# EE430 Digital Signal Processing
# HW 1

1. A DT $x[n]$ is obtained by sampling the $x_c(t) = 4\sin(20000\pi t + \frac{\pi}{13})$ at sampling rate of 3 $kHz$.

   (a) The same DT signal can be constructed by sampling the set of signals in a form
   $$x_c(t) = 4\sin((20000 + 3000k)\pi t + \frac{\pi}{13}) \quad , \quad k = [-6, \infty)$$
   or equivalently,
   $$x_c(t) = 4\sin((2000 + 3000k)\pi t + \frac{\pi}{13}) \quad , \quad k = [0, \infty)$$

   (b) Let $\Omega_0 = 2000$ for our signal, the sampled signal can be expressed as;
   $$x[n] = 4\sin(\Omega_0 n T_s + \pi/3)$$
   it should be equal to the signal sampled at $\widetilde{T_s} \triangleq T_s + \Delta T$
   $$x[n] = 4\sin(\Omega_0 n \widetilde{T_s} + \pi/3) = 4\sin(\Omega_0 n(T_s + \Delta T) + \pi/3)$$
   To satisfy the equation $\Omega_0 n \Delta T$ should be equal to $k2\pi$ and knowing that
   $$\Omega_0 = 2\pi f_0$$
   $$\Delta t = \frac{k}{f_0} = kT_0$$
   $$\widetilde{T_s} = T_s + \Delta T$$
   using the equations above, the new set sampling frequencies that give the same $x[n]$ can be found as follows;
   $$\boxed{f_s' = \frac{f_s f_0}{f_0 + k f_s}} \quad , \quad k = 0, 1, 2...$$
   for our case $f_0 = 1000$ and $f_s = 3000$, from there other sampling frequencies that yield $x[n]$ from $x_c(t)$ can be calculated.

2. For any DT sinusoidal $cos(w_0 n + \phi)$ or complex exponential $e^{w_0 n + \phi}$ to be periodic with $N$, it has to satisfy the following,
   $$w_0 n = k2\pi$$
   or equivalently,
   $$\boxed{N = \frac{2\pi}{w_0} k} \quad , \quad k, N \in \mathcal{Z}$$
   For the given functions,

- $\sin(1.74\pi n + 3.1)$, periodic with $N_1 = \dfrac{2\pi}{1.74\pi}k = 100$ with $k = 87$
- $\sin(1.74\pi n + 31\pi)$, periodic with $N_2 = \dfrac{2\pi}{1.74\pi}k = 100$ with $k = 87$
- $\cos(15.74\pi n + \dfrac{3\pi}{8})$, periodic with $N_3 = \dfrac{2\pi}{15.74\pi}k = 100$ with $k = 787$
- $\cos(\sqrt{\pi}n)$, not periodic since there is no integer $k$ that makes $N_4 = \dfrac{2\pi}{\sqrt{\pi}}k$ an integer
- $\cos(\pi\sqrt{\pi}n)$, not periodic since there is no integer $k$ that makes $N_5 = \dfrac{2\pi}{\pi\sqrt{\pi}}k$ an integer
- $\cos(\pi\sqrt{2}n)$, not periodic since there is no integer $k$ that makes $N_6 = \dfrac{2\pi}{\pi\sqrt{2}}k$ an integer

3. For any linear system, the output can be calculated as,

$$y[n] = x[n] * h[n] \triangleq \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Response to a shifted response, can be calculated as follows,

$$h_k[n] = (u-k)u[n-k] = \delta[n-k] * h[n]$$

$$\delta[n-k] * h[n] \triangleq \sum_{a=-\infty}^{\infty} \delta[a-k]h[n-a] = h[n-a]$$

$$h[n-a]|_{a=k} \equiv h[n-k] = h_k[n] = (n-k)u[n-k]$$

From there, the impulse response $h[n]$ can be found as,

$$h[n] = nu[n]$$

Thus, $y[n]$ for any input can be found as follows,

$$y[n] = x[n] * h[n] \triangleq \sum_{k=-\infty}^{\infty} x[k](n-k)u[n-k]$$

$$\boxed{y[n] = \sum_{k=-\infty}^{n} x[k](n-k)}$$

To check time-invariance, let us find $y[n-m$ and the output $y_1[n]$ for an input $x_1[n] \triangleq x[n-m]$

$$y[n-m] = \sum_{k=-\infty}^{\infty} x[k]h[n-m-k]$$

$$y_1[n] = x[n-m] * h[n] \triangleq \sum_{k=-\infty}^{\infty} x[k-m]h[n-k]$$

Letting $\widetilde{k} \triangleq k-m$, $k = m + \widetilde{k}$

$$y_1[n] = \sum_{\widetilde{k}=-\infty}^{\infty} x[\widetilde{k}]h[n-m-\widetilde{k}]$$

It can be easily seen that $y[n-m] = y_1[n]$. Thus, the system is **Time-Invariant**.

4. The system basically up-samples the system, by adding the average of two consecutive samples between these samples. An example Input/Output pair for the system can be seen at *Figure 11*.

