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(1)

EE 430 HW #2

1) a) First find it for $y[n]-\frac{1}{2}y[n-1]=x[n]$

$\hat{y}_s[n]-\frac{1}{2}\hat{y}_s[n-1]=0$

$\rightarrow Az^n(1-\frac{1}{2}z^{-1})=0 \qquad z=\frac{1}{2}$

$\Rightarrow \hat{y}_s[n]=c_1\left(\frac{1}{2}\right)^n u[n]$

$\rightarrow h_s[n]-\frac{1}{2}h_s[n-1]=\delta[n] \quad ; \quad h_s[0]=1,\ h_s[1]=1/2$

$h_s[0]=c_1=1 \rightarrow c_1=1$

$h_s[n]=\left(\frac{1}{2}\right)^n u[n]$

and as LTI $\Rightarrow h[n]=h_s[n]-h[n-1]+h[n+1]$

$=\left(\frac{1}{2}\right)^n\left(\delta[n]-\frac{1}{2}\delta[n-1]+3u[n-2]\right) \qquad = h[n]$

b) $H(e^{j\omega})=\sum_{k=-\infty}^{\infty}h[k]e^{-j\omega k}$

$=1-\frac{1}{2}e^{-j\omega}+3\sum_{k=2}^{\infty}\left(\frac{1}{2}\right)^k\cdot e^{-j\omega k}$

$=1-\frac{1}{2}e^{-j\omega}+3\left[\sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k e^{-j\omega k}-1-\frac{1}{2}e^{-j\omega}\right]$

$=1-\frac{1}{2}e^{-j\omega}+3\frac{1}{1-\frac{1}{2}e^{-j\omega}}-3-\frac{3}{2}e^{-j\omega}$

$=-2-2e^{-j\omega}+\frac{6}{2-e^{-j\omega}}$

c) we need z transform

$$Y(z) - \frac{1}{2} Y(z) z^{-1} = X(z) - X(z) z^{-1} + X(z) z^{-2}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-1} + z^{-2}}{1 - \frac{1}{2} z^{-1}}$$

A and B vectors are such that

$A = [1 \ -0.5]$ $B = [1 \ -1 \ 1]$

freqz (B, A, 'whole')

Plots are given in the page 4.

h[n] is real, as expected, magnitude plot is even and phase plot is odd

d) $y[n] = x[n] * h[n]$

Convolution in time domain is not so easy here

$$x[n] = \cos\left(\frac{\pi}{3}n\right) + \sin\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)$$

$$= \cos\left(\frac{\pi}{3}n\right) + \sin\left(\frac{\pi}{2}n\right) \cdot \frac{1}{\sqrt{2}} + \cos\left(\frac{\pi}{2}n\right) \cdot \frac{1}{\sqrt{2}} \quad \text{(this is easier)}$$

$$X(e^{j\omega}) = \pi\left[\delta\left(\omega - \frac{\pi}{3}\right) + \delta\left(\omega + \frac{\pi}{3}\right)\right] + \frac{\pi}{j\sqrt{2}}\left[\delta\left(\omega - \frac{\pi}{2}\right) - \delta\left(\omega + \frac{\pi}{2}\right)\right]$$

$$+ \frac{\pi}{\sqrt{2}}\left[\delta\left(\omega - \frac{\pi}{2}\right) + \delta\left(\omega + \frac{\pi}{2}\right)\right]$$

From the convolution property $Y(e^{j\omega}) = X(e^{j\omega}) H(e^{j\omega})$

$$Y(e^{j\omega}) = \left(-2 - 2e^{-j\omega} + \frac{6}{2 - e^{-j\omega}}\right)\left[\frac{\pi(1-j)}{\sqrt{2}} \delta\left(\omega - \frac{\pi}{2}\right) + \pi\delta\left(\omega - \frac{\pi}{3}\right) + \pi\delta\left(\omega + \frac{\pi}{3}\right) + \left(\frac{(1+j)\pi}{\sqrt{2}}\right)\delta\left(\omega + \frac{\pi}{2}\right)\right]$$

