Aykut DEMİREC
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Section: 2

EE430 - HW3

2) a)

$$x[n] = \delta[n] + 3\delta[n-1] + \delta[n-2]$$

$$X_3[k] = \sum_{n=0}^{2} x[n] e^{-j\frac{2\pi}{3}kn} = 1 + 3e^{-j\frac{2\pi}{3}k} + e^{-j\frac{4\pi}{3}k} \quad , \text{for } k=0,1,2$$

$$0 \text{ , elsewhere}$$

$$X_5[k] = \sum_{n=0}^{4} x[n] e^{-j\frac{2\pi}{5}kn} = 1 + 3e^{-j\frac{2\pi}{5}k} + e^{-j\frac{4\pi}{5}k} \quad , \text{for } k=0,1,2,3,4$$

$$0 \text{ , elsewhere}$$

b)

$$\tilde{X}_3[k] = \sum_{n=-\infty}^{\infty} X_3[k-3n] = X_3[((k))_3]$$

$$\tilde{X}_5[k] = \sum_{n=-\infty}^{\infty} X_5[k-5n] = X_5[((k))_5]$$

c)

$$x[n] = \begin{cases} \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn} & , n=0,1,2 \ldots N-1 \\ 0 & , \text{else} \end{cases}$$

$$\Rightarrow x[n] = \frac{1}{3}\sum_{k=0}^{2} X[k] e^{j\frac{2\pi}{3}kn} \quad , \text{for } n=0,1,2$$

$$0 \text{ , else}$$

$$= \frac{1}{3}\sum_{k=0}^{2} \left(1 + 3e^{-j\frac{2\pi}{3}k} + e^{-j\frac{4\pi}{3}k}\right) e^{j\frac{2\pi}{3}kn}$$

$$= \frac{1}{3}\sum_{k=0}^{2} e^{j\frac{2\pi}{3}kn} + 3e^{j\frac{2\pi}{3}k(n-1)} + e^{j\frac{2\pi}{3}k(n-2)}$$

$$= \frac{1}{3}\left[X_3[0] + X_3[1] e^{j\frac{2\pi}{3}n} + X_3[2] e^{j\frac{4\pi}{3}n}\right] \quad , \text{for } n=0,1,2$$

$$0 \text{ , else}$$

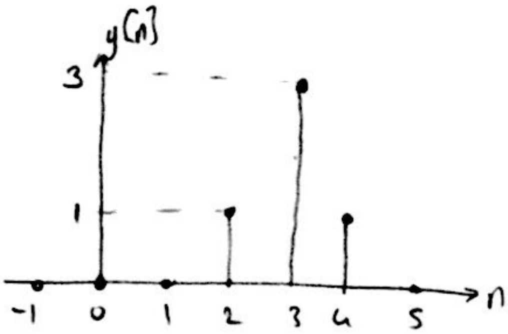
in the same way.: $x[n] = \frac{1}{5}\left[X_5[0] + X_5[1] e^{j\frac{2\pi}{5}n} + X_5[2] . e^{j\frac{4\pi}{5}n} + X_5[3] e^{j\frac{6\pi}{5}n} + X_5[4] . e^{j\frac{8\pi}{5}n}\right]$

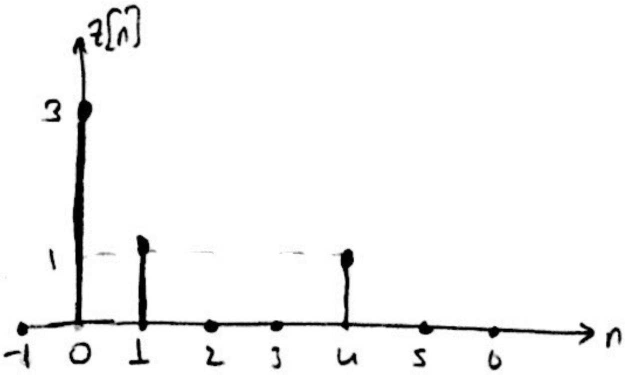
for $n=0,1,2,3,4$

0 , else

d) Matlab

3) a)





b) $y[n] = x[n-2]$ $\quad$ $z[n] = x[((n+1))_5]$

c) $Y[k] = X[k] \cdot e^{-j\frac{4\pi}{5}k}$ $\Rightarrow$ $y[n] = x[n-2] = x[((n-2))_5]$ for this case

$Z[k] = X[k]\, e^{j\frac{2\pi}{5}k}$

$z[n]$ and $y[n]$ do not have 3-point DFTs since they are length of 5.