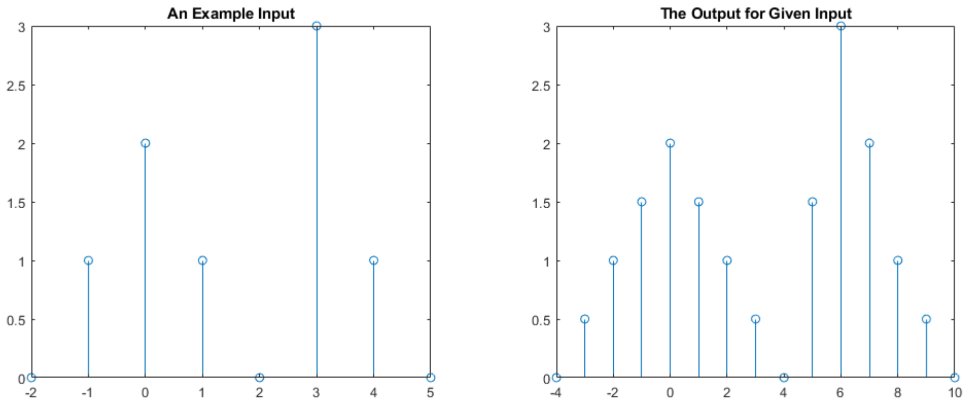



Figure 1: An Example Input/Output Pair

- For linearity, let us check the output $y[n]$ for the input $x[n] = ax_1[n] + bx_2[n]$

$$y[n] = \begin{cases} \dfrac{x[n]}{2} & if \quad n \quad is \quad even \\ \dfrac{x[\frac{n-1}{2}] + x[\frac{n+1}{2}]}{2} & if \quad n \quad is \quad odd \end{cases}$$

$$y[n] = \begin{cases} a\dfrac{x_1[n]}{2} + b\dfrac{x_2[n]}{2} & n \quad is \quad even \\ a\dfrac{x_1[\frac{n-1}{2}] + x_1[\frac{n+1}{2}]}{2} + b\dfrac{x_2[\frac{n-1}{2}] + x_2[\frac{n+1}{2}]}{2} & if \quad n \quad is \quad odd \end{cases}$$

Let us also find $y_1[n]$ and $y_2[n]$ for $x_1[n]$ and $x_2[n]$ respectively,

$$y_1[n] = \begin{cases} \dfrac{x_1[n]}{2} & n \;\; is \;\; even \\ \dfrac{x_1[\frac{n-1}{2}] + x_1[\frac{n+1}{2}]}{2} & if \;\; n \;\; is \;\; odd \end{cases}$$

$$y_2[n] = \begin{cases} \dfrac{x_2[n]}{2} & n \;\; is \;\; even \\ \dfrac{x_2[\frac{n-1}{2}] + x_2[\frac{n+1}{2}]}{2} & if \;\; n \;\; is \;\; odd \end{cases}$$

It can be clearly seen that $y[n] = ay_1[n] + y_2[n]$ for $x[n] = ax_1[n] + bx_2[n]$. Thus, the system is **Linear**.

- For time invariance, let us check $y[n-m]$ and $y_1[n]$ for the $x_1[n] = x[n-m]$

$$y[n-m] = \begin{cases} \dfrac{x[n-m]}{2} & if \;\; (n-m) \;\; is \;\; even \\ \dfrac{x[\frac{n-m-1}{2}] + x[\frac{n-m+1}{2}]}{2} & if \;\; (n-m) \;\; is \;\; odd \end{cases}$$

$$y_1[n] = \begin{cases} \dfrac{x_1[n]}{2} & if \;\; n \;\; is \;\; even \\ \dfrac{x_1[\frac{n-1}{2}] + x_1[\frac{n+1}{2}]}{2} & if \;\; n \;\; is \;\; odd \end{cases}$$

$$y_1[n] = \begin{cases} \dfrac{x[n-m]}{2} & if \;\; n \;\; is \;\; even \\ \dfrac{x[\frac{n-m-1}{2}] + x[\frac{n-m+1}{2}]}{2} & if \;\; n \;\; is \;\; odd \end{cases}$$

Due to condition difference $y[n-m] \neq y_[n]$. For different $m$, the result changes. Thus, the system is **not Time-Invariant**.

5. Let us check stability and causality for the following systems;

- $$y[n] = 2^{\delta[n+1]} + x[n-3]$$

  The system is **not casual** since the impulse response $h[n] \neq 0$ as $n < 0$;

  $$h[n] = 2^{\delta[n+1]} + \delta[n-3]$$

For BIBO stability, let us assume $|x[n]| < \beta_x < \infty$ and check $|y[n]|$;

$$|y[n]| = |2^{\delta[n+1]} + x[n-3]| = |c + x[n]| = \beta_y < \infty$$

where $c$ and $\beta_y$ are finite constants, thus, the system is **Stable**.

