Aykut DEMİREL
1813948
Section: 2.

EE-430 - Digital Signal Processing

Homework I.

1) $x_c(t) = 4\sin\left(20000\pi t + \frac{\pi}{13}\right)$ and $f_s = 3kHz = 3000\ Hz$.

$\Rightarrow x[n] = 4\sin\left(20000\pi \frac{n}{3000} + \frac{\pi}{13}\right) = 4\sin\left(\frac{20\pi n}{3} + \frac{\pi}{13}\right)$

a) Since $\sin(w_0 n + \Phi) = \sin(w_0 n + 2\pi k n + \Phi)$ for all n's;

$x[n] = 4\sin\left(\frac{20\pi n}{3} + \frac{\pi}{13}\right) = 4\sin\left(\frac{20\pi n}{3} + 2\pi k n + \frac{\pi}{13}\right) = 4\sin\left(\left(\frac{20\pi}{3} + 2\pi k\right)n + \frac{\pi}{13}\right)$

$\Rightarrow x_c(t) = 4\sin\left(\left(\frac{20\pi}{3} + 2\pi k\right)t \cdot f_s + \frac{\pi}{13}\right) = \underbrace{4\sin\left((20000\pi + 6000\pi k)t + \frac{\pi}{13}\right)}_{\text{set of all cont. time signals that yield } x[n] \text{ when sampled at 3 kHz sampling frequency.}}, \ k \in \mathbb{Z}$

$x_c(t) = 4\sin\left(2\pi f t + \frac{\pi}{13}\right) \Rightarrow \underline{f = f_0 + f_s k = 10^4 + 3000k\ Hz}$

b) $x[n] = 4\sin\left(\frac{20\pi n}{3} + \frac{\pi}{13}\right) = 4\sin\left(\frac{20\pi n}{3} + 2\pi k n + \frac{\pi}{13}\right) = 4\sin\left(\frac{20000\pi n}{f_s} + \frac{\pi}{13}\right)$

* $\Rightarrow \frac{20\pi \cancel{n}}{3} + 2\pi k \cancel{n} + \cancel{\frac{\pi}{13}} = \frac{20000\pi \cancel{n}}{f_s} + \cancel{\frac{\pi}{13}} \Rightarrow \frac{20\pi}{3} + 2\pi k = \frac{20000\pi}{f_s}$

$\Rightarrow f_s = \frac{20000\pi}{\frac{20\pi}{3} + 2\pi k} = \frac{10^4}{\frac{10}{3} + k} = \frac{3 \cdot 10^4}{3k + 10}\ Hz, \ k \in \mathbb{Z}$ } Set of all f_s's that yields $x[n]$ from $x_c(t)$.

** $\frac{20\pi n}{3} + 2\pi k n + \frac{\pi}{13} = \frac{12\pi}{13} - \frac{20000\pi n}{f_s}$ } There is no solution of f_s (independent from n) for this situation.

2) i) $\sin(1.74\pi n + 3.1) \Rightarrow \frac{w_0}{2\pi} = \frac{1.74\pi}{2\pi} = \frac{1.74}{2} \Rightarrow$ rational $\Rightarrow$ periodic.

$N = \frac{2\pi}{w} k_{min} = \frac{2\pi}{1.74\pi} k_{min} = \frac{1}{0.87} k_{min} \Rightarrow N = 100$ (fundamental period)

$k_{min} \to 87$

ii) $\sin(1.74\pi n + 3.1\pi) \Rightarrow \frac{\omega}{2\pi} = \frac{1.74}{2} \Rightarrow$ rational $\Rightarrow$ periodic.

$$N = \frac{2\pi}{\omega_0} k_{min} = \frac{1}{0.87} \underset{\substack{\downarrow \\ 87}}{k_{min}} = 100 \Rightarrow N = 100.$$

iii) $\cos(15.74\pi n + 3\pi/8) \Rightarrow \frac{\omega}{2\pi} = \frac{15.74}{2} \Rightarrow$ rational $\Rightarrow$ periodic.

