# Middle East Technical University

## Department of Electrical and Electronics Engineering

EE498 CONTROL SYSTEM DESIGN AND SIMULATION
Term Project

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**Project Type 1: Linear Quadratic Regulator Design for a Differential Drive Robot Modelled in State Space Using Inverse Kinematics Model**

# Final Report

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# Contents

# 1 Introduction

As the term project of EE498 course, we have decided to focus on the control system design, simulation and implementation based on our graduation project. To successfully extract the model of the differential drive robot, we have conducted a thorough literature research and got advice from our instructor. After the modelling is complete, we have moved on to the Linear Quadratic Regulator Design. The aim in LQR design is to have a proper and well-functioning controller for the lane keeping project. In the following parts of this report, one may examine the detailed quantitative work related to the modelling, design procedure and implementation of the EE498 Control System Design and Simulation project. In *Figure 1*, the overall appearance of the vehicle to be controlled is shown.

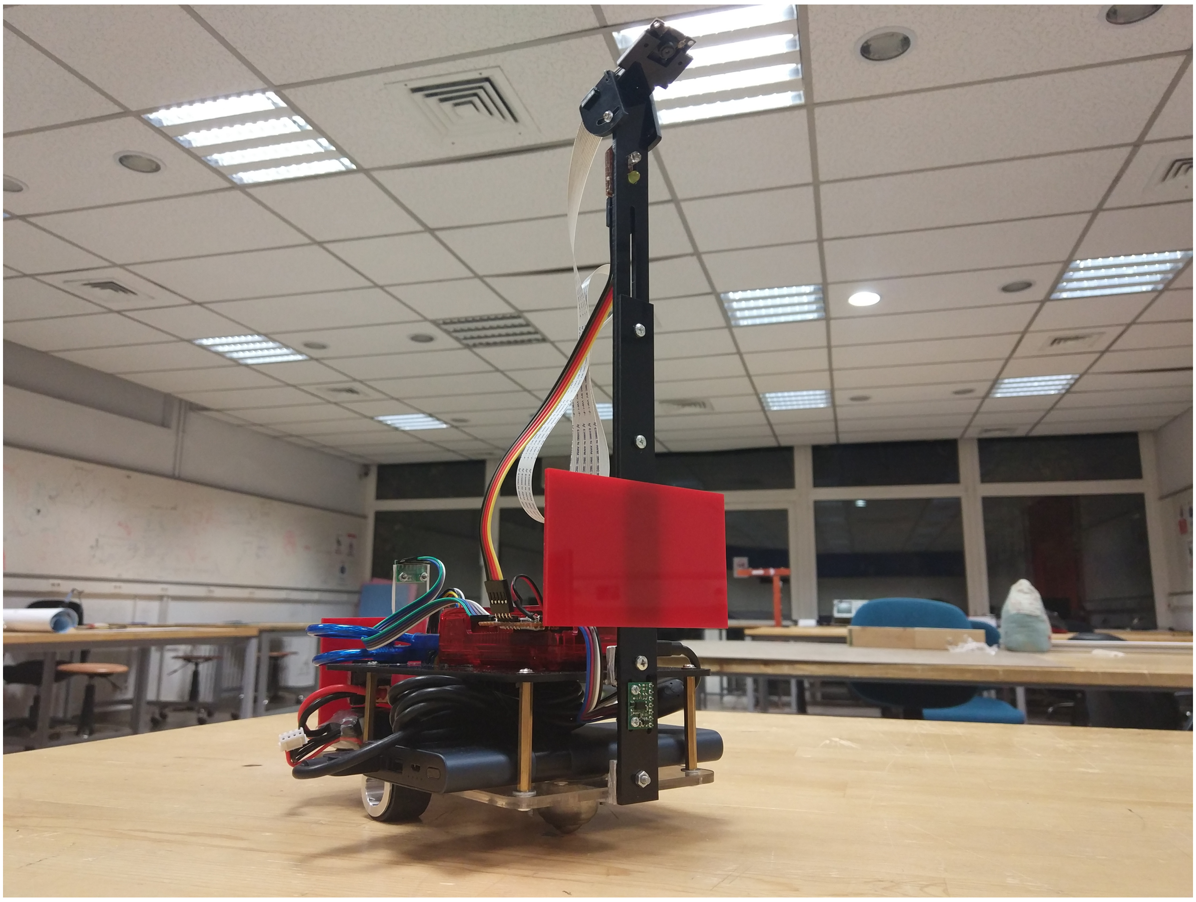

Figure 1: The Overall Appearance of the Vehicle

## 2 Problem Definition

For the project of EE498 course, our main aim is to drive the differential drive robot that we have constructed for the graduation project. To do so, first we have to find a way to model our system. Then according to our model, a controller is to be designed. This controller is responsible of producing the PWM difference between two motors of the vehicle as can be seen in *Figure 2*.

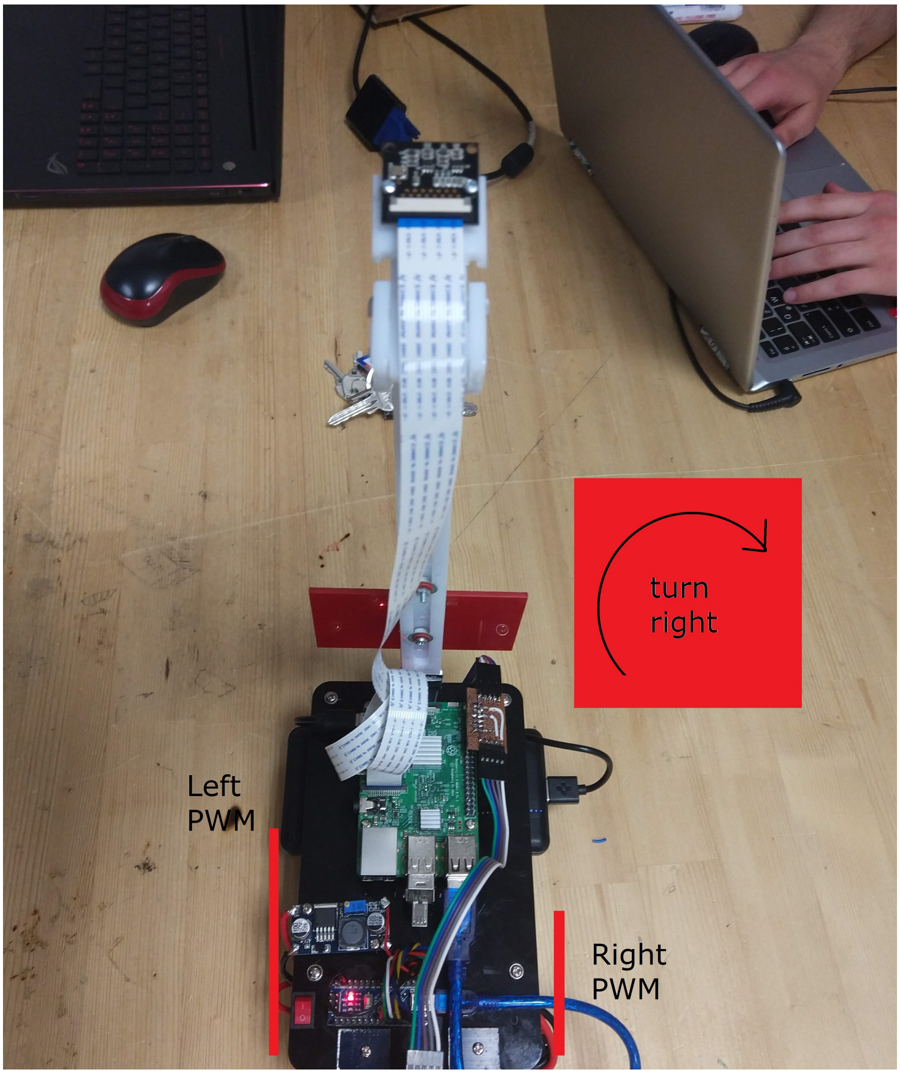


Figure 2: Differential Drive Vehicle on the Path

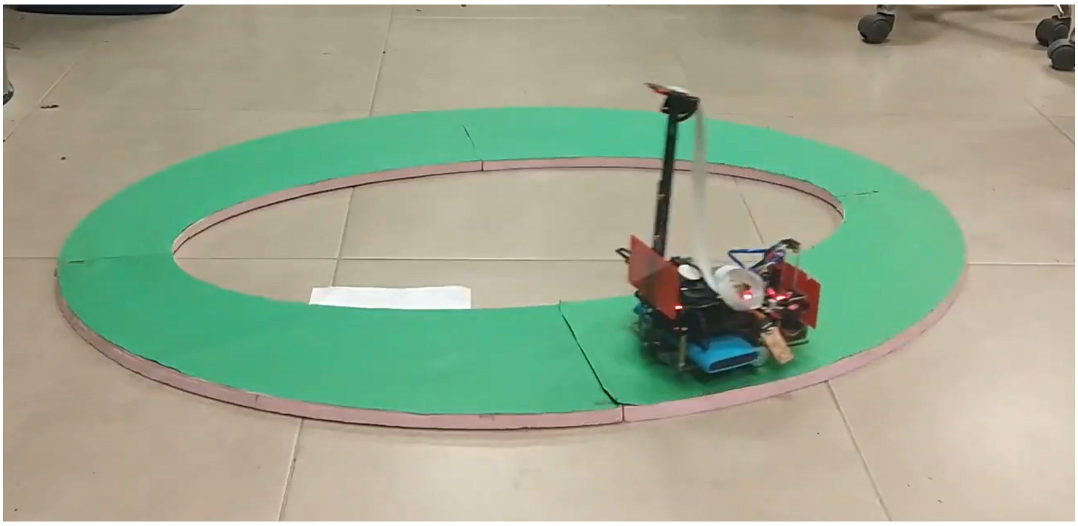

Figure 3: Differential Drive Vehicle on the Path

# 3 Solution

## 3.1 Modelling the Plant Basics

Sub-problem of the project is extraction of state state space model of the vehicle using Inverse Kinematic Model and then using this state space model as our plant for the implementation of the differential drive motor control. There exist two main design constraints in project which are;

- Completing a full tour without falling out of the lane.
- Executing that tour under 20 seconds.

In the model, following speed parameters were used at the modelling [2]

- $v_r$ is current position linear velocity
- $w_r$ is current angular velocity
- $v_R$ and right wheel linear velocity
- $v_L$ and left wheel linear velocity
- $w_R = \dfrac{v_R}{r}$ and right wheel angular velocity
- $w_L = \dfrac{v_L}{r}$ and left wheel angular velocity

where $r$ is the half-radius of the wheels and the relation between linear speed and angular velocity of the vehicle with respect to right and left wheel are as follows;

$$v_{vehicle} = \frac{v_R + v_L}{2}$$

$$w_{vehicle} = \frac{v_R - v_L}{r}$$

In our case we could use constant base speed $V_0 = \dfrac{v_R + v_L}{2}$ and $\Delta V = v_R - v_L$.

$$v_R = V_0 + \Delta V/2$$
$$v_L = V_0 - \Delta V/2$$

Thus, the state rule that gives current position is follows

$$\dot{q_r} = \begin{bmatrix} \dot{x_r} \\ \dot{y_r} \\ \dot{\theta_r} \end{bmatrix} = \begin{bmatrix} \cos(\theta_r) & 0 \\ \sin(\theta_r) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_r \\ w_r \end{bmatrix}$$

Where with a small-angle approximation and Base speed approximation that is Base is $V_0$ and $\Delta_V$ is input applied

$$\dot{q_r} = \begin{bmatrix} \dot{x_r} \\ \dot{y_r} \\ \dot{\theta_r} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & V_0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ \theta_d \end{bmatrix} + \begin{bmatrix} 0 & V_0 \\ 0 & 0 \\ \frac{1}{r} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \Delta V_r \end{bmatrix}$$

## 3.2 Addition of Extra States

$$\dot{x} = \begin{bmatrix} \dot{x_r} \\ \dot{y_r} \\ \dot{\theta_r} \\ \dot{y_1} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & V & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & V & 0 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ \theta_d \\ y_1 \\ \beta \end{bmatrix} + \begin{bmatrix} V & 0 \\ 0 & 0 \\ 0 & \frac{K}{2L} \\ -V/R & \frac{K}{2L} \\ 0 & \frac{Kl_s}{2L} \end{bmatrix} \begin{bmatrix} 1 \\ \Delta p \end{bmatrix}$$

where $\beta$ and $y_1$ are measurable quantities from *Figure 4*.

