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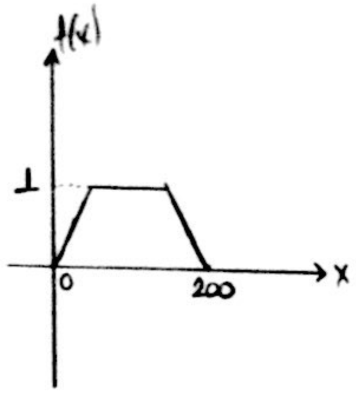
Section 2

EE430 - HW4

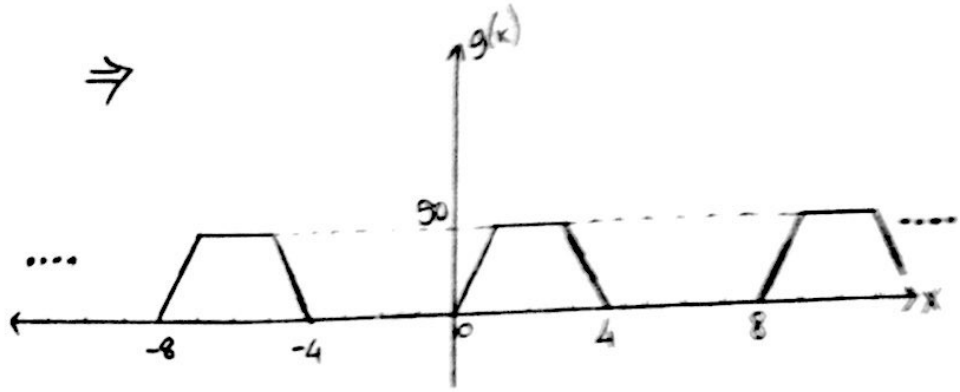
1)



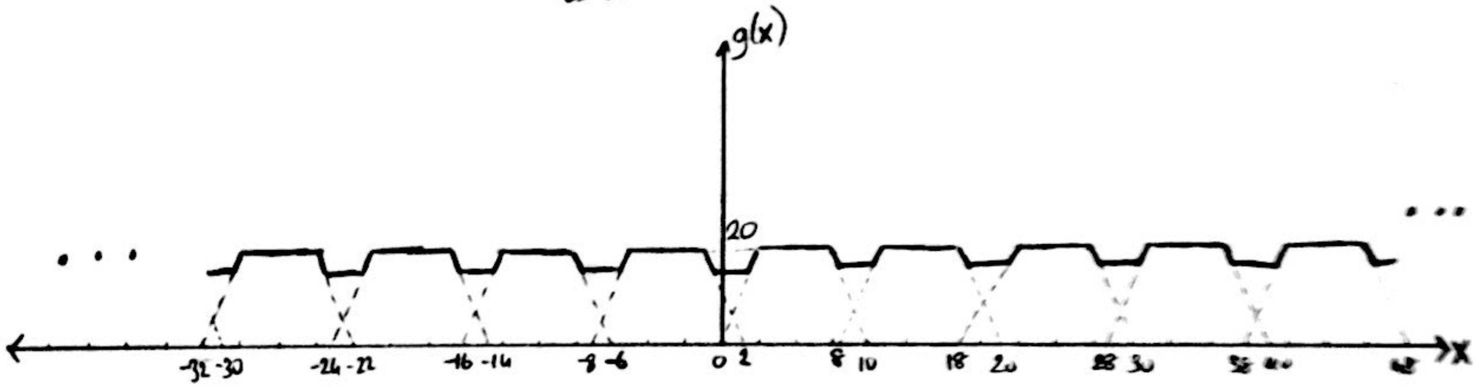
$\Rightarrow$ i) for $A = 50$;

$$g(x) = 50\sum_{k=-\infty}^{\infty} f(50x - k400)$$

$\Rightarrow$



ii) for $A = 20$; $g(x) = 20\sum_{k=-\infty}^{\infty} f(20x - k160)$

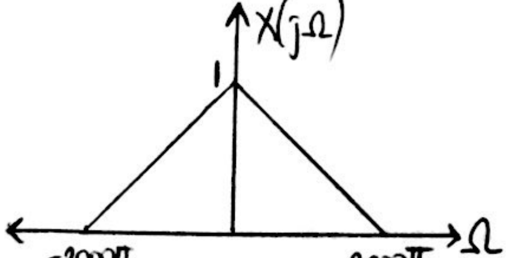


iii) for $f(Ax - k8A), k \in \mathbb{Z}$, to not overlap ; $8A > 200 \Rightarrow \boxed{A > 25}$

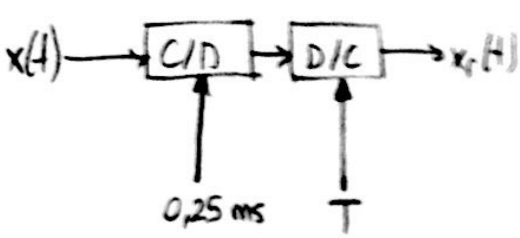
2) $X(\Omega) = 0$ for $\Omega \leq -100\pi$ and $\Omega \geq 1000\pi$.