$$N = \frac{2\pi}{\omega_0} k_{min} = \frac{1}{7.87} \underset{\substack{\downarrow \\ 787}}{k_{min}} = 100 \Rightarrow N = 100$$

iv) $\cos(\sqrt{\pi} n) \Rightarrow \frac{\omega}{2\pi} = \frac{\sqrt{\pi}}{2\pi} = \frac{1}{2\sqrt{\pi}} \Rightarrow$ is not rational $\Rightarrow$ NOT periodic.

v) $\cos(\pi\sqrt{\pi} n) \Rightarrow \frac{\omega}{2\pi} = \frac{\pi\sqrt{\pi}}{2\pi} = \frac{\sqrt{\pi}}{2} \Rightarrow$ is not rational $\Rightarrow$ NOT periodic

vi) $\cos(\pi\sqrt{2} n) \Rightarrow \frac{\omega}{2\pi} = \frac{\pi\sqrt{2}}{2\pi} = \frac{1}{\sqrt{2}} \Rightarrow$ is not rational $\Rightarrow$ NOT periodic.

3) The highest frequency means the smallest fundamental period N.

Since N is an integer $\Rightarrow$ smallest value is $N = 1$

But when $N = 1$, $x[n]$ is constant and has no frequency

$\Rightarrow$ take $N = 2 \Rightarrow$ the highest frequency discrete time signal can be $\cos(\pi n)$

$$\cos(\pi n) = \begin{cases} 1 & , n \text{ even} \\ -1 & , n \text{ odd.} \end{cases}$$

4)
$$y[n] = \begin{cases} x\left[\frac{n}{2}\right] & , n \text{ even} \\ \dfrac{x\left[\frac{n+1}{2}\right] + x\left[\frac{n-1}{2}\right]}{2} & , n \text{ odd.} \end{cases}$$

The system extends the input by 2 and calculates the averages of the next and previous values for the missing elements (when n = odd).

System is linear : $x[n] \longrightarrow y[n]$. $\Downarrow$ $ax[n] \longrightarrow ay[n]$ and $x_1[n] \to y_1[n]$ and $x_2[n] \to y_2[n]$ $\Downarrow$ $x_1[n] + x_2[n] \longrightarrow y_1[n] + y_2[n]$.

System is not time-invariant.:

suppose
$$x_1[n] = \begin{cases} 3 & \text{for } n \text{ even} \\ 5 & \text{for } n \text{ odd} \end{cases} \Rightarrow y_1[n] = 3$$

$$x_2[n] = x_1[n-1] = \begin{cases} 5 & \text{for } n \text{ even} \\ 3 & \text{for } n \text{ odd.} \end{cases} \Rightarrow y_2[n] = 5 \Rightarrow \text{Time-varying}$$

$\Rightarrow y_2[n] \neq y_1[n-1]$

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5) i) $y[n] = 2^{\delta[n+1]} + x[n-3]$ ⇒ First of all, the system is neither linear nor time-invariant.

System is causal, since the output $y[n]$ is independent from the future values of $x[n]$. In fact, $y[n]$ only depends on $x[n-3]$.

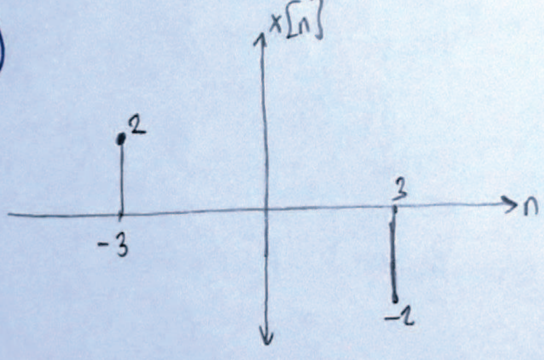
System is stable, since it just shifts the input to the right by 3 units and adds $\begin{cases} 1, & \text{for } n \neq -1 \\ 2, & \text{for } n = -1 \end{cases}$, so any bounded input $x[n]$ yields a bounded output.

