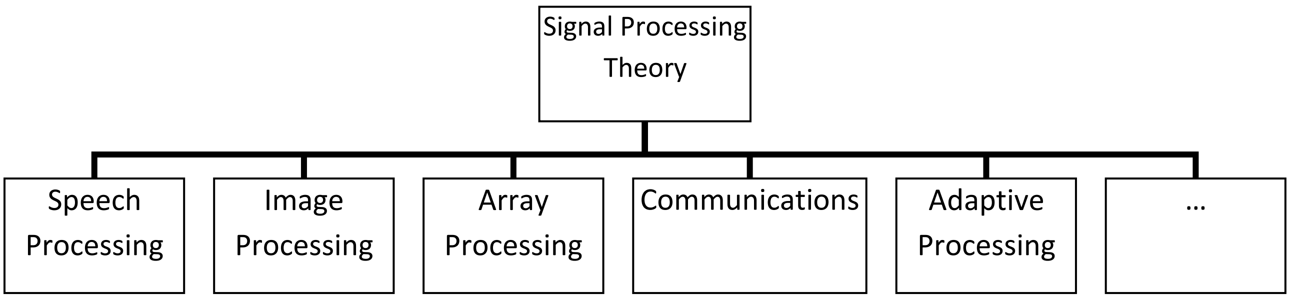
EE430 Digital Signal Processing

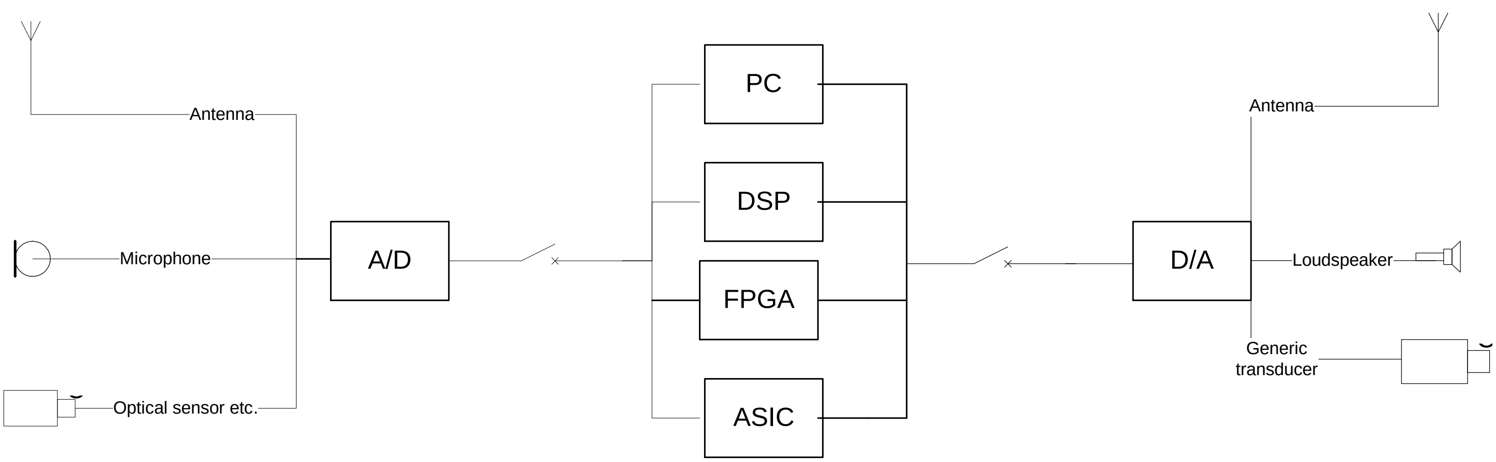
- Signal processing is concerned with the representation, transformation and manipulation of signals as well as the information they contain.



Applications: Radar, sonar, acoustics, robotics, seismology, electronics etc.

Reference for DSP Fields: IEEE Transactions on Signal Processing

Goal of the Course: Learn about the fundamentals of DSP, to be able to use different tools of DSP in different applications.



