Due Date: 21 November 2014, Friday (17:00).

Homework 3

1) Suppose we have two 4-point sequences $x[n]$ and $h[n]$ as follows,

$x[n]=\sin(\pi n/2)$, $n=0,1,2,3$. $h[n]=2^n$ $n=0,1,2,3$.

a.) Calculate the 4-point DFTs, $X[k]$. & $H[k]$
b.) Calculate $y[n]=x[n]$ ④ $h[n]$ by computing the circular convolution directly.
c.) Calculate $y[n]$ of part-b by multiplying the DFT's of $x[n]$ and $h[n]$, and performing an inverse DFT.
d.) We need to find the linear convolution x[n]*h[n], by using the DFT's of the x[n] and h[n], (not necessarily the 4-point). Explain how this can be obtained.

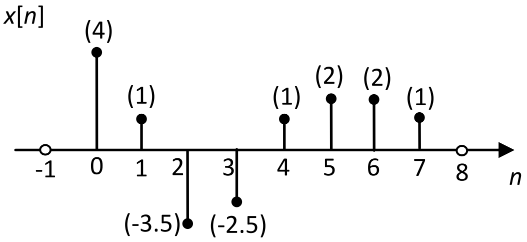
2) $x[n]$ is a DT sequence which is nonzero for $0 \le n \le 7$ and it is given as below, where $X[k]$ is the 8-point DFT of $x[n]$.

a) $Y[k]=e^{j\frac{2\pi}{8}3k}X[k]$ is given. Find the 8-point inverse DFT (IDFT) of $Y[k]$ and write the sample values of the sequence $y[n]$.

b) $G_1[k]=X[2k]$, $0 \le k \le 3$ is given. Find the 4-point IDFTof $G_1[k]$ and write the sample values of the sequence $g_1[n]$.

c) $G_2[k]=X[k]-(-1)^k X[((k+2))_8]$, $0 \le k \le 7$ is given. Find the 8-point IDFT of $G_2[k]$ and express $g_2[n]$ in terms of $x[n]$.

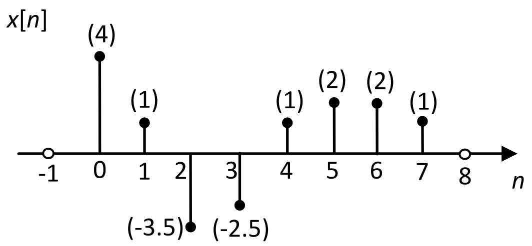
d) $h[n]$ is a 3-point sequence which is nonzero for $0 \le n \le 2$. $g[n]$ is the 8-point circular convolution of $x[n]$ and $h[n]$. How many samples of $g[n]$ would be the same as the samples of linear convolution of $h[n]$ and $x[n]$?

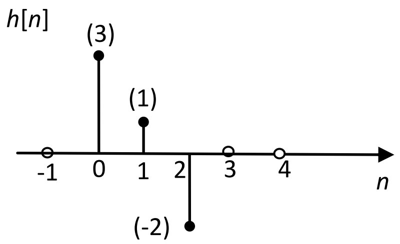


3) Prove the following formula for N-point signals $x[n]$, y[n] and their N-point DFT $X[k]$,Y[k] (namely *Parseval relation* for DFT).

$$\sum_{n=0}^{N-1} x[n]y^*[n]=\frac{1}{N}\sum_{k=0}^{N-1} X[k]Y^*[k]$$

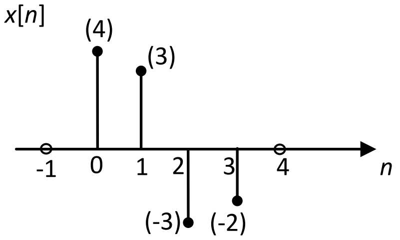
4) Given x[n] and h[n] as below,





a)Apply OLA method when L=5,P=3 and find the linear convolution of x[n] and h[n].
b)Apply OLS method when L=5,P=3 and find the linear convolution of x[n] and h[n].