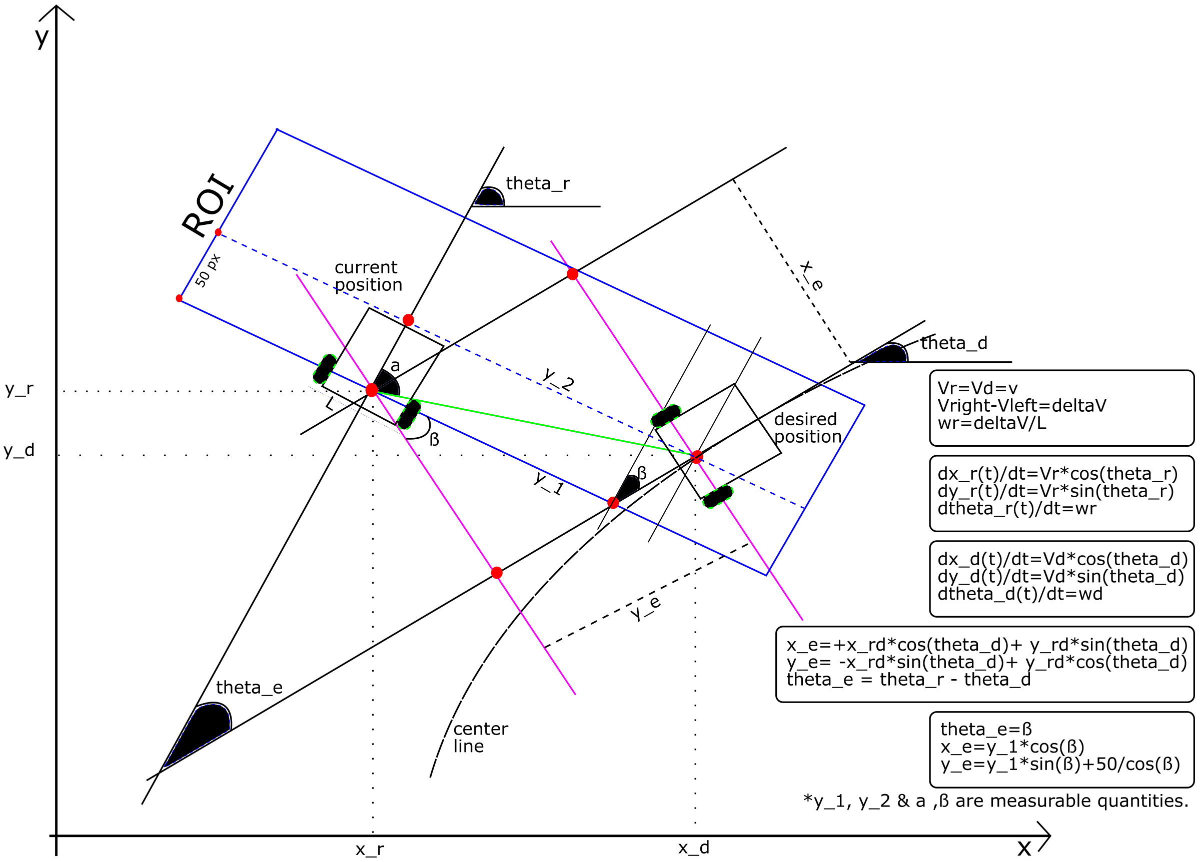


Figure 4: Inverse Kinematic Model for the Differential Drive Motor

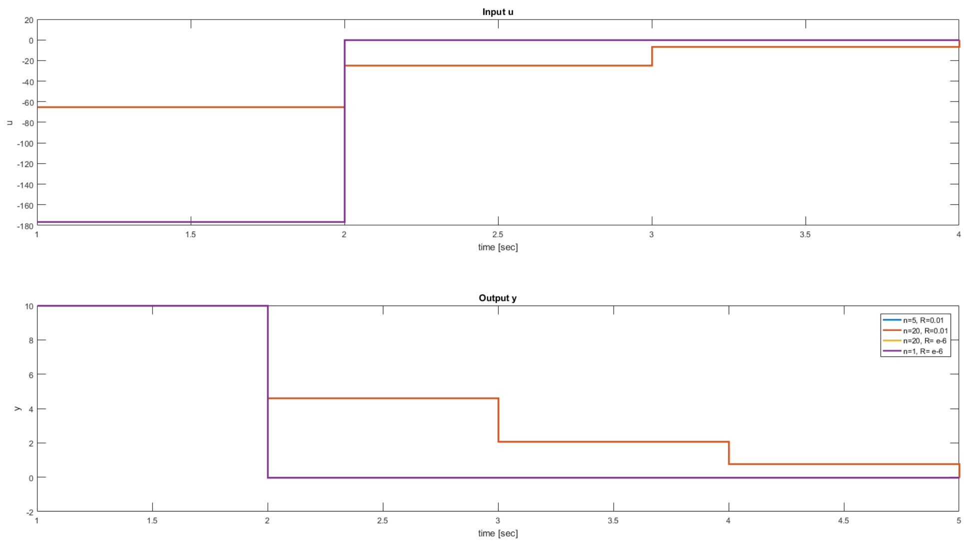


Figure 5: LQR simulation of 5 state system

Even though we have started with 3 state, according to advise from the supervisor, we try to implement the system with 5 state. At this stage, we obtain simulation result such *Figure 5*. Although this simulation results are promising, at the end, we decided to reduce the states. This is because of the fact that only the forth and the fifth states are the observable states that can be measured from the camera.

## 3.3 State Reduction and Discretization

```
%% Check Controllability
Co = ctrb(Sysd)
unco = length(Ad) - rank(Co) % Not Controlllable if not equal to zero %currently Co=0
```

$$\dot{x} = \begin{bmatrix} \dot{y_1} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ V & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ \beta \end{bmatrix} + \begin{bmatrix} -V/R & \dfrac{K}{2L} \\ 0 & \dfrac{Kl_s}{2L} \end{bmatrix} \begin{bmatrix} 1 \\ \Delta p \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} \dot{y_1} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ V & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ \beta \end{bmatrix} + \begin{bmatrix} \dfrac{K}{2L} \\ \dfrac{Kl_s}{2L} \end{bmatrix} \Delta p + \begin{bmatrix} -V/R \\ 0 \end{bmatrix}$$

```
%% DT Conversion 2*2
sysc=ss(A,B,C,0);
Ts=63*10^(-3);
Sysd=c2d(sysc,Ts);
[Ad,Bd,Cd,Dd,Ts] = ssdata(Sysd)
```

$$\dot{x} = \begin{bmatrix} \dot{y_1} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} 1.0000 & 0 \\ 15.7500 & 1.0000 \end{bmatrix} \begin{bmatrix} y_1 \\ \beta \end{bmatrix} + \begin{bmatrix} 0.0005 \\ 0.0829 \end{bmatrix} \Delta p + \begin{bmatrix} -0.5263 \\ 0 \end{bmatrix}$$

# 4 Observations

After the modelling part of the project is done, we have moved on to the LQR design. Using the dlqr function of the MATLAB, we have constructed our Q matrix and R value. After construction, we have manipulated the values of the Q and R according to the response of the plant and minimization of the state y1 and input values.

When we sweep the R value low to high, we obtain lower input values while output performance going worse, as expected. On the other hand, while Q2 is increasing, $y_1$ is decreasing. As a trade off input is getting worse. As we know from our course, the change in Q1 puts a restraint to the first state which is $y_1$, distance of the vehicle to the midpoint of the road, for our system. These results are clearly illustrated in the following figures from *Figure 6* to *Figure 47*.

Demo videos for the vehicle can be watched at CW Direction Video & CCW Direction Video .

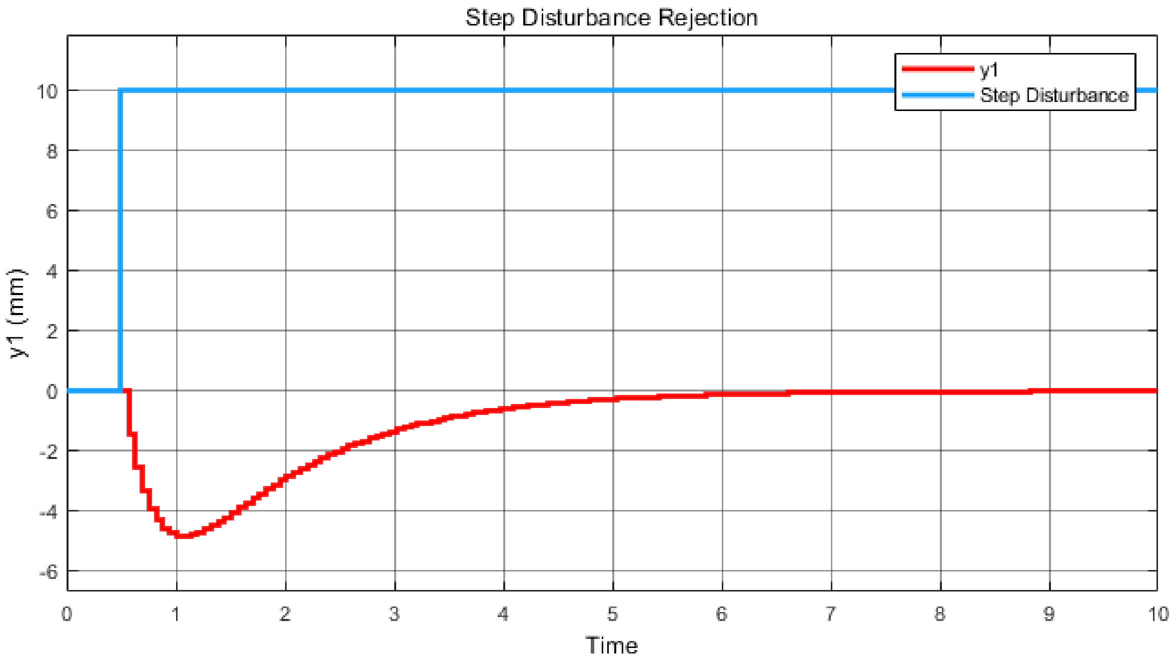


Figure 6: LQR simulation of 2 state system for K1=280

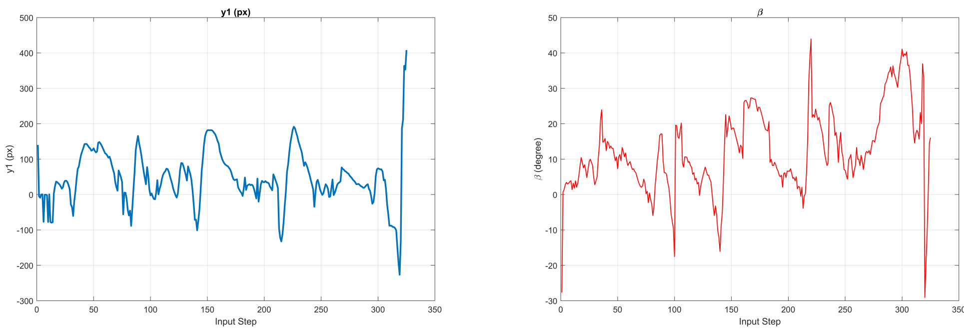


Figure 7: Real-time values of states

Table 1: LQR parameters vs K1 and K2 parameter relations

| LQR Parameters | | | K1 Values | K2 Values |
|---|---|---|---|---|
| Q(1,1)=1 | Q(2,2)= 1 | R=1 | K1= 134.7446 | K2= 0.9274 |
| Q(1,1)=1 | Q(2,2)= 1 | R=5 | K1= 108.3453 | K2= 0.4269 |
| Q(1,1)=1 | Q(2,2)= 1 | R=10 | K1= 96.9857 | K2= 0.3044 |
| Q(1,1)=1 | Q(2,2)= 1 | R=20 | K1= 86.0554 | K2= 0.2166 |
| Q(1,1)=1 | Q(2,2)= 1 | R=40 | K1= 75.7631 | K2= 0.1540 |
| Q(1,1)=1 | Q(2,2)= 1 | R=100 | K1= 63.3645 | K2= 0.0980 |
| Q(1,1)=1 | Q(2,2)= 1 | R=250 | K1= 52.4754 | K2= 0.0622 |
| Q(1,1)=1 | Q(2,2)= 5 | R=1 | K1= 157.7470 | K2= 1.9630 |
| Q(1,1)=1 | Q(2,2)= 10 | R=1 | K1= 165.7295 | K2= 2.6714 |
| Q(1,1)=1 | Q(2,2)= 25 | R=1 | K1= 174.2154 | K2= 0.9274 |
| Q(1,1)=1 | Q(2,2)= 50 | R=1 | K1= 179.1096 | K2= 3.9236 |
| Q(1,1)=1 | Q(2,2)= 100 | R=1 | K1= 182.8159 | K2= 6.4830 |
| Q(1,1)=1 | Q(2,2)= 250 | R=1 | K1= 186.1682 | K2= 8.3523 |
| Q(1,1)=10 | Q(2,2)= 1 | R=1 | K1= 134.7586 | K2= 0.9274 |
| Q(1,1)=100 | Q(2,2)= 1 | R=1 | K1= 134.8985 | K2= 0.9274 |
| Q(1,1)=750 | Q(2,2)= 0.01 | R=0.01 | K1= 229.8575 | K2= 0.9012 |
| Q(1,1)=750 | Q(2,2)= 0.01 | R=0.005 | K1= 286.1342 | K2= 1.2312 |
| Q(1,1)=750 | Q(2,2)= 0.01 | R=1 | K1= 67.8060 | K2= 0.0978 |
| Q(1,1)=1000 | Q(2,2)= 0.1 | R=1 | K1= 63.9722 | K2= 0.0979 |

