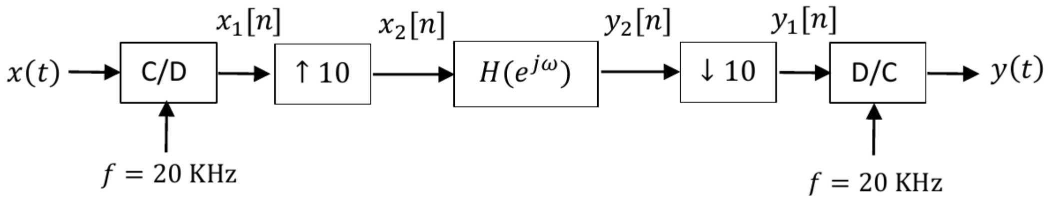
EE430 Homework 3

Fall 2018

Sampling of Continuous-time Signals and Transform Analysis of LTI Systems

1. A signal *x(t)* that is bandlimited to 10 KHz (X(jΩ) = 0, |Ω| ≥ 2 × 10^4 π) is processed as follows. Let N be an integer and $H(e^{j\omega}) = \begin{cases} e^{-jN\omega}, & |\omega| < \frac{\pi}{10} \\ 0, & \text{otherwise} \end{cases}$.



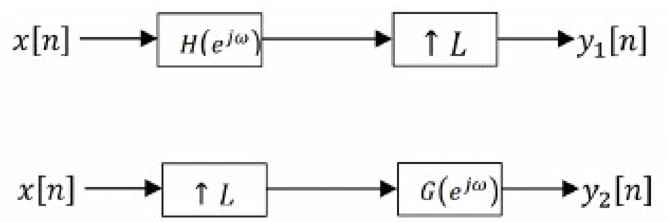
   a. Express $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$ in terms of $X(j\Omega)$ for $|\omega| < \pi$.

   b. Express $Y_1(e^{j\omega})$ and $Y_2(e^{j\omega})$ in terms of $X_1(e^{j\omega})$.

   c. Express y*(t)* in terms of *x(t)*.

   d. Plot magnitudes of spectra for all signals in the above diagram assuming some spectrum for *x(t)*.

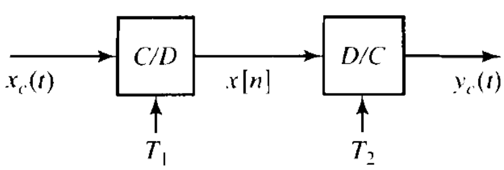
2. Consider the two systems shown below.



   a. Can you find a relationship between $H(e^{j\omega})$ and $G(e^{j\omega})$ so that the two systems are equivalent, i.e. $y_1[n] = y_2[n]$.

   b. Repeat part-a after replacing the expanders by *L* in the systems with compressors by *M.*

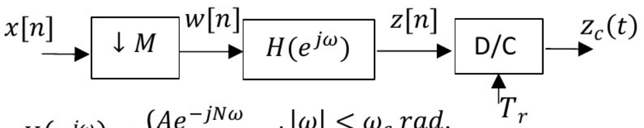
3. In the figure below, assume $X_c(j\Omega) = 0$ for $|\Omega| > \frac{\pi}{T_1}$.



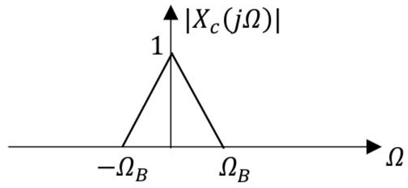
   a. Find the relation between $x_c(t)$ and $y_c(t)$ when $T_1 = T_2$. Would this relation hold if $X_c(j\Omega) \neq 0$ for $|\Omega| > \frac{\pi}{T_1}$.

b. For the general case in which $T_1 \neq T_2$, express $y_c(t)$ in terms of $x_c(t)$. Is the basic relationship different for $T_1 > T_2$ and $T_1 < T_2$?

