

Dividing the problem in functions:

$$\begin{aligned}\mathbf{f(x)} &= \mathbf{f} \\ \mathbf{g(x)} &= \mathbf{x} \\ \mathbf{h(x)} &= \sqrt{x}\end{aligned}$$

The problem can be factored by the product rule:

$$\begin{aligned}xf' + f &= xf' + 1f = g(x)f'(x) + g'(x)f(x) = (g(x)f(x))' \\ xf' + f &= (\mathbf{xf})'\end{aligned}$$

Replacing the result in the original equation:

$$\begin{aligned}(\mathbf{xf})' &= \sqrt{x} \\ \mathbf{xf} &= \int \sqrt{x} \\ \mathbf{f} &= \frac{\int \sqrt{x}}{x}\end{aligned}$$

Since  $\sqrt{x} = x^{\frac{1}{2}}$  :

$$\begin{aligned}\int x^{\frac{1}{2}} &= \frac{2x^{\frac{3}{2}}}{3} + C \\ \mathbf{f} &= \frac{\frac{2x^{\frac{3}{2}}}{3} + C}{x} = \frac{2x^{\frac{3}{2}}}{3x} + \frac{C}{x} = \frac{2x^{\frac{1}{2}}}{3} + \frac{C}{x} = \frac{2\sqrt{x}}{3} + \frac{C}{x}\end{aligned}$$

Since  $\mathbf{f(x) = f}$ , the solution is:

$$\mathbf{f(x) = \frac{2\sqrt{x}}{3} + \frac{C}{x}}$$