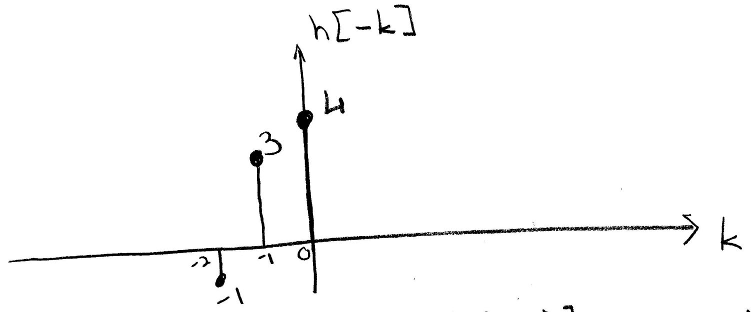
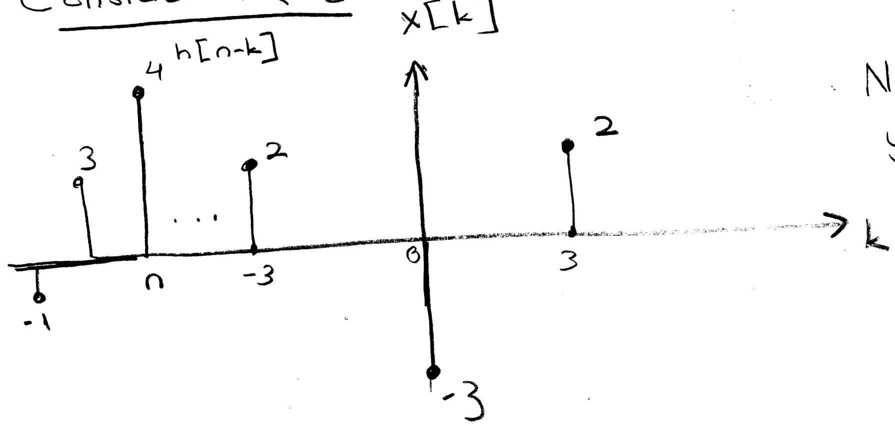
EE 430
Fall 2014
HW 2 (Sec. 1 and 3)
Solutions

(1) $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n] * h[n]$



$h[n-k] = h[-(k-n)] \rightarrow$ shift to the right.

Consider $n < -3$



No overlapping
$y[n] = 0$ for $n < -3$

$-3 \leq n \leq -1$

$y[-3] = 4 \times 2 = 8$

$y[-2] = 3 \times 2 = 6$

$y[-1] = -1 \times 2 = -2$

$y[0] = 4 \times -3 = -12$

$y[1] = 3 \times -3 = -9$

$y[2] = -1 \times -3 = 3$

$y[3] = 4 \times 2 = 8$

$y[4] = 3 \times 2 = 6$

$y[5] = -1 \times 2 = -2$

$y[n] = 0$ for $n > 5$

$y[n] = [8 \ \ 6 \ \ -2 \ \ -12 \ \ -9 \ \ 3 \ \ 8 \ \ 6 \ \ -2]$

$\uparrow$
$n=-3$

(2) a) A finite impulse response (FIR) system is the system whose impulse response is of finite duration. A infinite impulse response (IIR) system has an impulse response with infinite duration. For a FIR causal system the following difference equation can be given,

$$y[n] = \sum_{k=0}^{M} \left(\frac{b_k}{a_0}\right) x[n-k]$$

Here we need no feedback $(y[n-k], k \geq 1)$
For a given input $y[n]$ can be directly found from $x[n]$.