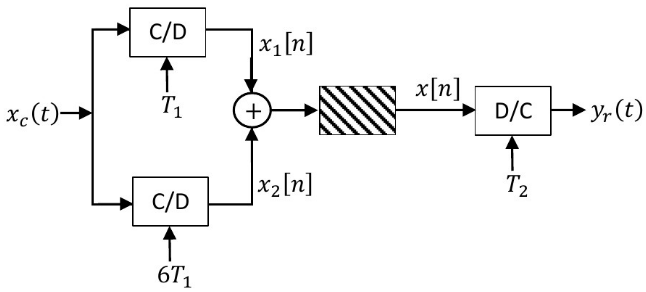
4. A continuous-time (CT) signal $x_c(t)$ that is bandlimited to 20 KHz ($X_c(j\Omega) = 0$, $|\Omega| \geq 4\pi 10^4$ rad/sec) is sampled at a sampling frequency of f_s = 80 KHz to obtain a discrete-time (DT) signal $x[n]$, which is processed with the system shown below. If $z_c(t) = x_c(2t - 2)$, find suitable values for M, N, ω_c , A, T_r and plot magnitudes of spectrums of $x[n]$, $w[n]$, $z[n]$ and $z_c(t)$ for your values of M, N, ω_c , A, T_r assuming the given shape below for the magnitude spectrum of $x_c(t)$.



$$H(e^{j\omega}) = \begin{cases} Ae^{-jN\omega} & , |\omega| < \omega_c \; rad. \\ 0 & , \quad \text{otherwise.} \end{cases}$$



5. Consider the given system below and let $x_c(t) = \cos(200\pi t)$ and $T_1 = 10^{-3}$.



a. Find $x_1[n]$ and $x_2[n]$.

b. Find suitable discrete-time processing elements for the striped box and the value of T_2 so that $y_r(t) = x_c(t)$.

6. A <u>causal</u> <u>generalized linear phase</u> system with <u>real</u> impulse response $h[n]$ has the following system function. It is also known that the system's frequency response is zero at $\omega = \pi$, i.e. $H(e^{j\pi}) = 0$.

$$H(z) = (1 - e^{-j\pi/2} z^{-1})(1 - 0.25z^{-1})(1 - az^{-1})(1 - bz^{-1})(1 - cz^{-1})$$

a. Find a, b, and c and plot the pole-zero plot of $H(z)$. Explain clearly and briefly how you found a, b, and c.

b. If $h[n]$ is expressed as $h[n] = \pm h[M - n]$, $0 \leq n \leq M$, find M. Is $h[n]$ symmetric or anti-symmetric?

c. Determine the group delay of this system.

d. For input $x[n] = \cos(\pi n/4)$, the output is $y[n] = A\cos(\pi(n - n_d)/4)$, where A is a constant. Find n_d .

7. Answer following questions.

a. Can you make a minimum-phase and all-pass decomposition of the system function below.

$$H(z)=\frac{(1-2z^{-1})(z+2)(z-\frac{1}{2})}{(1-\frac{1}{2}z^{-1}+\frac{1}{2}z^{-2})}$$

b. The squared magnitude response of $H(e^{j\omega})$ is given as, $H(e^{j\omega})=\dfrac{\frac{5}{4}-\cos(\omega)}{\frac{10}{9}-\frac{2}{3}\cos(\omega)}$.

Find the minimum-phase system $H_{min}(e^{j\omega})$ which has the same magnitude response as $H(e^{j\omega})$.

c. Assume $H(z)$ is given as $H(z)=\dfrac{(1-0.5z^{-1})(1+4z^{-2})}{1-0.64z^{-2}}$. Find the minimum-phase $H_{min}(z)$ and linear-phase $H_{lin}(z)$ filters such that $H(z)=H_{min}(z)H_{lin}(z)$. Plot the approximate magnitude response of $H_{min}(z)$ indicating only the critical values.

d. Assume $h_{lp}(z)$ is the impulse response of and FIR generalized linear-phase low-pass filter of length N. A generalized linear-phase high-pass FIR filter $h_{hp}(z)$ will be obtained from $h_{lp}(z)$ by multiplication with a cosine.

i. Determine the requirement for ω_0 of the frequency of the cosine.

ii. Write the expression for $h_{hp}(z)$. Also write the expression for $H_{hp}(e^{j\omega})$.

