Date: 25.11.2014 Time: 08:40 Duration: **110 minutes** Attempt all questions Closed books and notes

Middle East Technical University
Electrical-Electronics Engineering Department

EE 430 Digital Signal Processing

Midterm Examination I

CLOSED BOOKS
110 MINUTES

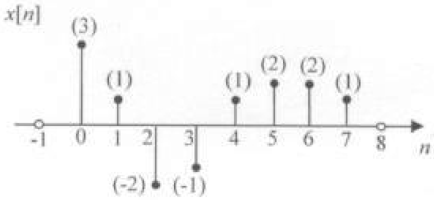
LASTNAME	
NAME	
STUDENT ID.	

Question	Grade
Q1 **(25 pts)**	
Q2 **(25 pts)**	
Q3 **(25 pts)**	
Q4 **(25 pts)**	
TOTAL	

Warning: Plagiarism is defined as the action of using or copying someone else's idea or work and pretending that you thought of it, or created it. Cheating is defined as lying or behaving dishonestly in order to reach your goal. In grading the exam papers in this course, occurrences of plagiarism and cheating will be seriously dealt with, leading to punishment through disciplinary procedures as indicated in University Catalog.

I have read and fully understood the warning, and I pledge to comply with the exam rules.
SIGNATURE :

Q1) a) x[n] is a finite-length sequence with DTFT $X(e^{j\omega})$. Define $Y[k]=X(e^{j\omega})\big|_{\omega=\frac{2\pi}{4}k}$ where $0\le k\le 3$. Find and sketch the 4-point sequence y[n], which is the IDFT(Y[k]).



b) h[n] is a 5-point sequence given below. y[n] is the 8-point inverse DFT of H[k]X[k] where H[k] and X[k] are the 8-point DFTs. Sketch y[n] and determine the number of samples which are the same as the linear convolution of h[n] and x[n].

c) y[n] is a 16-point sequence obtained from the 8-point sequence x[n] as,

$$y[n]=\begin{cases} x\left[\dfrac{n}{2}\right], & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

Find and write 16-point DFT Y[k] for the values of k in $0\le k\le 15$, in terms of 8-point DFT X[k].

a) $y[n]=\sum_{k=-\infty}^{\infty} x[n+4k]\,,\quad 0\le n\le 3$

$$\begin{aligned} x(n) &= [\underset{n=0}{3}\ 1\ -2\ -1\ 1\ 2\ 2\ 1] \\ + \quad x[n+4] &= [1\ 2\ 2\ 1\ 0\ 0\ 0\ 0] \\ \hline y[n] &= [4\ 3\ 0\ 0],\quad 0\le n\le 3 \end{aligned}$$

b) $y[n]=x[n] \circledast_8 h[n]$

$y[n]=[\underset{n=0}{7}\ 4\ -2\ -1\ 5\ 5\ 2\ 1]$

$\bar{y}[n]_{\text{linear}}=[\underset{n=0}{6}\ 2\ -4\ -2\ 5\ 5\ 2\ 1\ 1\ 2\ 2\ 1]$

h[n]: 5-point seq. $P-1=4$ samples of y[n] are corrupted

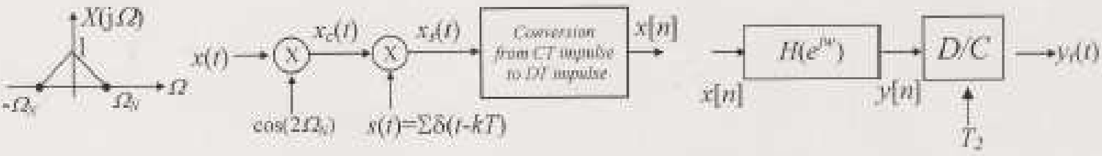
Hence in general $L-(P-1)=8-4=4$ samples are same. But for this specific example 5 samples are same.

c) $Y[k]=\sum_{\substack{n=0\\ n\ \text{even}}}^{15} y[n]e^{-j\frac{2\pi}{16}kn}+\underbrace{\sum_{\substack{n=0\\ n\ \text{odd}}}^{15} y[n]e^{-j\frac{2\pi}{16}kn}}_{0}=\sum_{m=0}^{7} y[2m]e^{-j\frac{2\pi}{16}k2m}$

$=\sum_{m=0}^{7} x[m]e^{-j\frac{2\pi}{8}km}=X[k]$

$$Y[k]=\begin{cases} X[k], & 0\le k\le 7 \\ X[((k))_8], & 8\le k\le 15 \end{cases}$$

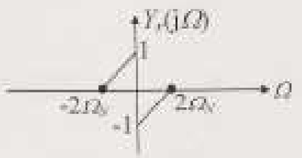
Q2) Assume the following sampling system with a continuous-time input, $x_c(t)$.