<u>Ex</u> $h[n] = \frac{1}{2}\delta[n] + \frac{1}{4}\delta[n-2]$

It is a FIR system. The following difference equation describes the system relation,

$$y[n] = \frac{1}{2}x[n] + \frac{1}{4}x[n-2]$$

For a IIR system, both feedback and input terms occur, i.e,

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{m=0}^{M} b_m x[n-m]$$

<u>Ex</u> $h[n] = \left(\frac{1}{2}\right)^n u[n]$

It is a IIR system. $H(e^{jw})$ exists and $H(e^{jw}) = \frac{1}{1-\frac{1}{2}e^{jw}}$

$$\frac{Y(e^{jw})}{X(e^{jw})} = H(e^{jw}) = \frac{1}{1-\frac{1}{2}e^{-jw}}$$

$$\Downarrow$$

$$Y(e^{jw}) - \frac{1}{2}Y(e^{jw})e^{-jw} = X(e^{jw})$$

$\Downarrow$ inverse DTFT

$$y[n] - \frac{1}{2}y[n-1] = x[n].$$

We can find $y[n]$ recursively if we work in time domain

b) $y[n] = \frac{3}{4} y[n-1] - \frac{1}{8} y[n-2] + x[n]$

i) Initial rest conditions should be satisfied

$\equiv$ If the input $x[n]$ is zero for n less than some time n_0, then the output $y[n]$ is constrained to be zero for n less than n_0.

ii) Since we don't know whether the system is stable or not consider z transform.

$$Y(z) - \frac{3}{4} Y(z) z^{-1} + \frac{1}{8} Y(z) z^{-2} = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}} = \frac{1}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})}$$

$$= \frac{A}{1-\frac{1}{4}z^{-1}} + \frac{B}{1-\frac{1}{2}z^{-1}}$$

$A + B = 1$
$\frac{-A}{2} - \frac{B}{4} = 0$ $\Rightarrow$ $B = 2$, $A = -1$

$|z| > \frac{1}{2}$ (since the system is causal)

$$h[n] = -\left(\frac{1}{4}\right)^n u[n] + 2\left(\frac{1}{2}\right)^n u[n]$$

iii) $y[n] = y_h[n] + y_p[n]$

$y_h[n] = A_1 z_1^n + A_2 z_2^n$

where z_1 and z_2 are the roots of the

$$1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2} = 0.$$

$$\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{2} z^{-1}\right) \Rightarrow z_1 = \frac{1}{4}, \quad z_2 = \frac{1}{2}$$

$$y_h[n] = A_1\left(\frac{1}{4}\right)^n + A_2\left(\frac{1}{2}\right)^n$$

Consider $y_p[n]$ which is the unit step response of $h[n]$.

FIRST APPROACH

$$y_p[n] = h[n] * u[n]$$
$$= \sum_{m=0}^{\infty} h[m]u[n-m]$$
$$= \sum_{m=0}^{n} h[m] \qquad n \geq 0 \qquad \text{o.w } y_p[n]=0$$

$$y_p[n] = \sum_{m=0}^{n}\left(-\left(\frac{1}{4}\right)^m + 2\left(\frac{1}{2}\right)^m\right) \qquad n \geq 0$$

$$= \left[-\frac{1-\left(\frac{1}{4}\right)^{n+1}}{1-\frac{1}{4}} + 2\,\frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}\right]u[n]$$

$$= \left[\frac{-4}{3} + \frac{4}{3}\left(\frac{1}{4}\right)^{n+1} + 4 - 4\left(\frac{1}{2}\right)^{n+1}\right]u[n]$$

$$= \frac{8}{3}u[n] + \frac{1}{3}\left(\frac{1}{4}\right)^{n}u[n] - 2\left(\frac{1}{2}\right)^{n}u[n]$$

SECOND APPROACH

$$Y_p(e^{jw}) = U(e^{jw})H(e^{jw})$$

$$= \frac{1}{\left(1-\frac{1}{4}e^{-jw}\right)\left(1-\frac{1}{2}e^{-jw}\right)}\underbrace{\left[\frac{1}{1-e^{-jw}} + \sum_{k=-\infty}^{\infty}\pi\delta(w+2\pi k)\right]}_{U(e^{jw})}$$

$$= \frac{A}{1-\frac{1}{4}e^{-jw}} + \frac{B}{1-\frac{1}{2}e^{-jw}} + \frac{C}{1-e^{-jw}} + \sum_{k=-\infty}^{\infty}\pi H(e^{j(-2\pi k)})\delta(w+2\pi k)$$

$$H(e^{j(-2\pi k)}) = \frac{1}{\left(1-\frac{1}{4}\right)\left(1-\frac{1}{2}\right)} = \frac{8}{3}$$