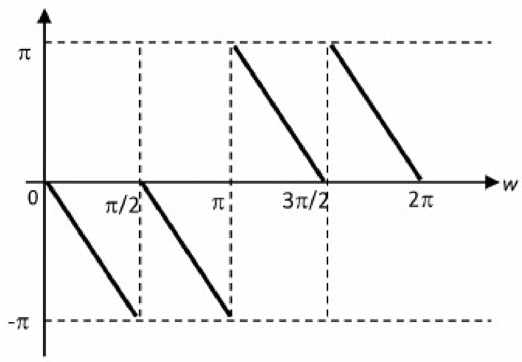
8. Answer following questions.

a. Assume a real casual FIR filter, *g[n]*, with the system function $G(z) = 1 + z^{-1}$. Find pole(s) and zero(s) of *G(z)* and plot only the principal value of the phase for this system function, namely $\angle G(e^{jw})$.

b. Another real casual FIR filter, *h[n]*, has the following principal value of its phase, $\angle H(e^{jw})$, presented in the figure below.

i. Explain reasons of discontinuities in $\angle H(e^{jw})$ for π/2, π, and 3π/2.

ii. Determine group delay of this filter. State whether this filter has a generalized linear phase; if so, try to determine its type, when $h[n] = 0, n > 4$.



c. Apart from the properties of *h[n]* given in part (b), *H(z)* also contains a zero at $z = 1/2$, and $H(1) = 1$. Find *H(z)* and indicate its poles and zeros on the z-plane.

d. Determine $|H(e^{jw})|$ as a closed form expression and approximately plot $|H(e^{jw})|$.

9. Answer the follow questions about the systems having the pole-zero plots shown below.

   a. Which systems are IIR systems ?

   b. Which systems are FIR systems ?

   c. Which systems are stable systems ?

   d. Which systems are minimum-phase systems ?

   e. Which systems are generalized linear-phase systems ?

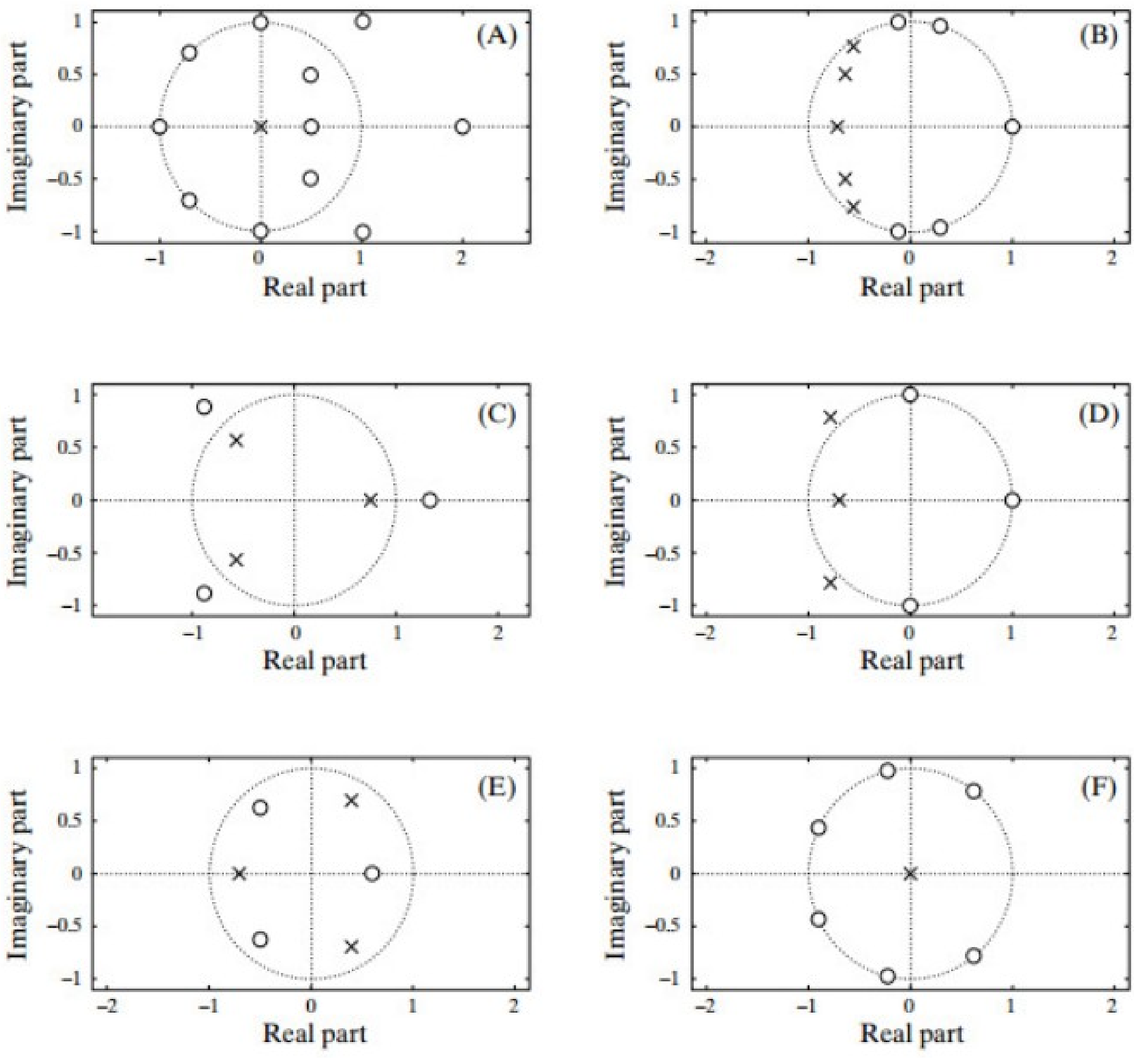
   f. Which systems have $|H(e^{jw})|$=constant for all ω ?