For $x(t)$ to be reconstructable from its samples, the impulse train that samples $x_c(t)$ must be at least $f_s = \dfrac{100\pi + 1000\pi}{2\pi} = 550$ Hz so that there is no aliasing.

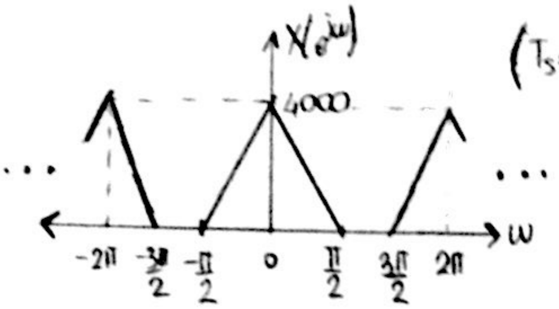
3)



and the system is



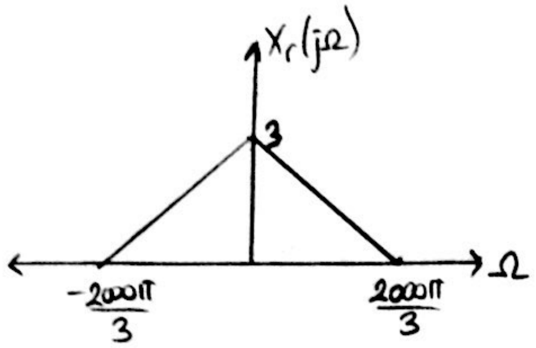
⇒ Lets call the discrete time signal obtained by sampling $x(t)$ with $T_s = 0{,}25$ ms $x[n]$. Then; $X(e^{jw})$ (DTFT of $x[n]$):



($T_s = 0{,}25$ ms ⇒ $f_s = 4$ kHz)

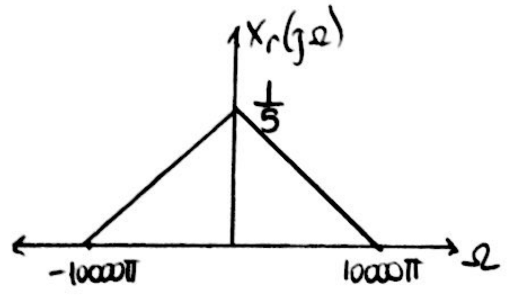
i) if reconstruction freq is $\frac{4}{3}$ kHz ($T = 0{,}75$ ms)

$$\Rightarrow X_r(j\Omega) = 3X(3j\Omega) \Rightarrow$$

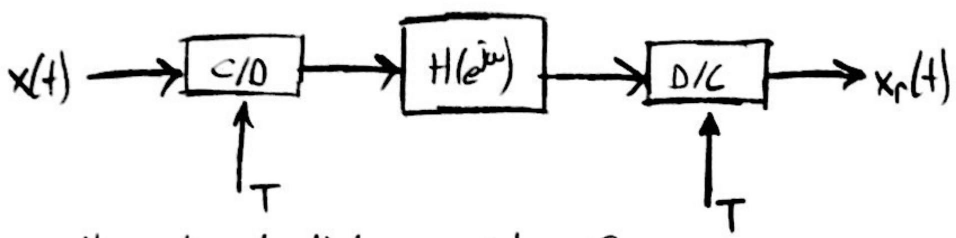


ii) if reconstruction freq is 20 kHz ($T = 0{,}05$ ms)

$$\Rightarrow X_r(j\Omega) = \frac{1}{5} X\left(j\frac{\Omega}{5}\right) \Rightarrow$$



5)



It is desired that
$$H_{eff}(\Omega) = \begin{cases} 1 , & 9000\pi \leq |\Omega| \leq 10000\pi \\ 0 , & \text{o.w.} \end{cases}$$

and it is known that $H(e^{jw})$ is a high pass filter.

a) We know that $H_{eff}(\Omega) = H(e^{j\Omega T})$ for $\Omega \leq \frac{\pi}{T}$