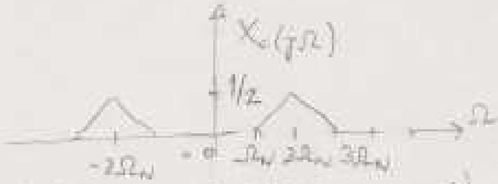


a) Determine and plot $X_c(j\Omega)$. Find the sampling periods, T, for obtaining alias-free reconstruction of $x_c(t)$ and $x(t)$ based on Nyquiest theorem.

b) Let $T = \pi/\Omega_N$, find and plot $X_s(j\Omega)$ and $X(e^{jw})$. Obtain a block diagram including a D/C converter and some additional system blocks to reconstruct modulated input, $x_c(t)$ (not $x(t)$).

c) The output of system in part-b, $x[n]$, is applied to the discrete system, $H(e^{jw})$, as shown in the figure above. If the resulting continuous-time signal , $y_r(t)$, has the following form, determine $H(e^{jw})$ and T_2 that yields this result.

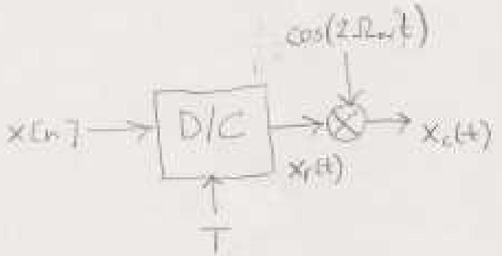


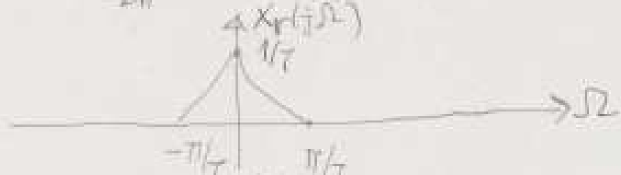


2 a. $x_c(t) = x(t)\cos(2\Omega_N t) \Rightarrow X_c(j\Omega) = \frac{1}{2}\left(X(j(\Omega-2\Omega_N)) + X(j(\Omega+2\Omega_N))\right)$

For $x(t)$ $\quad \frac{2\pi}{T} \geq 2\Omega_N \quad$ & $\quad x_c(t) \quad \frac{2\pi}{T} \geq 6\Omega_N$

b) If $T = \frac{\pi}{\Omega_N} \Rightarrow \frac{2\pi}{T} = 2\Omega_N$



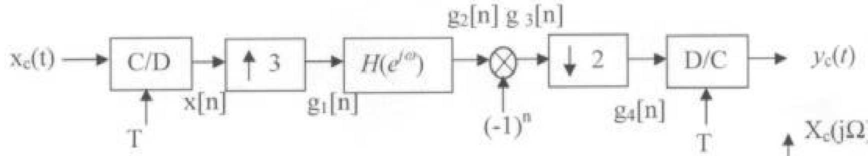


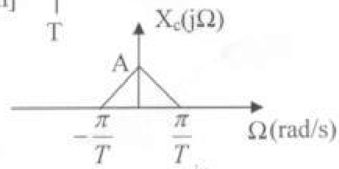
c) If

& $T_2 = \frac{\pi}{2\Omega_N}$

$K T_2 \frac{1}{T} = 1 \Rightarrow K = 2$

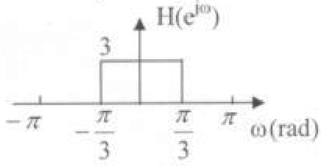
Q2)





Given the above block diagram, and figures,

a) Plot $X(e^{j\omega})$, $G_1(e^{j\omega})$, $G_2(e^{j\omega})$, $G_3(e^{j\omega})$, $G_4(e^{j\omega})$, and $Y_c(j\Omega)$.



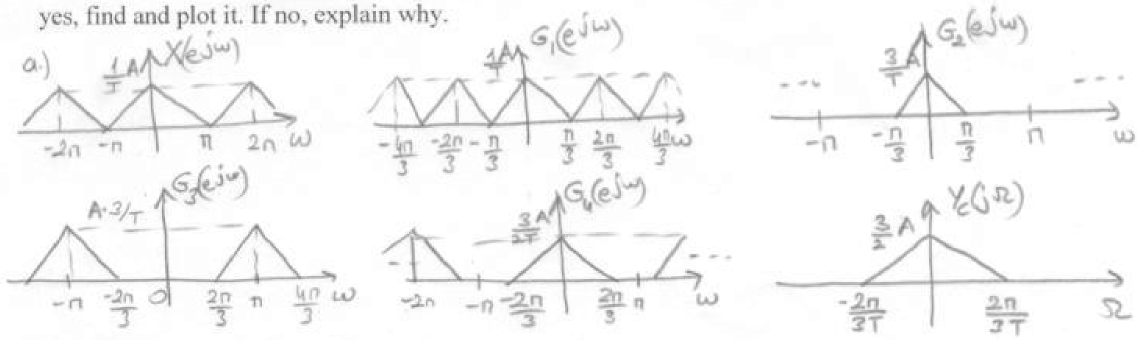
b) i) Write $X(e^{j\omega})$ in terms of $X_c\left(j\frac{\omega}{T}\right)$.