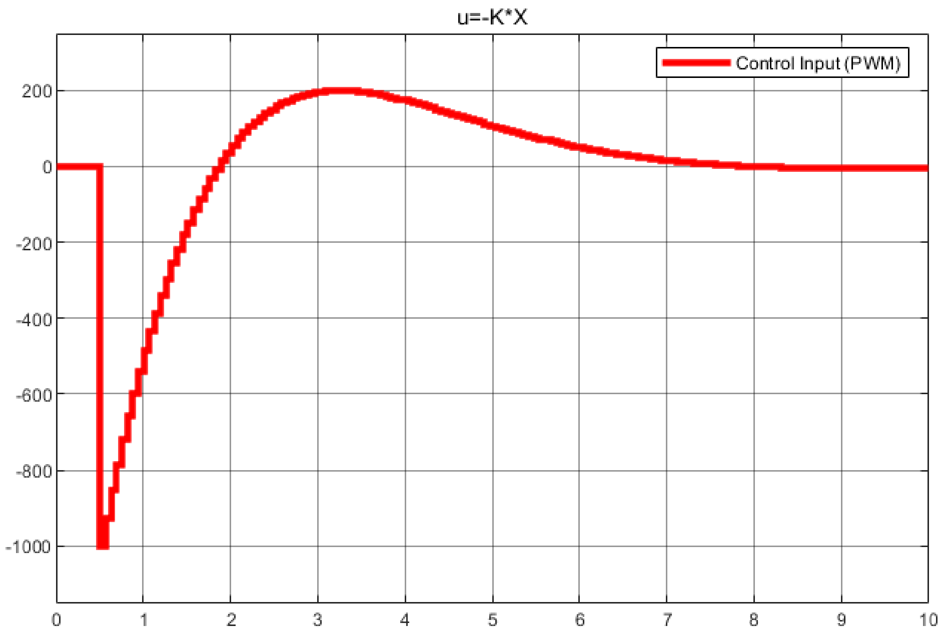


Figure 8: Control Input for Q1=1000, Q2=0.1, R=1

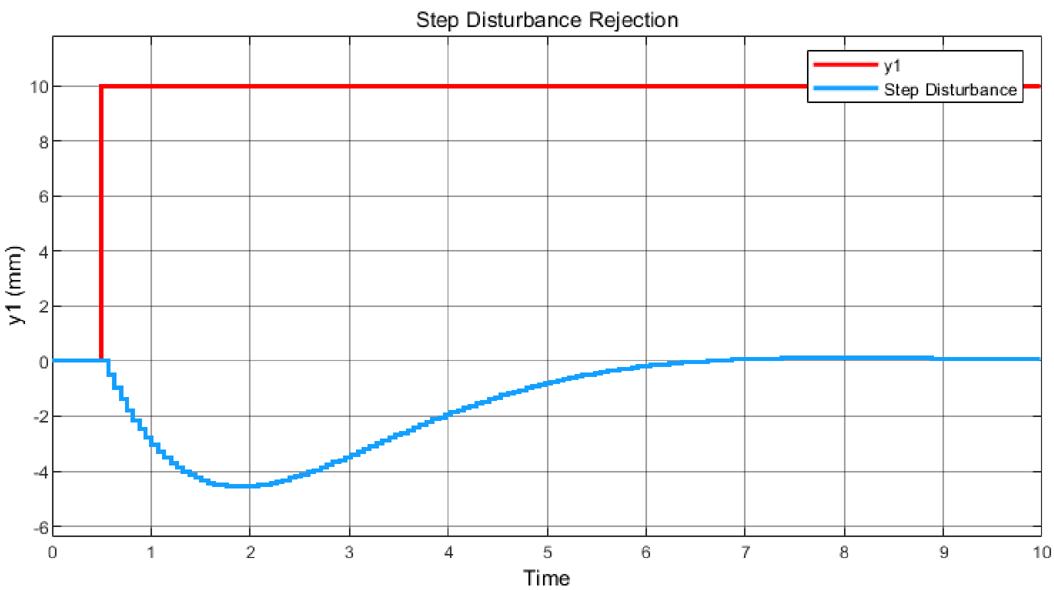


Figure 9: Output for Q1=1000, Q2=0.1, R=1

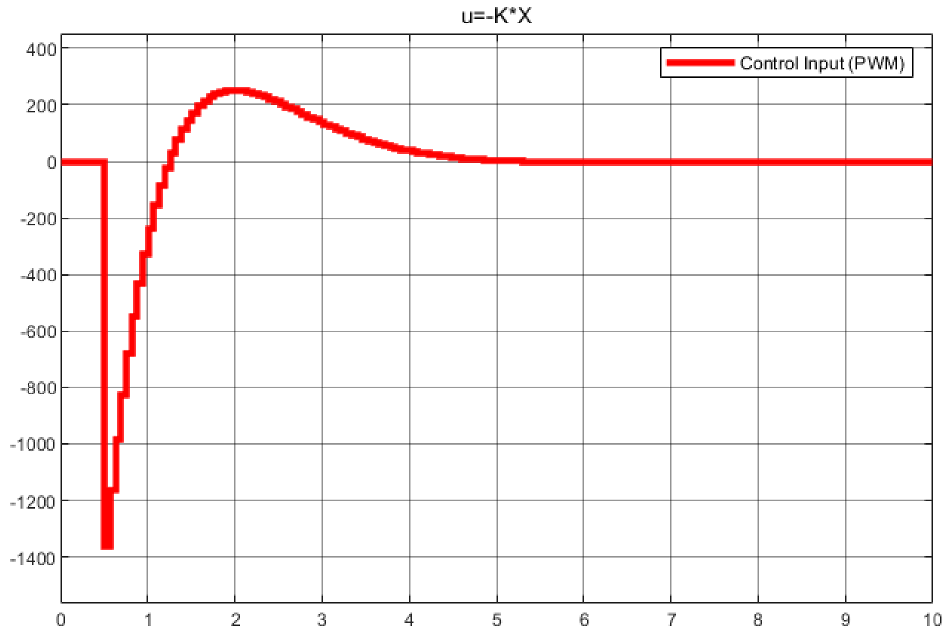


Figure 10: Control Input for Q1=1000, Q2=1, R=1

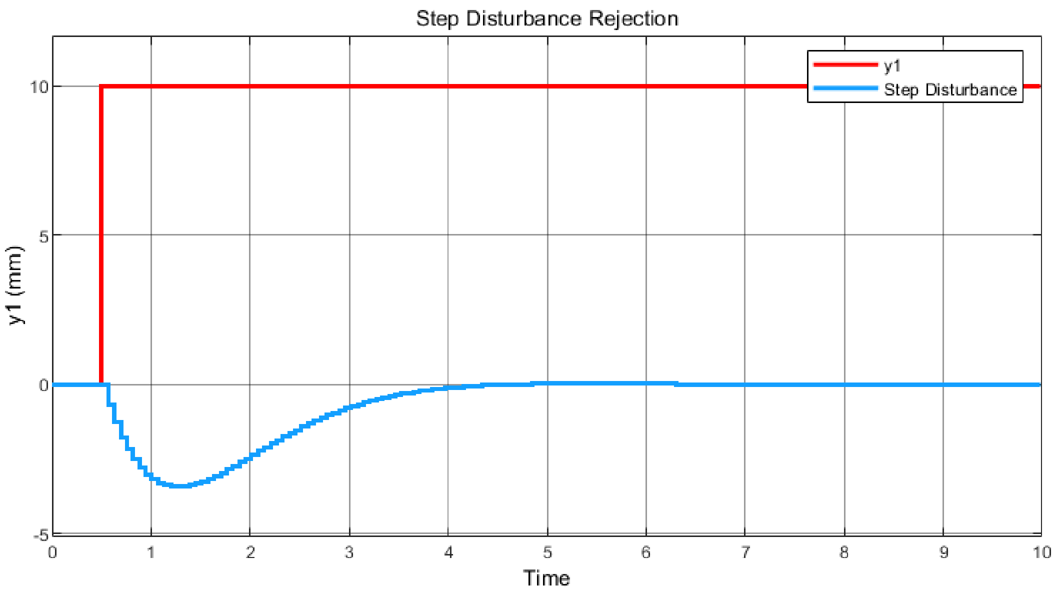


Figure 11: Output for Q1=1000, Q2=1, R=1

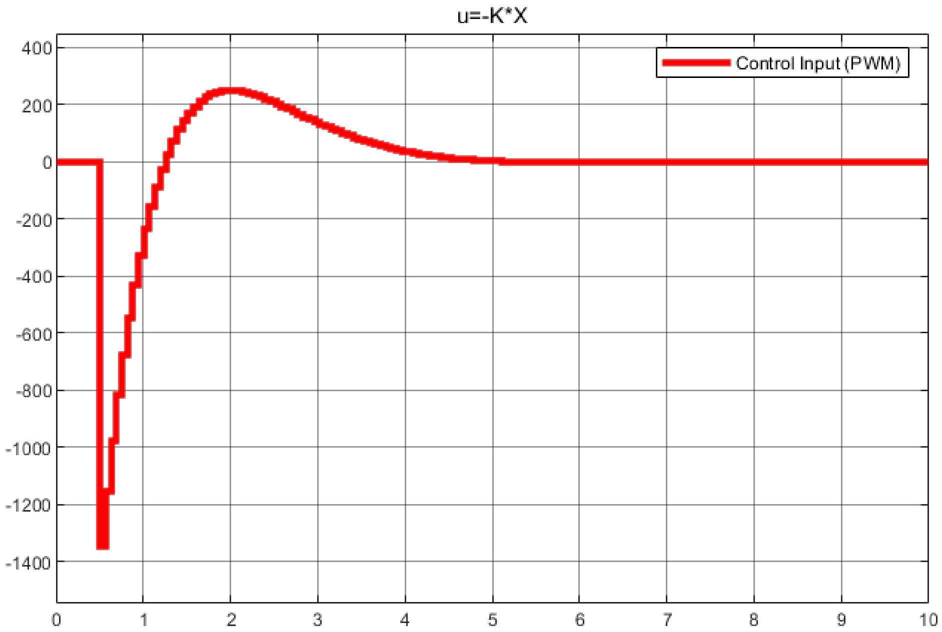


Figure 12: Control Input for Q1=100, Q2=1, R=1

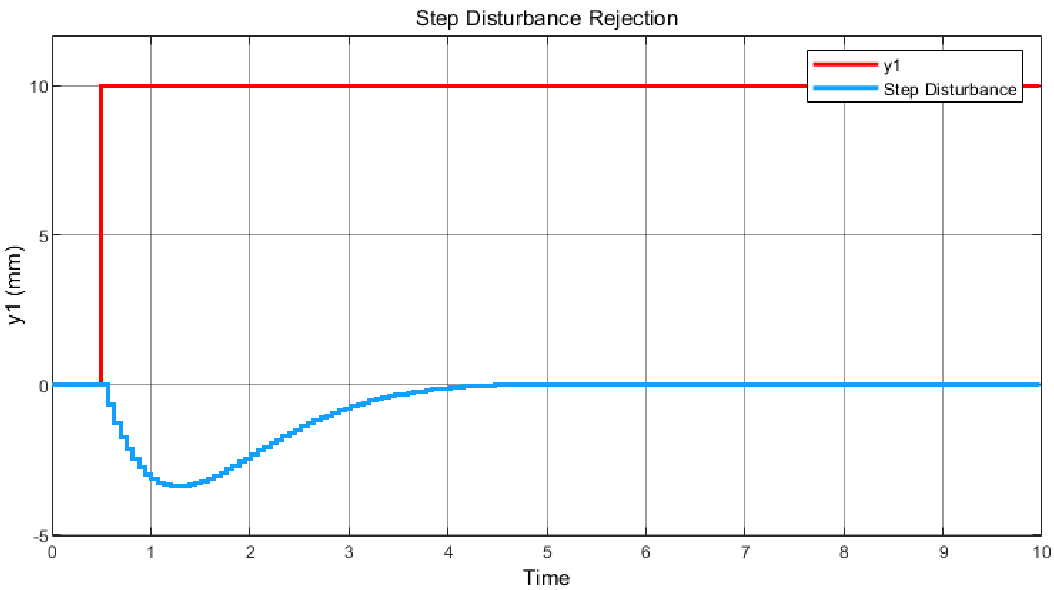


Figure 13: Output for Q1=100, Q2=1, R=1

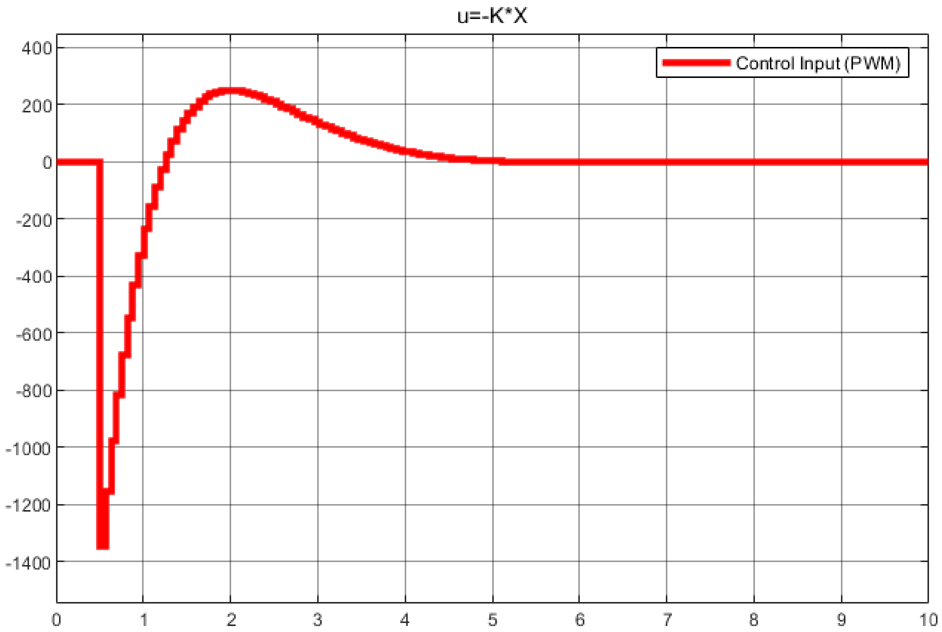


Figure 14: Control Input for Q1=10, Q2=1, R=1

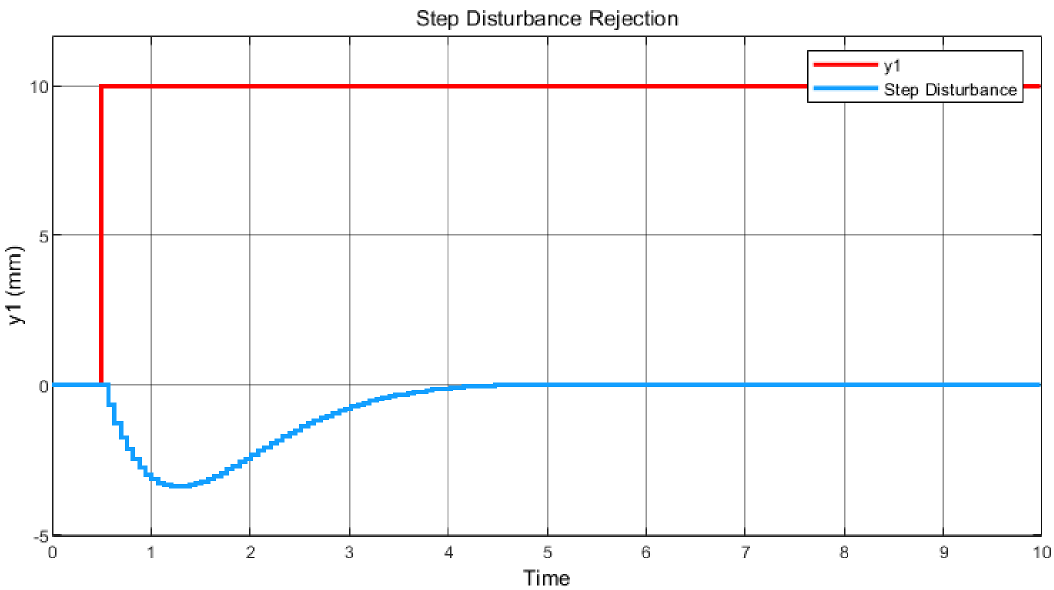


