Summary draft

    We begin by discussing the history of gauge theory.  Although it was Weyl who eventually gave the freedom inherent in the scalar and vector potentials its name, gauge invariance had been noted as early as Maxwell.  Maxwell worked solely in the Coulomb gauge, noting the freedom in the vector potential but deciding that the simplest gauge was sufficient for his goals.  It was Lorenz who first suggested that other choices may be of use.  He used the Lorenz gauge to derive a general solution to Maxwell’s equations, reproduced below.  Weyl then continued the work by noting that the wave function solution to Schrödinger’s equation was also invariant, and derived an equation for transforming the wave function from one gauge to another (Jackson, 2001).

    We now discuss the general solution to Maxwell's equations.  This is done in three parts.  First, we simplify Maxwell's equations to two equations.  Then we choose the Lorenz gauge to work in, and using this we simplify Maxwell's equations to two inhomogeneous wave equations, which can finally be solved by guessing an answer and checking that it works (Griffiths, 1999).  We may conclude this section by discussing some other gauges and their uses in physics (Jackson, 2002), and finally discuss the gauge freedom inherent in the wave function solution to Schrödinger’s equation (Fock, 1926).

    At this point, further discussion of gauge theory necessitates some background in differential geometry.  We will begin by discussing the basics - the definition of a manifold, as a surface (shape) which can be covered by bijective functions mapping from the surface (shape) to Euclidean space (Baez, 1994).  We will provide some examples, as clarity is important at this early stage.  We will then discuss vector fields and tangent vectors before introducing the concept of differential forms, making sure to relate this to elementary multivariable calculus whenever possible.  A final goal for this section may be to rewrite Maxwell’s equations in terms of differential geometry, using a formulation provided by Baez (1994).

    We now delve further into the mathematics behind gauge theory.  The next step is to introduce groups, Lie groups, group actions, and perhaps the more general idea of a representation as space permits.  The most important idea here is Lie groups.  Covering Lie algebras would be nice, but it does not appear strictly necessary for the formulation of a gauge theory.  Rather more necessary is the idea of vector bundles and connections, on which we may need to spend some time.  Our final goal in this section is to formulate the electromagnetic force as a gauge, in terms of the Lie group U(1) (Baez, 1994).  We will provide as complete as possible a discussion of the action of U(1) on a manifold which permits Maxwell’s equations, and finally aim to use this formulation to show that the Coulomb and Lorenz gauges are equivalent by way of finding an explicit gauge transformation (Jackson, 2002).

    Having formulated the theory of electromagnetism in terms of gauge theory, we are now free to discuss its generalizations.  We will introduce the Yang-Mills theory and discuss the structure of the Standard Model from the perspective of gauges, and separately attempt to describe the role of gauges in general relativity.  Information for this section will be drawn from both Baez (1994) and Griffiths (1987).  If space permits, we may then discuss the search for the Grand Unified Theory, although a new reference may be necessary here.

References:

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