- $$y[n] = \begin{cases} y[-\delta[n-1]] + x[n-3] & if \quad n > 0 \\ 2^n x[n-3] & if \quad n \leq 0 \end{cases}$$

  The system is **Casual** since the impulse response $h[n] = 0$ as $n < 0$;

  $$h[n] = \begin{cases} h[-\delta[n-1]] + \delta[n-3] & if \quad n > 0 \\ 2^n \delta[n-3] & if \quad n \leq 0 \end{cases}$$

  For BIBO stability, let us assume $|x[n]| < \beta_x < \infty$ and check $|y[n]|$;

  $$|y[n]| = \begin{cases} |y[-\delta[n-1]] + x[n-3]| & if \quad n > 0 \\ |2^n x[n-3]| & if \quad n \leq 0 \end{cases}$$

  Let us assume $|y[n]| < \beta_z < \infty$. Checking the two conditions, the assumption holds, thus, it can be seen that the system is **Stable**.

6. $$y[n] = x[n] * x[n] = \sum_{k=-\infty}^{\infty} x[k]x[n-k]$$

   If the first non-zero element of $x[n]$ is $x[-6] = -3$ and the last non-zero element of $x[n]$ is equal to $x[24] = -4$, the first and last non-zero elements of $y[n]$ will be $y[-12]$ from $(-6 = 6 + n)$ and $y[48]$ from $(24 = -24 + n)$. These values can be calculated from the formula above as,

   $$\boxed{y[-12] = x[-6]x[-6] = 9}$$

   $$\boxed{y[48] = x[24]x[24] = 16}$$

7. Let us calculate $y[n]$ as $n \to \infty$, given that $x[n] = u[n]$ and $h[n] = 3(\frac{1}{2})^n u[n] + 2(\frac{1}{3})^{n-1} u[n]$

   $$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

   $$y[n] = \sum_{k=-\infty}^{\infty} u[k](3(\frac{1}{2})^{(n-k)} u[n-k] + 2(\frac{1}{3})^{n-k-1} u[n-k])$$

$$y[n] = \sum_{k=0}^{n} 3(\frac{1}{2})^{(n-k)} + 6(\frac{1}{3})^{(n-k)}$$

with simple change of variables, let $m \triangleq n - k$

$$= \sum_{m=n}^{0} 3(\frac{1}{2})^{m} + 6(\frac{1}{3})^{m}$$

or as $n \to \inf$

$$y[n] = \sum_{m=0}^{\infty} 3(\frac{1}{2})^{m} + 6(\frac{1}{3})^{m}$$

$$\boxed{\lim_{n\to\infty} y[n] = 3\frac{1}{1-1/2} - 6\frac{1}{1-1/3} = -3}$$

8. 8

**a)** y[n]- $\frac{1}{2}$ y[n-1]=0 (homogenous equation)

y[n]=A$r^n$

A$r^n$- $\frac{1}{2}$A$r^{n-1}$=0

A$r^{n-1}$(r- $\frac{1}{2}$)=0 =>r= $\frac{1}{2}$

y[n]=A$(\frac{1}{2})^n$

Assume y[n]- $\frac{1}{2}$ y[n-1]=x[n] where x[n]= $\delta$[n]

y[n]= $\frac{1}{2}$ y[n-1]+ $\delta$ [n]

y[0]= $\frac{1}{2}$ y[-1]+ $\delta$ [0]=1

y[0]=A=1 => y[n]=$(\frac{1}{2})^n$ u[n]

We evaluate that for x[n]. However, we have a linear combination of shifted inputs. Thus in order to find h[n] we must use linearity and time-invariance properties.

h[n]=y[n]-y[n-1]+y[n-2]= $(\frac{1}{2})^n$ u[n]- $(\frac{1}{2})^{n-1}$ u[n-1]+ $(\frac{1}{2})^{n-2}$ u[n-2]= $(\frac{1}{2})^n$($\delta$[n]- $\delta$[n-1]+3u[n-2])

Hence;

h[n]= $(\frac{1}{2})^n$($\delta$[n]- $\delta$[n-1]+3u[n-2])

Figure 2: An Example Input/Output Pair

**b)** $H(e^{j\omega})=\sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}=$

$$=\sum_{n=-\infty}^{\infty}(\tfrac{1}{2})^n\delta[n]e^{-j\omega n}-\sum_{n=-\infty}^{\infty}(\tfrac{1}{2})^n\delta[n-1]e^{-j\omega n}+3\sum_{n=-\infty}^{\infty}(\tfrac{1}{2})^n u[n-2]e^{-j\omega n}=$$

$$=1-\tfrac{1}{2}\,e^{-j\omega}+3\sum_{n=2}^{\infty}(\tfrac{e^{-j\omega}}{2})^n = 1-\tfrac{1}{2}\,e^{-j\omega}+3(\sum_{n=0}^{\infty}(\tfrac{e^{-j\omega}}{2})^n - 1 - \tfrac{1}{2}\,e^{-j\omega})=$$

$$= -2-2e^{-j\omega}+\frac{6}{2-e^{-j\omega}} \qquad \text{for } -\pi<\omega<\pi$$

Figure 3: An Example Input/Output Pair

**c)** "freqz" command operates by the coefficients of the z-transform of a sequence. So, first transform should be evaluated.