Figure 15: Output for Q1=10, Q2=1, R=1

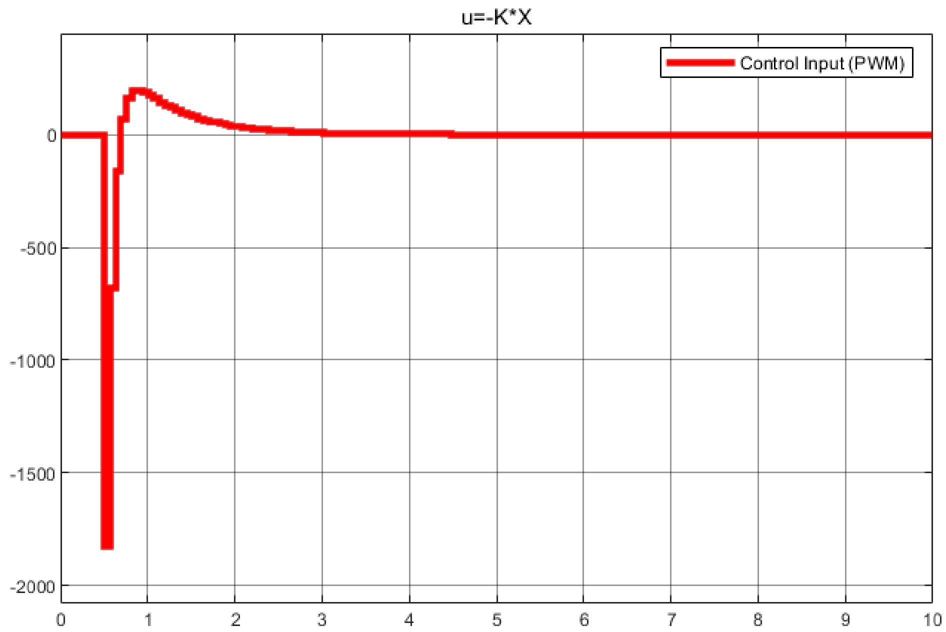


Figure 16: Control Input for Q1=1, Q2=100, R=1

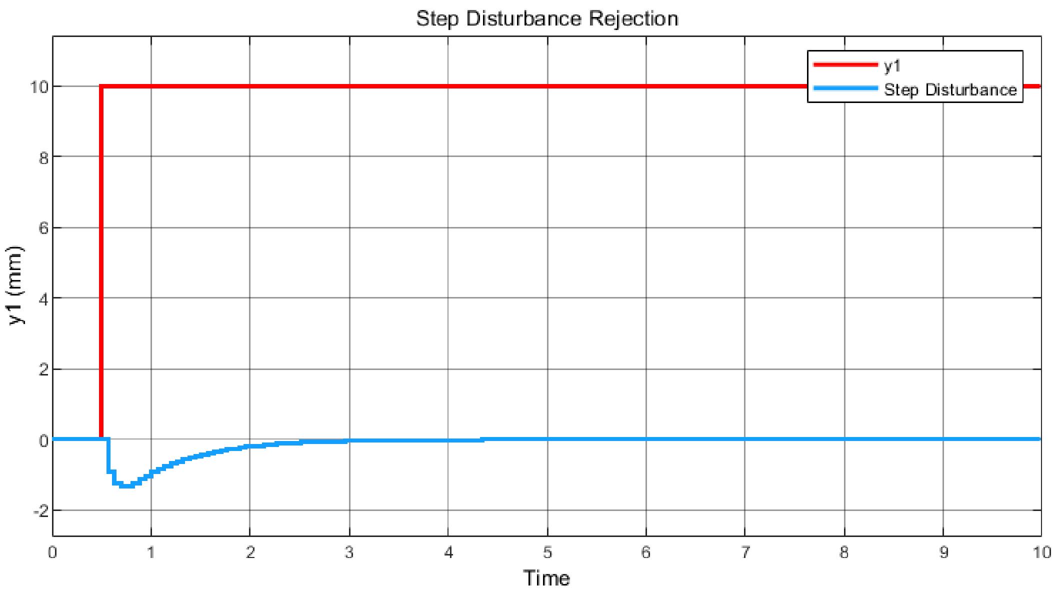


Figure 17: Output for Q1=1, Q2=100, R=1

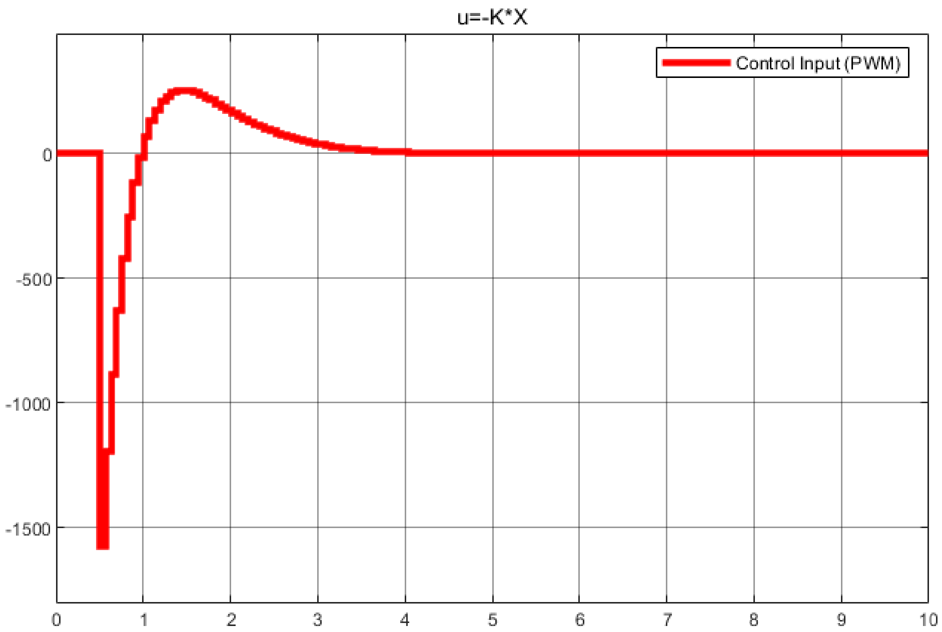


Figure 18: Control Input for Q1=1, Q2=10, R=1

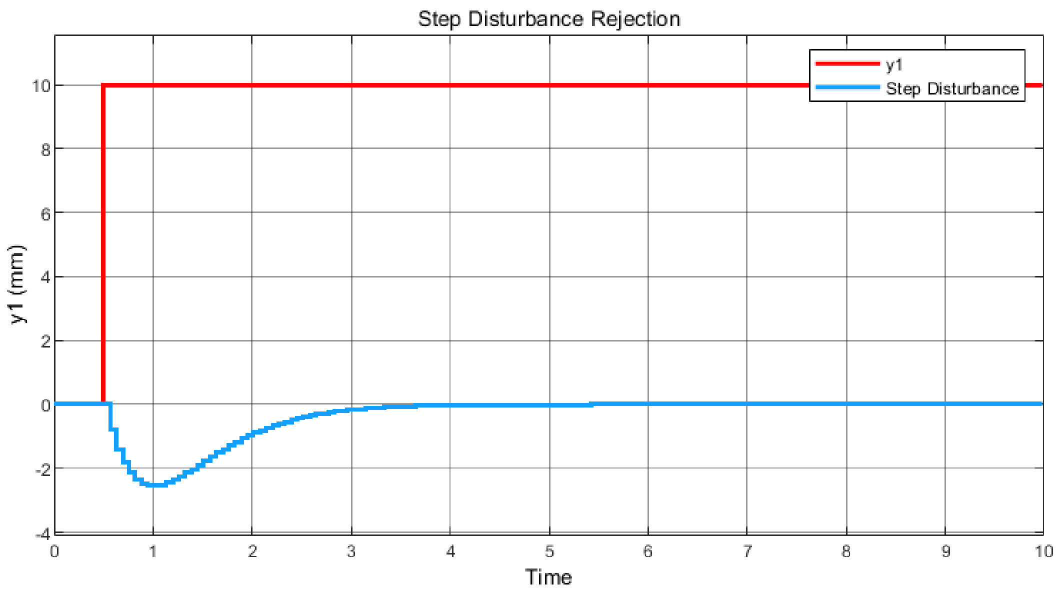


Figure 19: Output for Q1=1, Q2=10, R=1

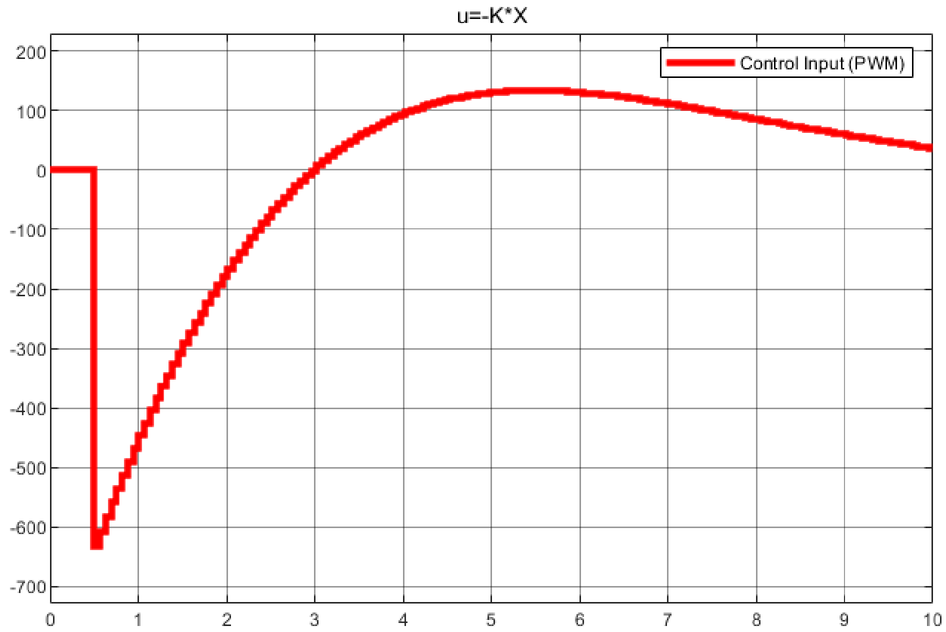


Figure 20: Control Input for Q1=1, Q2=1, R=100


Figure 21: Output for Q1=1, Q2=1, R=100

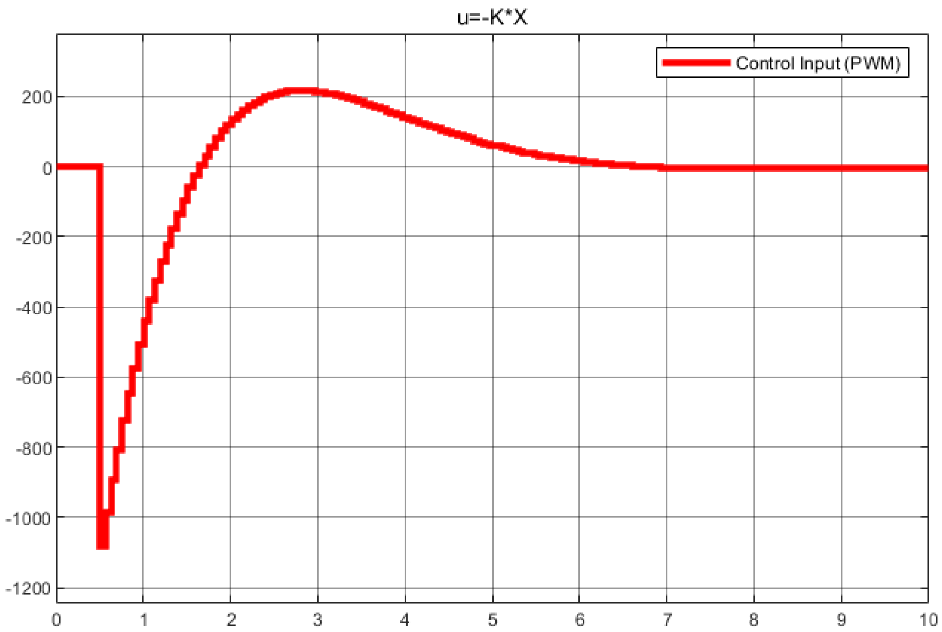


Figure 22: Control Input for Q1=1, Q2=1, R=10

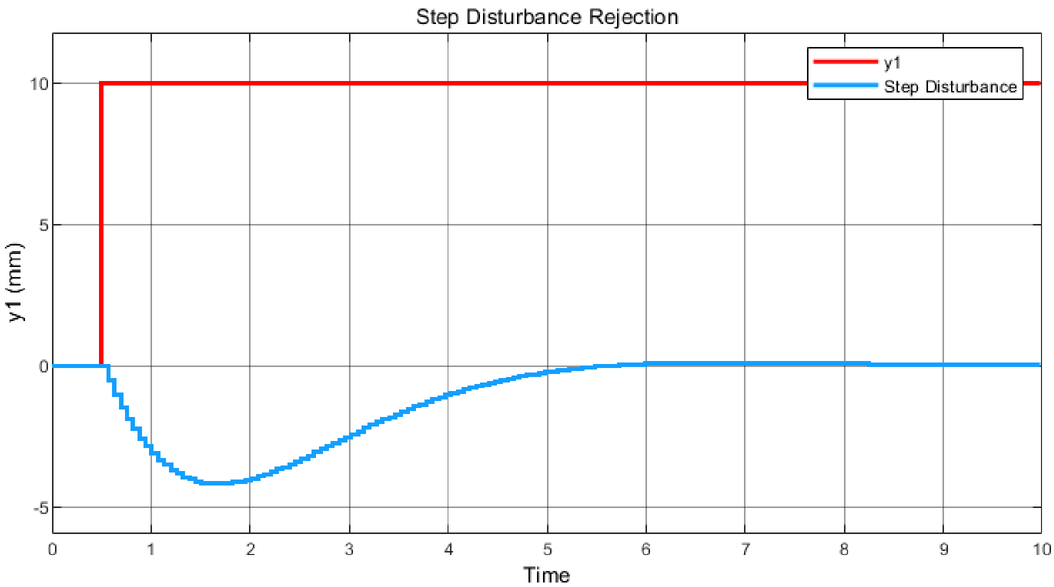


Figure 23: Output for Q1=1, Q2=1, R=10