   g. Which systems have corresponding stable and causal inverse systems ?

   h. Which system has the shortest (least number of nonzero samples) impulse response ?

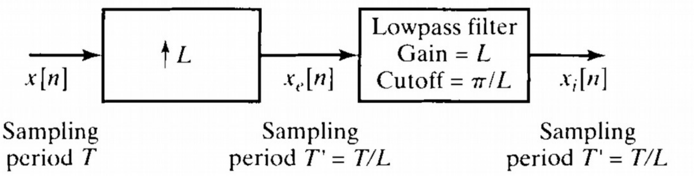
   i. Which systems have low-pass frequency responses ?

   j. Which systems have minimum group delay ?



10. (**Matlab question**) We would like to use sampling rate change concepts discussed in class to obtain a 1024x1024-pixel version of the 256x256-pixel cameraman image. (Type in the Matlab prompt the following to get it : [I] = imread('cameraman.tif'); figure, imshow(I);).

Notice that “pixel” is just another name for a sample of a 2-D DT signal. Consider the interpolator system we discussed in class and shown below. To obtain the desired 1024x1024-pixel version of the cameraman image, you can apply the interpolator first to each row of the input 256x256-pixel cameraman image (each row can be seen asa 1-D signal *x[n]*), which should give you a 1024x256-pixel output. The output can be again processed with the same interpolator system for each column now (which should give you the desired 1024x1024-pixel output).



a. Implement a Matlab function that performs the desired operation. In the interpolator system, use an ideal LPF. Determine approximately the number of multiplications required to obtain the result. Provide the written Matlab function and a picture of the obtained 1024x1024-pixel cameraman image.

b. Implement a Matlab function that performs the desired operation. In the interpolator system, use a simpler low-pass filter to achieve linear interpolation, i.e. using a low-pass filter with the impulse response given as follows. $h_{lin}[n] = \begin{Bmatrix} 1-|n|/L, & |n|<L \\ 0, & \text{otherwise} \end{Bmatrix}$

   Determine approximately the number of multiplications required to obtain the result. Provide the written Matlab function and a picture of the obtained 1024x1024-pixel cameraman image.

c. Compare the systems in a and b briefly in terms of computation complexity and the output image’s visual quality.

d. Propose a system to obtain a 128x128-pixel version of the cameraman image from the 256x256-pixel cameraman image. (No need for Matlab implementation.)