A generic structure for the implementation of a digital signal processing system

Advantages of digital systems:

a) Better accuracy
b) Identical blocks, circuits
c) Easy storage without loss (except A/D converters)
d) Low cost?

Main disadvantage is the speed of A/D converters and DSP units (BW < 100 MHz)

History of DSP:

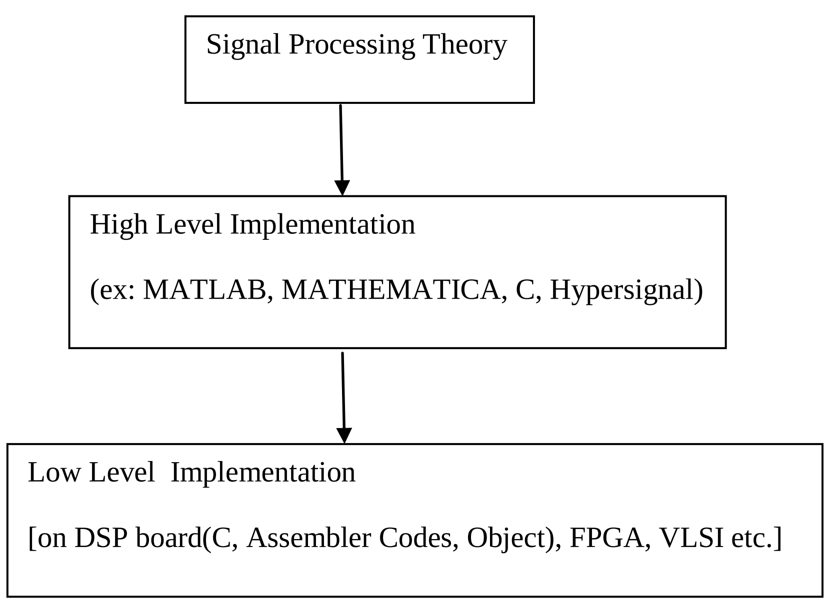
During 1960's, computers proved to be useful in signal processing. FFT is rediscovered. (Gauss laid out the fundamentals of FFT algorithm in 1805. Cooley-Tukey rediscovered it) Digital computers efficiently implemented the signal processing algorithms using FFT.

The rapid development of integrated circuit technology lead to the development of powerful, smaller, faster and cheaper computers and special-purpose digital hardware (FPGA). These systems are capable of performing complex digital signal processing tasks which are hard or difficult for analog systems.

Limitations of Analog Signal Processing:

a) Accuracy : Not accurate enough, low precision
b) Cannot build all identical circuits due to drift and aging
c) Diagnostics is comparably harder

Digital Signal Processing Layers:



Signals, Systems and Signal Processing

A signal is a physical quantity that varies with time, space etc. A signal is a function of one or more variables.

Signal Types

Analog Signals
(continuous-time signals)

CT

(cont. amp – cont. time)

Discrete-time signals

DT

(cont. amp. – discrete time)

Digital signals

(discrete amp. – discrete time)

Further Classification of Signals:

Multichannel Signals:

$$s(t) = \begin{pmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{pmatrix}$$ signals coming from an array of receivers

Multidimensional Signals:

$s(t) = s_1(t_1, t_2)$ image signal

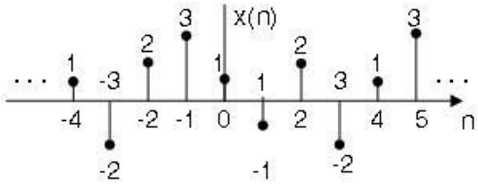
Deterministic Signals:

Any signal that can be uniquely described by an explicit mathematical expression, a table or a rule is called deterministic.

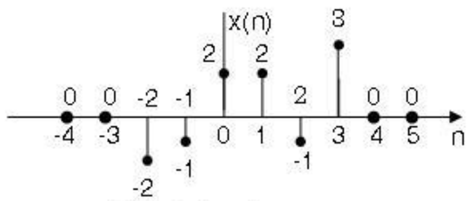
Random Signals:

Signals that evolve in time according to a statistical law or in an unpredictable manner.