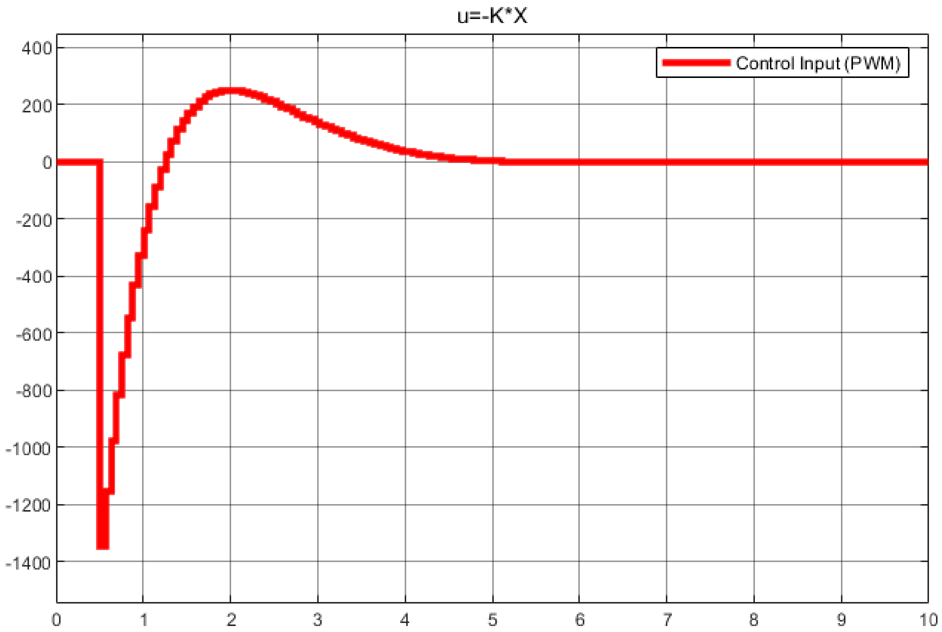


Figure 24: Control Input for Q1=1, Q2=1, R=1

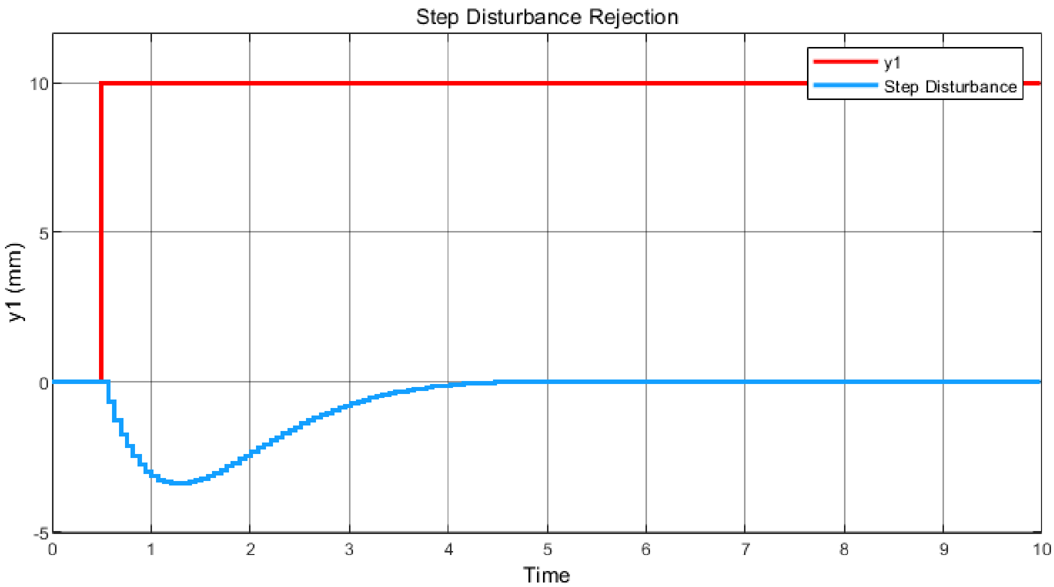


Figure 25: Output for Q1=1, Q2=1, R=1

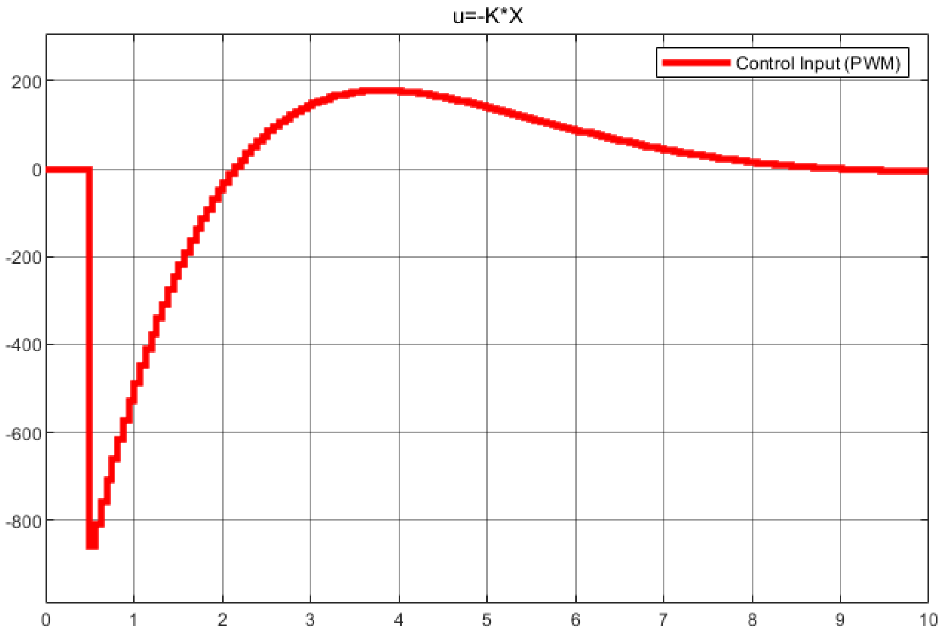


Figure 26: Control Input for Q1=1, Q2=1, R=20


Figure 27: Output for Q1=1, Q2=1, R=20

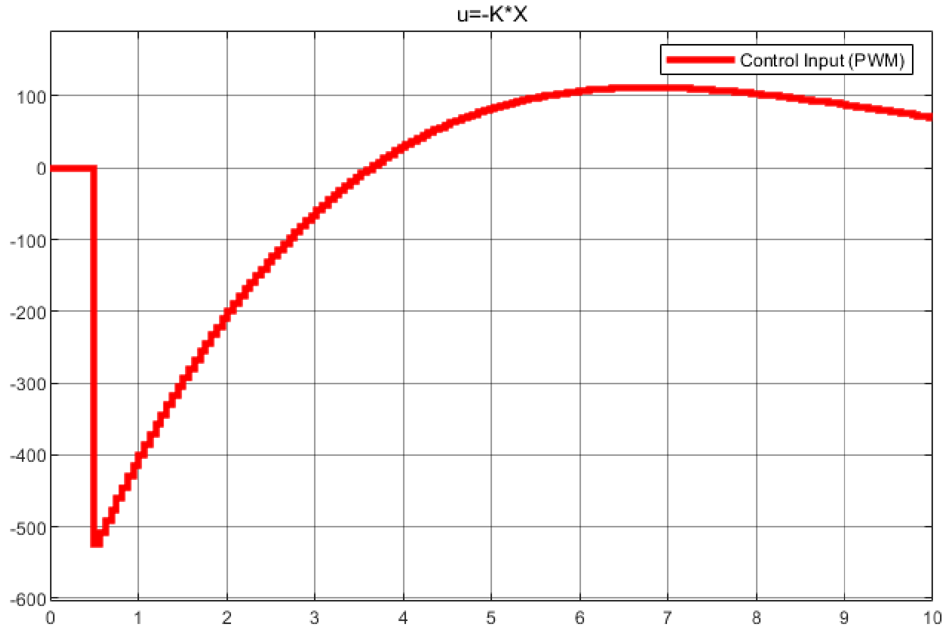


Figure 28: Control Input for Q1=1, Q2=1, R=250

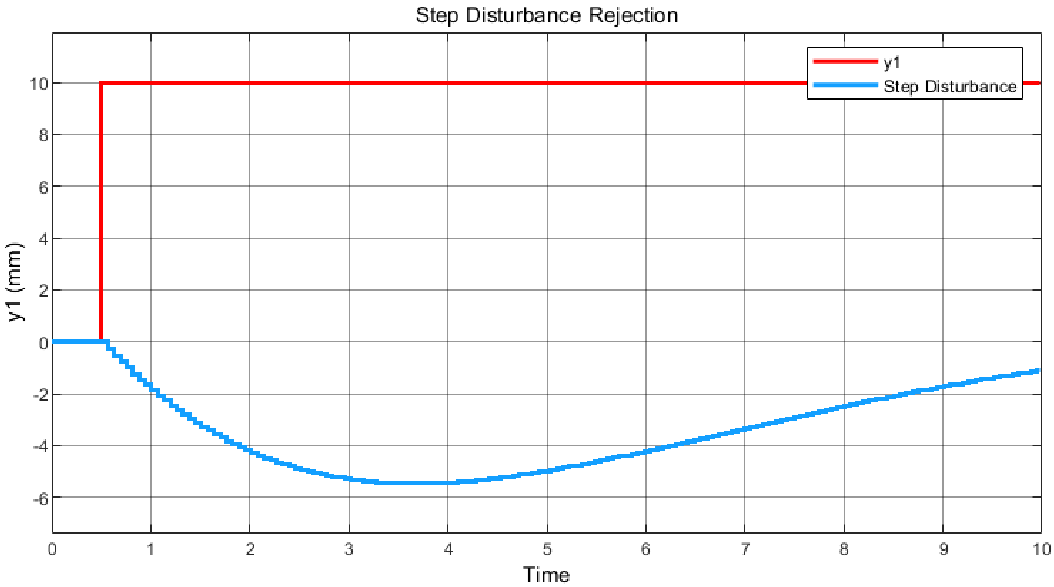


Figure 29: Output for Q1=1, Q2=1, R=250

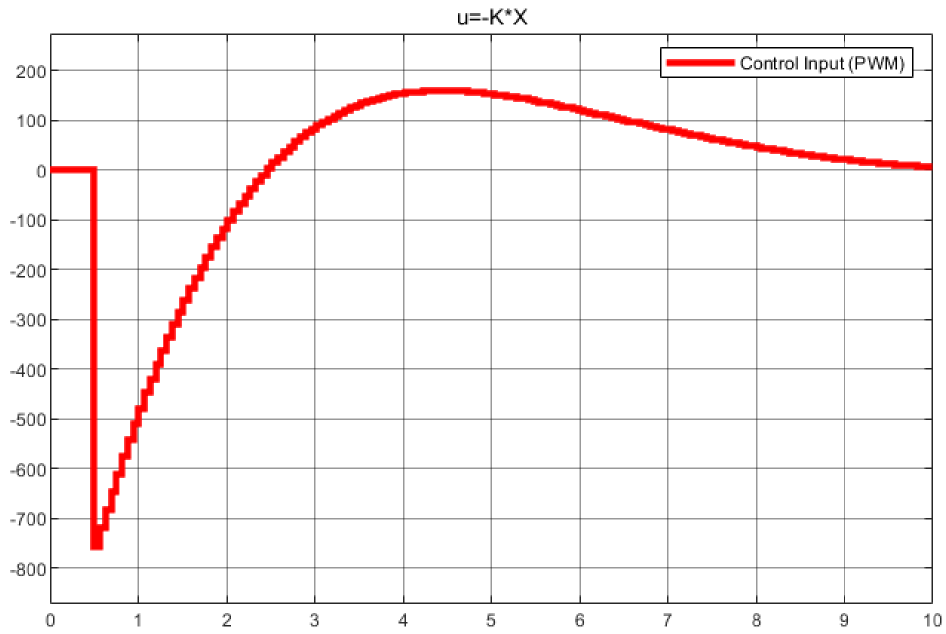


Figure 30: Control Input for Q1=1, Q2=1, R=40

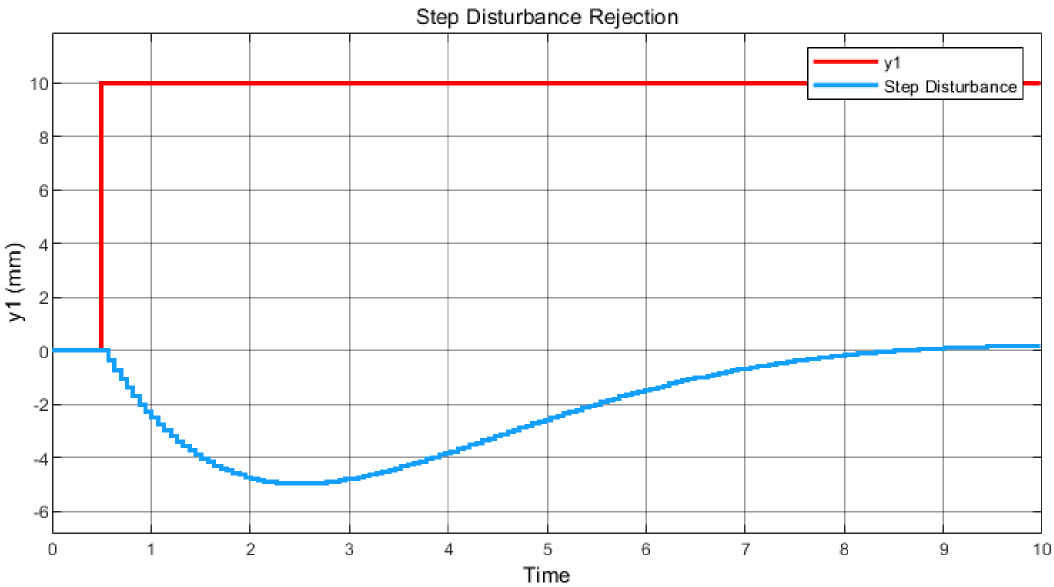


Figure 31: Output for Q1=1, Q2=1, R=40

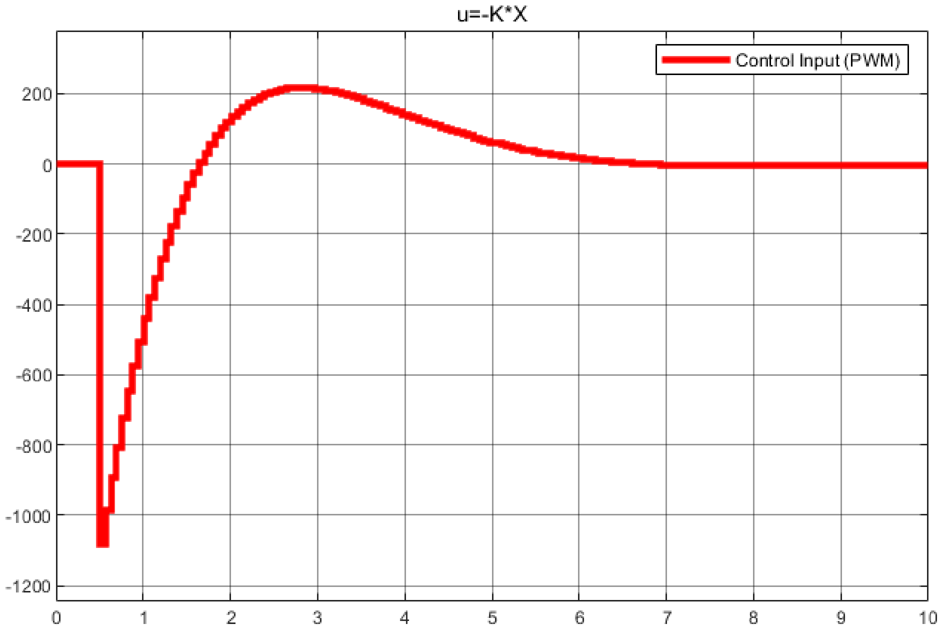