$$A+B+C=1$$

$$\left(-1-\tfrac{1}{2}\right)A+\left(-1-\tfrac{1}{4}\right)B+\left(\tfrac{-1}{4}-\tfrac{1}{2}\right)C=0$$

$$\frac{1}{2}A+\frac{1}{4}B+\frac{1}{8}C=0$$

$$A+B+C=1$$

$$\frac{-3}{2}A-\frac{5}{4}B-\frac{3}{4}C=0$$

$$3A+\frac{6}{4}B+\frac{3}{4}C=0 \implies \frac{3}{2}A+\frac{1}{4}B=0$$

$$B=-6A$$

$$\Downarrow$$

$$-5A+C=1$$

$$-6A+\frac{3}{4}C=0 \implies A=\frac{1}{3}$$

$$B=-2$$

$$C=\frac{8}{3}$$

$$Y_p(e^{jw})=\frac{1/3}{1-\frac{1}{4}e^{-jw}}+\frac{-2}{1-\frac{1}{2}e^{-jw}}+\frac{8}{3}\left[\frac{1}{1-e^{-jw}}+\sum_{k=-\infty}^{\infty}\pi\,\delta(w+2\pi k)\right]$$

$$y_p[n]=\frac{1}{3}\left(\frac{1}{4}\right)^n u[n]-2\left(\frac{1}{2}\right)^n u[n]+\frac{8}{3}u[n]$$

$$y[n]=A_1\left(\frac{1}{4}\right)^n+A_2\left(\frac{1}{2}\right)^n+\frac{1}{3}\left(\frac{1}{4}\right)^n u[n]-2\left(\frac{1}{2}\right)^n u[n]+\frac{8}{3}u[n]$$

$$y[-2]=16A_1+4A_2=-1$$

$$y[-1]=4A_1+2A_2=1 \implies A_1=\frac{-3}{8}$$

$$A_2=\frac{5}{4}$$

$$y[n]=\left(\frac{-3}{8}\right)\left(\frac{1}{4}\right)^n+\left(\frac{5}{4}\right)\left(\frac{1}{2}\right)^n+\frac{1}{3}\left(\frac{1}{4}\right)^n u[n]-2\left(\frac{1}{2}\right)^n u[n]+\frac{8}{3}u[n]$$

3) a)

$$\sum_{n=-\infty}^{\infty} x[n]\, y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, Y^*(e^{j\omega})\, d\omega$$

$$\sum_{n=-\infty}^{\infty} \underbrace{\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega}_{x[n]}\; y^*[n]$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) \underbrace{\sum_{n=-\infty}^{\infty} y^*[n]\, e^{j\omega n}}_{Y^*(e^{j\omega}) = \left[\sum_{n=-\infty}^{\infty} y[n]\, e^{-j\omega n}\right]^*} d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, Y^*(e^{j\omega})\, d\omega$$

b) i) $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$

$$X^*(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x^*[n]\, e^{j\omega n} = \sum_{n=-\infty}^{\infty} x[n]\, e^{j\omega n} = X(e^{-j\omega})$$

$\Rightarrow X^*(e^{j\omega}) = X(e^{-j\omega})$

$$|X(e^{-j\omega})| = |X^*(e^{j\omega})| = |X(e^{j\omega})|$$

ii) $\angle X(e^{j\omega}) = \tan^{-1} \dfrac{X_I(e^{j\omega})}{X_R(e^{j\omega})}$

$$\angle X(e^{-j\omega}) = \angle X^*(e^{j\omega}) = -\tan^{-1} \frac{X_I(e^{j\omega})}{X_R(e^{j\omega})}$$

iii) $Re\{X(e^{-j\omega})\} = Re\{X^*(e^{j\omega})\} = X_R(e^{j\omega})$

iv) $\operatorname{Im}\{X(e^{-j\omega})\} = \operatorname{Im}\{X^*(e^{j\omega})\} = -X_I\{e^{j\omega}\}$

v) $X(e^{-j\omega}) = X^*(e^{j\omega})$

$\theta = -\omega$

$X(e^{j\theta}) = X^*(e^{-j\theta})$

4) a) $X(e^{j\omega}) = \dfrac{1-2e^{-j\omega}}{(1-0.5e^{-j\omega})(1-0.3e^{-j\omega})}$