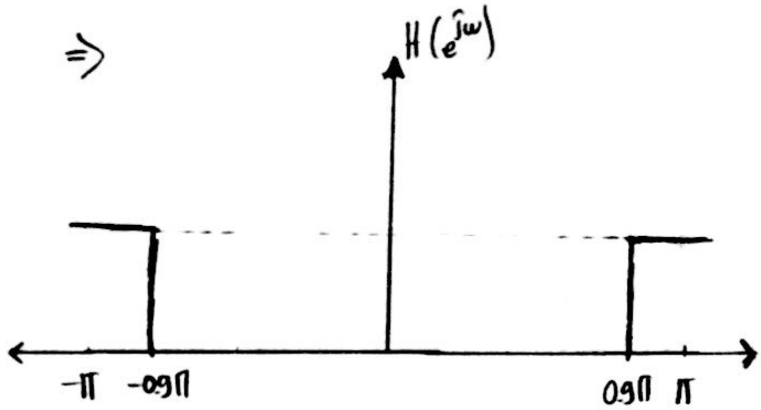
provided that the input is bandlimited to $\frac{\pi}{T}$.

So, our constrain is that $x(t)$ must be bandlimited to $\frac{\pi}{T}$ for us to use $H(e^{jw})$ as a bandpass filter.

$\Rightarrow \quad \frac{\pi}{T} = 10000\pi \Rightarrow \boxed{T = 0.1 \text{ ms}}$

and cut-off frequency of $H(e^{jw})$ must be $w_c = 9000\pi . T = 0.9\pi$

$\Rightarrow$



b) for this case it is desired that;

$$H_{eff}(j\Omega) = \begin{cases} 1 & , 900\pi \leq |\Omega| \leq 1000\pi \\ 0 & , \text{o.w} \end{cases}$$

$\Rightarrow \frac{\pi}{T} = 1000\pi \Rightarrow T = 1 \text{ ms}$ and $w_c = 900\pi . T = 0.9\pi$

and $x(t)$ must be bandlimited to $\frac{\pi}{T} = 1000\pi$

7)

$$x[n] \longrightarrow \boxed{\uparrow 2} \xrightarrow{z_1[n]} \boxed{\downarrow 5} \longrightarrow y_1[n]$$

$$x[n] \longrightarrow \boxed{\downarrow 5} \xrightarrow{z_2[n]} \boxed{\uparrow 2} \longrightarrow y_2[n]$$

a) $z_1[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-2k] \Rightarrow Z_1(e^{jw}) = \sum_{k=-\infty}^{\infty} x[k] e^{-jwkL}$

$$\boxed{\Rightarrow Z_1(e^{jw}) = X(e^{jwL})}$$

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and $z_1[n] = z_c(nT) \Rightarrow y_1[n] = z_c(n5T)$

$$\Rightarrow Z_1(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} Z_c\left(\frac{1}{T}(\omega - k2\pi)\right) \Rightarrow \frac{1}{5}Z_1(e^{j\frac{\omega}{5}}) = \frac{1}{5T}\sum_{k=-\infty}^{\infty} Z_c\left(\frac{1}{5T}(\omega - 2k\pi m)\right) \quad (1)$$

$$\Rightarrow Y_1(e^{j\omega}) = \frac{1}{5T}\sum_{k=-\infty}^{\infty} Z_c\left(\frac{1}{5T}(\omega - k2\pi)\right) \quad (2)$$

by (1) and (2) we see that

$$\boxed{Y_1(e^{j\omega}) = \frac{1}{5}\sum_{k=0}^{4} Z\left(e^{j\left(\frac{\omega}{5} - \frac{k2\pi}{5}\right)}\right) = \frac{1}{5}\sum_{k=0}^{4} X\left(e^{j2\left(\frac{\omega}{5} - \frac{k2\pi}{5}\right)}\right)}$$

b) Similarly;

$$\boxed{Z_2(e^{j\omega}) = \frac{1}{5}\sum_{k=0}^{4} X\left(e^{j\left(\frac{\omega}{5} - \frac{k2\pi}{5}\right)}\right)}$$

and

$$\boxed{Y_2(e^{j\omega}) = Z_2(e^{j\omega 2}) = \frac{1}{5}\sum_{k=0}^{4} X\left(e^{j\left(\frac{2\omega}{5} - \frac{k2\pi}{5}\right)}\right)}$$

c) $y_1[n] = y_2[n]$ since $Y_1(e^{j\omega}) = Y_2(e^{j\omega})$ found in part a) and part b), simply.

$$Y_1(e^{j\omega}) = \frac{1}{5}\left[X(e^{j\frac{2\omega}{5}}) + X\left(e^{j\left(\frac{2\omega}{5} - \frac{4\pi}{5}\right)}\right) + X\left(e^{j\left(\frac{2\omega}{5} - \frac{8\pi}{5}\right)}\right) + X\left(e^{j\left(\frac{2\omega}{5} - \frac{12\pi}{5}\right)}\right) + X\left(e^{j\left(\frac{2\omega}{5} - \frac{16\pi}{5}\right)}\right)\right]$$

and $$Y_2(e^{j\omega}) = \frac{1}{5}\left[X(e^{j\frac{2\omega}{5}}) + X\left(e^{j\left(\frac{2\omega}{5} - \frac{2\pi}{5}\right)}\right) + X\left(e^{j\left(\frac{2\omega}{5} - \frac{4\pi}{5}\right)}\right) + X\left(e^{j\left(\frac{2\omega}{5} - \frac{6\pi}{5}\right)}\right) + X\left(e^{j\left(\frac{2\omega}{5} - \frac{8\pi}{5}\right)}\right)\right]$$