Figure 32: Control Input for Q1=1, Q2=1, R=5

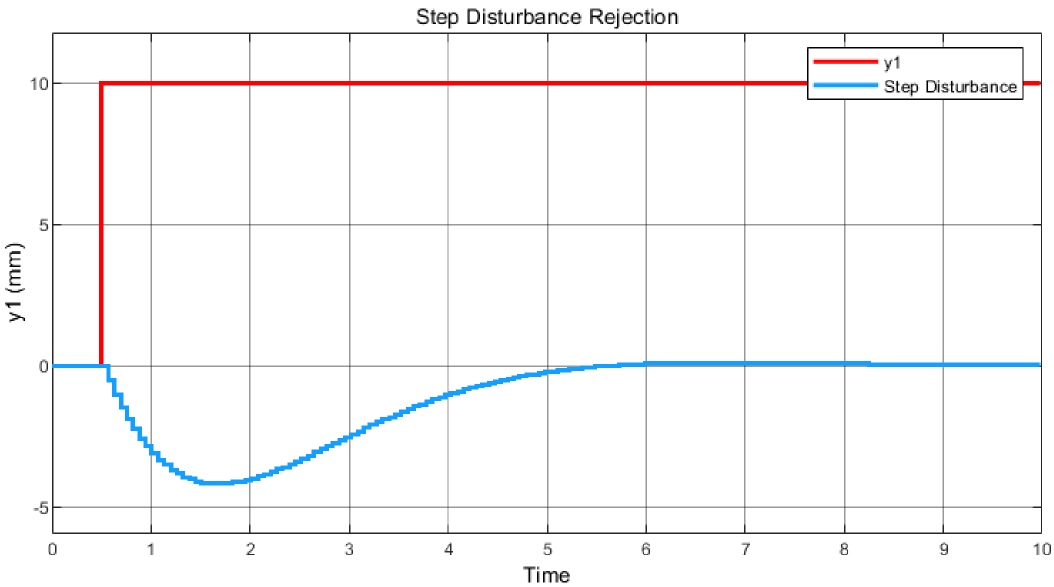


Figure 33: Output for Q1=1, Q2=1, R=5

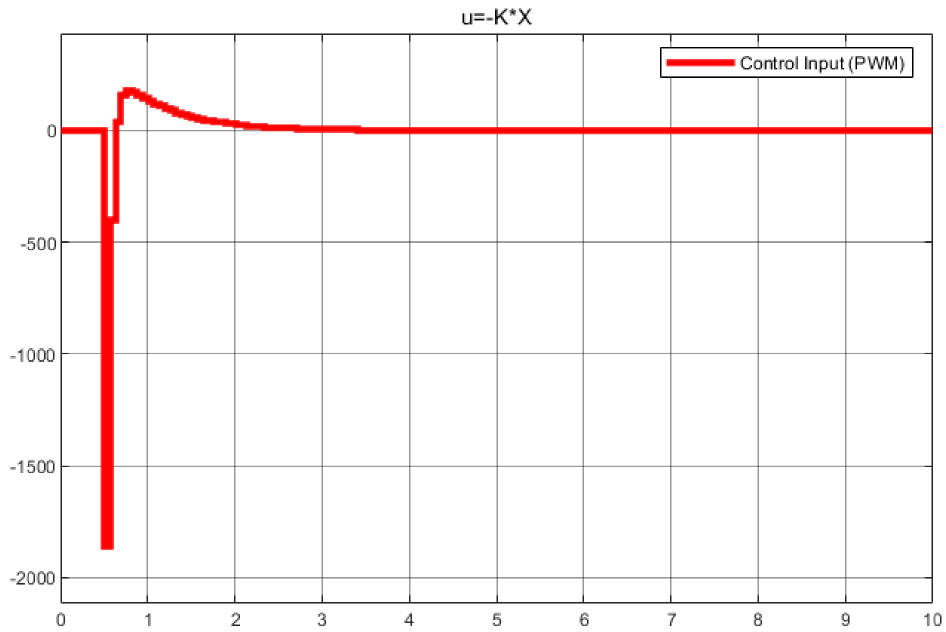


Figure 34: Control Input for Q1=1, Q2=250, R=1

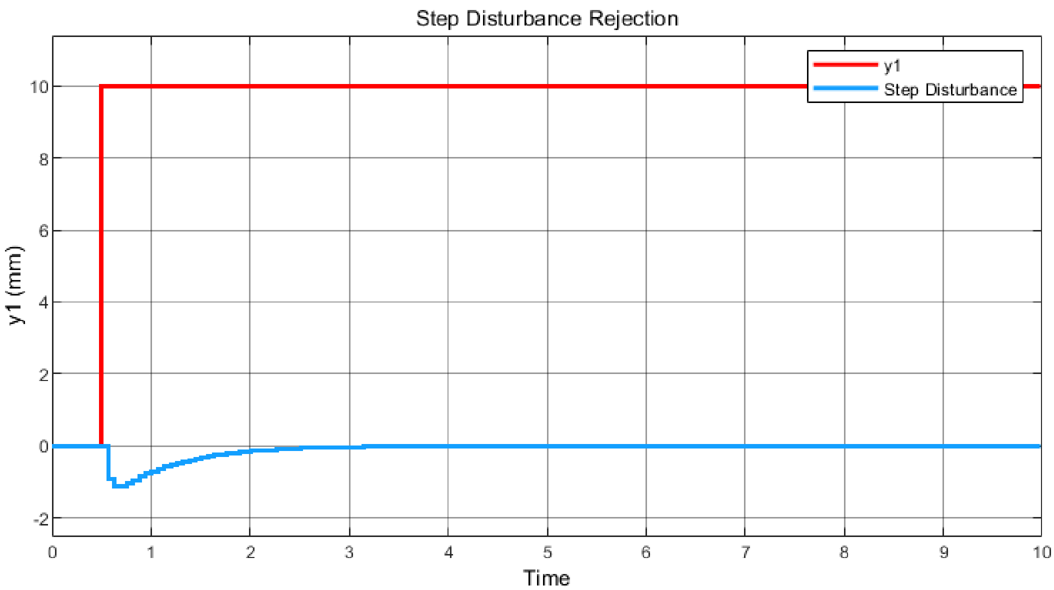


Figure 35: Input for Q1=1, Q2=250, R=1

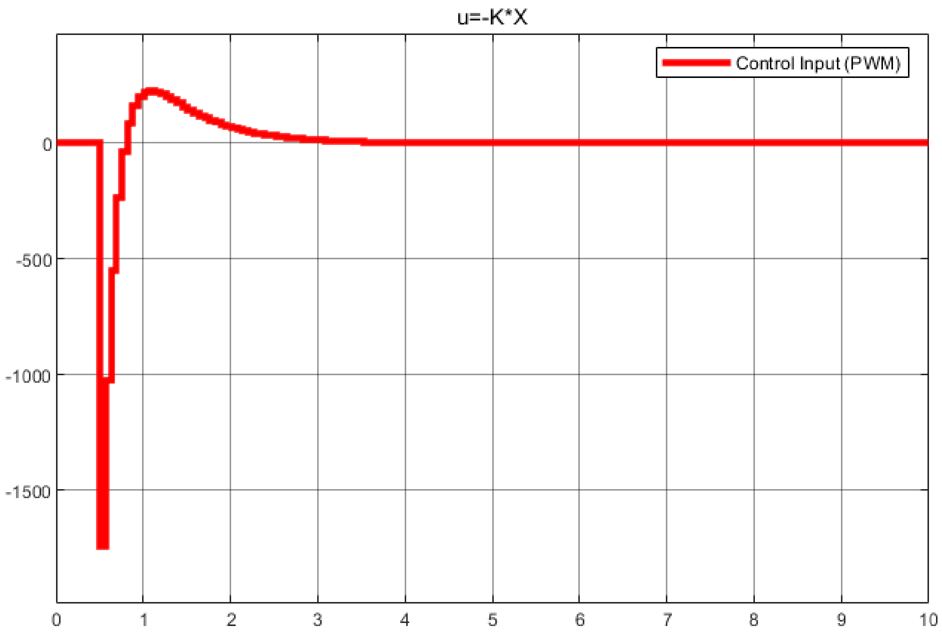


Figure 36: Input for Q1=1, Q2=25, R=1

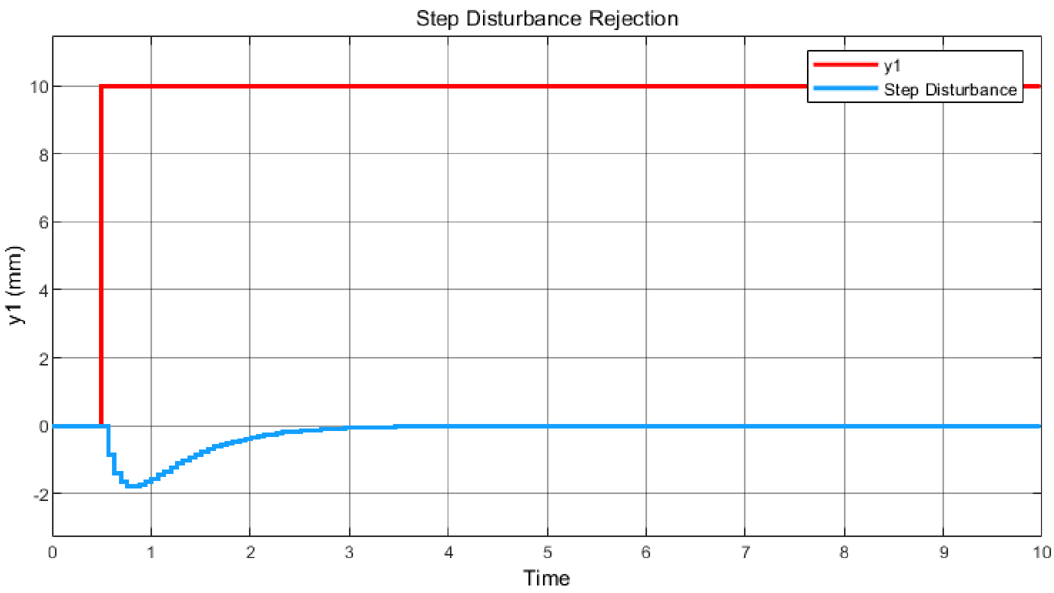


Figure 37: Output for Q1=1, Q2=25, R=1

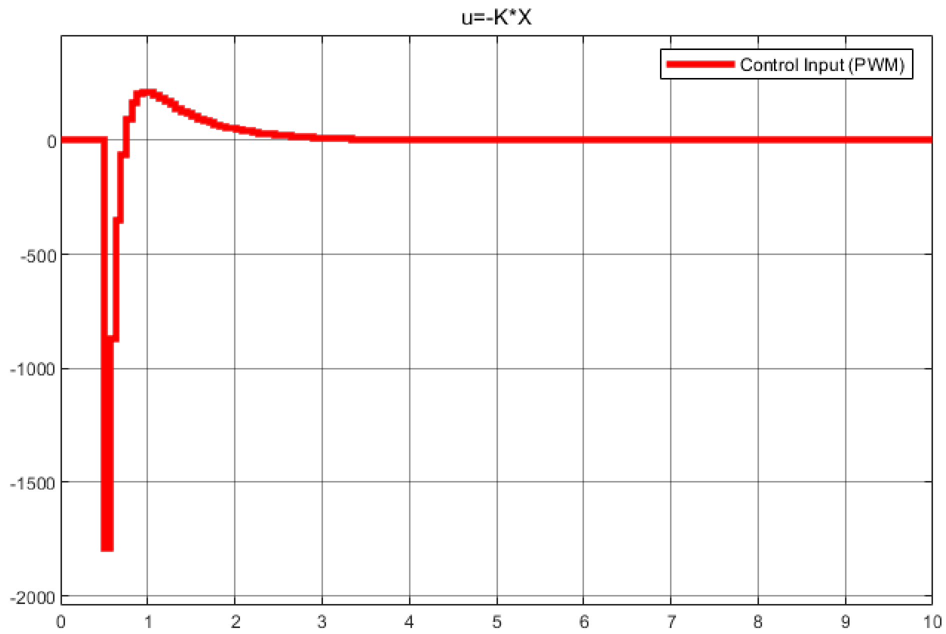


Figure 38: Input for Q1=1, Q2=50, R=1

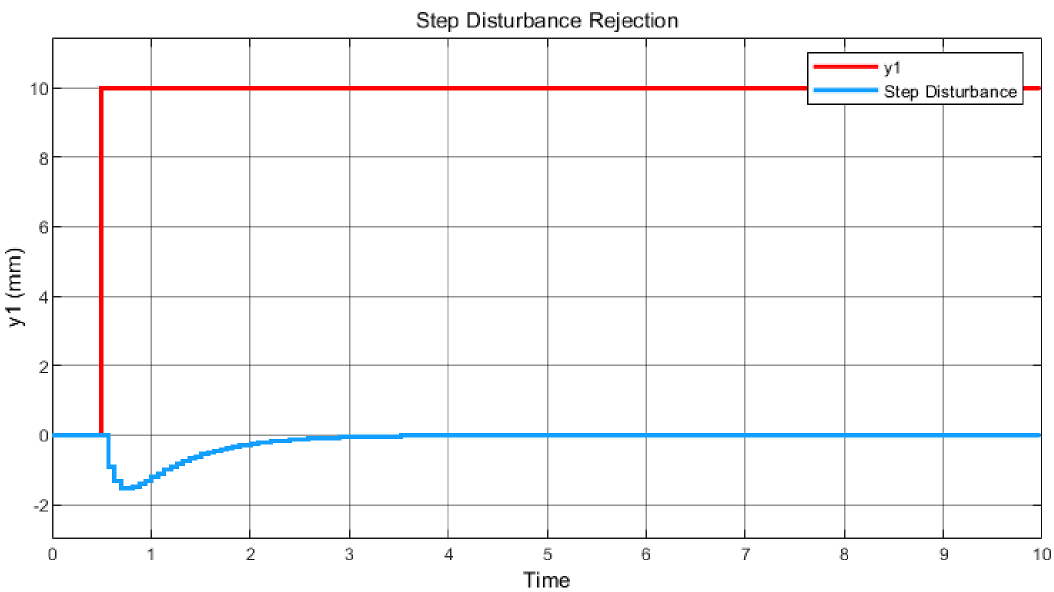


Figure 39: Output for Q1=1, Q2=50, R=1

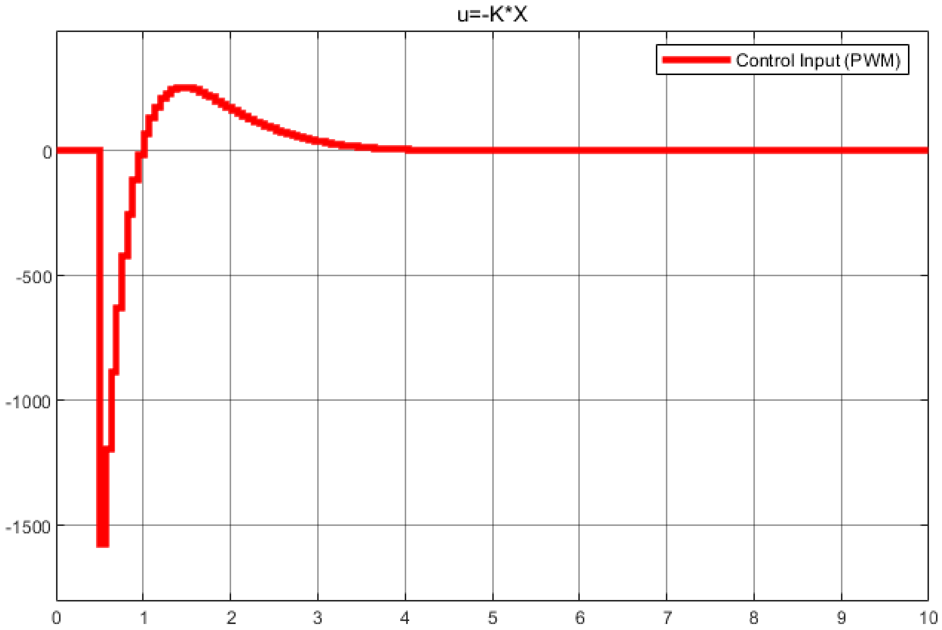