$= \dfrac{A}{1-0.5e^{-j\omega}} + \dfrac{B}{1-0.3e^{-j\omega}}$

$A+B=1$

$-0.3A-0.5B=-2$

$0.4A=-3$

$A=\dfrac{-15}{2}$

$B=\dfrac{17}{2}$

$x[n] = \dfrac{-15}{2}\left(\dfrac{1}{2}\right)^n u[n] + \dfrac{17}{2}(0.3)^n u[n]$

$\displaystyle\int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega = ?$

$x[n] = \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\,d\omega$

$2\pi x[0] = \displaystyle\int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega = 2\pi \times 1 = 2\pi$

b) $H(e^{j\omega}) = \cos(\omega)$

From the eigenfunction property of complex exponentials,

$x[n] = e^{j0n} + \dfrac{1}{2}e^{j\frac{\pi}{2}n} + \dfrac{1}{2}e^{-j\frac{\pi}{2}n} \longrightarrow y[n] = H(e^{j0})e^{j0n} + \dfrac{1}{2}H(e^{j\frac{\pi}{2}})e^{j\frac{\pi}{2}n} + \dfrac{1}{2}H(e^{-j\frac{\pi}{2}})e^{-j\frac{\pi}{2}n}$

$H(e^{j0}) = \cos(0) = 1$

$H(e^{j\frac{\pi}{2}}) = \cos\left(\frac{\pi}{2}\right) = 0$

$H(e^{-j\frac{\pi}{2}}) = \cos\left(\frac{-\pi}{2}\right) = 0$

$y[n] = e^{j0n} = 1$

* The filter $h[n]$ has a zero frequency response at $w = \frac{\pi}{2}$ so at the output, $\cos(\frac{\pi}{2}n)$ diminishes.

5) a) $x[n]$ should be absolutely summable such that its DTFT exists and converges uniformly to a continuous function of w.

Absolute Summability

$$\sum_{n=-\infty}^{\infty} |x[n]| < \infty.$$

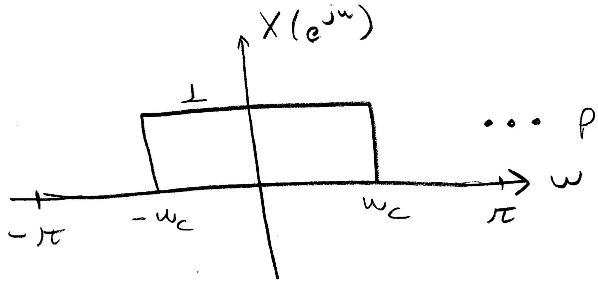
b) If $x[n]$ is square summable, its DTFT exists. However its DTFT does not converge uniformly. It has mean-square convergence, i.e

$$\lim_{N \to \infty} \int_{-\pi}^{\pi} \left| X(e^{jw}) - \sum_{n=-N}^{N} x[n] e^{-jwn} \right|^2 dw = 0$$

The error $\left| X(e^{jw}) - \sum_{n=-N}^{N} x[n] e^{-jwn} \right|$ may not approach zero at each value of w as $N \to \infty$ but the total energy in the error does

Ex. $x[n] = \frac{\sin w_c n}{\pi n}$

$$X(e^{jw}) = \begin{cases} 1 & |w| < w_c \\ 0 & w_c < |w| \leq \pi \end{cases}$$ with periodicity 2π.



... periodic with 2π

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} \left|\frac{\sin w_c n}{\pi n}\right|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{jw})|^2 dw = \frac{2w_c}{2\pi} < \infty$$

(From Parseval's relation)

★ $x[n] = \frac{\sin w_c n}{\pi n}$ is square summable.