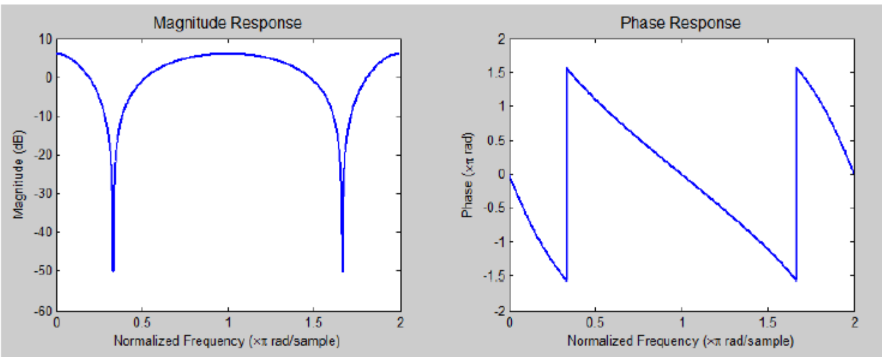
$Y(z) - \frac{1}{2}\,Y(z)\,z^{-1} = X(z) - X(z)\,z^{-1} + X(z)\,z^{-2}$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1-z^{-1}+z^{-2}}{1-\frac{1}{2}z^{-1}}$

Corresponding MATLAB code and plots are given below.

```
b=[1 -1 1];
a=[1 -0.5];

[h,w] = freqz(b,a,'whole',2001);
```




Magnitude response is even symmetric and phase response is odd symmetric as we expect from a real h[n] sequence.

Figure 4: An Example Input/Output Pair

**d)** $x[n]=\cos(\frac{\pi}{3}n)+\sin(\frac{\pi}{2}n+\frac{\pi}{4})= \cos(\frac{\pi}{3}n)+\sin(\frac{\pi}{2}n)\cos(\frac{\pi}{4})+\cos(\frac{\pi}{2}n)\sin(\frac{\pi}{4})=$

$= \cos(\frac{\pi}{3}n)+\frac{1}{\sqrt{2}}\sin(\frac{\pi}{2}n)+\frac{1}{\sqrt{2}}\cos(\frac{\pi}{2}n)$

$X(e^{j\omega})= \pi[\delta(\omega-\frac{\pi}{3})+\delta(\omega+\frac{\pi}{3})]-\frac{j\pi}{\sqrt{2}}[\delta(\omega-\frac{\pi}{2})-\delta(\omega+\frac{\pi}{2})]+\frac{\pi}{\sqrt{2}}[\delta(\omega-\frac{\pi}{2})+\delta(\omega+\frac{\pi}{2})]=$

$= \pi\,\delta(\omega-\frac{\pi}{3})+\pi\,\delta(\omega+\frac{\pi}{3})+\frac{\pi}{\sqrt{2}}(1-j)\,\delta(\omega-\frac{\pi}{2})+\frac{\pi}{\sqrt{2}}(1+j)\,\delta(\omega+\frac{\pi}{2})$

$Y(e^{j\omega})= X(e^{j\omega})\,H(e^{j\omega})= \pi\,(-2-2e^{-j\frac{\pi}{3}}+\frac{6}{2-e^{-j\frac{\pi}{3}}})\,\delta(\omega-\frac{\pi}{3})+\pi\,(-2-2e^{j\frac{\pi}{3}}+\frac{6}{2-e^{j\frac{\pi}{3}}})\,\delta(\omega+\frac{\pi}{3})$

$+\frac{\pi}{\sqrt{2}}(1-j)(-2-2e^{-j\frac{\pi}{2}}+\frac{6}{2-e^{-j\frac{\pi}{2}}})\,\delta(\omega-\frac{\pi}{2})+\frac{\pi}{\sqrt{2}}(1+j)(-2-2e^{j\frac{\pi}{2}}+\frac{6}{2-e^{j\frac{\pi}{2}}})\,\delta(\omega+\frac{\pi}{2})$

Here $-2-2e^{-j\frac{\pi}{3}}+\frac{6}{2-e^{-j\frac{\pi}{3}}} = -2-2e^{j\frac{\pi}{3}}+\frac{6}{2-e^{j\frac{\pi}{3}}} =0.$

Thus two terms with delta functions vanish. Also;

$-2-2e^{-j\frac{\pi}{2}}+\frac{6}{2-e^{-j\frac{\pi}{2}}}=\frac{2}{5}+j\frac{4}{5}$ $\qquad$ $-2-2e^{j\frac{\pi}{2}}+\frac{6}{2-e^{j\frac{\pi}{2}}}=\frac{2}{5}-j\frac{4}{5}$

$Y(e^{j\omega})=\frac{\pi}{\sqrt{2}}(\frac{6}{5}+j\frac{2}{5})\,\delta(\omega-\frac{\pi}{2})+\frac{\pi}{\sqrt{2}}(\frac{6}{5}-j\frac{2}{5})\,\delta(\omega+\frac{\pi}{2})$