6) $x[n] = 3\delta[n] - 2\delta[n-1] + \delta[n-2] + \delta[n-3] - 2\delta[n-4] - \delta[n-5]$

a) $W_3[k] = X(e^{j\omega})\Big|_{\omega = \frac{2\pi k}{3}}$ $k = 0,1,2$ $\Rightarrow$ $W_3[k]$ is 3 point DFT of $x[n]$ but $x[n]$ is of 6 in length.

$\Rightarrow$ we find $w[n]$ by overlaping the remaining 3 value.

that is;

$$\left.\begin{array}{l} w[0] = x[0] + x[3] \\ w[1] = x[1] + x[4] \\ w[2] = x[2] + x[5] \\ w[n] = 0 \quad \text{else} \end{array}\right\} \quad w[n] = 4\delta[n] - 4\delta[n-1]$$

b) in a similar way: $w[n] = 2\delta[n] - 2\delta[n-1] + \delta[n-2] + \delta[n-3] - 2\delta[n-4]$

c) $w[n] = x[n] = 3\delta[n] - 2\delta[n-1] + \delta[n-2] + \delta[n-3] - 2\delta[n-4] - \delta[n-5]$

d) $h[n] = 2\delta[n] - \delta[n-1]$

$y_3[k] = H_3[k] W_3[k] \Rightarrow y[n] = h[n] ⊛ w[n] \Rightarrow$

$$\begin{array}{r} [4 \ -4 \ 0] \\ [2 \ -1 \ 0] \end{array} ⊛ \quad \begin{array}{r} 8 \ -8 \ 0 \\ 0 \ -4 \ 4 \\ + \ 0 \ 0 \ 0 \\ \hline \boxed{8 \ -12 \ 4} \end{array} \leftarrow y[n]$$

$$\boxed{\Rightarrow y[n] = 8\delta[n] - 12\delta[n-1] + 4\delta[n-2]}$$

e) $w[n] = [2 \ -2 \ 1 \ 1 \ -2]$
$h[n] = [2 \ -1 \ 0 \ 0 \ 0]$ ⊛

$$\begin{array}{r} 4 \ -4 \ 2 \ 2 \ -4 \\ + \ 2 \ -2 \ 2 \ -1 \ -1 \\ \hline 6 \ -6 \ 4 \ 1 \ -5 \end{array}$$

$$\boxed{\Rightarrow y[n] = 6\delta[n] - 6\delta[n-1] + 4\delta[n-2] + \delta[n-3] - 5\delta[n-4]}$$

f) $w[n] = [3 \ -2 \ 1 \ 1 \ -2 \ -1 \ 0 \ 0]$
$h[n] = [2 \ -1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$ ⊛

$$\begin{array}{r} 6 \ -4 \ 2 \ 2 \ -4 \ -2 \ 0 \ 0 \\ + \ 0 \ -3 \ 2 \ -1 \ -1 \ 2 \ 1 \ 0 \\ \hline 6 \ -7 \ 4 \ 1 \ -5 \ 0 \ 1 \ 0 \end{array}$$

$$\boxed{\Rightarrow y[n] = 6\delta[n] - 7\delta[n-1] + 4\delta[n-2] + \delta[n-3] - 5\delta[n-4] + \delta[n-6]}$$

g) $x[n]$ is of length 6 and $h[n]$ is of length 2

$\Rightarrow$ N-point circular convolution = linear convolution ; if $N \geq 7$

$\Rightarrow y[n] = y_8[n] \neq y_3[n] \neq y_5[n]$

7) a) $X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}$; $k = 0, 1, 2 \ldots N-1$ where $W_N = e^{-j\frac{2\pi}{N}}$

$$= \sum_{n=\text{even}} x[n] W_N^{kn} + \sum_{n=\text{odd}} x[n] W_N^{kn} = \sum_{m=0}^{\frac{N}{2}-1} \underbrace{x[2m]}_{e[n]} W_N^{2mk} + \sum_{m=0}^{\frac{N}{2}-1} \underbrace{x[2m+1]}_{o[n]} W_N^{(2m+1)k}$$