$$= \left(-2 + j2 + \frac{6}{2+j}\right)\frac{\pi(1-j)}{\sqrt{2}}\delta\left(\omega - \frac{\pi}{2}\right) + \left(-2 - j2 + \frac{6}{2-j}\right)\frac{\pi(1+j)}{\sqrt{2}}\delta\left(\omega + \frac{\pi}{2}\right)$$

$$+ \underbrace{\left(-2 - 2e^{-j\pi/3} + \frac{6}{2 - e^{-j\pi/3}}\right)}_{0}\pi\delta\left(\omega - \frac{\pi}{3}\right) + \underbrace{\left(-2 - 2e^{j\pi/3} + \frac{6}{2 - e^{j\pi/3}}\right)}_{0}\pi\delta\left(\omega + \frac{\pi}{3}\right)$$

$$= \left(\frac{3\sqrt{2}}{5} + j\frac{\sqrt{2}}{5}\right)\pi\delta\left(\omega - \frac{\pi}{2}\right) + \left(\frac{3\sqrt{2}}{5} - j\frac{\sqrt{2}}{5}\right)\pi\delta\left(\omega + \frac{\pi}{2}\right)$$

(3)

Now inverse transform

$$y[n] = \left(\frac{3\sqrt{2}}{5} + j\frac{\sqrt{2}}{5}\right)\pi \cdot \frac{1}{2\pi} e^{j\frac{\pi}{2}n} + \left(\frac{3\sqrt{2}}{5} - j\frac{\sqrt{2}}{5}\right)\pi \cdot \frac{1}{2\pi} e^{-j\frac{\pi}{2}n}$$

$$y[n] = \left(\frac{3\sqrt{2}}{10} + j\frac{\sqrt{2}}{10}\right)(j)^n + \left(\frac{3\sqrt{2}}{10} - j\frac{\sqrt{2}}{10}\right)(-1)^n \cdot (j)^n$$

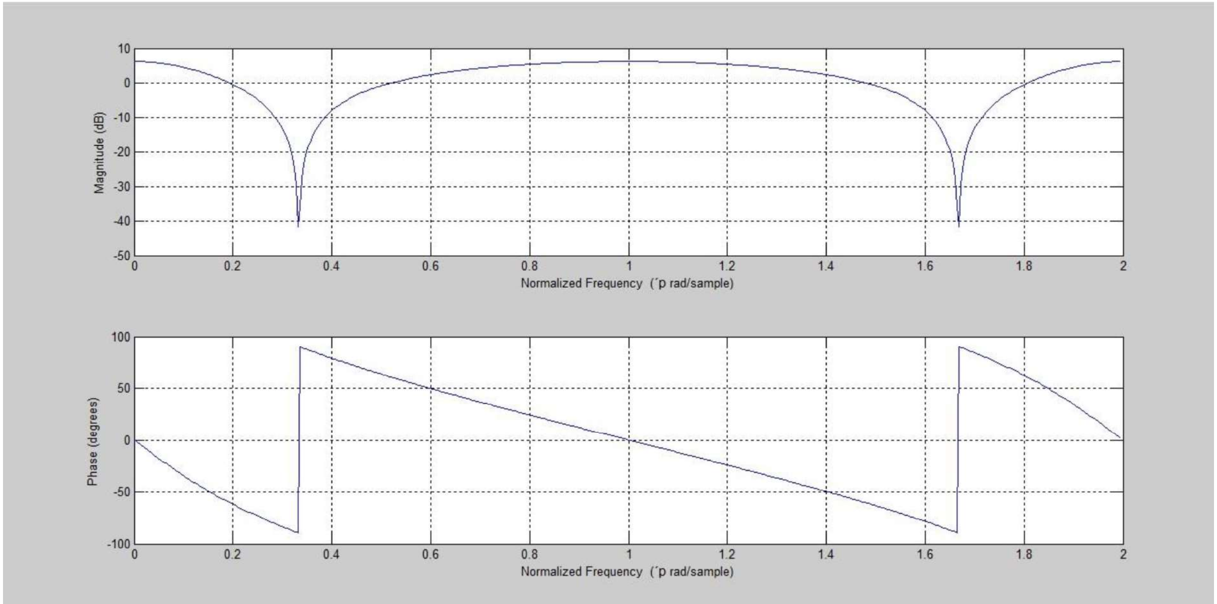
$$\Rightarrow y[n] = \begin{cases} \frac{3\sqrt{2}}{5} & 0 \equiv \mathrm{mod}(n) \\ -\frac{\sqrt{2}}{5} & 1 \equiv \mathrm{mod}(n) \\ -\frac{3\sqrt{2}}{5} & 2 \equiv \mathrm{mod}(n) \\ \frac{\sqrt{2}}{5} & 3 \equiv \mathrm{mod}(n) \end{cases}$$