ii) Write $G_1(e^{j\omega})$ in terms of $X(e^{j\omega})$.

iii) Write $G_4(e^{j\omega})$ in terms of $G_3(e^{j\omega})$. → LTI

c) Can you find an equivalent analog filter $H_c(j\Omega)$ such that $Y_c(j\Omega) = H_c(j\omega)X_c(j\omega)$? If yes, find and plot it. If no, explain why.

a.)



b.) i) $X(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega-2\pi k}{T}\right)\right)$

ii) $G_1(e^{j\omega}) = X(e^{j3\omega})$

iii) $G_4(e^{j\omega}) = \frac{1}{2}\sum_{k=0}^{1} G_3\left(e^{j\left(\frac{\omega-2\pi k}{2}\right)}\right)$

c.) No, we can not find an equivalent LTI $H_c(j\Omega)$ analog filter since the original spectrum, $X_c(j\Omega)$, is compressed by $\frac{2}{3}$ facto which cannot be realized with a LTI filter.

Q4) Discrete Cosine Transform (DCT) of sequence, $x[n]$, (length-N) is defined as

$$X_{DCT}[k] = \sum_{n=0}^{N-1} 2x[n]\cos(\frac{\pi k(2n+1)}{2N}), \quad k = 0, \ldots, N-1$$

a) Assume $x[n]$ is extended to a sequence, $v[n]$, of length $2N$ by padding N zeros to its end. Let $y[n] = v[n] + v[2N\text{-}1\text{-}n]$. If $2N$-point DFT of $y[n]$ is equal to $Y[k]$, obtain $Y[k]$ in terms of $X_{DCT}[k]$.

b) Determine DCT of $x^*[n]$ in terms of $X_{DCT}[k]$.

c) The following relation is equal to a simple result; find this result for m and k integers [Hint: You may use complex exponentials].

$$\alpha[m,k] = \frac{1}{N}\sum_{n=0}^{N-1}\cos(\frac{\pi k(2n+1)}{2N})\cos(\frac{\pi m(2n+1)}{2N})$$

a) $$Y[k] = \sum_{n=0}^{2N-1} y[n]W_{2N}^{nk} = \sum_{n=0}^{N-1} y[n]W_{2N}^{nk} + \sum_{n=N}^{2N-1} y[n]W_{2N}^{nk} = \sum_{n=0}^{N-1} x[n]W_{2N}^{nk} + \sum_{n=N}^{2N-1} x[2N-1-n]W_{2N}^{nk}$$

$$= \sum_{n=0}^{N-1} x[n]W_{2N}^{nk} + \sum_{n=0}^{N-1} x[n]W_{2N}^{-kn}W_{2N}^{k(2N-1)} = W_{2N}^{-k/2}\sum_{n=0}^{N-1} x[n]\left(W_{2N}^{kn}W_{2N}^{k/2} + W_{2N}^{-kn}W_{2N}^{-k/2}\right)$$

$$= W_{2N}^{-k}\sum_{n=0}^{N-1} 2x[n]\cos\left(\frac{\pi k(2n+1)}{2N}\right) = W_{2N}^{-k}X_{DCT}[k], \quad 0 \le k \le 2N-1$$

b) $$X_{DCT}[k] = \sum_{n=0}^{N-1} 2x[n]\cos\left(\frac{\pi k(2n+1)}{2}\right) \longrightarrow X^*_{DCT}[k] = \sum_{n=0}^{N-1} 2x^*[n]\cos\left(\frac{\pi k(2n+1)}{2N}\right)$$

c) $$\alpha[m,k] = \frac{1}{4N}\sum_{n=0}^{N-1}\left(e^{j\frac{\pi kn}{N}}.e^{j\frac{\pi k}{2N}} + e^{-j\frac{\pi kn}{N}}.e^{-j\frac{\pi k}{2N}}\right)\left(e^{j\frac{\pi mn}{N}}.e^{j\frac{\pi m}{2N}} + e^{-j\frac{\pi mn}{N}}.e^{-j\frac{\pi m}{2N}}\right)$$

$$= \frac{1}{4N}\left[e^{j\frac{\pi(k+m)}{2N}}\underbrace{\sum_{n=0}^{N-1} e^{j\frac{\pi(m+k)n}{N}}}_{\substack{=0\text{, if } m\neq k\\ =1\text{, if } m=k=0}} + e^{-j\frac{\pi(m+k)}{2N}}\underbrace{\sum_{n=0}^{N-1} e^{-j\frac{(m+k)n}{N}}}_{\substack{=0\text{, if } m\neq k\\ =1\text{, if } m=k\neq 0\\ =1\text{, if } m=k=0}} + e^{j\frac{\pi(m-k)}{2N}}\underbrace{\sum e^{j\frac{\pi(m-k)n}{N}}} + e^{-j\frac{\pi(m-k)}{2N}}\sum \ldots\right]$$

$$\Rightarrow \alpha[m,k] = \begin{cases} 0 & m \neq k \\ 1/2 & k=m\neq 0 \\ 1 & k=m=0 \end{cases}$$