$$= \underbrace{\sum_{m=0}^{\frac{N}{2}-1} e[n] W_{\frac{N}{2}}^{mk}}_{\frac{N}{2}\text{ point DFT of } e[n]} + W_N^k \underbrace{\sum_{m=0}^{\frac{N}{2}-1} o[n] W_{\frac{N}{2}}^{mk}}_{\frac{N}{2}\text{ point DFT of } o[n]} \quad \Big\} \text{ here we used } W_N^{2km} = W_{\frac{N}{2}}^{km}$$

$$\Rightarrow X[k] = E\left[((k))_{\frac{N}{2}}\right] + e^{-j\frac{2\pi}{N}k} O\left[((k))_{\frac{N}{2}}\right] \ ; \text{ for } \underline{k = 0, 1, 2 \ldots N-1}$$

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b)

i) $X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn} = \sum_{n=0}^{N-1} x[n]\left(\cos\left(\frac{2\pi}{N}kn\right) - j\sin\left(\frac{2\pi}{N}kn\right)\right)$ } here we are given that $x[n]$ is a real sequence

Direct computation involves $2N$ real multiplications and $2N-2$ real additions for each k.

$\Rightarrow$ For all values of $X[k]$; direct computation involves $2N^2$ real multiplications and $2N^2-2N$ real additions.

ii)

$X[k] = E[((k))_{\frac{N}{2}}] + W_N^k\, O[((k))_{\frac{N}{2}}]$ we know that $E[k] = E[k+\frac{N}{2}]$, $O[k] = O[k+\frac{N}{2}]$ } periodicity with $\frac{N}{2}$.

and we also know that $W_N^{k+\frac{N}{2}} = -W_N^k$

$\Rightarrow$ we can divide $X[k]$ into two as follows:

$$X[k] = E[k] + W_N^k O[k] \quad \text{for } k=0,1,2 \ldots \tfrac{N}{2}-1$$

$$X[k+\tfrac{N}{2}] = E[k] - W_N^k O[k] \quad \text{for } k=0,1,2 \ldots \tfrac{N}{2}-1$$

$\Rightarrow$ We need to do $2\left(\frac{N}{2}\right)^2$ real multiplications and $2\left(\frac{N}{2}\right)^2 - 2\left(\frac{N}{2}\right)$ real additions for $E[k]$ and $O[k]$. Also $W_N^k . O[k]$ involves $4(N)$ real multiplications and $2(N)$ real additions. To find $E[k] + W_N^k O[k]$, we also need to do $2(N)$ real additions.

$\boxed{\text{Method is called Split-Radix FFT algorithm}}$

} Computation involves total of $2\left(\frac{N}{2}\right)^2 + 2\left(\frac{N}{2}\right)^2 + 4(N) = N^2 + 4N$ real mults. and $N^2 + 2N$ real additions.

iii) Second computation method is better especially for large N's (Includes less comps)

8) $h[n] = 2\delta[n] - \delta[n-1] + \delta[n-2]$ $\quad x[n] = [1\ 2\ 3\ 4\ -1\ -2\ -3\ -4\ 1\ 2\ 3\ 4]$ $\quad 0 \le n < 11$

a) $P=3$ $\quad L=4$ $\Rightarrow$ $N = 3+4-1 = 6$-point DFT, we will use.

b) input is length of 12 $\Rightarrow$ there are 3 input segments $\Rightarrow$ $x_1 = [1\ 2\ 3\ 4]$

$x_2 = [-1\ -2\ -3\ -4] = -x_1$

$x_3 = [1\ 2\ 3\ 4] = x_1$

c) $y_1 = [1\ 2\ 3\ 4] \circledast [2\ -1\ 1\ ] = [2\ 3\ 5\ 7\ -1\ 4]$

```
  1  2  3  4
  2 -1  1
-------------
  2  4  6  8
    -1 -2 -3 -4
        1  2  3  4
------------------
  2  3  5  7 -1  4
```

$\Rightarrow y_2 = [-2\ -3\ -5\ -7\ 1\ -4]$

$y_3 = y_1$ (also verified with MATLAB)

d)

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y =  2  3  5  7 -1  4
                -2 -3 -5 -7  1 -4
                             2  3  5  7 -1  4
```

$y[n] = [2\ 3\ 5\ 7\ -3\ 1$ [illegible]$]$