5) Parts (a) and (b) are independent.

a) Let $a[n] = \delta[n-1] - 2\delta[n-2] + \frac{3}{2}\delta[n-3]$ and $A[k]$ denote its 6-point DFT.

i) Find and plot the 6-point DFT, $B[k]$, of $A[n]$.

$$x[n] \xleftrightarrow{N\text{-point DFT}} X[k] \xleftrightarrow{N\text{-point DFT}} Nx\left[\left((-m)\right)_N\right]$$

$$a[n] = \begin{bmatrix} 0 & 1 & -2 & \frac{3}{2} & 0 & 0 \end{bmatrix} \Rightarrow B[k] = [0 \quad 0 \quad 0 \quad 9 \quad -12 \quad 6]$$

ii) Find and plot the 6-point DFT, $C[k]$, of $A\left[\left((n+3)\right)_6\right]$.

$$C[k] = e^{j3\frac{2\pi}{6}k}B[k] = (-1)^k B[k] = [0 \quad 0 \quad 0 \quad -9 \quad -12 \quad -6]$$

b) Let $x[n]$ be a 6-point signal that is zero outside the interval $0 \le n \le 5$. Let $X[k]$ be its 6-point DFT.

i) Evaluate the following expression for $n = 8$ and find the result in terms of $x[n]$:

$$\frac{1}{6}\sum_{k=0}^{5} X[k]e^{jk\frac{2\pi}{6}n}$$

$$\frac{1}{6}\sum_{k=0}^{5} X[k]e^{jk\frac{2\pi}{6}8} = \frac{1}{6}\sum_{k=0}^{5} X[k]\left(e^{jk\frac{2\pi}{6}2}e^{jk\frac{2\pi}{6}6}\right) = \frac{1}{6}\sum_{k=0}^{5} X[k]e^{jk\frac{2\pi}{6}2} = x[2]$$

ii) Assume in this part that $x[n]$ is real. If $X[k]$, for $k = 0,1,2,3$ are given as follows,

$$X[0] = a + jb$$
$$X[1] = c + jd$$
$$X[2] = e + jf$$
$$X[3] = g + jh,$$

find the value of b and h. Find $X[4]$ and $X[5]$ in terms of c, d, e, f.

$x[n]$ is real $\Rightarrow$ 1) $X[0]$ *is real*

2) $X[k] = X^*\left[\left((-k)\right)_6\right], k = 1,2,\ldots,5 \quad \Rightarrow \quad X[3]$ *is real*

$$\Rightarrow b = h = 0,\ X[4] = X^*[2] = e - jf,\ X[5] = X^*[1] = c - jd$$

iii) Determine a procedure to compute N samples of the DTFT of $x[n]$ at the equally spaced frequencies given by $\omega_k = \frac{2\pi}{N}k$ for $k = 0,1,\ldots,N-1$ by using only one N-point DFT for

(1) $N = 9$

$$X(e^{j\omega})\Big|_{\omega=\frac{2\pi}{9}k} = X_9[k],\ \ k = 0,1,\ldots,8$$

$X_9[k]$: 9-point DFT of $x[n]$

(2) $N = 4$.

We need $\sum_{n=0}^{5} x[n]e^{-jk\frac{2\pi}{4}n}$, $k = 0,1,2,3$.

$$\sum_{n=0}^{5} x[n]e^{-jk\frac{2\pi}{4}n} = \sum_{n=0}^{3} x[n]e^{-jk\frac{2\pi}{4}n} + \sum_{n=4}^{5} x[n]e^{-jk\frac{2\pi}{4}n} =$$
$$= \sum_{n=0}^{3} x[n]e^{-jk\frac{2\pi}{4}n} + \sum_{n=0}^{1} x[n+4]e^{-jk\frac{2\pi}{4}n}.$$

Therefore, compute the 4-point DFT, $Y[k]$, of the 4-point sequence

$$y[n] = \begin{cases} x[n] + x[n+4] & n = 0,1,2,3 \\ 0 & otherwise \end{cases},$$

then,

$$X(e^{j\omega})\Big|_{\omega=\frac{2\pi}{4}k} = Y[k], \ \ k = 0,1,2,3.$$