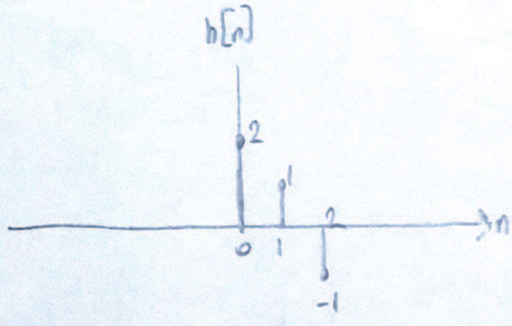
ii) $y[n] = \begin{cases} y[-\delta[n-1]] + x[n-3] & , n > 0 \\ 2^n x[n-3] & , n \leq 0 \end{cases}$ System is not linear and not time-invariant.

System is causal, since for any n, the output $y[n]$ is independent from the future values of $x[n]$.

System is stable since it shifts the input to the right by 3 units and adds some finite number for $n > 0$ and multiplies by $2^n \leq 1$ for $n \leq 0$. Then the output $y[n]$ is definitely bounded for a bounded input $x[n]$.

6)

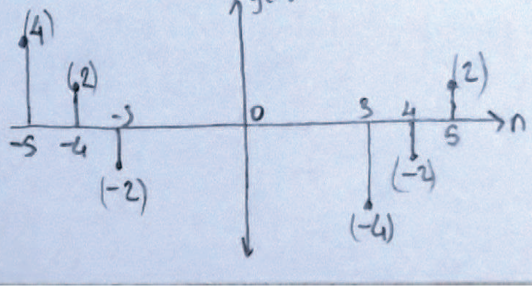




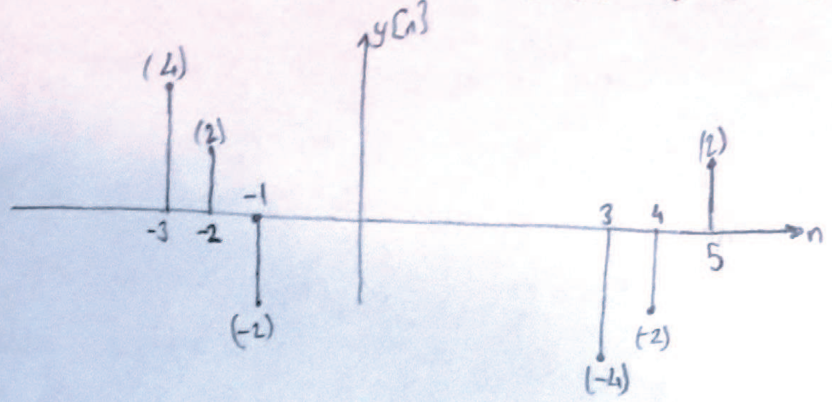
$$\Rightarrow x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] = x[3] h[n-3] + x[-3] h[n+3] = -2h[n-3] + 2h[n+3] = y[n]$$

⇓ only take values for $k=-3$ and $k=3$

⇒



$$x[n] * h[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] = h[0]x[n] + h[1]x[n-1] + h[2]x[n-2]$$
$$= 2x[n] + x[n-1] - x[n-2] = y[n]$$



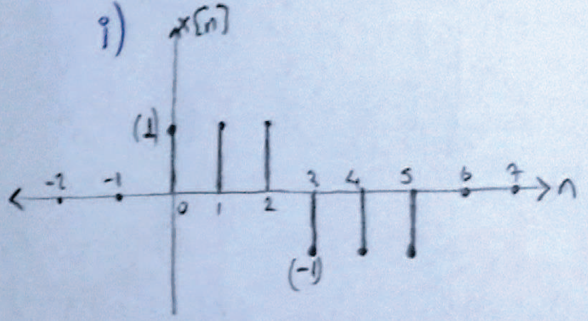
7) $$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