e) $H(e^{j\omega}) = -2 - 2e^{-j\omega} + \dfrac{6}{2 - e^{-j\omega}}$

$$H^*(e^{j(2\pi-\omega)}) = \left(-2 - 2\underbrace{e^{-j2\pi}}_{1} e^{j\omega} + \frac{6}{2 - \underbrace{e^{-j2\pi}}_{1} \cdot e^{j\omega}}\right)^*$$

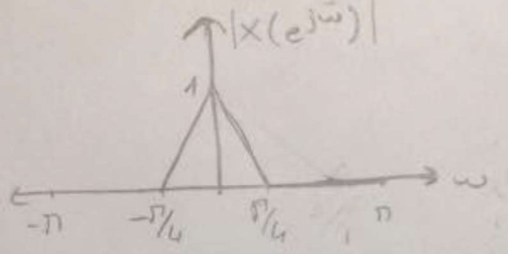
$$= \left(-2 - 2e^{j\omega} + \frac{6}{2 - e^{j\omega}}\right)^* = -2 - 2e^{-j\omega} + \frac{6}{2 - e^{-j\omega}}$$

At the last step we can see that, the equality $H(e^{j\omega}) = H^*(e^{j(2\pi-\omega)})$ because $H(e^{j\omega})$ is even symmetric. If it wasn't, this wouldn't satisfy. That means, for this equation to hold $h[n]$ must be real.


Magnitude (dB)
Normalized Frequency (´p rad/sample)
Phase (degrees)
Normalized Frequency (´p rad/sample)

⑦

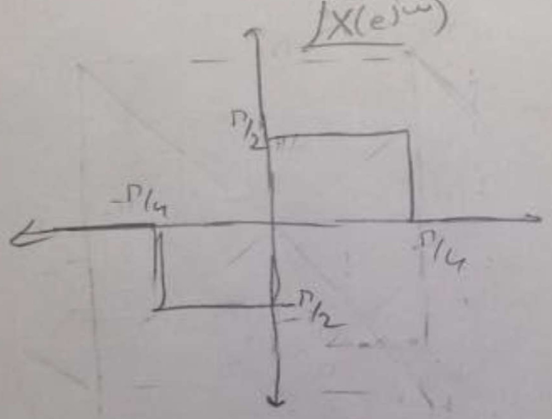
4) a)



$a = -\frac{\pi}{4}$

$b = \frac{3\pi}{4}$

must be even as $x[n]$ is real. However, phase diagram must be odd.



b) • $\cos\left(\frac{\pi}{5}n\right) \leftrightarrow \left(\delta\left(\omega-\frac{\pi}{5}\right)+\delta\left(\omega+\frac{\pi}{5}\right)\right)\times\pi$