c) $x[n] = u[n]$ which is neither absolute nor square summable since

$$\sum_{n=0}^{\infty} 1 = \infty \; , \; \sum_{n=0}^{\infty} 1^2 = \infty$$ but it has DTFT,

$$U(e^{jw}) = \frac{1}{1-e^{-jw}} + \sum_{k=-\infty}^{\infty} \pi\delta(w+2\pi k)$$

d) $x[n] = \cos(w_c n)$ has DTFT but does not have z-transform

$$X(e^{jw}) = \sum_{k=-\infty}^{\infty} \pi\delta(w-w_c+2\pi k) + \pi\delta(w+w_c+2\pi k)$$

$$x[n] = \underbrace{\left[\frac{1}{2}e^{jw_c n} + \frac{1}{2}e^{-jw_c n}\right]u[n]}_{\substack{\text{ROC of z transform} \\ |z|>1}} + \underbrace{\left[\frac{1}{2}e^{jw_c n} + \frac{1}{2}e^{-jw_c n}\right]u[-n-1]}_{\substack{\text{ROC of z transform} \\ |z|<1}}$$

no intersection So $X(z)$ does not exist.

e) $x[n] = 2^n u[n]$

$X(z) = \dfrac{1}{1-2z^{-1}}$ $\quad |z|>2 \Rightarrow$ It has z transform

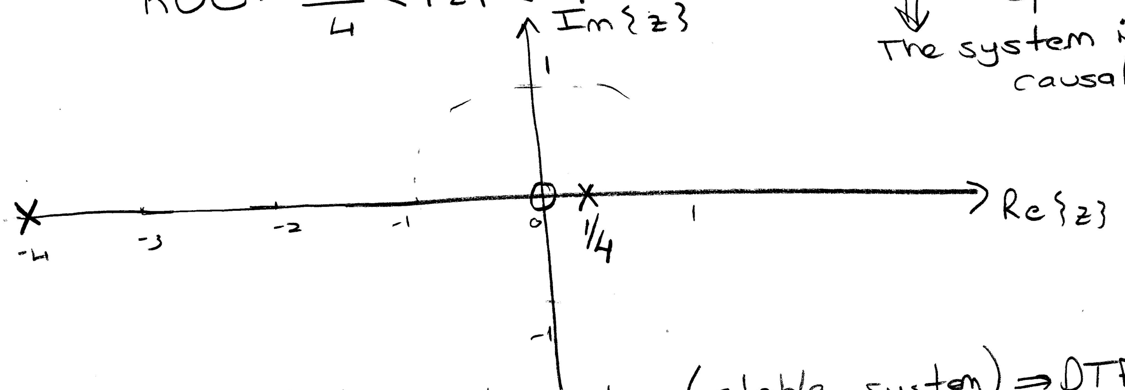
Since ROC does not include unit circle $x[n]$ does not have DTFT.

6) a) $H(z) = \dfrac{2z^{-1}+1}{\left(1-\frac{1}{4}z^{-1}\right)\left(1+2z^{-1}\right)\left(1+\frac{1}{4}z\right)} = \dfrac{z\cancel{(2+z)}}{\left(z-\frac{1}{4}\right)\cancel{(z+2)}\left(1+\frac{1}{4}z\right)}$

zeros : $z_1 = 0$, $z_2 = \infty$
poles : $p_1 = \frac{1}{4}$, $p_2 = -4$

Since the system is stable, ROC must include unit circle.

ROC: $\frac{1}{4} < |z| < 4$ $\longrightarrow$ $h[n]$ is a two sided sequence
$\Downarrow$
The system is not causal.



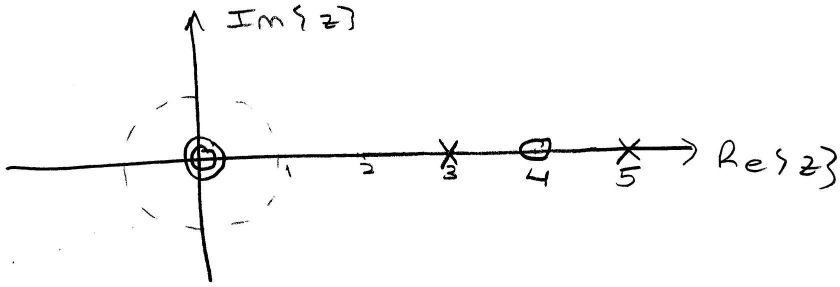
ROC includes unit circle (stable system) $\Rightarrow$ DTFT exists

b) $H(z) = \dfrac{z-4}{(1-3z^{-1})(1-5z^{-1})} = \dfrac{z^2(z-4)}{(z-3)(z-5)}$

zeros : $z_1 = z_2 = 0$, $z_3 = 4$
poles : $p_1 = 3$, $p_2 = 5$, $p_3 = \infty$

ROC $|z| < 3$ $\rightarrow$ left sided sequence
$\Downarrow$
The system is not causal

Stable system $\Rightarrow$ DTFT exists.