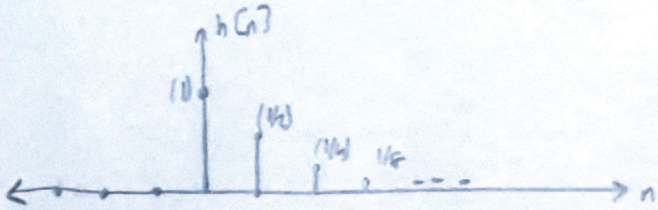
$\Rightarrow$ i) $$x[-n]*h[-n] = \underbrace{\sum_{k=-\infty}^{\infty} x[-k]\underbrace{h[-n-k]}_{-k=m} = \sum_{m=\infty}^{-\infty} x[m]\,h[m-n]}_{=y[-n]} = h[-n] * x[-n]$$

$\Rightarrow$ ii) $$x[n-4]*h[n] = \underbrace{\sum_{k=-\infty}^{\infty} x[k-4]\,h[n-k]}_{k-4=m} = \sum_{m=-\infty}^{\infty} x[m]\,h[\underbrace{n-4-m}_{p}] = x[n]*h[n-4]$$
$$= y[p] = y[n-4]$$

8) $x[n] = u[n] - 2u[n-3] + u[n-6]$ $\qquad h[n] = \left(\frac{1}{2}\right)^n u[n]$

i)

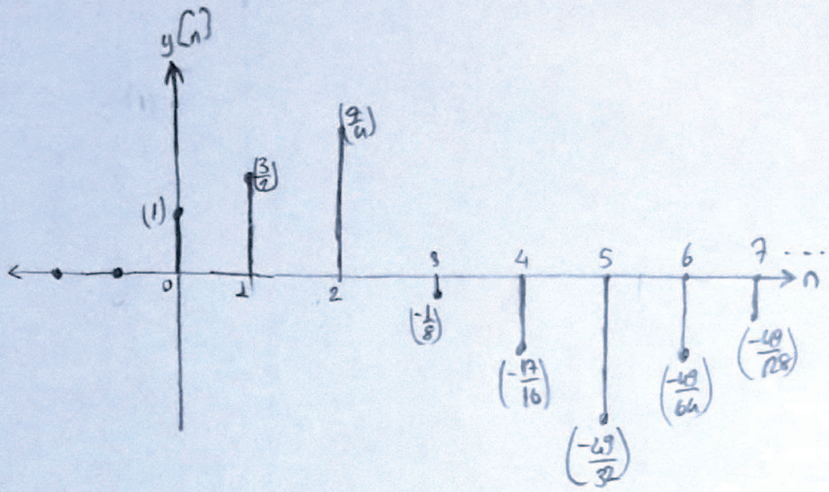




ii) $$y[n] = x[n] * h[n] = \left(\delta[n] + \delta[n-1] + \delta[n-2] - \delta[n-3] - \delta[n-4] - \delta[n-5]\right) * h[n]$$
$$= h[n] + h[n-1] + h[n-2] - h[n-3] - h[n-4] - h[n-5]$$
$$= \left(\tfrac{1}{2}\right)^n u[n] + \left(\tfrac{1}{2}\right)^{n-1} u[n-1] + \left(\tfrac{1}{2}\right)^{n-2} u[n-2] - \left(\tfrac{1}{2}\right)^{n-3} u[n-3] - \left(\tfrac{1}{2}\right)^{n-4} u[n-4] - \left(\tfrac{1}{2}\right)^{n-5} u[n-5]$$

$$\Rightarrow \quad y[n] = \begin{cases} 0 & , \; n<0 \\ 1 & , \; n=0 \\ 3/2 & , \; n=1 \\ 9/4 & , \; n=2 \\ -1/8 & , \; n=3 \\ -17/16 & , \; n=4 \\ -49/32 & , \; n=5 \\ \left(\frac{1}{2}\right)^{n}+\left(\frac{1}{2}\right)^{n-1}+\left(\frac{1}{2}\right)^{n-2}-\left(\frac{1}{2}\right)^{n-3}-\left(\frac{1}{2}\right)^{n-4}-\left(\frac{1}{2}\right)^{n-5} = \left(\frac{1}{32}+\frac{1}{16}+\frac{1}{8}-\frac{1}{4}-\frac{1}{2}-1\right)\left(\frac{1}{2}\right)^{n-5} & \\ = -\frac{49}{32}\left(\frac{1}{2}\right)^{n-5} & , \; n>5 \end{cases}$$

$\Rightarrow$



9) Question is answered using MATLAB and necessary files are added at the end.