5) Given the following 4-point sequence,

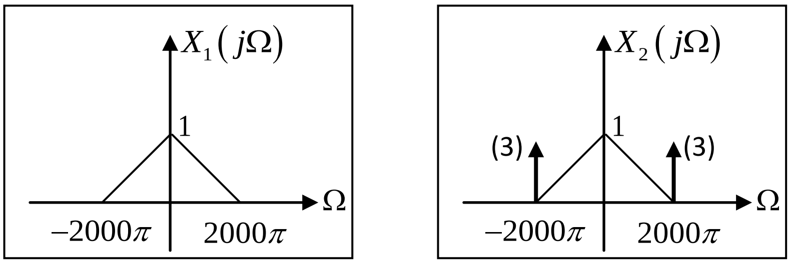


a) Sketch the sequence $x_1[n]=x[((n-2))_4],\ 0\le n\le 3$

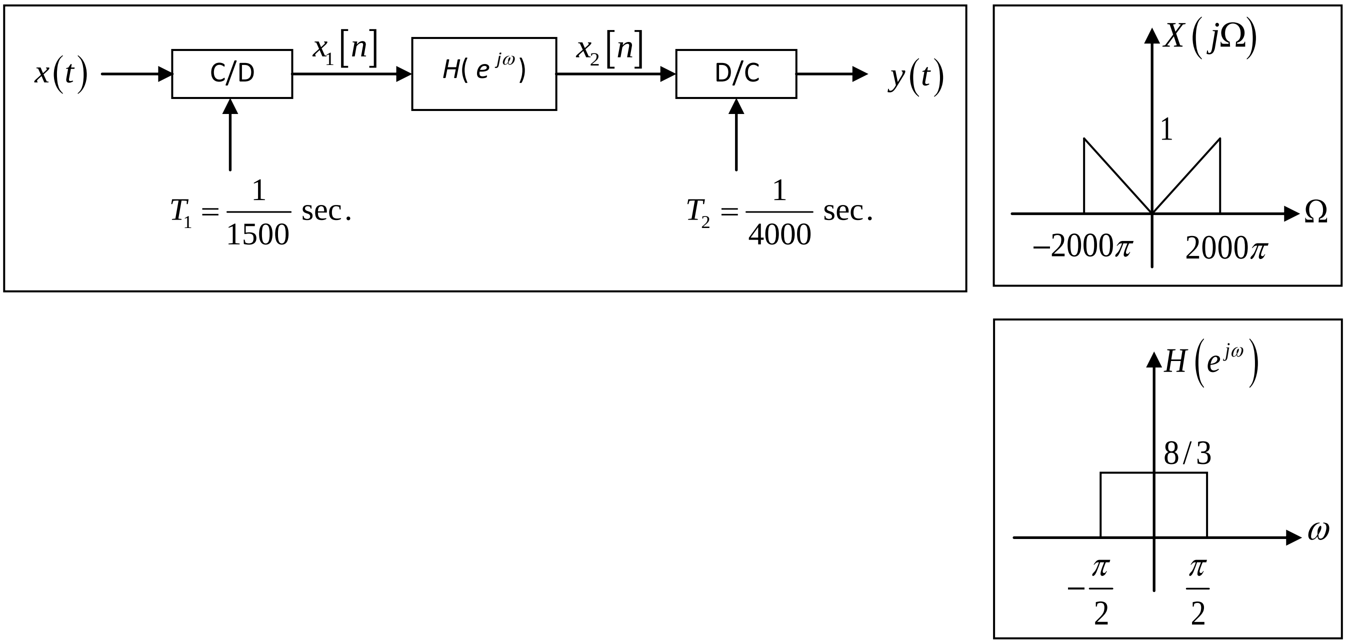
b) Sketch the sequence $x_2[n]=x[((-n))_4],\ 0\le n\le 3$

6)

a - Define *Nyquist frequency*. Considering the signals shown below, what are the minimum sampling frequencies for these signals so that they can be perfectly reconstructed by bandlimited interpolation?



b - Consider the system below. The Fourier transform of the input, *x*(*t*), and the frequency response, $H(e^{j\omega})$, of the discrete-time LTI filter are also shown. Find and plot $X_1(e^{j\omega})$, $X_2(e^{j\omega})$ and $Y(j\Omega)$. Show all your work clearly.

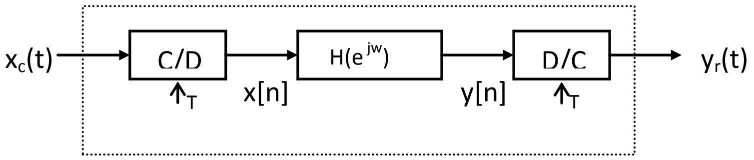


7) Given the system below, the frequency response of the digital filter is equal to $H(e^{jw})=1-e^{-j5w}$. Assume the input is given to the system, as $x_c(t)=2+3\cos(\Omega_1 t)+4\sin(\Omega_2 t)$ for $-\infty<t<\infty$.

a) Given $\Omega_1=200\pi$ and $\Omega_2=440\pi$, state and explain the Nyquist rate of this signal

b) Let the sampling rate be 1000Hz. Given $\Omega_1=500\pi$ and $\Omega_2=1500\pi$, sketch $|X(e^{jw})|$, $|Y(e^{jw})|$ and $|Y_r(j\Omega)|$

c) For the same sampling rate 1000 Hz, now, let Ω_1 and Ω_2 to be unknown and not equal to each other. Determine non-zero analog frequencies for Ω_1 and Ω_2 which causes output signal to be zero for the input $x_c(t)$. Pick Ω_1 and Ω_2, so that there is no aliasing.



MATLAB PART

$$X(k)=\sum_{n=0}^{N-1} x[n]W_N^{nk}, \qquad W_N = e^{-j2\pi/N} \quad k=0,1,..N-1$$

$$x[n]=\frac{1}{N}\sum_{k=0}^{N-1} X(k)W_N^{-nk}, \qquad n=0,1,..N-1$$

1) Write MATLAB code for the DFT equations. Then find and plot the N-point DFT's of the following signals, (N=32) (you are expected to plot the magnitude and phase separately)

a) $x[n]=\delta[n]$, (or x=[1 0 0 0 0 0 0 … 0])

b) x[n]=u[n]-u[n-N] (or x=[1 1 1 1 … 1])

c)$x[n]=\mathrm{Sin}(\pi n/5+\pi/8)$, n=0,1…N-1

d) $x[n]=\mathrm{Sin}((n-N/2)\,\pi/8)/(\pi(n-N/2))$, n=0,1,…N-1

2) Now, compare the N-point DFT's found in the first part with the results of "fft" command in MATLAB, i.e. y=fft(x,N). Are they the same? Use "tic" and "toc" commands to compare the implementation time of your DFT code with that of fft. Comment on the results.