5\ 7\ -1\ 4]$

$$\begin{array}{r} 2\ 4\ 6\ 8 \\ -1\ -2\ -3\ -4 \\ \quad 1\ 2\ 3\ 4 \\ \hline 2\ 3\ 5\ 7\ -1\ 4 \end{array}$$

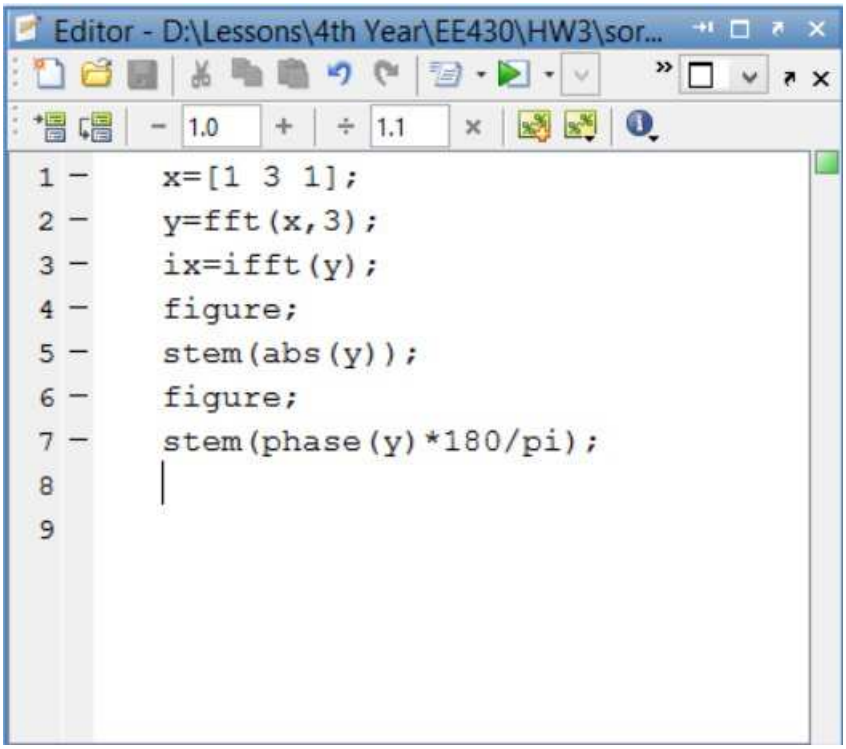
$\Rightarrow y_2 = [-2\ -3\ -5\ -7\ 1\ -4]$

$y_3 = y_1$ (also verified with MATLAB)

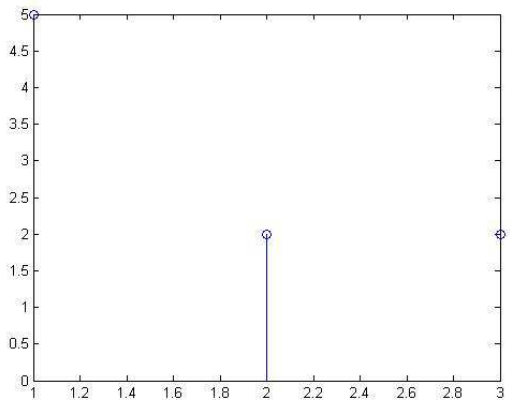
d) $y = 2\ 3\ 5\ 7\ -1\ 4$
$-2\ -3\ -5\ -7\ 1\ -4$
$y[n] = [2\ 3\ 5\ 7\ -3\ 1\ 5\ 7\ -1\ 4]$

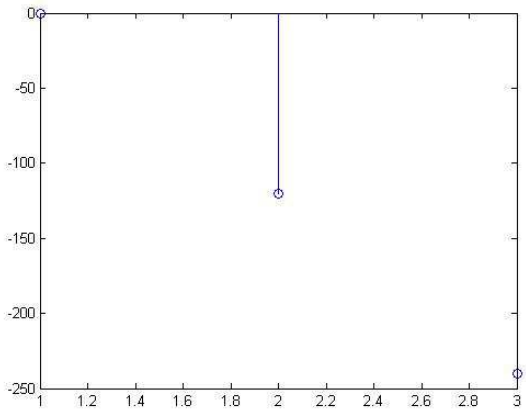
APPENDIX:

2)d) MATLAB code for 3-point DFT:

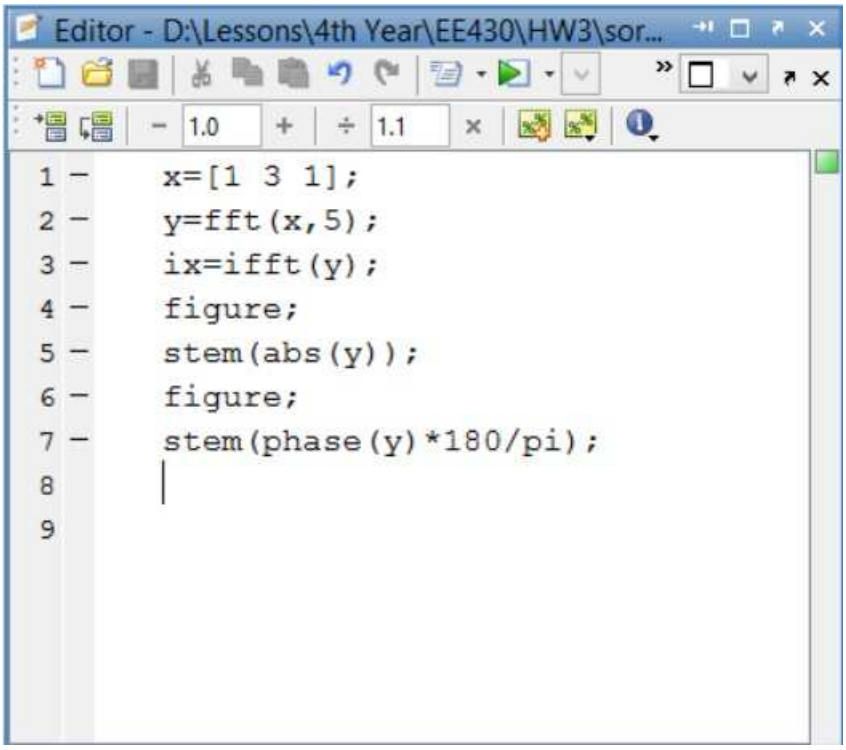


Magnitude and phase(degree) of X[k], respectively:

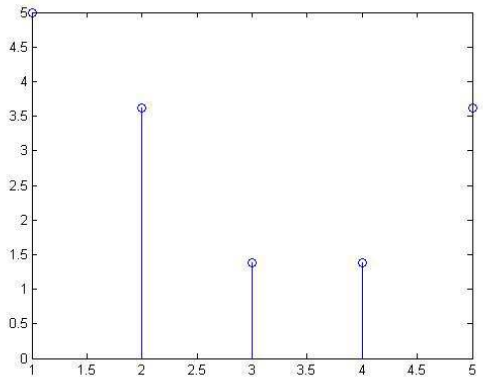


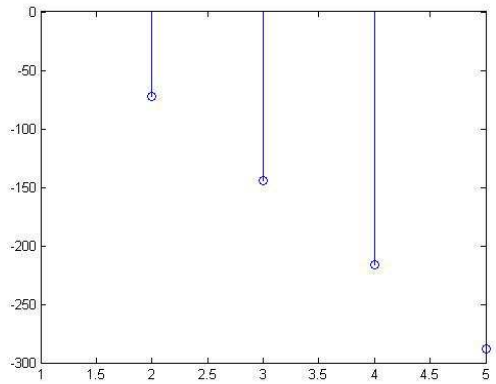


MATLAB code for 5-point DFT:



Magnitude and phase(degree) of X[k], respectively:





8)c) MATLAB code for obtaining the outputs corresponding to each input segment:

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```
x1=[1 2 3 4];
x2=[-1 -2 -3 -4];
x3=x1;
h=[2 -1 1];
X1=fft(x1,6);
X2=fft(x2,6);
X3=fft(x3,6);
H=fft(h,6);
Y1=X1.*H;
Y2=X2.*H;
Y3=X3.*H;
y1=ifft(Y1);
y2=ifft(Y2);
y3=ifft(Y3);
stem(y1,'color','red');
hold on;
stem(y2,'color','blue');
hold on;
stem(y3,'color','green');
```

Outputs, y1 is red, y2 is green and y3 is blue: (since y1=y3 red line is not seen)