10) a) Question is answered using MATLAB and necessary files are added at the end.

b) $y[n] = [1 \;\; 1 \;\; 2 \;\; 3 \;\; 4 \;\; -1 \;\; 5]$ for $n = 0, 1, 2, 3 \ldots 6$

$x[n] = [1 \;\; 2 \;\; 3 \;\; 4 \;\; 5]$ for $n = 0, 1, 2, 3, 4$.

$$\Rightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k].h[n-k] \quad \Rightarrow \quad y[0] = \sum_{k=-\infty}^{\infty} x[k]h[-k] = x[0]h[0] = 1 = 1.h[0].$$

$$\Rightarrow h[0] = 1$$

$$y[1] = \sum_{k=-\infty}^{\infty} x[k]h[1-k] = \underbrace{x[0]}_{1}h[1] + \underbrace{x[1]}_{2}\underbrace{h[0]}_{1} = h[1]+2 = 1 \Rightarrow h[1] = -1$$

$$y[2] = \sum_{k=-\infty}^{\infty} x[k]h[2-k] = \underbrace{x[0]}_{1}h[2] + \underbrace{x[1]}_{2}\underbrace{h[1]}_{-1} + \underbrace{x[2]}_{3}\underbrace{h[0]}_{1} = h[2]+1 = 2$$

$$\Rightarrow h[2] = 1$$

since $y[n]$ is of length 7, $x[n]$ is of length 5

$\Rightarrow h[n]$ is of length $7-5+1 = 3$

$\Rightarrow h[n] = [1 \;\; -1 \;\; 1]$ for $n = 0, 1, 2$.

The answer is verified via MATLAB and result is added to the end of the document.

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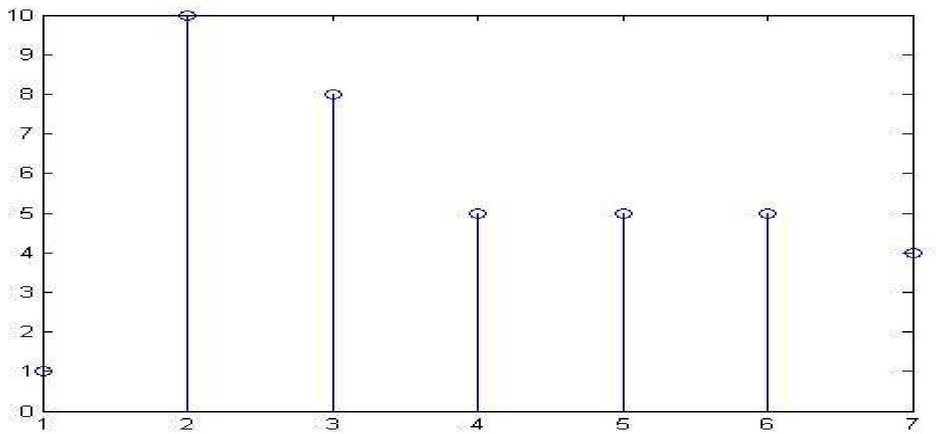
c) The question is answered using MATLAB and necessary documents are added to the end.