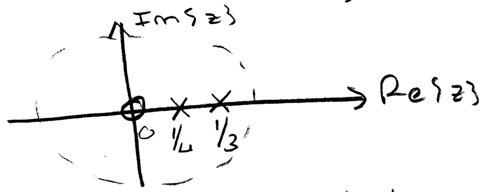


c)

$$H(z) = \frac{z^{-1}}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)} = \frac{z}{\left(z-\frac{1}{4}\right)\left(z-\frac{1}{3}\right)}$$

zeros: $z_1 = 0$, $z_2 = \infty$

poles: $p_1 = \frac{1}{4}$, $p_2 = \frac{1}{3}$



ROC: $|z| > \frac{1}{3}$ $\Rightarrow$ It is a right sided sequence. Furthermore there is no pole at $z=\infty$. So the system is causal.

If there existed pole/poles at $z=\infty$, then $H(z)$ would include z^k $k>0$ terms. This means $h[n] \neq 0$ for $n<0$. For example consider $h[n] = \delta[n+1] + \delta[n-1]$

Then $H(z) = z + z^{-1}$ when $z \uparrow \infty$ $H(z)$ blows up.

Therefore there is a pole at $z = \infty$.

★ Stable system $\Longrightarrow$ DTFT exists.

d) $$H(z) = \frac{1}{\left(1-\frac{1}{2}z^{-1}\right)^3} = \frac{z^3}{\left(z-\frac{1}{2}\right)^3}$$

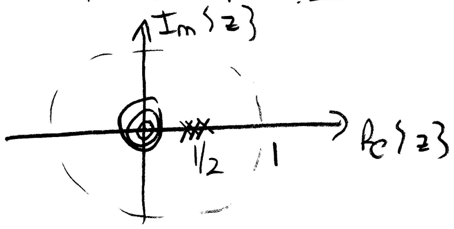
Zeros: $z_1 = z_2 = z_3 = 0$

poles. $p_1 = p_2 = p_3 = \frac{1}{2}$

ROC: $|z| > \frac{1}{2}$

$\Rightarrow$ causal system

stable system $\Rightarrow$ DTFT exists.



$\boxed{7}$, $x[n] = \delta[n+1] + \left(\frac{1}{2}\right)^n u[n]$

a) $X(z) = \underset{ROC_1 : |z| < \infty}{z} + \underbrace{\frac{1}{1-\frac{1}{2}z^{-1}}}_{ROC_2 : |z| > \frac{1}{2}}$

$$X(z) = \frac{z - \frac{1}{2} + 1}{1-\frac{1}{2}z^{-1}} = \frac{z\left(z+\frac{1}{2}\right)}{z - \frac{1}{2}}$$

Zeros: $z_1 = 0$, $z_2 = \frac{-1}{2}$

Poles: $p_1 = \frac{1}{2}$, $p_2 = \infty$ ROC: $\frac{1}{2} < |z| < \infty$

b) $y[n] = x[n-5]$

$$Y(z) = X(z)\, z^{-5} = \frac{\left(z+\frac{1}{2}\right)}{z^4\left(z-\frac{1}{2}\right)}$$

Zeros: $z_1 = \frac{-1}{2}$, $z_2 = z_3 = z_4 = z_5 = \infty$ (4 zeros at ∞)

poles $p_1 = \frac{1}{2}$, $p_2 = p_3 = p_4 = p_5 = 0$ (4 poles at 0)