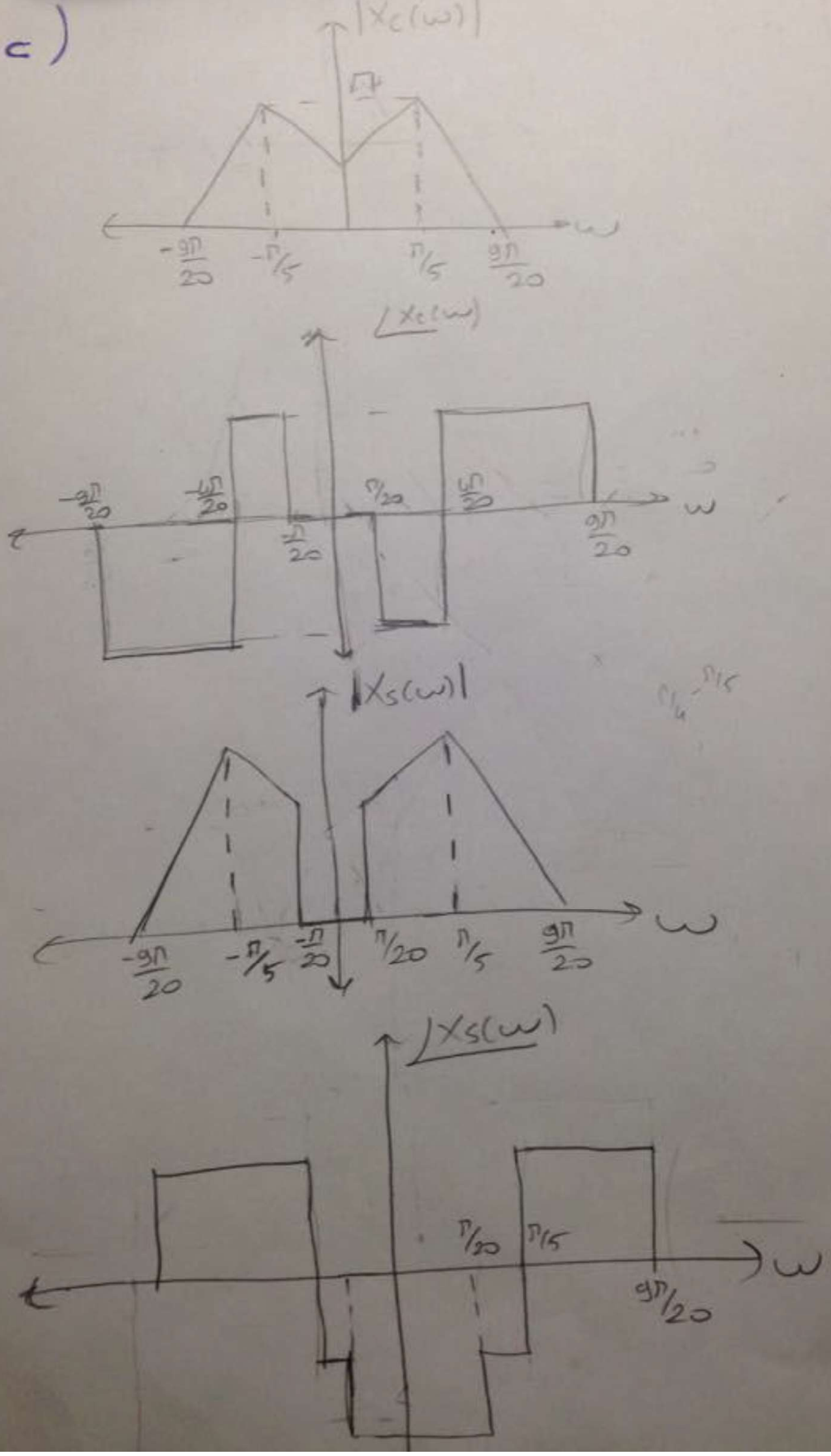
$\Rightarrow \cos\left(\frac{\pi}{5}n\right)x[n] \leftrightarrow \mathcal{F}\left\{\cos\left(\frac{\pi}{5}n\right)\right\} * X(e^{j\omega})$

$X_c(\omega)=\left[X\left(\omega-\frac{\pi}{5}\right)+X\left(\omega+\frac{\pi}{5}\right)\right]\times\pi$

• $\sin\left(\frac{\pi}{5}n\right) \leftrightarrow \frac{\pi}{j}\left(\delta\left(\omega-\frac{\pi}{5}\right)-\delta\left(\omega+\frac{\pi}{5}\right)\right)$

$X_s(\omega)=\frac{\pi}{j}\left[X\left(\omega-\frac{\pi}{5}\right)-X\left(\omega+\frac{\pi}{5}\right)\right]$

c)



|Xc(ω)|
ω
-9π/20
-π/5
π/5
9π/20
∠Xc(ω)
ω
-9π/20
-4π/20
-π/20
π/20
4π/20
9π/20
|Xs(ω)|
ω
-9π/20
-π/5
-π/20
π/20
π/5
9π/20
∠Xs(ω)
ω
π/20
π/5
9π/20