Discrete-time Signal (Sequence)



(a) Infinite duration



(b) Finite duration

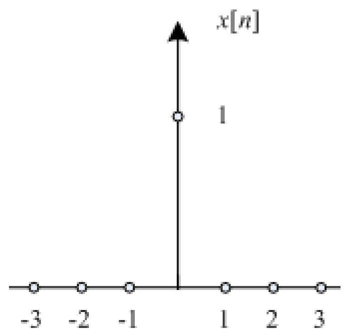
x[n] is a sequence of numbers which constitute discrete-time signal at different times. n is the address where the value *x[n]* is stored in the computer and it should be integer. Noninteger indexed values are not defined.

- Discrete-time signals can be real or complex.
- Discrete-time signal processing covers digital signal processing. In digital signal processing all the operations are done digitally and the results are digital numbers.
- For a digital signal, the numbers are represented by only finite number of bits. Therefore there are finite number of permissible values for *x[n]*.
- For discrete-time signals, *x[n]* can have continuous amplitude values in discrete time.
  Ex: CCD charged-coupled devices.

Certain DT Sequences

Unit-sample sequence:

$$\delta[n] = \begin{Bmatrix} 0, n \neq 0 \\ 1, n = 0 \end{Bmatrix}$$

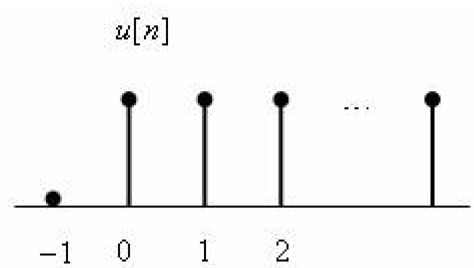


Unit sample sequence is important since any discrete-time sequence can be represented by unit-sample sequence. It does not suffer from the mathematical complications of the continuous-time impulse or (Dirac delta function)

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Unit-step sequence:

$$u[n] = \begin{Bmatrix} 1, n \geq 0 \\ 0, n < 0 \end{Bmatrix}$$



$$u[n] = \sum_{k=-\infty}^{n} \delta[k] = \sum_{k=0}^{\infty} \delta[n-k]$$

$$\delta[n] = u[n] - u[n-1]$$

Exponential Sequences:

$$x[n] = A\alpha^n$$

- If A and ∝ are real, then *x[n]* is real.
- If |∝|<1, then $\begin{Bmatrix} 0 < \alpha < 1 \rightarrow x[n] \text{ decreases in time} \\ -1 < \alpha < 0 \rightarrow x[n] \text{ decreases in time with alternating sign} \end{Bmatrix}$
- If A and ∝ are complex,

$$\alpha = |\alpha| e^{jw_0}, A = |A| e^{j\phi}$$

$$x[n] = |A||\alpha|^n \, e^{j(\underbrace{w_0 n}_{\text{Frequency}} + \underbrace{\phi}_{\text{phase}})}$$

$$x[n] = |A||\alpha|^n \, [\cos(w_0 n + \phi) + j\sin(w_0 n + \phi)]$$

$$\text{Euler's relation: } e^{j\theta} = \cos\theta + j\sin\theta$$

- $|\propto|=1 \rightarrow x[n]$ is a complex exponential sequence

Complex exponentials (or sinusoids)

$$x[n] = Ae^{jw_0 n} \qquad \text{(real A)}$$

- Periodicity of complex exponentials in frequency

$$x[n] = Ae^{j(w_0+2\pi)n} = Ae^{jw_0 n}\underbrace{e^{jw_0 2\pi n}}_{1} = Ae^{jw_0 n}$$

$$e^{j2\pi n} = \cos(2\pi n) + j\sin(2\pi n) = 1$$

$$e^{j(2n+1)\pi} = -1$$

$$e^{j\frac{\pi}{2}} = j$$

$$e^{j\frac{3\pi}{2}} = -j$$

- Periodicity in time.