ROC: $|z| > \frac{1}{2}$ ($x[n-5]$ is right sided sequence)

c) $y[n] = n\,x[n]$

$$Y(z) = -z\,\frac{dX(z)}{dz} = -z\left[1 + \frac{1\cdot\left(z-\frac{1}{2}\right) - z}{\left(z-\frac{1}{2}\right)^2}\right]$$

$$= -z + \frac{\frac{1}{2}z}{z^2 - z + \frac{1}{4}} = \frac{-z^3 + z^2 + \frac{1}{4}z}{\left(z-\frac{1}{2}\right)^2} = \frac{z\left(-z^2 + z + \frac{1}{4}\right)}{\left(z-\frac{1}{2}\right)^2}$$

$$= \frac{-z\left(z - \left(\frac{1}{2} + \frac{1}{\sqrt{2}}\right)\right)\left(z - \left(\frac{1}{2} - \frac{1}{\sqrt{2}}\right)\right)}{\left(z-\frac{1}{2}\right)^2}$$

zeros: $z_1 = 0$, $z_2 = \frac{+1}{2} + \frac{1}{\sqrt{2}}$, $z_3 = \frac{1}{2} - \frac{1}{\sqrt{2}}$

poles: $p_1 = p_2 = \frac{1}{2}$, $p_3 = \infty$ ROC $\frac{1}{2} < |z| < \infty$

($nx[n]$ is a right sided sequence)

d) $y[n] = \cos\left(\frac{\pi}{2}n\right) x[n]$

$$= \cos\left(\frac{\pi}{2}n\right) \delta[n+1] + \left(\frac{1}{2}\right)^n \cos\left(\frac{\pi}{2}n\right) u[n]$$

$$= \underbrace{\cos\left(\frac{\pi}{2}(-1)\right) \delta[n+1]}_{0} + \left(\frac{1}{2}\right)^n \cos\left(\frac{\pi}{2}n\right) u[n]$$

$$= \left(\frac{1}{2}\right)^n \left[\frac{1}{2} e^{j\frac{\pi}{2}n} + \frac{1}{2} e^{-j\frac{\pi}{2}n}\right] u[n]$$

$$= \left[\frac{1}{2}\left(\frac{1}{2} e^{j\frac{\pi}{2}}\right)^n + \frac{1}{2}\left(\frac{1}{2} e^{-j\frac{\pi}{2}}\right)^n\right] u[n]$$

$$Y(z) = \frac{1/2}{1 - \frac{1}{2}jz^{-1}} + \frac{1/2}{1 + \frac{1}{2}jz^{-1}} = \frac{z^2}{\left(z - \frac{1}{2}j\right)\left(z + \frac{1}{2}j\right)}$$

zeros: $z_1 = z_2 = 0$

poles: $p_1 = \frac{1}{2}j$, $p_2 = -\frac{1}{2}j$ ROC: $|z| > \frac{1}{2}$

($\cos\left(\frac{\pi}{2}n\right) x[n]$ is a right sided sequence)

8) a) $X(z) = \dfrac{Kz}{z-2}, \quad |z|<2$

$X(1) = \dfrac{K1}{1-2} = -1 \Longrightarrow K=1$

$X(z) = \dfrac{z}{z-2}, \quad |z|<2$

b) $Y(z) = \dfrac{2-\frac{1}{6}z^{-1}}{1-\frac{13}{6}z^{-1}+\frac{1}{6}z^{-2}+\frac{1}{3}z^{-3}}, \quad \frac{1}{2}<|z|<2$

$$Y(z) = \frac{\left(2-\frac{1}{6}z^{-1}\right)}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)\left(1-2z^{-1}\right)}$$

$$= \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1+\frac{1}{3}z^{-1}} + \frac{C}{1-2z^{-1}}$$

$A+B+C=2$

$\frac{-5}{3}A - \frac{5}{2}B - \frac{1}{6}C = \frac{-1}{6}$

$\frac{-2}{3}A + B - \frac{1}{6}C = 0$

$\frac{5}{3}A + \frac{7}{6}C = 2 \qquad \frac{5}{6}A + \frac{7}{3}C = \frac{29}{6}$

$10A+7C=12 \qquad 5A+14C=29$

$A=\frac{-1}{3} \qquad C=\frac{46}{21} \qquad B=\frac{3}{21}$

$$y[n] = -\frac{1}{3}\left(\frac{1}{2}\right)^n u[n] + \frac{1}{7}\left(\frac{-1}{3}\right)^n u[n] - \frac{46}{21}(2)^n u[-n-1]$$