Figure 40: Control Input for Q1=1, Q2=5, R=1

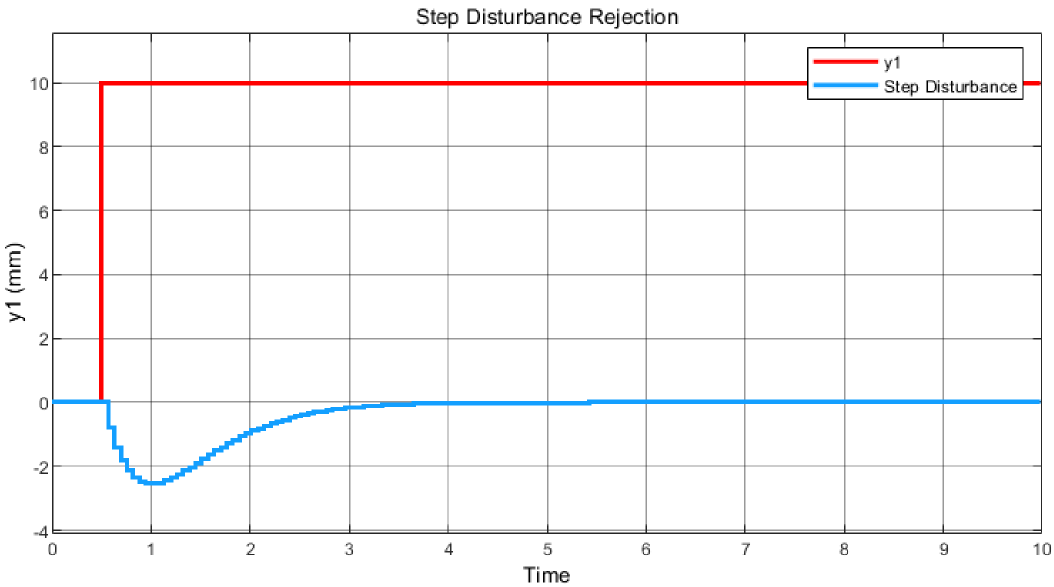


Figure 41: Output for Q1=1, Q2=5, R=1

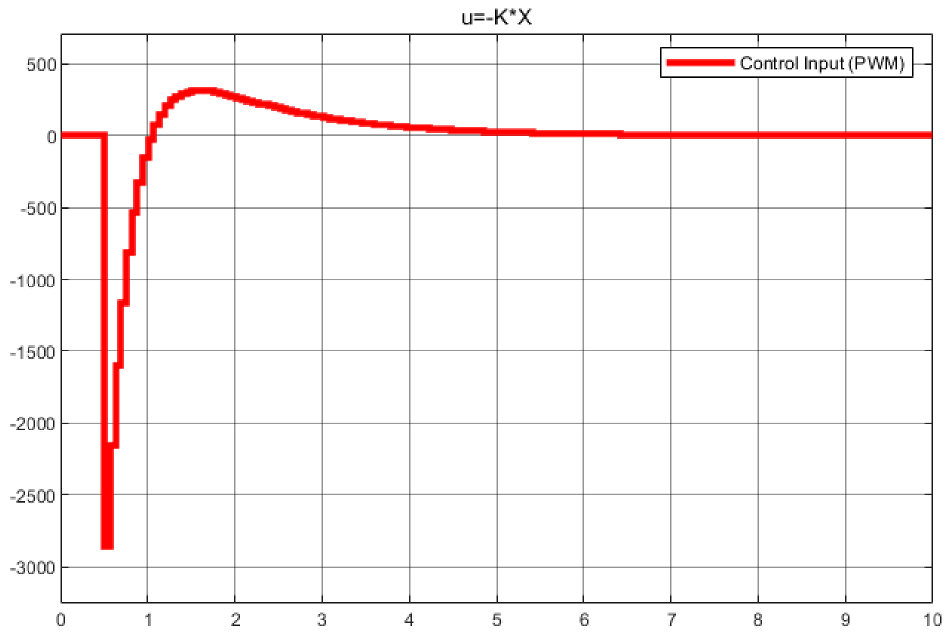


Figure 42: Control Input for Q1=750, Q2=0.01, R=0.005

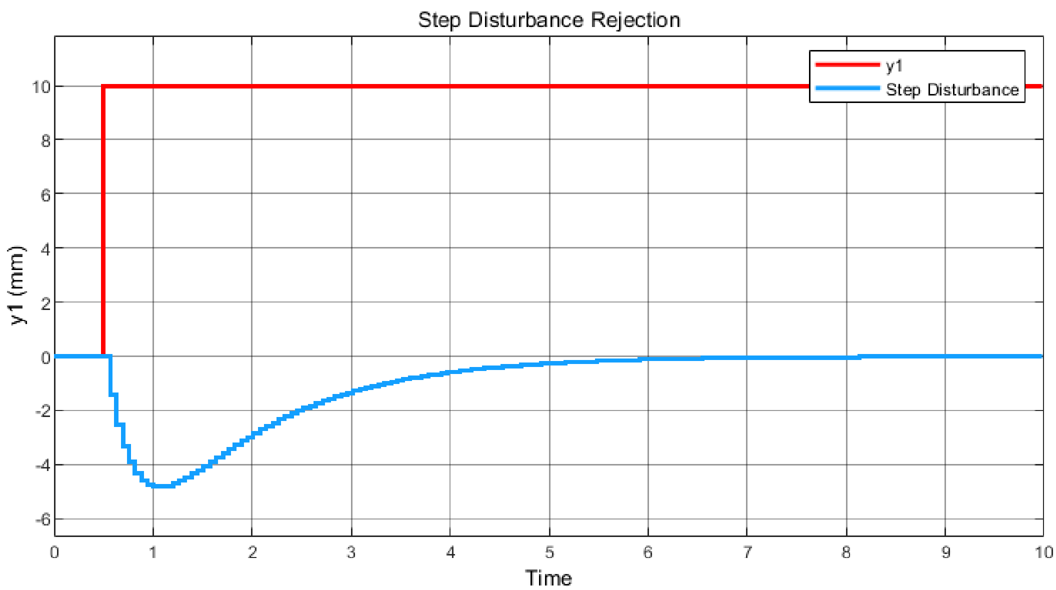


Figure 43: Output for Q1=750, Q2=0.01, R=0.005

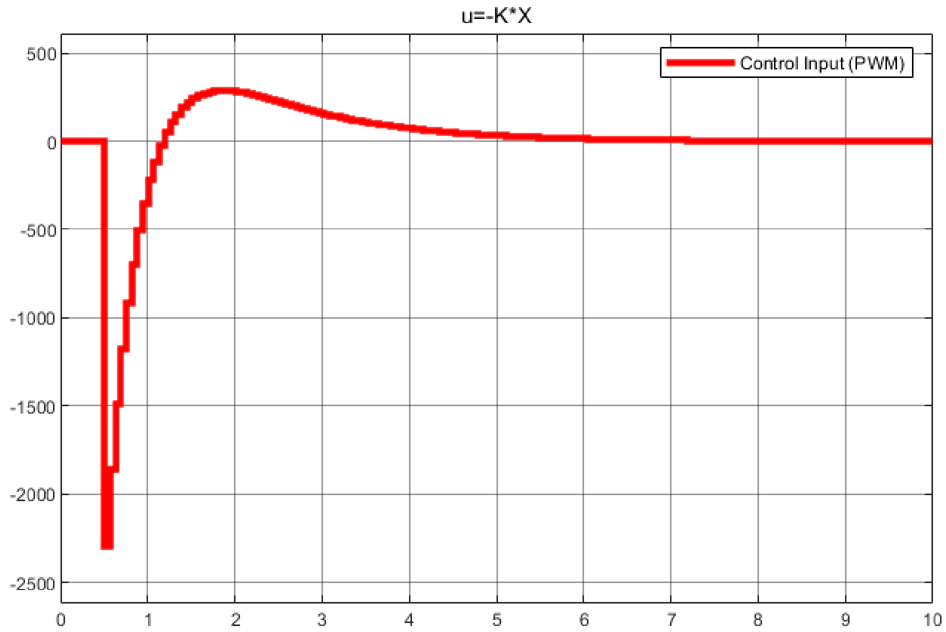


Figure 44: Control Input for Q1=750, Q2=0.01, R=0.01


Figure 45: Output for Q1=750, Q2=0.01, R=0.01

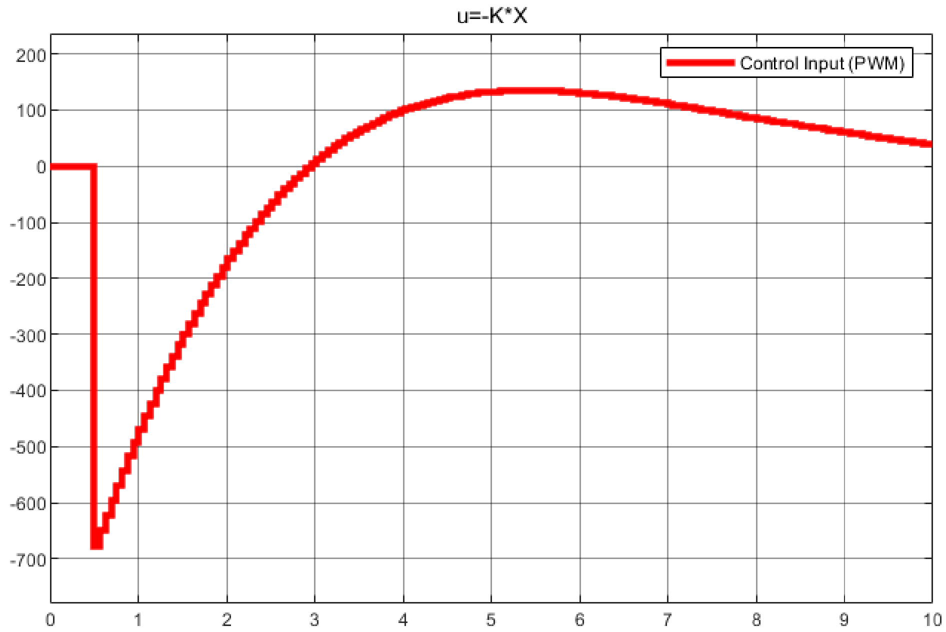


Figure 46: Control Input for Q1=750, Q2=0.01, R=1

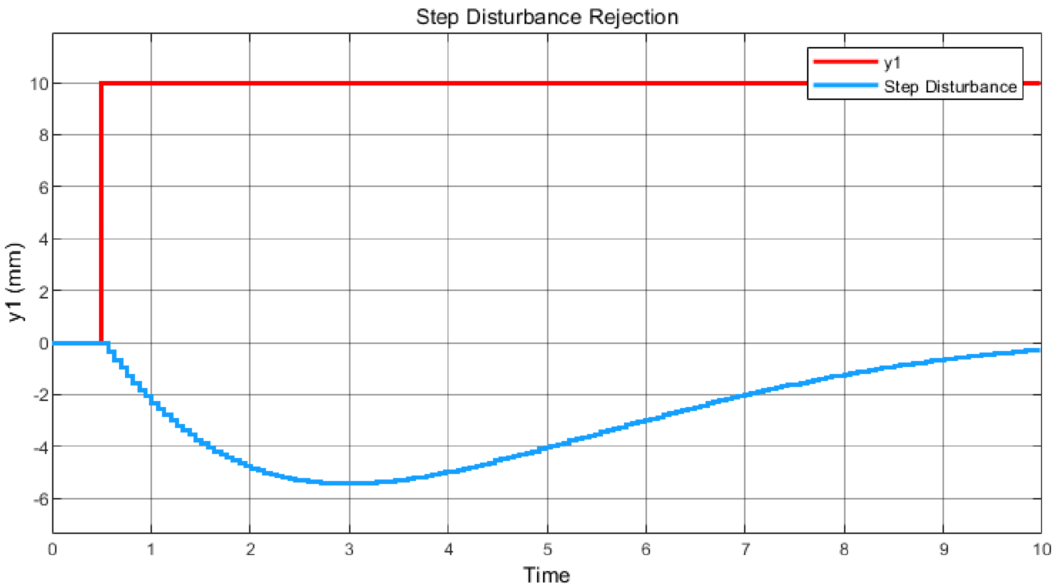


Figure 47: Output for Q1=750, Q2=0.01, R=1

# 5 Conclusion

In this report, problem definition of the chosen the term project of the EE498, our solution approach using LQR Design Method and obtained results have been delivered. Thanks to the systematic methodology of this approach, without a lot of effort, the vehicle can follow the lane without going out of the track. This project was a excellent opportunity for us to apply the theoretical knowledge acquired in EE498 course to a real life application. Especially, we have gathered knowledge about LQR design from first hand experience. Overall, as a prospective control engineers, we had a glimpse of what the professional life may offer to us.

# References

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