$y[n]=\frac{\pi}{\sqrt{2}}(\frac{6}{5}+j\frac{2}{5})\frac{1}{2\pi}e^{j\frac{\pi}{2}n}+\frac{\pi}{\sqrt{2}}(\frac{6}{5}-j\frac{2}{5})\frac{1}{2\pi}e^{-j\frac{\pi}{2}n}=\frac{1}{\sqrt{2}}(\frac{3}{5}+j\frac{1}{5})e^{j\frac{\pi}{2}n}+\frac{1}{\sqrt{2}}(\frac{3}{5}-j\frac{1}{5})e^{-j\frac{\pi}{2}n}$

Hence;

$y[n]=\frac{1}{\sqrt{2}}(\frac{3}{5}+j\frac{1}{5})e^{j\frac{\pi}{2}n}+\frac{1}{\sqrt{2}}(\frac{3}{5}-j\frac{1}{5})e^{-j\frac{\pi}{2}n}$

Figure 5: An Example Input/Output Pair

**e)** $H(e^{j\omega}) = -2-2e^{-j\omega}+\frac{6}{2-e^{-j\omega}}$

$H^{*}(e^{j(2\pi-\omega)}) = (-2-2e^{-j(2\pi-\omega)}+\frac{6}{2-e^{-j(2\pi-\omega)}})^{*} = (-2-2e^{-j2\pi}\, e^{j\omega}+\frac{6}{2-e^{-j2\pi}e^{j\omega}})^{*} =$

$=(-2-2e^{j\omega}+\frac{6}{2-e^{j\omega}})^{*} = -2-2e^{-j\omega}+\frac{6}{2-e^{-j\omega}}$

Hence; $H(e^{j\omega})=H^{*}(e^{j(2\pi-\omega)})$ for this question. This is not always the case. This equation is only valid when h[n] is a real sequence.

Figure 6: An Example Input/Output Pair

(a)

(b)

(c)

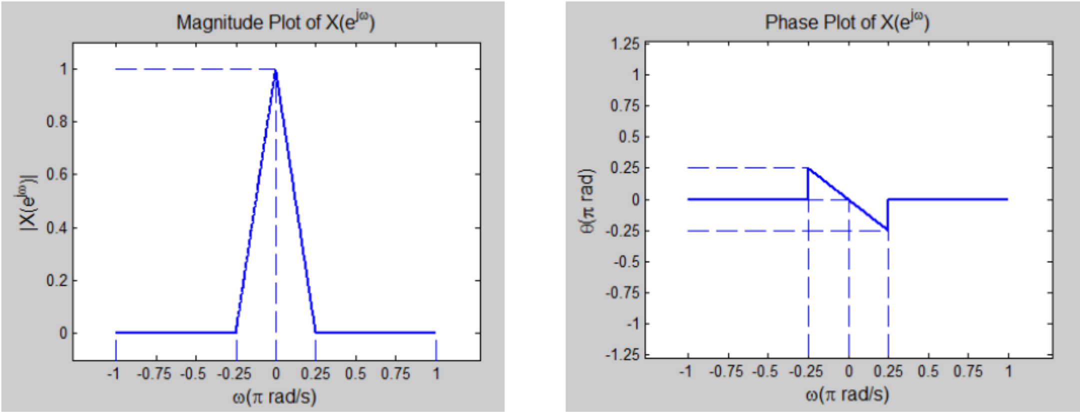
(d)

(e)

9. 9

**4)**

**a)** Since x[n] is a real sequence its magnitude response must be even symmetric and phase response must be odd symmetric.




**b)** $F\{\cos(\frac{\pi}{5}n)\} = \pi\,[\delta(\omega-\frac{\pi}{5})+\delta(\omega+\frac{\pi}{5})]$

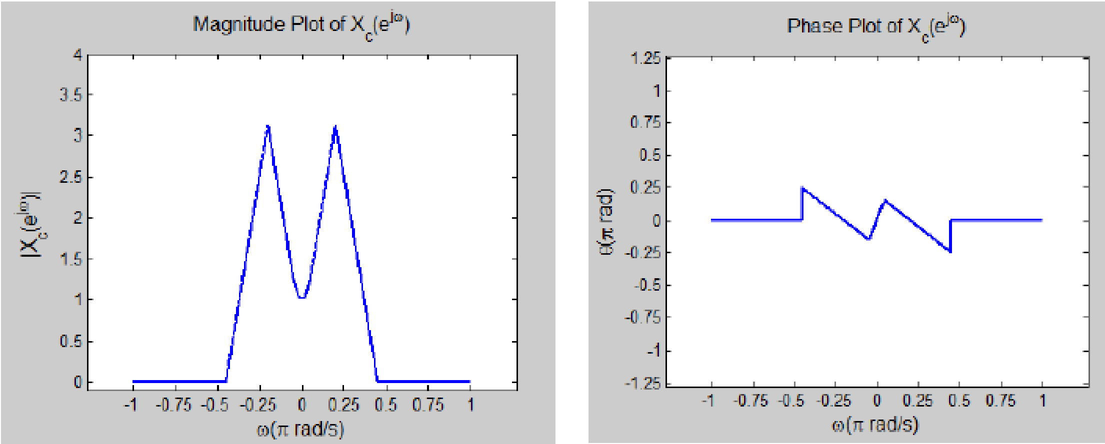
$F\{\sin(\frac{\pi}{5}n)\} = -j\pi\,[\delta(\omega-\frac{\pi}{5})-\delta(\omega+\frac{\pi}{5})]$