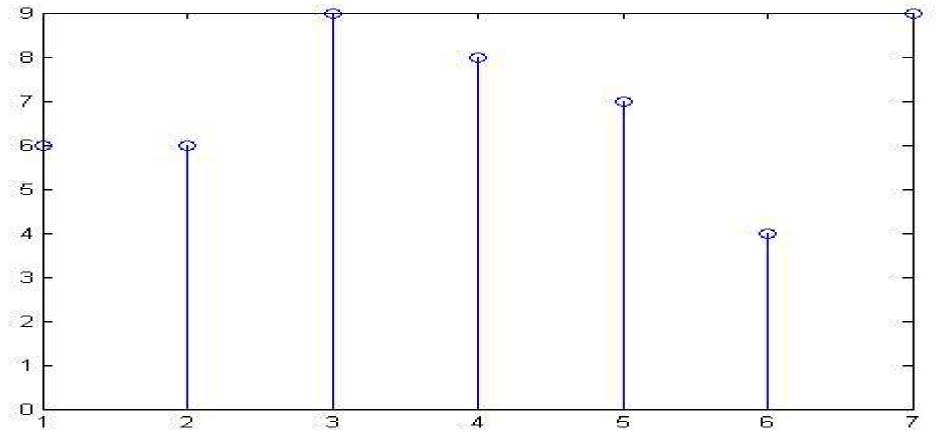
Q9) The executed code;

```
x=random('unid', 10,1,7);
h=random('unid', 10,1,7);
y=conv(x,h);
figure
stem(x);
figure
stem(h);
figure
stem(y);
```

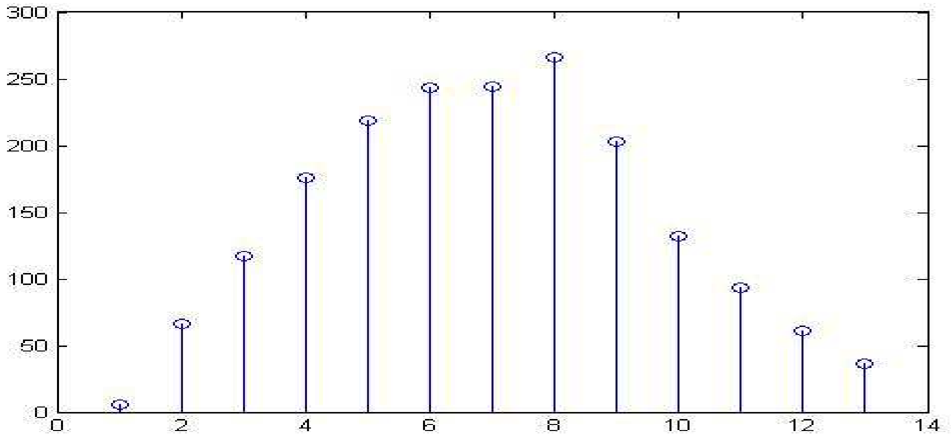
x[n]



h[n]



y[n]



The convolution is resulted as expected.

Q10)

a)

```
%Take the coefficients of the polynomials as the vector
elements.
p1=[23 45 21 67]; %23x^3+45x^2+21x+67

p2=[12 23 1 0  0 9]; %12x^5+23x^4+x^3+9

p3=conv(p1,p2); %p3 is the multiplication of p1 and p2
```

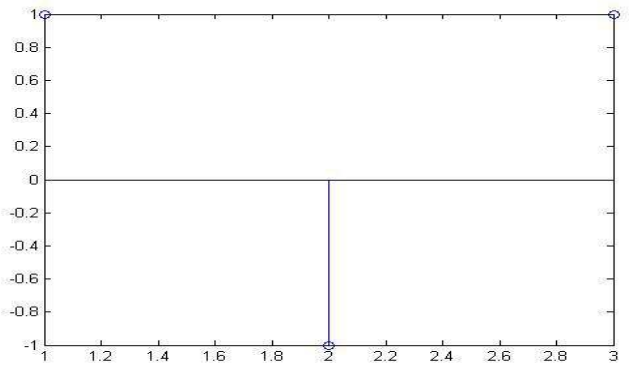
The results is;

```
p3 =

  Columns 1 through 6

276          1069          1310          1332          1562         274



  Columns 7 through 9



405           189           603
```

b)

```
y=[1 1 2 3 4 -1 5];
x=1:5;

h=deconv(y,x);
stem(h);
```

and the result is;



We see that our computations are correct.

c)

```
y=[1 2 2 3 4 -1 5];
x=1:5;

h=deconv(y,x);
stem(h);
```

and the result is;