

a) The degree distribution of this network, for any N, is:

Degree	Probability
1	(N-1)/N
N	1/N

b) First, calculate the mean degree:

$$\langle k \rangle = \frac{1 \times N + (N-1) \times 1}{N} \approx 2 \frac{N}{N} = 2$$

After that, it's possible to calculate  $q_k$  of 1 and N

$$q_1 = \frac{k p_k}{\langle k \rangle} = \frac{1 \times \frac{N-1}{N}}{2} = \frac{N-1}{2N} \quad q_N = \frac{k p_k}{\langle k \rangle} = \frac{N \times \frac{1}{N}}{2} = \frac{N}{2N} = \frac{1}{2}$$

As N is much larger than 1, we can assume that  $N-1 \approx N$  and :

$$q_1 = \frac{N}{2N} = \frac{1}{2}$$

Finally, as  $q_1$  and  $q_N$  are the same,  $q_k = \frac{1}{2}$  for any k in this graph.

c)

$$\begin{aligned} \sigma^2 &= \sum_k k q_k^2 - \left[ \sum_k k q_k \right]^2 = \frac{(N+1)}{4} - \frac{(N+1)^2}{4} = \frac{(N+1) - (N^2 + 2N + 1)}{4} = \frac{-(N^2 + N)}{4} \\ r &= \sum_{jk} \frac{jk(e_{jk} - q_j q_k)}{\sigma^2} = \frac{1 \times 1(e_{11} - q_1 q_1)}{\sigma^2} + \frac{1 \times N(e_{1N} - q_1 q_N)}{\sigma^2} + \frac{N \times 1(e_{N1} - q_N q_1)}{\sigma^2} + \frac{N \times N(e_{NN} - q_N q_N)}{\sigma^2} \\ r &= \frac{1 \times 1(\frac{1}{2} - \frac{1}{4})}{\sigma^2} + \frac{1 \times N(0 - \frac{1}{4})}{\sigma^2} + \frac{N \times 1(0 - \frac{1}{4})}{\sigma^2} + \frac{N \times N(\frac{1}{2} - \frac{1}{4})}{\sigma^2} = \frac{\frac{2}{4} - \frac{N}{4} - \frac{N}{4} + \frac{2N^2}{4}}{\sigma^2} \end{aligned}$$

As N is much larger than 1, we can assume that  $N \rightarrow \infty$  and :

$$r = \frac{\frac{2 - 2N + 2N^2}{4}}{\frac{-(N^2 + N)}{4}} = \frac{2(N^2 - N + 1)}{-(N^2 + N)} \Rightarrow \frac{\infty}{-\infty} = -1$$

d) The star network is disassortative, mainly because of two reasons:

- By concept, the network hub connects only with nodes of degree 1, meaning that it prefers to link with low-degree nodes than other main hubs.
- The value of r is minor than 0.