$X_c(e^{j\omega}) = X(e^{j\omega}) * F\{\cos(\frac{\pi}{5}n)\} = \pi\,[X(e^{j(\omega+\pi/5)}) + X(e^{j(\omega-\pi/5)})]$

$X_s(e^{j\omega}) = X(e^{j\omega}) * F\{\sin(\frac{\pi}{5}n)\} = -j\pi\,[X(e^{j(\omega+\pi/5)}) - X(e^{j(\omega-\pi/5)})]$

Figure 7: An Example Input/Output Pair

**c)**



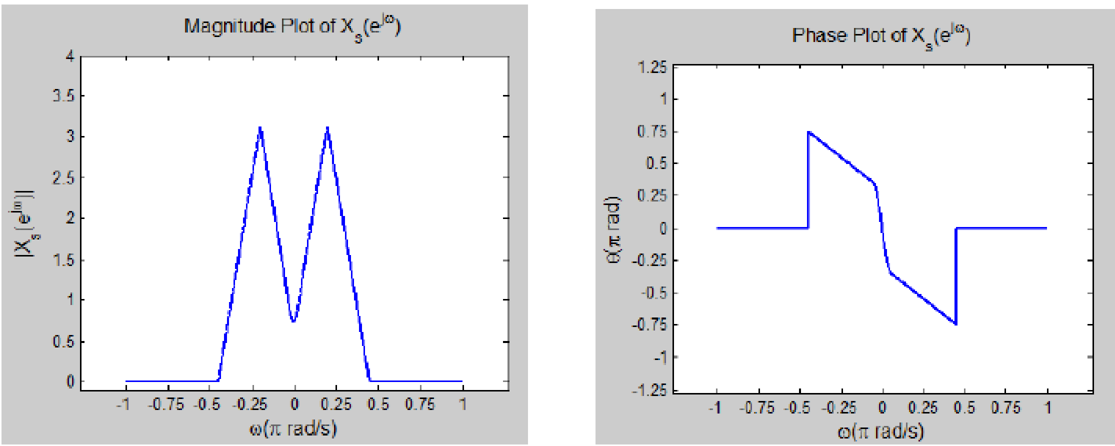



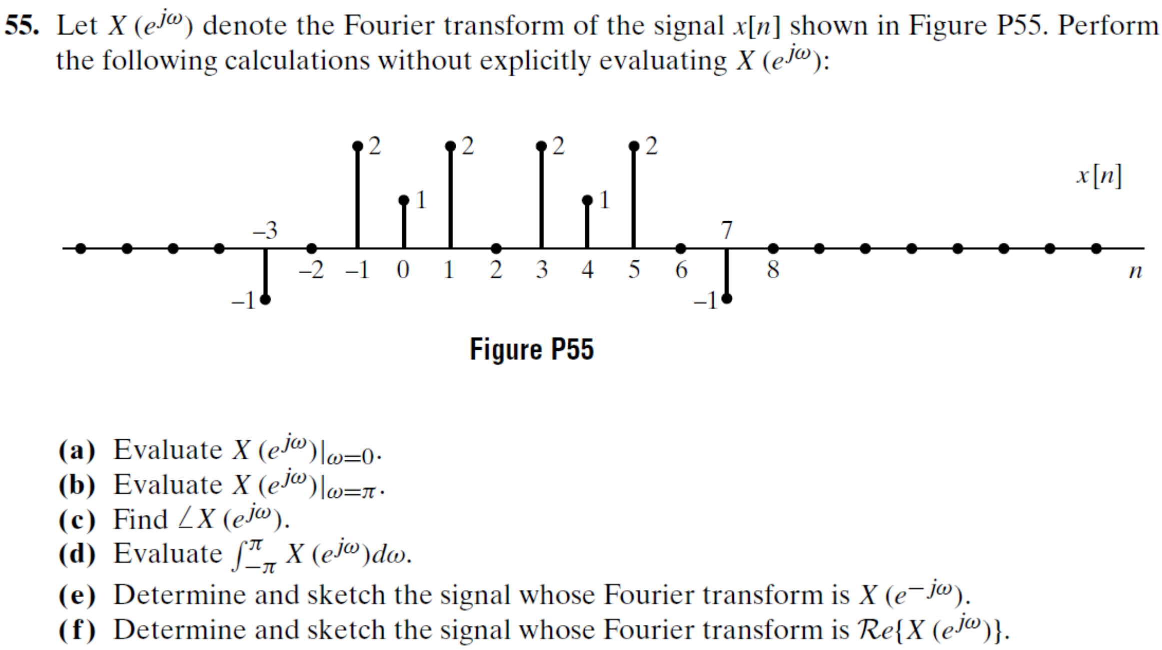

Figure 8: An Example Input/Output Pair

(a)