$$x[n] = x[n+N], \qquad \forall n$$

$$Ae^{jw_0 n} = Ae^{jw_0(n+N)} = Ae^{jw_0 n}\underbrace{e^{jw_0 N}}_{1}$$

$$e^{jw_0 N} = 1$$

$$\rightarrow w_0 N = 2\pi k$$

$$\rightarrow \frac{2\pi}{w_0} = \underbrace{\frac{N}{k}}_{\text{rational}} \qquad \text{requirement for periodicity in time}$$

- There are only N distinct periodic complex exponentials which are periodic by N. They are called as <u>harmonically related</u> complex exponentials.

$$\underbrace{s_k[n]}_{k^{th}\ \text{harmonic}} = e^{j\frac{2\pi}{N}kn}, \quad k = 0,1,2,\ldots,N-1$$

Fundamental period: N

Fundamental frequency: $f_0 = \frac{1}{N}$

Let x[n] be a periodic sequence with period N,

$$x[n]=\sum_{k=0}^{N-1} c_k e^{j\frac{2\pi}{N}kn} \quad \rightarrow \text{Fourier series representation}$$

Examples:

$$x[n]=\sin(\underbrace{5\pi n}_{w_0 n}), \qquad \frac{2\pi}{N}kn=5\pi n$$

$$w_0=5\pi \qquad \frac{2}{N}k=5 \quad \rightarrow N=\frac{2}{5}k \quad \rightarrow N \text{ should be integer so}$$

$$k=5,10,15...$$

$$\frac{2\pi}{w_0}=\frac{2\pi}{5\pi}=\frac{2}{5} \quad \text{is rational}$$

fundamental period: <u>N=2</u>

$$x[n]=\sin(2n), \quad \frac{2\pi}{2}=\pi \quad \text{not rational, N=?}$$

Even & Odd (Conjugate Symmetric, Conjugate Anti-symmetric) Sequences

Even(Conjugate Symmetric) Sequence:

$$x[n]=x^*[-n]$$

Odd Sequence:

$$x[n]=-x^*[-n], \qquad x[n]\xleftrightarrow{DTFT} X(e^{jw})$$

$$x[n]=x_e[n]+x_o[n\}, \qquad X(e^{jw})=X_R(e^{jw})+jX_I(e^{jw})$$

$$x_e[n]=\frac{1}{2}\left[x[n]+x^*[-n]\right]\xleftrightarrow{DTFT} X_R(e^{jw})=\frac{1}{2}\left[X[e^{jw}]+X^*[e^{jw}]\right]$$

$$x_o[n]=\frac{1}{2}\left[x[n]-x^*[-n]\right]\xleftrightarrow{DTFT} jX_I(e^{jw})=\frac{1}{2}\left[X[e^{jw}]-X^*[e^{jw}]\right]$$

Energy and Power Signals:

Energy of a signal,

$$E=\sum_{n=-\infty}^{\infty}|x[n]|^2 \qquad \text{(real or complex signals)}$$

If E is finite, then x[n] is an energy signal.

Average power,

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Define

$$E_N = \sum_{n=-N}^{N} |x[n]|^2$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} E_N$$

- If E is finite, P = 0
- If E is infinite, P may be finite or infinite.
- If P is finite, x[n] is a power signal.

Transformation of the independent variable (time):

Time-shift:

$$x[n] \to x[n-k] = \begin{cases} k \text{ positive } \to \text{ time delay} \\ k \text{ negative } \to \text{ time advance} \end{cases}$$

$$x[n] \to \boxed{z^{-1}} \to x[n-1] \qquad \text{unit delay}$$

Time reversal:

$$FD\{x[n]\} = x[-n] \qquad \text{time reversal}$$

$$TD_k\{x[n]\} = x[n-k] \qquad \text{k>0 time delay}$$

$$\left.\begin{aligned} TD_k\{FD\{x[n]\}\} &= TD_k\{x[-n]\} = x[-n+k] \\ FD\{TD_k\{x[n]\}\} &= FD\{x[n-k]\} = x[-n-k] \end{aligned}\right\} \text{Not commutative}$$