Homework 1

Due date: 29/10/2018, until 23.55

1) A discrete-time signal $x[n]$ is obtained by sampling a continuous-time sinusoidal signal

$$x_c(t) = 4\sin\left(20000\pi t + \frac{\pi}{13}\right)$$

at a sampling rate of 3 kHz.

a) Describe the set of all other continuous-time sinusoidal signals (their frequencies) that yield $x[n]$ when sampled at 3 kHz.

b) Describe the set of all other sampling frequencies that yield $x[n]$ from $x_c(t)$.

2) Among the following sinusoidal signals, identify periodic ones and find their fundamental periods.

$$\sin(1.74\pi n + 3.1), \sin(1.74\pi n + 3.1\pi), \cos\left(15.74\pi n + \frac{3\pi}{8}\right), \cos(\sqrt{\pi}n), \cos(\pi\sqrt{\pi}n), \cos(\pi\sqrt{2}\, n)$$

3) A linear sistem whose response to a shifted impulse, $\delta[n-k]$, is $h_k[n] = (n-k)u[n-k]$. Find the output, $y[n]$ of this system for an arbitrary input, $x[n]$. Is this system time-invariant? Explain clearly by using $y[n-m]$ and $x[n-m]$.

4) What does the following system do? Is it linear, time-invariant? Prove your answer.

$$y[n] = \begin{cases} x\left[\frac{n}{2}\right] & if\ n\ is\ even \\ \dfrac{x\left[\frac{n-1}{2}\right] + x\left[\frac{n+1}{2}\right]}{2} & if\ n\ is\ odd \end{cases}$$

5) Are the following systems causal, stable? Justify/discuss/prove.

$$y[n] = 2^{\delta[n+1]} + x[n-3]$$

$$y[n] = \begin{cases} y[-\delta[n-1]] + x[n-3] & n > 0 \\ 2^n x[n-3] & n \leq 0 \end{cases}$$

6) Let $y[n] = x[n] * x[n]$ where $*$ is the convolution operator. If the first and the last non-zero elements of $x[n]$ are equal to $x[-6] = -3$ and $x[24] = -4$, respectively, find the first and the last non-zero elements of $y[n]$.

7) Calculate the value for output, $y[n]$, as $n \rightarrow \infty$ for the LTI system, whose input is $x[n]$= $u[n]$ and its impulse response is equal to $h[n] = 3\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{3}\right)^{n-1} u[n]$

8) Consider the following difference equation of a causal LTI system,

$$y[n] - \frac{1}{2}y[n-1] = x[n] - x[n-1] + x[n-2]$$

a) Find the impulse response, $h[n]$.

b) Find the frequency response, $H(e^{j\omega})$.

c) Plot the magnitude response, $\left|H\left(e^{j\omega}\right)\right|$ and phase response, $\measuredangle H\left(e^{j\omega}\right)$, in MATLAB using "freqz" command.

d) Let $x[n] = \cos\left(\frac{\pi}{3}n\right) + \sin\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)$ be the system input. Find the output $y[n]$.

e) Prove that $H\left(e^{j\omega}\right) = H^*\left(e^{j(2\pi-\omega)}\right)$. Does this equality hold for an arbitrary $h[n]$? Explain.

9) In the $[-\pi, \pi)$ interval, the DTFT, $X\left(e^{j\omega}\right)$, of a real sequence $x[n]$ is nonzero only in $[a, b], a < 0 < b, b - a = \frac{\pi}{2}$; otherwise it is arbitrary.

a) Plot the magnitude and phase of a typical $X\left(e^{j\omega}\right)$ for $-\infty < \omega < \infty$.

b) Express the DTFTs, $X_C\left(e^{j\omega}\right)$ and $X_S\left(e^{j\omega}\right)$, respectively, of $\cos\left(\frac{\pi}{5}n\right)x[n]$ and $\sin\left(\frac{\pi}{5}n\right)x[n]$ in terms of $X\left(e^{j\omega}\right)$.

c) Assume that $X\left(e^{j\omega}\right)$ is real valued and still complies with the specifications above. Plot typical magnitudes and phases for $X\left(e^{j\omega}\right)$, $X_C\left(e^{j\omega}\right)$ and $X_S\left(e^{j\omega}\right)$.

10) Textbook question 2.55

11) Textbook question 2.60

12) Textbook question 2.65

The following is a computer problem.

13) **MATLAB PART**

a) Create a MATLAB function named `myconv` in a new m-file. This function will compute and return the convolution of two finite length sequences using multiplications and delay operations.

b) Generate the sequences x[n]=[1 2 3 4 5 4 3 2 1] and h[n]=[1 1 1].

↑ ↑

n=0 n=0

Convolve these sequences using `myconv`. Compute the convolution output and give the graph of it (using `stem` command). Verify the result using `conv` command. Plot the result on the same graph you plot `myconv` output.

c) Read the description of `freqz` command of MATLAB. Using this command, plot the 1024-point sampled magnitude and phase response of the filter h[n]. Comment on its magnitude and phase chracteristics. Based on your comment and the convolved sequence in part b), explain the reason of smoothing effect of this filter.

d) Now, consider the filter h2[n]=[1 -2 1]. Convolve x[n] with h2[n] and plot the result using `stem` command.

↑

n=0

e) Using `freqz` command, plot the 1024-sampled magnitude and phase response of the filter h2[n]. Comment on its magnitude and phase chracteristics. Based on your comment and the convolved sequence in part d), explain the effect of this filter.

f) Generate the following sequence in MATLAB

$$z[n] = \begin{cases} 1 + \sin\left(\frac{\pi}{3}n\right) + \sin\left(\frac{2\pi}{3}n\right), & n = 0, 1, \dots 59 \\ 0, & \text{otherwise .} \end{cases}$$

Plot z[n]. Using `freqz` command, plot the 1024-point sampled magnitude and phase response of it. Comment on the results.

g) Convolve the sequence z[n] with the filter h[n]. Denote the output by y[n]. Plot y[n]. Using `freqz` command, plot the 1024-point sampled magnitude and phase response of it. Comment on the results.