(b)

(c)

10. 10



**55.** Let $X(e^{j\omega})$ denote the Fourier transform of the signal $x[n]$ shown in Figure P55. Perform the following calculations without explicitly evaluating $X(e^{j\omega})$:



**Figure P55**

**(a)** Evaluate $X(e^{j\omega})|_{\omega=0}$.
**(b)** Evaluate $X(e^{j\omega})|_{\omega=\pi}$.
**(c)** Find $\angle X(e^{j\omega})$.
**(d)** Evaluate $\int_{-\pi}^{\pi} X(e^{j\omega})d\omega$.
**(e)** Determine and sketch the signal whose Fourier transform is $X(e^{-j\omega})$.
**(f)** Determine and sketch the signal whose Fourier transform is $\mathcal{R}e\{X(e^{j\omega})\}$.

Figure 9: An Example Input/Output Pair

**2.44.** (a)

$$\begin{aligned} X(e^{j\omega})|_{\omega=0} &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}|_{\omega=0} \\ &= \sum_{n=-\infty}^{\infty} x[n] \\ &= 6 \end{aligned}$$

(b)

$$\begin{aligned} X(e^{j\omega})|_{\omega=\pi} &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\pi n} \\ &= \sum_{n=-\infty}^{\infty} x[n](-1)^n \\ &= 2 \end{aligned}$$

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(c) Because $x[n]$ is symmetric about $n = 2$ this signal has linear phase.

$$X(e^{j\omega}) = A(\omega)e^{-j2\omega}$$

$A(\omega)$ is a zero phase (real) function of $\omega$. Hence,

$$\angle X(e^{j\omega}) = -2\omega, \quad -\pi \leq \omega \leq \pi$$

(d)

$$\int_{-\pi}^{\pi} X(e^{j\omega})e^{-j\omega n}d\omega = 2\pi x[n]$$

for $n = 0$:

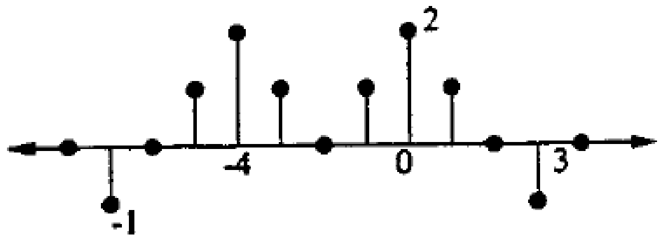
$$\int_{-\pi}^{\pi} X(e^{j\omega})d\omega = 2\pi x[0] = 4\pi$$

Figure 10: An Example Input/Output Pair

(e) Let $y[n]$ be the unknown sequence. Then

$$\begin{aligned} Y(e^{j\omega}) &= X(e^{-j\omega}) \\ &= \sum_n x[n]e^{j\omega n} \\ &= \sum_n x[-n]e^{-j\omega n} \\ &= \sum_n y[n]e^{-j\omega n} \end{aligned}$$

Hence $y[n] = x[-n]$.




(f) We have determined that:

$$X(e^{j\omega}) = A(\omega)e^{-j2\omega}$$

$$\begin{aligned} X_R(e^{j\omega}) &= \mathcal{R}e\{X(e^{j\omega})\} \\ &= A(\omega)\cos(2\omega) \\ &= \frac{1}{2}A(\omega)\left(e^{j2\omega} + e^{-j2\omega}\right) \end{aligned}$$

Taking the inverse transform, we have

$$\frac{1}{2}a[n+2] + \frac{1}{2}a[n-2] = \frac{1}{2}x[n+4] + \frac{1}{2}x[n]$$

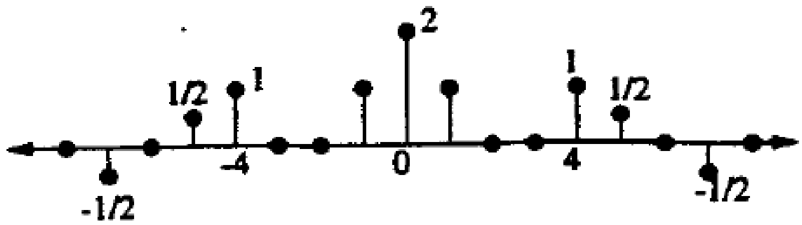



Figure 11: An Example Input/Output Pair

(a)

(b)

(c)

(d)

(e)

(f)

11. 11

**60.** Consider a discrete-time LTI system with frequency response $H(e^{j\omega})$ and corresponding impulse response $h[n]$.

**(a)** We are first given the following three clues about the system:

(i) The system is causal.

(ii) $H(e^{j\omega}) = H^*(e^{-j\omega})$.