$Y_2(e^{j\omega}) = Y_1(e^{j\omega})$ term by term since $X\left(e^{j\left(\frac{2\omega}{5} - \frac{2\pi}{5}\right)}\right) = X\left(e^{j\left(\frac{2\omega}{5} - \frac{12\pi}{5}\right)}\right)$

and $X\left(e^{j\left(\frac{2\omega}{5} - \frac{6\pi}{5}\right)}\right) = X\left(e^{j\left(\frac{2\omega}{5} - \frac{16\pi}{5}\right)}\right)$

due to periodicity of $X(e^{j\omega})$ with 2π.

d) If downsampling factor is 4;

$$y_1(e^{j\omega}) = \frac{1}{4}\sum_{k=0}^{3} X\left(e^{j2\left(\frac{\omega}{4}-\frac{k2\pi}{4}\right)}\right) \quad \text{and} \quad y_2(e^{j\omega}) = \frac{1}{4}\sum_{k=0}^{3} X\left(e^{j\left(\frac{2\omega}{4}-\frac{k2\pi}{4}\right)}\right)$$

$$\Rightarrow y_1(e^{j\omega}) = \frac{1}{4}\left[X\left(e^{j\frac{\omega}{2}}\right) + X\left(e^{j\left(\frac{\omega}{2}-\pi\right)}\right) + X\left(e^{j\left(\frac{\omega}{2}-2\pi\right)}\right) + X\left(e^{j\left(\frac{\omega}{2}-3\pi\right)}\right)\right]$$

$$y_2(e^{j\omega}) = \frac{1}{4}\left[X\left(e^{j\frac{\omega}{2}}\right) + X\left(e^{j\left(\frac{\omega}{2}-\frac{\pi}{2}\right)}\right) + X\left(e^{j\left(\frac{\omega}{2}-\pi\right)}\right) + X\left(e^{j\left(\frac{\omega}{2}-\frac{3\pi}{2}\right)}\right)\right]$$

$$\Rightarrow y_1(e^{j\omega}) \neq y_2(e^{j\omega}) \quad \text{since} \quad X\left(e^{j\left(\frac{\omega}{2}-2\pi\right)}\right) + X\left(e^{j\left(\frac{\omega}{2}-3\pi\right)}\right)$$

$$\neq X\left(e^{j\left(\frac{\omega}{2}-\frac{\pi}{2}\right)}\right) + X\left(e^{j\left(\frac{\omega}{2}-\frac{3\pi}{2}\right)}\right)$$

Hence $y_1[n] \neq y_2[n]$ for downsampling factor of 4.

8)

$$x[n] \longrightarrow \boxed{H(e^{j\omega})} \longrightarrow \boxed{\downarrow M} \longrightarrow y_1[n]$$

$$x[n] \longrightarrow \boxed{\downarrow M} \longrightarrow \boxed{G(e^{j\omega})} \longrightarrow y_2[n]$$

$$\Rightarrow y_1(e^{j\omega}) = \frac{1}{M}\sum_{k=0}^{M-1} X\left(e^{j\left(\frac{\omega}{M}-\frac{2\pi k}{M}\right)}\right) H\left(e^{j\left(\frac{\omega}{M}-\frac{2\pi k}{M}\right)}\right) = \underbrace{\frac{1}{M}\cdot\sum_{k=0}^{M-1} X\left(e^{j\left(\frac{\omega}{M}-\frac{2\pi k}{M}\right)}\right)\sum_{k=0}^{M-1} H\left(e^{j\left(\frac{\omega}{M}-\frac{2\pi k}{M}\right)}\right)}$$

all cross-products are zero since $x[n]$ is bandlimited to $\frac{\pi}{M}$

Therefore equality holds.

$$y_2(e^{j\omega}) = \left[\frac{1}{M}\sum_{k=0}^{M-1} X\left(e^{j\left(\frac{\omega}{M}-\frac{2\pi k}{M}\right)}\right)\right] G(e^{j\omega})$$

$$\Rightarrow \text{for } y_1[n] = y_2[n] \Rightarrow \boxed{G(e^{j\omega}) = \sum_{k=0}^{M-1} H\left(e^{j\left(\frac{\omega}{M}-\frac{2\pi k}{M}\right)}\right)}$$