(iii) The DTFT of the sequence $h[n+1]$ is real.

Using these three clues, show that the system has an impulse response of finite duration.

**(b)** In addition to the preceding three clues, we are now given two more clues:

(iv) $\dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} H(e^{j\omega})d\omega = 2.$

(v) $H(e^{j\pi}) = 0.$

Is there enough information to identify the system uniquely? If so, determine the impulse response $h[n]$. If not, specify as much as you can about the sequence $h[n]$.

Figure 12: An Example Input/Output Pair

**2.49.** (a) We start by interpreting each clue.

(i) The system is causal implies

$$h[n] = 0 \text{ for } n \leq 0.$$

(ii) The Fourier transform is conjugate symmetric implies $h[n]$ is real.

(iii) The DTFT of the sequence $h[n+1]$ is real implies $h[n+1]$ is even.

From the above observations, we deduce that $h[n]$ has length 3, therefore it has finite duration.

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(b) From part (a) we know that $h[n]$ is length 3 with even symmetry around $h[1]$. Let $h[0] = h[2] = a$ and $h[1] = b$, from (iv) and using Parseval's theorem, we have

$$2a^2 + b^2 = 2.$$

From (v), we also have

$$2a - b = 0.$$

Solving the above equations, we get

$$\begin{aligned} h[0] &= \frac{1}{\sqrt{3}} \\ h[1] &= \frac{2}{\sqrt{3}} \\ h[2] &= \frac{1}{\sqrt{3}} \end{aligned}$$

or

$$\begin{aligned} h[0] &= -\frac{1}{\sqrt{3}} \\ h[1] &= -\frac{2}{\sqrt{3}} \\ h[2] &= -\frac{1}{\sqrt{3}}. \end{aligned}$$

Figure 13: An Example Input/Output Pair

(a)

(b)

12. 12

**65.** The LTI system

$$H(e^{j\omega}) = \begin{cases} -j, & 0 < \omega < \pi, \\ j, & -\pi < \omega < 0, \end{cases}$$

is referred to as a 90° phase shifter and is used to generate what is referred to as an analytic signal $w[n]$ as shown in Figure P65-1. Specifically, the analytic signal $w[n]$ is a complex-valued signal for which

$$\mathcal{R}e\{w[n]\} = x[n],$$

$$\mathcal{I}m\{w[n]\} = y[n].$$

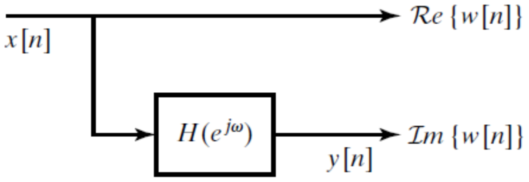


Figure P65–1

If $\mathcal{R}e\{X(e^{j\omega})\}$ is as shown in Figure P65-2 and $\mathcal{I}m\{X(e^{j\omega})\} = 0$, determine and sketch $W(e^{j\omega})$, the Fourier transform of the analytic signal $w[n] = x[n] + jy[n]$.

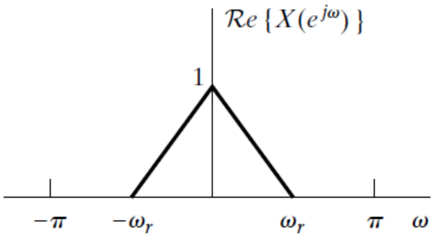


Figure P65–2

Figure 14: An Example Input/Output Pair

**2.58.** Note that $X(e^{j\omega})$ is real, and $Y(e^{j\omega})$ is given by:

$$Y(e^{j\omega}) = \begin{cases} -jX(e^{j\omega}) & , \quad 0 < \omega < \pi \\ +jX(e^{j\omega}) & , \quad -\pi < \omega < 0. \end{cases}$$

$w[n] = x[n] + jy[n]$, therefore:

$$W(e^{j\omega}) = X(e^{j\omega}) + jY(e^{j\omega}).$$

Using the above, we get:

$$jY(e^{j\omega}) = \begin{cases} X(e^{j\omega}) & , \quad 0 < \omega < \pi \\ -X(e^{j\omega}) & , \quad -\pi < \omega < 0. \end{cases}$$

We thus conclude:

$$W(e^{j\omega}) = \begin{cases} 2X(e^{j\omega}) & , \quad 0 < \omega < \pi \\ 0 & , \quad -\pi < \omega < 0. \end{cases}$$

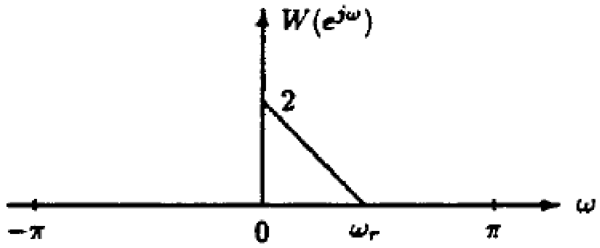


Figure 15: An Example Input/Output Pair

13. [MATLAB]

(a)

(b)

(c)

(d)

(e)

(f)

(g)